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FAST SPEED EXPANSION TECHNIQUE FOR THE TRANSIENT ANALYSIS OF AUTOMOTIVE CLUTCHES

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ABSTRACT

The transient modal analysis method (TMA) has been used to solve the inhomogeneous (loaded) transient thermoelastic contact problem (ITTEC). In the TMA method, the solution of the inhomogeneous transient problem is expressed in modal coordinates, corresponding to eigenfunctions of the homogeneous (unloaded) problem. However, for the large-scale ITTEC problem, this method is found to be extremely time-consuming, because of the computation-intensive of the eigen-solutions. This paper describes a new approach to solve the large-scale ITTEC problem with a dramatic reduction in computational complexity. The method is referred to as fast speed expansion method (FSE). With the FSE method, full eigen-solutions are performed only at a limited number of sparsely located speeds. For speeds between these speeds, eigenvectors are solved by linear interpolation, while the eigenvalues are computed from Taylor series. The method is illustrated with application to an automotive clutches.

1 INTRODUCTION

Upon clutch application, the steel disks are squeezed against the friction disks by hydraulic pressure. Rotational speed of the steel disks is produced by friction between the steel and friction disks. The overwhelming majority of the frictional energy is converted to heat. Because the heat generation is non-uniform, it produces non-uniform deformations in disks. These deformations affect the contact pressure distribution, which, in turn, further affects the temperature distribution. Hence, a feedback process is established. When a particular sliding speed, called the 'Critical Speed', is exceeded, this feedback process becomes instable, dut to at least one of real part of the eigenvalues is positive. This phenomenon is known as thermoelastic instability(TEI) [1]. Yi [2] developed a custom software, named 'Hotspotter', using the finite element method to solve the critical speed of each model.

However, 'Hotspotter' assumes that the sliding speed is constant, whereas the actual engagement takes place at variable sliding speed, which probably is higher than the critical speed initially, but falls below it finally. Because engagement occurs over a very short period of time, detrimental high local temperatures may not be developed before the sliding speed has fallen below the critical value [3]. Therefore, it is necessary to solve the transient behavior of the thermoelastic contact problem.

For a large-scale problem, transient simulation is extremely time-consuming. Al-Shabibi and Barber [4] has developed a reduced order model by using a truncated eigenfunction expansion to represent the temperature. The truncated eigenfunction series contain only those terms corresponding to several dominant eigenvalues. However, when the sliding speed falls below the critical speed, the eigenvalues tend to cluster together and no subset can be regarded as dominant. At same time, all of the real part of the eigenvalues are negative and the stiffness ratio is large, in other words, the system is stiff [5]. Thus, it is also hard to solve the ITTEC problem by traditional numerical methods when the sliding speed is below the critical speed. Zagrodzki [6] has developed transient modal analysis method(TMA) to solve the ITTEC problem for the whole speed range. In the TMA method, the transient nodal temperatures are expressed in modal coordinates, corresponding to the eigenfunctions of the corresponding homogeneous(unloaded) problem. However, for the large-scale ITTEC problem, this method is time-consuming, too.

To overcome this difficulty, we propose a new method based on the asymptotic waveform evaluation(AWE) [7, 8]. This method is referred to as fast speed expansion(FSE). With the known eigenvalues and eigenfunctions at a limited number of sparsely speeds, the FSE technique employs an efficient algorithm to interpolate and expand the eigenfunctions and eigenvalues over a speed band. Consequently, the FSE method can greatly reduce the transient problem computational time by dramatically reducing the number of full eigenvalue solutions required.

2 FORMULATION

2.1 Finite Element Discretization

The temperature field can be expanded by the Fourier series

$$T(r,\phi,z,t) = \sum_{m=0}^{\infty} \mathrm{Re}\left[\theta_m(r,z,t)\, e^{jm\phi}\right] , \tag{1}$$

where m is the wave numbers. The temperature field must satisfy the heat conduction equation

$$K^{\beta}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right) - \rho^{\beta} c^{\beta}\left(\frac{\partial T}{\partial t} + \omega\frac{\partial T}{\partial \phi}\right) = 0 \tag{2}$$

where K^{β}, ρ^{β} and c^{β} are the thermal conductivity, density and specific heat of material β, respectively, and ω is the angular velocity in the chosen frame of the reference. As long as no intermittent contact is experienced in the circumferential direction, the thermoelastic contact problem for each wave number will be independent. Thus, we obtain the matrix form by applying the finite element method to Eqn. (2)

$$\mathbf{M}\dot{\Theta}_m + (\mathbf{C} + jm\omega\mathbf{M}_{\omega})\,\Theta_m + \mathbf{q}_c = \mathbf{0} , \tag{3}$$

where Θ_m and $\mathbf{q}_c$ are the vectors of nodal temperatures and nodal heat sources, respectively. The nodal heat sources $\mathbf{q}_c = 0$ at all nodes, except the ones on the contact interface due to frictional heat generation

$$\mathbf{q}_c = f\omega\mathbf{R}\mathbf{P}_c , \tag{4}$$

where

$$R_{ji} = r_i \delta_{ji} , \tag{5}$$

r_i is the radial coordinate for the ith node, f is the friction coefficient, and $\mathbf{P}_c$ are nodal contact forces, which is a linear function of the nodal temperatures, Θ_m, and the external nodal forces, $\mathbf{F}$ — i.e.

$$\mathbf{P}_c = \mathbf{A}\Theta_m + \mathbf{G}\mathbf{F} , \tag{6}$$

where $\mathbf{A}$, $\mathbf{G}$ are matrices which can be obtained using a routine finite element discretization of the thermoelastic problem.

Substituting Eqn. (4) and Eqn. (6) into Eqn. (3), we then have

$$\dot{\Theta}_m - \tilde{\mathbf{B}}\Theta_m = \tilde{\mathbf{F}} , \tag{7}$$

2.2 Transient Modal Analysis(TMA) Mehod

If the sliding speed is constant, the TMA method permits us to write the solution of Eqn. (7) as an integral of the possibly time varying external loads $\mathbf{F}(t)$ and the eigenvalues of the corresponding homogeneous equation $\dot{\Theta}_m - \tilde{\mathbf{B}}\Theta_m = 0$,

$$\Theta_m(t) = \mathbf{V}\exp(\Lambda t)\,\mathbf{V}^{-1}\Theta_m(0) + \int_0^t \mathbf{V}\exp[\Lambda(t-\tau)]\,\mathbf{V}^{-1}\tilde{\mathbf{F}}(\tau)\,d\tau . \tag{8}$$

If the sliding speed varies, we need to discretize the sliding speed in the time domain. During each small time step, the sliding speed can be regarded as constant and solution (8) can be employed for each time step.

$$\begin{aligned}\Theta_m(t_j) = {} & \mathbf{V}_j\exp(\Lambda_j\Delta t_j)\,\mathbf{V}_j^{-1}\Theta(t_{j-1}) + \\ & \int_0^{\Delta t}\mathbf{V}_j\exp[\Lambda_j(\Delta t_j-\tau)]\,\mathbf{V}_j^{-1}\tilde{\mathbf{F}}_j(\tau)\,d\tau,\ j = 1\ldots M ,\end{aligned} \tag{9}$$

where

$$\Delta t_j = t_j - t_{j-1} \tag{10}$$

and $\mathbf{V}_j$, Λ_j are respectively the eigenvector matrix and the diagonal eigenvalue matrix of $\tilde{\mathbf{B}}_j$, at sliding speed ω_j.

2.3 Fast Speed Expansion (FSE) Method

Eqn. (9) requires to solve the full eigenvalue problem for each matrix $\tilde{\mathbf{B}}_j$, which makes the TMA method extremely time-consuming for the large-scale problem. However, the computation time can be dramatically reduced by interpolating the solution to the eigenvalue problem, using FSE method. The eigenvalue equation of the matrix $\tilde{\mathbf{B}}$ is

$$\tilde{\mathbf{B}}(\omega)\,\mathbf{X}(\omega) = \lambda(\omega)\,\mathbf{X}(\omega) , \tag{11}$$

Expanding the matrices in Eqn. (11) as Taylor series over the speed domain in the vicinity of some speed ω_0, we obtain

$$\tilde{\mathbf{B}}(\omega) = \tilde{\mathbf{B}}_0 + \tilde{\mathbf{B}}_1(\omega-\omega_0) + \ldots + \tilde{\mathbf{B}}_n(\omega-\omega_0)^n \quad (12)$$
$$\lambda(\omega) = \lambda_0 + \lambda_1(\omega-\omega_0) + \ldots + \lambda_n(\omega-\omega_0)^n \quad (13)$$
$$\mathbf{X}(\omega) = \mathbf{X}_0 + \mathbf{X}_1(\omega-\omega_0) + \ldots + \mathbf{X}_n(\omega-\omega_0)^n \ . \quad (14)$$

For our problem, the expansion of $\tilde{\mathbf{B}}(\omega)$ contains only the two terms $\tilde{\mathbf{B}}_0$ and $\tilde{\mathbf{B}}_1$. Substituting Eqn. (12-14) into Eqn. (11) and matching coefficients for each power of $(\omega-\omega_0)$, we obtain

$$\left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_1 + \left(\tilde{\mathbf{B}}_1-\lambda_1\mathbf{I}\right)\mathbf{X}_0 = \mathbf{0} \quad (15)$$
$$\left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_2 + \left(\tilde{\mathbf{B}}_1-\lambda_1\mathbf{I}\right)\mathbf{X}_1 - \lambda_2\mathbf{X}_0 = \mathbf{0} \quad (16)$$
$$\left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_3 + \left(\tilde{\mathbf{B}}_1-\lambda_1\mathbf{I}\right)\mathbf{X}_2 - \lambda_2\mathbf{X}_1 - \lambda_3\mathbf{X}_0 = \mathbf{0} \quad (17)$$
$$\cdots$$
$$\left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_n + \left(\tilde{\mathbf{B}}_1-\lambda_1\mathbf{I}\right)\mathbf{X}_{n-1} - \lambda_2\mathbf{X}_{n-2} - \ldots - \lambda_n\mathbf{X}_0 = \mathbf{0} \ . \quad (18)$$

where λ_0 and $\mathbf{X}_0$ are the eigenvalues and eigenvectors of the matrix $\tilde{\mathbf{B}}_0$. Numerical solution shows that the eigenvectors preserve approximately the same form over a quite wide speed range, thus, they can be expanded by the only first two terms,

$$\mathbf{X}(\omega) = \mathbf{X}_0 + \mathbf{X}_1(\omega-\omega_0) \ , \quad (19)$$

where

$$\mathbf{X}_1 = \frac{\mathbf{X}(\omega_1) - \mathbf{X}_0}{\omega_1-\omega_0} \quad (20)$$

and $\mathbf{X}(\omega_1)$ represents the eigenvectors of the matrix $\tilde{\mathbf{B}}(\omega_1)$. Because $\mathbf{X}(\omega)$ in Eqn. (19) is an approximation, Eqns. (15-18) cannot be satisfied exactly. The error measures

$$\mathbf{E}_1 = \left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_1 - \left(\lambda_1\mathbf{I}-\tilde{\mathbf{B}}_1\right)\mathbf{X}_0 \quad (21)$$
$$\mathbf{E}_2 = \left(\tilde{\mathbf{B}}_1-\lambda_1\mathbf{I}\right)\mathbf{X}_1 - \lambda_2\mathbf{X}_0 \quad (22)$$
$$\mathbf{E}_3 = \lambda_2\mathbf{X}_1 + \lambda_3\mathbf{X}_0 \quad (23)$$
$$\cdots$$
$$\mathbf{E}_n = \lambda_{n-1}\mathbf{X}_1 + \lambda_n\mathbf{X}_0 \quad (24)$$

can be minimized by using the least square method upon Eqns. (21-24). From Eqn. (21), the optimal λ_1 is found from

$$\frac{\partial\left(\mathbf{E}_1^*\mathbf{E}_1\right)}{\partial\lambda_1^*} = \mathbf{X}_0^*\left[\left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_1 - \left(\lambda_1\mathbf{I}-\tilde{\mathbf{B}}_1\right)\mathbf{X}_0\right] = 0 \ , \quad (25)$$

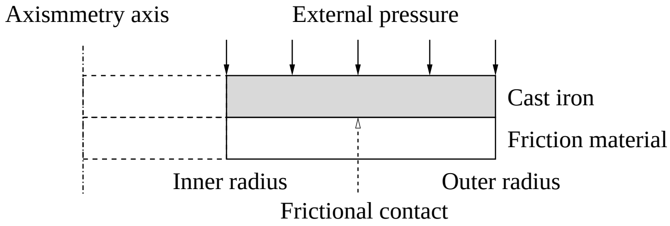

Figure 1. Two layers clutch system.

where * means conjugate and transpose. From Eqn. (25), we obtain

$$\lambda_1 = \frac{\mathbf{X}_0^*\left[\left(\tilde{\mathbf{B}}_0-\lambda_0\mathbf{I}\right)\mathbf{X}_1 + \tilde{\mathbf{B}}_1\mathbf{X}_0\right]}{\mathbf{X}_0^*\mathbf{X}_0} \quad (26)$$

and the other coefficients of $\lambda(\omega)$ are obtained in the same way as

$$\lambda_2 = \frac{\mathbf{X}_0^*\left(\tilde{\mathbf{B}}_1-\lambda_1\mathbf{I}\right)\mathbf{X}_1}{\mathbf{X}_0^*\mathbf{X}_0} \quad (27)$$
$$\lambda_3 = -\frac{\lambda_2\mathbf{X}_0^*\mathbf{X}_1}{\mathbf{X}_0^*\mathbf{X}_0} \quad (28)$$
$$\cdots$$
$$\lambda_n = -\frac{\lambda_{n-1}\mathbf{X}_0^*\mathbf{X}_1}{\mathbf{X}_0^*\mathbf{X}_0} \ . \quad (29)$$

These results permit us to approximate the eigenvalues and eigenfunctions at any speed ω using the full eigenvalue solution at only a limited number of 'main' speeds. In particular, we can use this method to obtain the eigensolution at the 'minor' speeds ω_j in Eqn (9).

3 Results

The FSE method was applied to the two-layer transmission clutch problem shown in Fig.1. The layers are pressed against each other by a uniform hydraulic pressure, and the lower plate is supported by frictionless contact with a rigid plane. The thickness of each layer is 2 mm. The inner radius and the outer radius are 60 mm and 80 mm, respectively. Sliding friction occurs at the interface between two layers. The sliding speed is decreased linearly from 170 r/s to 0 r/s in a period of 1 second. The magnitude of the initial perturbation temperature field is from -40°C to +40°C.

3.1 Convergence of FSE Method

The 'exact' solution to the problem is established by the TMA method with 160 time steps. To reduce the computational

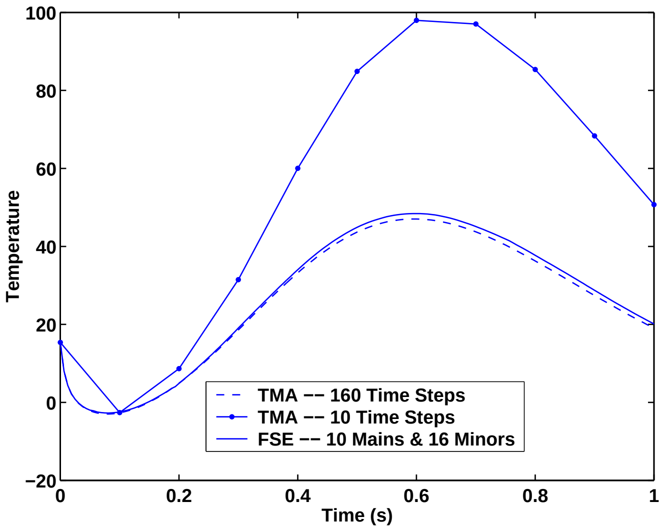

Figure 2. Comparision between the FSE method and the TMA method

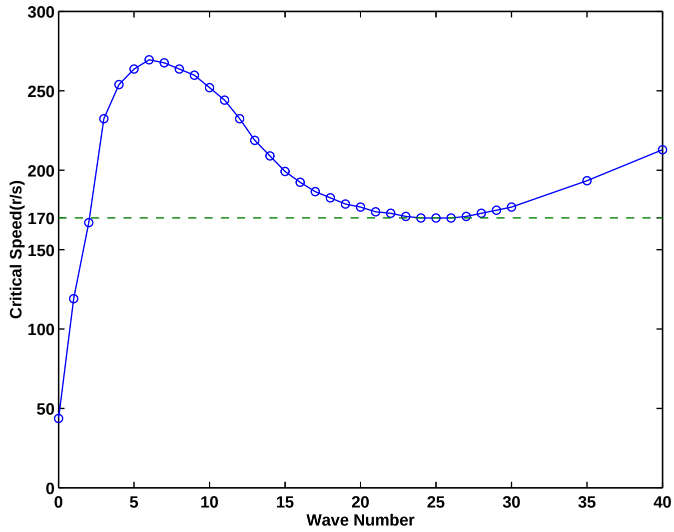

Figure 3. Critical speeds with different wave numbers

effort involved, we applied the FSE method to the same problem. We solved the full eigenvalue problem at 10 main speeds and used interpolation and expansion to approximate the eigensolution at 16 minor speeds in each main speed interval. Fig.2 compares the evolution of a typical nodal temperature near the mid-point of the contact interface by the proposed FSE method with the TMA method. The predicted temperatures are almost indistinguishable from those obtained with the TMA method using 160 time steps. By contrast, significant errors are obtained if the TMA method is used alone with only ten time steps.

3.2 Temperature Field by FSE Method

Fig.3 shows the critical speeds for this clutch problem with different wave numbers. We notice only when the wave number is 0, 1 or 2, which are defined as mode0, mode1 and mode2 respectively, the critical speed is below the initial sliding speed, 170r/s. In other words, only these three modes are instable at the current initial sliding speed. Therefore, the effect of the modes with other wave numbers to the temperature field can be ignored. The temperature field is

$$T(r,\phi,z,t) = \sum_{m=0}^{2} \mathrm{Re}\left[\theta_m(r,z,t)\, e^{jm\phi}\right] , \qquad (30)$$

Fig.4 shows the contour of the temperature field from 0 to 0.45 seconds, using the FSE method. As we can see from Fig.3, the critical speeds for mode1 and mode2 locate closer to inital sliding speed than that of mode0. Therefore, corresponding temperature fields of mode1 and mode2 decay more rapidly than that of mode0. As a consequence, in Fig.4, the mode1 and mode2 components in temperature field are found to die out quickly, with only mode0 remaining in the end of the transient process.

4 CONCLUSIONS

The FSE method employs an efficient algorithm to perform full eigenvalue solutions only at a limited number of sparsely main speeds, whilst to interpolate and expand the eigenfunctions and eigenvalues at speeds between these main speeds. This efficiently reduces the number of eigenvalue solutions required to solve the ITTEC transient problem and results in a significant reduction in computation time.

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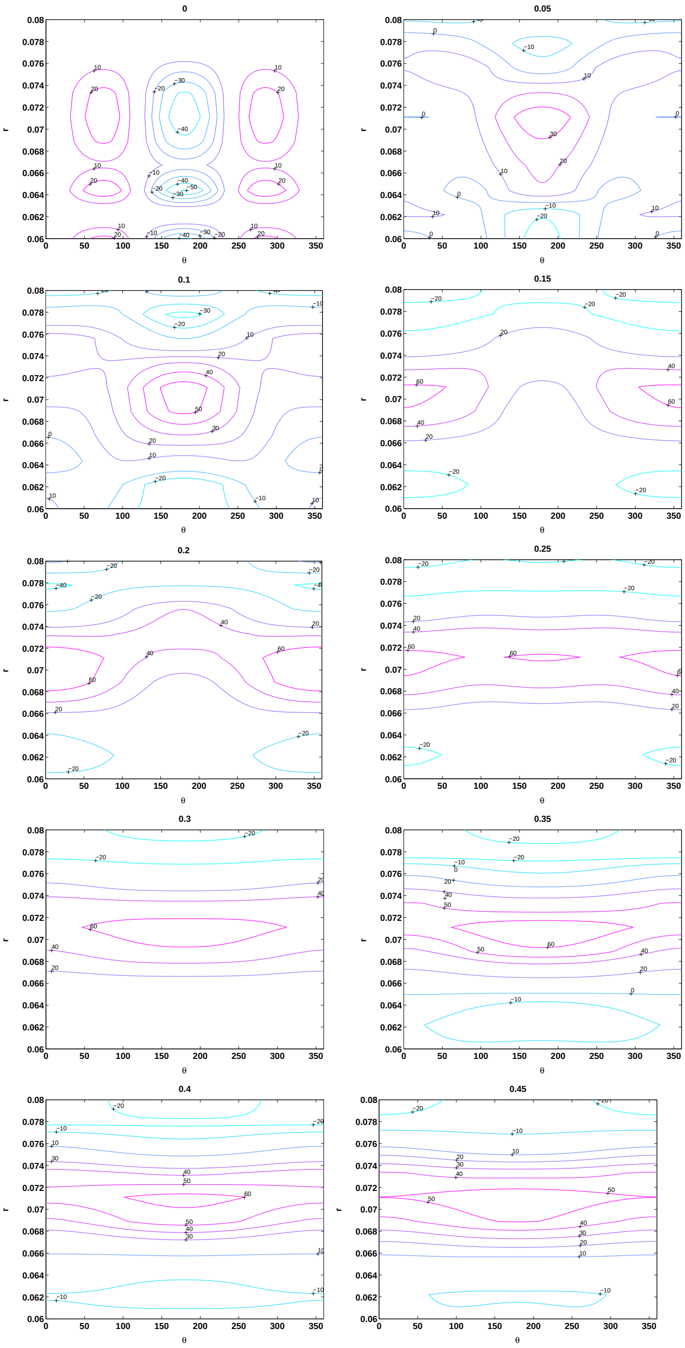

Figure 4. The contour of the temperature field at different time

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