

Apparent optical properties of the sea illuminated by Sun and sky: case of the optically deep sea

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The influence of illumination by direct sunlight and the diffuse light of the sky on the apparent optical properties of seawater are studied. This study is based on the earlier self-consistent approach for solution of the radiative transfer equation. The resulting set of equations couples diffuse reflectance and diffuse attenuation coefficients and other apparent optical properties of the sea with inherent optical properties of seawater and parameters of illumination by the Sun and the sky. The resulting equations in their general form are valid for any possible values of absorption and backscattering coefficients.

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1. Introduction

The study of the spectral signatures of upwelling light from the sea and its dependencies on dissolved and suspended matter in seawater are important for the creation and enhancement of satellite and aircraft algorithms for processing optical information. Precision of analytical expressions that couple apparent optical properties with inherent ones is also important for retrieval of concentrations of dissolved and suspended matter in seawater from optical remote measurements. The diffuse reflection coefficient of the sea, the upward and downward diffuse attenuation coefficients, are apparent optical properties.¹ They depend not only on inherent optical properties (absorption and scattering coefficients and phase function of scattering) but also on conditions of illumination of the sea surface, including radiance distribution, sea surface properties, and geometry of measurement.²⁻⁴ Current algorithms for processing optical remote data usually ignore the dependence of the diffuse reflection coefficient on conditions of natural illumination. This assumption might introduce some systematic error that degrades the precision of the retrieval of inherent optical properties from *in situ* and remotely measured apparent optical properties.

My main objective in this paper is to calculate apparent optical properties of the sea as functions of the

inherent optical properties and the parameters of natural illumination. This study develops previous models.⁵⁻¹⁰

2. Solutions for Downward and Upward Irradiances

Let a homogeneous and optically infinite deep sea be illuminated by the Sun, elevated at h_s deg above the horizon, and by the light of the sky. According to Snell's law direct solar rays enter the water at the angle $\theta_s = \cos^{-1} \mu_s$ to nadir, where

$$\mu_s = \left[1 - \left(\frac{\cos h_s}{n_w} \right)^2 \right]^{1/2} \geq (1 - 1/n_w^2)^{1/2} \approx 0.6656 \quad (1)$$

and $n_w \approx 1.34$ is the refractive index of seawater. Following Ref. 10 the total radiance \tilde{L} is split into two components: the renormalized scattered radiance L and the renormalized direct solar radiance L_s :

$$\tilde{L}(z, \mu, \phi) = L(z, \mu, \phi) + L_s(z, \mu, \phi), \quad (2)$$

$$L_s(z, \mu, \phi) = \begin{cases} L_q(\mu, \phi) \exp(-\alpha z / \mu), & 0 < \mu \leq 1, \\ 0, & -1 \leq \mu \leq 0, \end{cases} \quad (3)$$

$$L_q(\mu, \phi) = E_s^\perp \delta(\mu - \mu_s) \delta(\phi), \quad (4)$$

where $\mu = \cos \theta$; θ and ϕ are polar and azimuth angles, respectively, to the direction of light propagation; z is a depth coordinate directed from the surface to the bottom; $\delta(x)$ is the Dirac delta function; and E_s^\perp is an irradiance by direct solar light on the plane placed normal to the Sun's rays just below the sea surface. This renormalization means that the forward-scattered light is excluded from the scattered light component and included into the direct light. This is considered acceptable, because the forward-

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scattered light for all practical purposes is principally undistinguishable from the unscattered light. The renormalized component of the direct light L_s attenuates with the renormalized attenuation coefficient

$$\alpha = a + 2b_B < c, \quad (5)$$

where a is the absorption coefficient, b_B is the backscattering coefficient, and $c = a + b$ is the beam attenuation coefficient. The component L_s is treated as an energy source that produces the renormalized scattered component L . The diffuse light of the sky that penetrates into the sea is taken into account in the form of boundary conditions for renormalized irradiances.

According to Eq. (2), $L(z, \mu, \phi)$ is the radiance of the renormalized scattered light in the water column. Here downward and upward irradiances E_1 and E_2 are introduced with respect to renormalized scattered light. They are expressed through radiances $L(z, \mu, \phi)$ by the following formulas:

$$\begin{aligned} E_1(z) &= \int_0^{2\pi} d\phi \int_0^1 L(z, \mu, \phi) \mu d\mu, \\ E_2(z) &= - \int_0^{2\pi} d\phi \int_{-1}^0 L(z, \mu, \phi) \mu d\mu. \end{aligned} \quad (6)$$

The total irradiances E_d and E_u are

$$\begin{aligned} E_d(z) &= \int_0^{2\pi} d\phi \int_0^1 \tilde{L}(z, \mu, \phi) \mu d\mu \\ &= E_1(z) + \mu_s E_s^\perp \exp\left(-\frac{\alpha z}{\mu_s}\right), \end{aligned} \quad (7)$$

$$\begin{aligned} E_u(z) &= - \int_0^{2\pi} d\phi \int_{-1}^0 \tilde{L}(z, \mu, \phi) \mu d\mu \\ &= E_2(z). \end{aligned} \quad (8)$$

Here z is a depth coordinate that originates from the sea surface and is directed to the sea bottom.

Let us start from the system of two-flow equations for renormalized scattered irradiances¹¹ derived in Ref. 10:

$$\begin{aligned} \left[\frac{d}{dz} + (2 - \bar{\mu})(a + b_B) \right] E_1(z) - (2 + \bar{\mu})b_B E_2(z) &= b_B E_s^\perp \exp(-\alpha z / \mu_s), \\ -(2 - \bar{\mu})b_B E_1(z) + \left[-\frac{d}{dz} + (2 + \bar{\mu})(a + b_B) \right] E_2(z) &= b_B E_s^\perp \exp(-\alpha z / \mu_s), \end{aligned} \quad (9)$$

where E_s^\perp is the irradiance by the direct solar light that penetrates into the sea on the surface that is perpendicular to the solar rays, $\alpha = a + 2b_B$, a is the absorption coefficient, $b_B = bB$ is the backscattering

coefficient, b is the scattering coefficient, and B is the backscattering probability determined by

$$B = 0.5 \int_{-1}^0 p(\mu) d\mu, \quad (10)$$

where $p(\mu)$ is the phase function of scattering^{12,13} to the angle $\vartheta = \cos^{-1} \mu$, $\bar{\mu}$ is an average cosine over radiance distribution $L_\infty(\mu) = \tilde{L}(z, \mu, \phi)|_{z \gg 1}$ in the sea depth

$$\begin{aligned} \bar{\mu} &= \int_0^{2\pi} d\phi \int_{-1}^1 L_\infty(\mu) \mu d\mu \Big/ \int_0^{2\pi} d\phi \int_{-1}^1 L_\infty(\mu) d\mu \\ &= \left\{ \frac{a}{a + 3b_B + [b_B(4a + 9b_B)]^{1/2}} \right\}^{1/2} \\ &\equiv \left\{ \frac{1 - g}{1 + 2g + [g(4 + 5g)]^{1/2}} \right\}^{1/2}, \end{aligned} \quad (11)$$

where g is the Gordon parameter,

$$g = \frac{b_B}{a + b_B} \equiv \frac{\omega_0 B}{1 - \omega_0 + \omega_0 B}, \quad (12)$$

and $\omega_0 = b/(a + b)$ is the single-scattering albedo.

The direct solar illumination is taken into account in Eqs. (9) by the source functions on the right-hand sides of these equations. The exclusion of renormalized direct light from the two-flow equations (9) makes this approach more accurate than the approach proposed by Aas.¹⁴ This is true because a significant portion of the total irradiances that correspond to the renormalized direct light L_s is calculated precisely (see Ref. 10).

The irradiance of the diffuse light of the sky that penetrates the water E_d^0 is taken into account by a boundary condition:

$$E_1(0) = E_d^0. \quad (13)$$

The contribution to irradiance by the skylight just below the sea surface E_d^0 is regarded as being q times weaker than the contribution to irradiance from the Sun $E_s^{\perp 7,15,16}$; consequently,

$$E_s^\perp = qE_d^0. \quad (14)$$

The solution of the system of Eqs. (9) can be treated as the sum of the general and the partial solutions:

$$E_i(z) = A_i \exp(-\alpha_s z) + C_i \exp(-\alpha z / \mu_s), \quad i = 1, 2, \quad (15)$$

where

$$\begin{aligned}
 -\alpha_\infty &= -\frac{a}{\bar{\mu}} \\
 &\equiv -\{[4a(a + 2b_B) + \bar{\mu}^2 b_B^2]^{1/2} \\
 &\quad - \bar{\mu}(a + b_B)\} < 0
 \end{aligned} \tag{16}$$

is the negative eigenvalue of the system of equations (9).¹⁰ The second positive eigenvalue of Eqs. (9) (Ref. 10) is

$$\alpha_0 = \bar{\mu}(a + b_B) + [4a(a + 2b_B) + \bar{\mu}^2 b_B^2]^{1/2} > 0. \tag{17}$$

By inserting Eq. (15) into Eqs. (9) and applying the boundary condition (13), we have the following solutions:

$$\begin{aligned}
 E_1(z) &= E_d^0 \exp(-\alpha_\infty z) + C_1[\exp(-\alpha z/\mu_s) \\
 &\quad - \exp(-\alpha_\infty z)],
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 E_2(z) &= E_d^0 R_\infty \exp(-\alpha_\infty z) + (C_2 - R_\infty C_1)\exp(-\alpha_\infty z) \\
 &\quad + C_2[\exp(-\alpha z/\mu_s) - \exp(-\alpha_\infty z)],
 \end{aligned} \tag{19}$$

where

$$\begin{aligned}
 C_1 &= \frac{\alpha b_B E_s^\perp}{\mu_s \Delta_s} [(2 + \bar{\mu})\mu_s + 1], \\
 C_2 &= \frac{\alpha b_B E_s^\perp}{\mu_s \Delta_s} [(2 - \bar{\mu})\mu_s - 1],
 \end{aligned} \tag{20}$$

$$\begin{aligned}
 \Delta_s &= \left(\alpha_0 + \frac{\alpha}{\mu_s}\right) \left(\alpha_\infty - \frac{\alpha}{\mu_s}\right) \\
 &\equiv \frac{\alpha^2}{\mu_s} [1 + \bar{\mu}\mu_s(4 - \bar{\mu}^2)] \\
 &\quad \times \left(\frac{1}{\mu_0} - \frac{1}{\mu_s}\right),
 \end{aligned} \tag{21}$$

$$\mu_0 = \frac{\alpha}{\alpha_\infty} \equiv \frac{1 + \bar{\mu}^2}{\bar{\mu}(3 - \bar{\mu}^2)}, \tag{22}$$

$$C_2 - R_\infty C_1 = \mu_s E_s^\perp R_s, \tag{23}$$

$$R_\infty = \left(\frac{1 - \bar{\mu}}{1 + \bar{\mu}}\right)^2 \tag{24}$$

is the diffuse reflectance by the sea illuminated with diffuse light (see the explanation in Section 3), and

$$R_s = \frac{(1 - \bar{\mu})^2}{1 + \bar{\mu}\mu_s(4 - \bar{\mu}^2)} \tag{25}$$

is the diffuse reflectance by the sea illuminated with directed solar light (see the explanation in Section 3).

By taking into account the relationships in Eqs. (7) and (8) between the total irradiances and the solutions in Eqs. (18) and (19) to Eqs. (9) and by making further simplifications, we obtain the following final

equations for the downward and the upward irradiances:

$$\begin{aligned}
 E_d(z) &= E_d^0 \exp(-\alpha_\infty z) + \mu_s E_s^\perp \exp(-\alpha z/\mu_s) \\
 &\quad + E_s^\perp h R_s [(2 + \bar{\mu})\mu_s + 1] F_s(z) \exp(-\alpha_\infty z),
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 E_u(z) &= E_d^0 R_\infty \exp(-\alpha_\infty z) + \mu_s E_s^\perp R_s \exp(-\alpha z/\mu_s) \\
 &\quad + E_s^\perp h R_s [(2 - \bar{\mu})\mu_s - 1] F_s(z) \exp(-\alpha_\infty z),
 \end{aligned} \tag{27}$$

where

$$F_s(z) = \begin{cases} \left(1 - \exp\left[-\alpha z \left(\frac{1}{\mu_s} - \frac{1}{\mu_0}\right)\right]\right) \left(\frac{1}{\mu_s} - \frac{1}{\mu_0}\right), & \mu_s \neq \mu_0, \\ \alpha z, & \mu_s = \mu_0; \end{cases} \tag{28}$$

$$h = \frac{(1 + \bar{\mu})^2}{2(1 + \bar{\mu}^2)}. \tag{29}$$

Equations (26) and (27) together with Eqs. (1), (5), (11), (16), (24), (25), (28), and (29) express downward and upward irradiances in the sea through the inherent optical properties a , b , and b_B ,^{1,2,17} irradiances by the sky E_d^0 and Sun E_s^\perp just below the sea surface; and depth coordinate z . Equations (26) and (27) are valid for any possible values of absorption and back-scattering coefficients.¹⁰ The irradiances E_d and E_u given by Eqs. (26) and (27) are the basis for calculation of apparent optical properties^{1,2} such as transmittance, diffuse reflectance, and diffuse attenuation coefficients. Let us calculate these properties.

3. Apparent Optical Properties

A. Transmittance

The transmittance of the layer $0 - z$ situated between the horizontal plane z is a constant and the surface is defined as $T(z) = E_d(z)/E_d(0)$. By using Eq. (26) with this definition, we have

$$\begin{aligned}
 T(z) &= \frac{1 + q\{\mu_s \epsilon(z) + h R_s [(2 + \bar{\mu})\mu_s + 1] F_s(z)\}}{1 + q\mu_s} \\
 &\quad \times \exp(-\alpha_\infty z),
 \end{aligned} \tag{30}$$

where

$$\epsilon(z) = \exp\left[-\alpha z \left(\frac{1}{\mu_s} - \frac{1}{\mu_0}\right)\right], \quad q = \frac{E_s^\perp}{E_d^0}. \tag{31}$$

B. Diffuse Reflectance

Diffuse reflectance measured at depth z is defined as $R(z) = E_u(z)/E_d(z)$. Using Eqs. (26) and (27) with this definition, we have

$$R(z) = \frac{R_\infty + q R_s \{\mu_s \epsilon(z) + h [(2 - \bar{\mu})\mu_s - 1] F_s(z)\}}{1 + q\{\mu_s \epsilon(z) + h R_s [(2 + \bar{\mu})\mu_s + 1] F_s(z)\}}. \tag{32}$$

Diffuse reflectance of the sea, illuminated by the diffuse light of the sky and by the direct solar rays and measured just below the sea surface, is obtained when we set $z = 0$ in Eq. (32):

$$R = \frac{R_\infty + \mu_s q R_s}{1 + \mu_s q} \quad (33)$$

or

$$R = \frac{E_d^0 R_\infty + E_s^\perp \mu_s R_s}{E_d^0 + E_s^\perp \mu_s}. \quad (34)$$

For diffuse ($E_s^\perp = 0$) and direct ($E_d^0 = 0$) illumination by use of Eqs. (24), (25), and (34) we obtain the following equalities:

$$R|_{E_s^\perp=0} = R_\infty \equiv \left(\frac{1 - \bar{\mu}}{1 + \bar{\mu}} \right)^2, \quad (35)$$

$$R|_{E_d^0=0} = R_s \equiv \frac{(1 - \bar{\mu})^2}{1 + \bar{\mu} \mu_s (4 - \bar{\mu}^2)}. \quad (36)$$

Equation (35) shows that R_∞ defined by Eq. (24) means the diffuse reflectance of a deep sea optically illuminated by diffuse light. Similarly, Eq. (36) shows that R_s defined by Eq. (25) means the diffuse reflectance of a deep sea optically illuminated by direct solar light.

For values of inherent optical parameters that are typical for most ocean waters ($b_B/a \ll 1$) the expressions for diffuse reflectances in Eqs. (33), (35), and (36) can be rewritten in the following simplified form:

(a) Diffuse reflectance of the sea illuminated by diffuse light of the sky:

$$R_\infty = \frac{b_B}{4a}, \quad \frac{b_B}{a} \ll 1. \quad (37)$$

(b) Diffuse reflectance of the sea illuminated by direct solar light:

$$R_s = \frac{b_B}{a + \mu_s(3a - \sqrt{ab_B})},$$

$$\mu_s = (1 - \cos^2 h_s / n_w^2)^{1/2}, \quad \frac{b_B}{a} \ll 1, \quad (38)$$

where h_s is the Sun elevation angle and n_w is the refractive index of water.

(c) Diffuse reflectance of the sea illuminated by the combined light of the sky and Sun:

$$R = k_c \frac{b_B}{a}, \quad \frac{b_B}{a} \ll 1, \quad (39)$$

where

$$k_c = \frac{1}{(1 + q_s)} \left[\frac{1}{4} + \frac{q_s}{1 + \mu_s(3 - \sqrt{b_B/a})} \right],$$

$$q_s = q \mu_s \equiv \frac{\mu_s E_s^\perp}{E_d^0} = \frac{E_s^0}{E_d^0}, \quad (40)$$

where $q_s \approx 0.8$ is the ratio of downward irradiance by the Sun to downward irradiance by the sky just below the sea surface. For many typical solar elevations and optical conditions $k_c \equiv 1/3$. Here Eq. (39) coincides with the formula proposed by Morel and Prieur.¹⁸

C. Diffuse Attenuation Coefficients

Using the following definitions for the downward and upward diffuse attenuation coefficients at depth z :

$$k_d(z) = -\frac{1}{E_d(z)} \frac{dE_d(z)}{dz}, \quad k_u(z) = -\frac{1}{E_u(z)} \frac{dE_u(z)}{dz}, \quad (41)$$

with the values for E_d and E_u given by Eqs. (26) and (27), we obtain the following equations for downward and upward diffuse attenuation coefficients:

$$k_d(z) = \alpha_\infty \frac{1 + q\{\mu_0 \epsilon(z) + h R_s [(2 + \bar{\mu}) \mu_s + 1] Y_s(z)\}}{1 + q\{\mu_s \epsilon(z) + h R_s [(2 + \bar{\mu}) \mu_s + 1] F_s(z)\}}, \quad (42)$$

$$k_u(z) = \alpha_\infty \frac{R_\infty + q R_s \{\mu_0 \epsilon(z) + h [(2 - \bar{\mu}) \mu_s - 1] Y_s(z)\}}{R_\infty + q R_s \{\mu_s \epsilon(z) + h [(2 - \bar{\mu}) \mu_s - 1] F_s(z)\}}, \quad (43)$$

where

$$Y_s(z) = \begin{cases} \mu_0 \mu_s \frac{1 - \epsilon(z)}{\mu_0 - \mu_s} - \mu_0 \epsilon(z), & \mu_s \neq \mu_0, \\ \alpha z - \mu_0, & \mu_s = \mu_0 \end{cases}. \quad (44)$$

4. Conclusion

An approach to calculating the apparent optical properties of homogeneous sea illuminated by direct sunlight and by the light of the sky has been proposed. This approach is valid for arbitrary values of the inherent optical properties of seawater b_B and a . All the resulting equations, except for the simplified ones in Eqs. (37)–(40), are valid for any type of water, including lake and coastal waters with an unusually high content of scattering material. The major results of this study are Eqs. (26) and (27), which express downward and upward irradiances in the sea through the inherent optical properties and parameters of illumination. Apparent optical properties such as transmittance $T(z)$, diffuse reflectance $R(z)$, and diffuse attenuation coefficients [$k_d(z)$ and $k_u(z)$] are calculated with $E_d(z)$ and $E_u(z)$ and are given by Eqs. (30)–(32) and by Eqs. (42) and (43), respectively. The diffuse reflectances by the sea illuminated by diffuse, directed, and combined light are given by Eqs. (35), (36), and (33), respectively, and, in simplified form, by Eqs. (37)–(40). The simplified equations (37)–(40) generalize the widely used equation

for diffuse reflectance [$R = kb_B/a$ ($0.25 \leq k \leq 0.5$), for arbitrary illumination], and they are valid for waters with $b_B/a \leq 0.3$.

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11. The approach of Ref. 10 and of this paper differs from that of Aas¹⁴ in that the values E_1 and E_2 in Eqs. (9) correspond to downward and upward irradiances by renormalized scattered components of light. In Ref. 14 irradiances E_d and E_u in two-flow equations (12) and (13) are total irradiances. In the current approach total irradiances are coupled with the renormalized irradiances E_1 and E_2 by the relationships (7) and (8). This means that the coefficients in Eqs. (9) should not be compared with the coefficients of Eqs. (12) and (13) by Aas.¹⁴ The comparable optical properties are only those that can be calculated with the total upward and downward irradiances E_d and E_u .
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