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Vehicle Routing with Stochastic Time-Dependent Travel Times

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Abstract Assigning and scheduling vehicle routes in a stochastic time-dependent environment is a crucial management problem. The assumption that in a real-life environment everything goes according to an a priori determined static schedule is unrealistic. Our methodology builds on earlier work in which the traffic congestion is captured based on queueing theory in an analytical way and applied to the VRP problem. In this paper, we introduce the variability in the traffic flows into the model. This allows for an evaluation of the routes based on the uncertainty involved. Different experiments show that the risk taking/avoiding behaviour of the planner can be taken into account during optimization. As more weight is contributed to the variability component, the resulting optimal route will take a slightly longer travel time, but be more reliable. We propose to evaluate the solution quality in terms of the 95^{th}-percentile of the travel time distribution (assumed lognormal) as this measure captures well the trade-off between the average travel time and its variance.

Keywords: vehicle routing, time-dependent travel times, travel time reliability

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1 Introduction

Most traffic networks in Europe face high utilization levels, and consequently, congestion occurs. For a sufficiently high utilization, the smallest

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stochastic events (both in arrival processes or service processes) cause waiting, which in the case of traffic systems, materialize in lower speeds. As speeds change, travel times vary for a given distance. As such, all transportation problems which intend to minimize total time used, are subject to these physical considerations of congestion. Consequently, the road traffic conditions and its resulting variability can not be ignored in order to carry out a good quality route optimization. Uncertainty about the traffic conditions represented in travel times is a pervasive aspect of routing and scheduling, especially in a just-in-time environment or in highly congested regions like Europe. As the cost impact due to this uncertainty can be substantial, risk sensitive planners may want to evaluate to what extent their routes and schedules are risky in terms of travel times. Indeed, a slightly longer route in terms of expected travel time might be more interesting for a planner if the associated variance is considerably smaller. In this paper, the VRP problem considered deals with stochastic time-dependent travel times. In a real-life environment the travel times on an individual link are stochastic in nature. Due to weather conditions, car accidents, congestion, time and spatial fluctuations of traffic flows can be observed. The key issue considered in this paper is then the variability of the travel times which we consider to be a good approximation of travel time reliability.

In Van Woensel et al. (2008), a dynamic vehicle routing problem with time-dependent travel times due to traffic congestion was presented. The approach developed introduced the traffic congestion component modeled using a queueing approach to traffic flows. Explicitly making use of the time-dependent congestion results in routes that are (considerably) shorter in terms of travel time. Moreover, a first rough expression for the variance of the travel time was obtained.

The main contributions of this paper are:

1. We extend the objective function of a VRP with time-dependent travel times with a variability component. As such, we control for the degree of travel time variability during the optimization. We show that extending the objective function with this extra information about the stochastic travel time distribution provides better results when considering the reliability of travel times. Depending on environmental and road conditions as well as the risk taking behavior of the planner, these improvements can be substantial.
2. The reduction of the travel time variability may come at the cost of an increase of the expected travel time. To evaluate a solution, we use the 95^{th}-percentile of the travel time distribution (assumed lognormal) as a quality measure. Using this measure, the solution quality improves, if the increase in expected travel time results in a lower travel time associated with the 95^{th}-percentile of its distribution. Results show that the reliability improves as more weight is given to the variance component

during optimization. Different environmental and road conditions are compared and evaluated using this quality measure.

3. The trade-off between expected travel time and standard deviation of the travel time and the resulting quality measurement is demonstrated using an Arena simulation. The simulation also validates the underlying assumption that the travel time over an entire tour is well approximated by a lognormal one.

This paper is organized as follows: in Section 2, the literature background on our VRP variant is presented, followed by a formal description of the VRP, the objective function and the time-dependency implementation in Section 3. Experimental results on solution quality are presented in Section 4. Finally, conclusions and future research are presented in Section 5.

2 Literature review

Stochastic Vehicle Routing

Travel times between any two customers are a stochastic process related to traffic congestion. Depending on the time of the day the traffic network will face a different level of congestion. The number of vehicles, the road capacity, road conditions, weather conditions, etc. influence the speed of the vehicles. There has been limited research on solving the VRP problem in the face of stochastic time-dependent travel times. One of the first approaches (Malandraki and Daskin, 1992) treated the travel time between two customers as a function of distance and the time of the day (if this temporal component causes more travel time variation than travel time variation due to accidents, weather conditions,etc.), resulting in a piecewise constant distribution of the travel time. Although they only incorporate the temporal component of traffic density variability, they acknowledge the importance of the traffic density variability due to accidents, weather conditions and other random events. However, the FIFO principle is not necessarily satisfied (Ichoua et al., 2003).

Ichoua et al. (2003) introduced a model that guarantees that if two vehicles leave the same location for the same destination (along the same path), the one that leaves first will never arrive later than the other. This is satisfied by working with step-like speed distributions and adjusting the travel speed whenever the vehicle crosses the boundary between two consecutive time periods. To reduce computational run times, they limited the number of time slices to three. The speed differences are then modeled using correction factors on the weights of the links. Donati et al. (2003) extended this line of research by indicating the importance of optimizing the starting time in addition to optimizing the routes in a time-dependent environment. They show that the degree of feasibility (defined as not violating a time constraint) and optimality decreases for the best solutions for the constant speed model when they are used in a time-dependent context with increasing variability of the traffic conditions. Similar results were also observed by Haghani and Jung (2005). In contrast with Ichoua et al. (2003) they present the travel time as a continuous function that can accept any kind of travel time variation. In their real time approach, (Haghani and Jung, 2005) propose to adjust the vehicle routes at certain times in the planning period to take new demands and new traffic information into account. They classify links into three types and each link type has two types of traffic flow characteristics. At any time during a day, the link travel speed is calculated based on the design speed of the link and the ratio of the travel speed to the design speed for that link type at that time. Travel times between the nodes are calculated using a time-dependent shortest path algorithm and are input to the vehicle routing problem algorithm. An important conclusion states that if the uncertainty in travel time forecasting increases, the dynamic routing strategy becomes increasingly superior. Uncertainty of travel time forecasting is inserted in 12 cases, in which they change the percentage of links that can change and the gap of that change. No information is provided on how to assess the uncertainty of the travel time forecasting.

As indicated by Ichoua et al. (2003) the literature on time-dependency in a VRP context is limited. Stochastic and time-dependent travel times are more extensively operated on in shortest path analysis (e.g. Hall (1986), Fu and Rilett (1998), Gao and Chabini (2002), Gao and Chabini (2006) and He et al. (2005)). He et al. (2005) indicates that although mean and variance contain the most important information about path travel time, finding the single route with expected shortest travel time is not appropriate for routing of planners who are not risk neutral in their behavior. The entire travel time distribution contributes to the routing choice. Chen et al. (2003) propose to using the standard deviation and the 90^{th} percentile travel time in addition to the mean to measure service quality.

Stochastic travel times are introduced in the vehicle routing problem by Laporte et al. (1992). Following Gendreau et al. (1996a) a stochastic VRP arises whenever some elements of the problem are random. A stochastic model is usually modeled in two stages. In the first stage, a planned a priori route is determined, followed by a realization of the random variables. In the second stage, a recourse or corrective action is then applied to the solution of the first stage. The cost/saving generated through the recourse may have to be considered when designing the first stage solution. Kenyon and Morton (2003) developed two models to tackle the stochastic VRP with random travel and service times with distribution assumed to be known. The first minimizes the expected completion time and the second maximizes the probability that the operation is complete before a preset target time T. After the routes have been constructed in the first phase, the actual travel times of those routes based on realizations of the random travel and service times are computed.

Modeling Traffic

Based on traffic counts, analytical queueing models model the behavior of traffic flows as a function of the most relevant physical and geographical determinants (i.e. free flow speed, maximum road capacity,variability due to the weather, etc.). The travel times can then be modeled much more realistically using these speeds (i.e. expressed in kilometer per hour) and are directly related to the physical characteristics and the geographical location on the arc.

An empirical validation of the queueing approach as well as parameter finetuning is provided in Van Woensel and Vandaele (2006) and validation based on simulation results is provided in Van Woensel et al. (2006). As the distribution of the speeds is calculated based on traffic counts using queueing models in which parameters can be finetuned as to represent current environmental conditions best, the link and consequently road travel time distributions can be modeled much more realistically. More specifically, the stochastic nature of travel times is captured using queueing theory applied to traffic flows (Vandaele et al. (2000) and Van Woensel (2003)). By making use of this analytical approach the necessary data to model congestion (i.e. traffic flow and some queueing parameters to capture road conditions) is limited which opens the door for real-life applications.

It must be noted that several other queueing models have been proposed in the literature, all aiming at improving traffic flow modeling accuracy and flexibility (see e.g. Jain and MacGregor Smith (1997) and Heidemann (1996)). For a detailed discussion refer to Van Woensel (2003).

3 Problem formulation

Formally, the routing problem considered can be represented by a complete directed graph $G = (V, A)$ where $V = \{0, 1, \ldots, n\}$ is a set of nodes representing the depot (0) and the customers $(1, \ldots, n)$, and $A = \{(i, j)|i, j \in V\}$ the set of directed links. For each customer, a fixed non-negative demand q_i is given ($q_0 = 0$). The aim is then to find routes with shortest travel time where the following conditions hold (Laporte, 1992): Every customer is visited exactly once by exactly one vehicle; all vehicle routes start and end at the single depot; every vehicle route has a total demand not exceeding the maximum vehicle capacity Q.

The basic objective function which needs to be minimized is similar to the one presented in (Gendreau et al., 1994) though expressed in terms of travel times. It is the sum of the expected travel time of all routes in each subtour and the excess capacity in each subtour. If the solution is infeasible with respect to capacity a penalty proportional to the excess capacity is added. Only taking into account the expected travel times ignores the risk profile of the planner. Note that the proposed approach is similar to mean-variance analysis used in financial planning of portfolios (Best and Grauer (1991) and Grauer and Hakansson (1993)). In this literature, it is argued that risk can be associated with the included variance term (Mulvey et al., 1995). An extension of the objective function thus involves adding the standard deviation (SD) of the travel times, where the weight of the latter component is controlled by a positive parameter β. Higher risk averseness will be reflected in an increase of the parameter β resulting in more weight attributed to the standard deviation in the objective function. The higher β, the less sensitive the solutions are to variability in the data, as such controlling the degree of travel time reliability.

In our time-dependent model, time is discretized into P time zones of equal length $\triangle p$ with a different travel speed distribution associated with each time zone p ($1 \leq p \leq P$). One of the earliest studies explicitly dealing with the travel speed distribution is that of (Berry and Belmont, 1951) who looked into the distribution of the measured speed of a vehicle as it crosses a particular point on the highway. Such speed distributions were found to be normally distributed. Travel times, taken as the reciprocal of speed, are shown to be also roughly normal, although slightly skewed indicating that a lognormal distribution might be interesting as an alternative (Kharoufeh and Gautam, 2004). Other empirical results (Taniguchi et al. (2001) and Kwon et al. (2000)) show that there is always a certain minimum time needed to cover the distance (i.e. it is impossible to traverse the distance in a time shorter than this minimum time). After this minimum, the probability increases rapidly to a maximum after which the probability slowly decreases with a long tail (i.e. skewed to the right). Due to these characteristics, Taniguchi et al. (2001) proposed to use a lognormal distribution rather than a normal distribution.

In our model the speed in each time zone is obtained by applying queueing theory to traffic flows. For a discussion of the queueing model, the interested reader is referred to Vandaele et al. (2000), Heidemann (1996) and Van Woensel et al. (2001).

It can be shown that the convolution of k lognormal distributions is again (approximative) lognormal (Beaulieu and Xie, 2004). We assumed a lognormal distribution of the travel time, but the analysis could also be applied if another distribution were chosen. Simulation results in Section 4 show that the travel time distribution over an entire tour is again well approximated by a lognormal one.

4 Experimental results

In this section, we first described the Tabu Search implementation and the used congestion information. We then extend the objective function with the standard deviation of the travel times. Depending on the road and environmental conditions, more substantial gain in terms of travel time reliability is found by contributing more weight to this component during optimization. The decrease in variability is offset by an increase in expected travel time, therefore, we introduce the 95^{th}-percentile of the travel time distribution as a solution quality measure. We show that contributing more weight to the standard deviation during optimization improves the 95^{th}-percentile of its distribution. Finally, an Arena (Rockwell Software Inc., 2005) based simulation is provided, which confirms the results.

4.1 Implementation

In this paper Tabu Search, first proposed by Glover (1989) and Glover (1990), is used to generate solutions as it has a number of advantages: general applicability of the approach, flexibility for taking into account specific constraints in real cases and ease of implementation (Pirlot, 1996). For this Tabu Search implementation the following references where used as a basis: Gendreau et al. (1994), Gendreau et al. (1996b), Hertz et al. (2000) and Van Woensel et al. (2008).

The first important change made to this basic algorithm consists of replacing distance by travel time. The main change consist of extending the basic objective function with the standard deviation of the route travel time. The objective function thus becomes $E(\tilde{TT})+\beta SD(\tilde{TT})$, with $E(\tilde{TT})$ $(SD(\tilde{TT}))$ the expected travel time (standard deviation of the travel time) and β a positive parameter to account for the risk averseness of the planner. Given an empirical dataset, they can be tuned to represent the relevant environment conditions as close as possible (Van Woensel and Vandaele, 2006). For the subsequent analysis, we use a speed profile with a congested flow during the entire day and one with heavy congestion only during the morning and evening peak hours.

As we start from the flow on a road segment and transform it into speed, we need to explain the setting of the queueing model parameters to do so. The flow on a road segment is assumed given as well as the free flow speed (s_f). The remaining queueing parameters ($c_a \in [0,1]$, $c_s \in [0,1]$ and k_j) are to be set properly. Given an empirical dataset, they can be tuned to represent the relevant environment conditions as close as possible ((Van Woensel and Vandaele, 2006)). As this is not the objective here, we choose $c_s = c_a$ and set them both to 0.99 if we want to represent weather conditions that cause a large variability of speeds or set them to 0.75 if the weather conditions are better. The jam density k_j will be set such that during rush hours the resulting speed is substantially lower than during off peak periods for a given observed flow q. As we see in Figures 1 and 2, a k_j of 40 for a free flow of 120 km/h results in a reasonable speed profile both when there is a congested flow during the entire day (Figure 1: *Congested flow*) as when there is only heavy congestion during the morning and evening peak hours (Figure 2: *Rush-hour flow*).

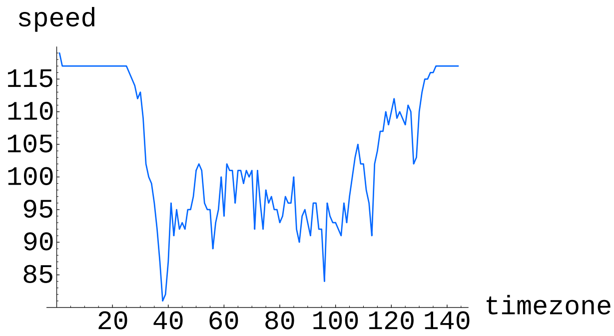

Fig. 1. Speeds per time zone with k_j 40, free flow 120 km/h and congested flows (almost) during the entire day.

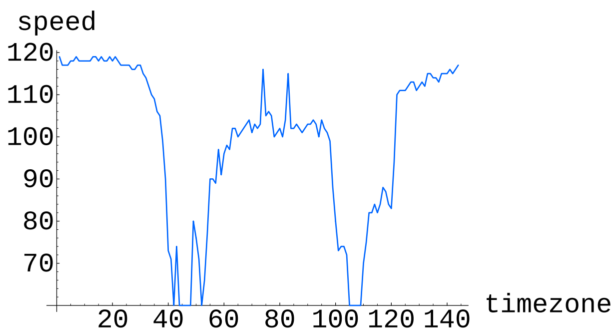

Fig. 2. Speeds per time zone with k_j 40, free flow 120 km/h and 2 heavy congested peak hour flows.

4.2 Impact of the variance component

Minimizing the expected total travel times assumes that the planner is risk neutral in his planning behavior, i.e. the planner does not care about the risk involved. Ignoring the associated variance of the travel time could be very costly since the variance could be unacceptably large from a managerial or planning point of view. Indeed, one might prefer to have a route that is on average slightly worse, but has a reduced variance, as such increasing the reliability of the predicted arrival times at all destinations. Depending on the risk profile a different route will be chosen. By adjusting the parameter β in the objective function, the planner can easily insert his personal risk profile. Higher values for this parameter indicate a risk avoiding preference of the planner and will result in routes that have more reliable travel times. From Table 1 it follows that the probability that the travel time is smaller than the travel time at TT_β (defined as $E(\tilde{TT}) + \beta SD(\tilde{TT})$) increases as β increases. In addition, the tail of the distribution to the right of TT_β contributes to the total mass of the distribution. The higher β, the less mass there is left that contributes to the total mass of the distribution ((Finkel, 1990)). For instance, for dataset $32k5$ from Augerat, the optimal route has a travel time distribution with σ (scale parameter of the lognormal travel time distribution) = 0.376 (E(TT): 1308.87 minutes, SD(TT): 509.62 minutes). When $\beta = 2.0$, 95.73 % of the population of travel times is below TT_β. The remaining 4.27 % however still contributes 8.93 % of the total mass. Therefore, we will examine β-values up to 3.0, where the remaining mass is about 3 % for this set.

β	β=0.0	β=0.5	β=1.0	β=1.5	β=2.0	β=2.5	β=3.0
$p(TT < TT_\beta)$ (%)	57.45	74.58	85.61	92.11	95.73	97.71	98.77
$mass(TT > TT_\beta)$ (%)	57.45	38.76	24.59	15.00	8.93	5.25	3.06

Table 1. Probability for $TT < TT_\beta$ and associated remaining mass in the tail of the travel time distribution for different β-values when σ (scale parameter of the lognormal travel time distribution) = 0.376.

The values in Table 2 indicate the relative decline of the standard deviation of the total travel time of the newly constructed route (with associated β) compared to the standard deviation of the travel time found by a minimization with $\beta = 0$. The values are an average over 27 Augerat datasets. As the weight of the standard deviation of the travel time adopts higher values, the standard deviation of the associated best route continues to decrease, regardless of the environmental and road conditions. The best improvement however is obtained by increasing the value of β from 0 to 0.5, whereas the additional improvement of further steps reduces in magnitude. It is thus crucially important to include the variability of the travel times in the objective function. Better improvements will be expected when the road conditions are bad (c_a=0.99, with c_a a parameter from the queueing model). If road conditions are bad, the speed will fluctuate more, which makes it more difficult to predict when we will complete a tour as opposed to better road conditions. By introducing this uncertainty factor in the objective function, larger improvements are expected when road conditions are bad. If the flow is congested during the entire day, the improvement is also more substantial as compared to a flow which is characterized by two rush-hours. This is due to the fact that between the two congestion periods, drivers are able to uphold free flow speed, which evidently is associated with less variability.

Test situation	β=0.5	β=1.0	β=1.5	β=2.0	β=2.5	β=3.0
congested flow, c_a=0.75	-1.61	-2.18	-2.52	-2.75	-3.14	-3.47
rush-hour flow, c_a=0.75	-1.45	-2.05	-2.50	-2.95	-3.32	-3.46
congested flow, c_a=0.99	-3.30	-4.18	-4.72	-5.17	-5.40	-5.56
rush-hour flow, c_a=0.99	-3.17	-3.83	-4.20	-4.55	-4.71	-5.04

Table 2. Impact of β on the standard deviation of the travel time, compared to the standard deviation of the travel time with $\beta = 0$ (%).

4.3 The 95^{th}-percentile as a quality measure

The reduction of the standard deviation comes at a certain cost, i.e. a likely increase of the average travel time. To check whether this cost is acceptable, we propose the use of the 95^{th}-percentile as a quality measure assuming a lognormal distribution for the travel time. The 95^{th}-percentile combines the expected total travel time and variance of total travel time into a single number. Figure 3 illustrates that if the 95^{th}-percentile of the solution with worse average travel time, but better standard deviation (Distribution 2) is lower than the one with best average travel time (Distribution 1), we have nevertheless managed to improve solution quality.

This can also be derived from our test cases. The impact of β on the improvement in the 95^{th}-percentile can be observed in Table 3. The travel time associated with the 95^{th}-percentile decreases when more weight (higher β) is given to the standard deviation in the objective function. The best improvement is observed in the first step, regardless of the test situation. The additional improvement of higher β values reduces in magnitude. This means that although the average travel time will become larger with increasing β, the total travel time will be better in 95% of all cases but with decreasing importance.

If the road conditions are good (low c_a), the relative improvement of the travel time of the 95^{th}-percentile is more substantial for the congested

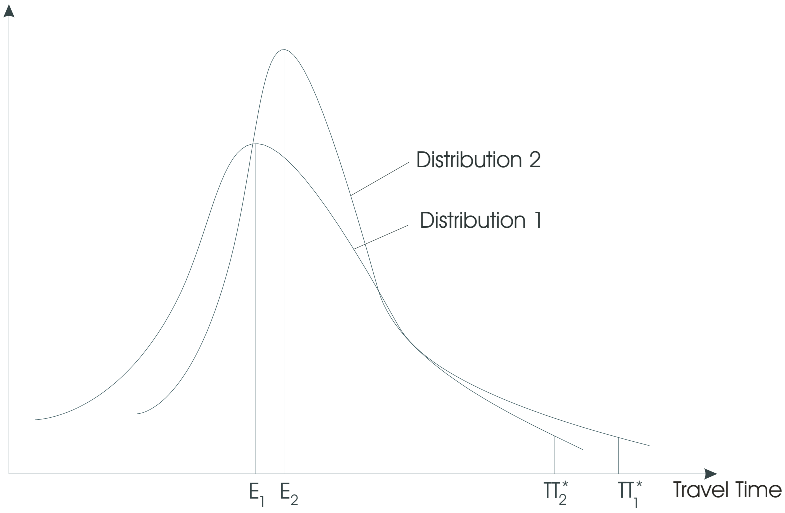

Fig. 3. Impact of 95^{th}-percentile on solution quality (Lognormal distribution)

flow throughout the day compared to a flow with two rush-hours for equal β values. From Table 4, we see that if weather conditions are good, the squared coefficient of the travel times of the two flow types are of the same magnitude. Therefore, since the standard deviation of the travel times is higher for the congested flow, better improvements can be expected for this flow type with increasing β.

On the other hand, if road conditions are bad, the best relative improvement is observed for the two rush-hour flow. In bad weather, the squared coefficient of the travel times for the flow with two rush-hours is larger than the congested flow (Table 4). This means that for the flow with two rush-hours the standard deviation is relatively large compared to the mean. Adding some weight to it will thus result in better relative results.

Test situation	β=0.5	β=1.0	β=1.5	β=2.0	β=2.5	β=3.0
congested flow, c_a=0.75	-0.62	-0.82	-0.94	-0.98	-1.12	-1.16
rush-hour flow, c_a=0.75	-0.52	-0.72	-0.84	-0.91	-1.06	-1.07
congested flow, c_a=0.99	-1.32	-1.62	-1.75	-1.94	-1.97	-1.99
rush-hour flow, c_a=0.99	-1.38	-1.66	-1.79	-1.90	-1.99	-2.07

Table 3. Impact of β on the 95^{th}-percentile of the travel time, compared to the 95^{th}-percentile of the travel time with $\beta = 0$ (%)(lognormal distribution).

In Table 5, the gain in travel time (in minutes) of the 95^{th}-percentile is presented for the test cases when comparing $\beta = 3$ with $\beta = 0$. For instance, the gain over all Augerat sets for the congested flow with c_a=0.99

Test situation	β=0.0	β=0.5	β=1.0	β=1.5	β=2.0	β=2.5	β=3.0
congested flow, c_a=0.75	0.038	0.037	0.037	0.036	0.036	0.036	0.036
rush-hour flow, c_a=0.75	0.038	0.037	0.037	0.036	0.036	0.036	0.036
congested flow, c_a=0.99	0.110	0.103	0.101	0.100	0.099	0.098	0.098
rush-hour flow, c_a=0.99	0.113	0.106	0.104	0.103	0.102	0.102	0.101

Table 4. Squared coefficient of variation of the travel times for given test situation and β values.

is on average 88.32 minutes. The minimum improvement for that test situation is 15.58 minutes and the maximum improvement is almost 3 hours (166.32 minutes). It is clear that the reduction of the standard deviation of the travel time is substantial enough to overcome the increase in average travel time. Extending the objective function to account for the travel time variability provides results with better overall reliability, especially when road conditions are bad.

Test situation	Average	Minimum	Maximum
congested flow, c_a=0.75	35.11	6.90	93.57
rush-hour flow, c_a=0.75	32.26	0.33	68.10
congested flow, c_a=0.99	88.32	15.58	166.32
rush-hour flow, c_a=0.99	81.61	15.85	259.98

Table 5. Improvement (in minutes) of the 95^{th}-percentile of the tour travel time when comparing the optimal routes with $\beta = 3$ and $\beta = 0$.

4.4 Simulation

To validate the approximations used when building the variance estimating model presented above, we constructed a simulation in (Rockwell Software Inc., 2005) in which we reconstructed the route as a sequence of lognormal distributions (representing the links) with mean and standard deviation as obtained through queueing theory. For set $32k5$, 3001 trucks completed the tour and their travel times have been plotted in Figure 4. Results indicate that the resulting tour travel time is indeed lognormally distributed (there clearly is a long tail to the right). In addition, the plotted results are close to the theoretical tour travel time distribution. The travel time associated with the 95^{th}-percentile is 2205.5 minutes (Figure 4, Table 6), which corresponds with what we expect from the theoretic travel time distribution (2262.71 minutes (95^{th}-percentile of lognormal travel time distribution with E(TT): 1308.87 minutes and SD(TT): 509.62 minutes)).

The positive impact in terms of travel time reliability when optimizing the VRP for a more heavily weighted standard deviation is validated by the simulation results provided in Table 6. The best solutions of a Tabu

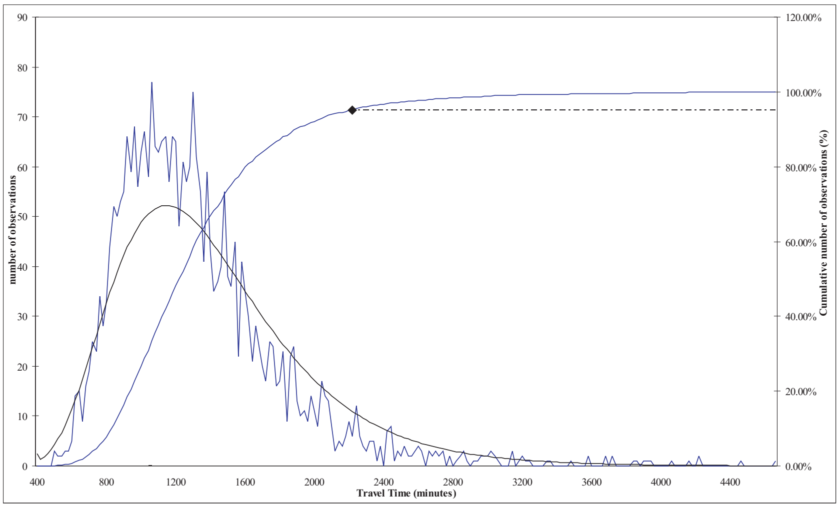

Fig. 4. Travel time distributions of set $32k5$ with congested flow and c_a=0.99. Results following a simulation with ARENA are plotted together with the expected lognormal distribution (parameters derived from Tabu Search solution).

Search optimization with β values $\in \{0.0; 3.0\}$ are reconstructed in ARENA. For each set the average travel time increases and the standard deviation of the travel time decreases, as such increasing the travel time reliability. The decrease in the standard deviation is substantial enough to improve the overall solution quality (better travel time associated with the 95^{th}-percentile). β values $\in \{0.0; 3.0\}$ are two extreme situations. The planner can use any value in between depending on his own risk avoidance behavior. From a planning point of view, it is better to have more predictability in the routing than a potentially faster route. The uncertainty about the actual arrival time will be avoided as the planner is more risk averse.

		Average TT	Standard Deviation TT	95^{th}-percentile TT
32k5	β= 0.0	1320.87	550.77	2205.5
	β= 3.0	1332.51	514.51	2169.5
38k5	β= 0.0	1286.60	480.72	2087.7
	β= 3.0	1296.14	471.02	2049.4
80k10	β= 0.0	2701.11	746.35	4007.0
	β= 3.0	2720.64	685.73	3972.9

Table 6. Comparing the best routes (through Tabu Search optimization with respective β-values) for three sets in congested flow with c_a=0.99 using ARENA. Average travel time, standard deviation of the travel time and 95^{th}-percentile are provided after 3001 trucks completed the best routes.

5 Conclusions and future research

The capability of taking into account time-dependent travel speeds is extremely valuable. Minimizing the expected travel time however still does not deal with the true stochastic nature of the travel times. As the real speed is a realization of a stochastic process, it is equally important to account for the variability of the speed and thus the travel time uncertainty when planning a route. This paper aims at obtaining more reliable routes in terms of travel time. These more realistic solutions have the potential to reduce real operating costs for a broad range of industries which face daily routing problems.

When including the variance of the travel time, the potential applications are vast: it gives a manager a powerful tool to incorporate and take into account congestion uncertainty in his optimization. The higher the risk averseness of the planner, the more weight is allowed to that factor while optimizing, as such making the resulting routes more reliable and predictable. Although the gain in terms of less travel time variability will be offset by a higher average travel time, the travel time associated with 95^{th}-percentile will improve. Depending on the road and environmental conditions, this improvement will be more or less substantial. These conclusions are confirmed by independent simulation studies.

It must be noted however, that in some cases the reduction in variability will not be substantial enough to compensate for the reduction of the expected travel time. If for instance the initial route has already a small travel time distribution (associated with a high speed), than it will be hard to find a new road/starting time with a better travel time distribution.

As there is not much informaton available on how to model the variance of the travel times in literature, most analyses are in terms of expected travel time, we have heuristically determined variances in the analysis presented in this paper. Hence, we are currently deriving general conditions for the speed-profiles that guarantee the validity of the conclusions derived here.

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