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VARIATIONAL LEVEL SET SEGMENTATION USING SHAPE PRIOR

S. Y. Yeo⋆, X. Xie⋆⋆, I. Sazonov⋆ and P. Nithiarasu⋆
⋆College of Engineering, Swansea University, Swansea SA2 8PP, U. K.,
{465186, i.sazonov, p.nithiarasu}@swansea.ac.uk
⋆⋆Department of Computer Science, Swansea University, Swansea SA2 8PP, U.K.,
x.xie@swansea.ac.uk

SUMMARY

We proposed a new level set segmentation model with statistical shape prior using a variational approach. The image attraction force is derived from the interactions of gradient vectors across the whole image domain. This gives the active contour a global representation of the geometric configuration, making it more robust to image noise, weak edges and initial configurations. Statistical shape information is incorporated using a nonparametric technique is used to model the shape distribution, which allows the model to handle relatively large shape variations.

Key Words: *level set, segmentation, shape prior.*

1 INTRODUCTION

Image segmentation is an important area in image processing and has a wide range of applications such as biomedical image analysis. Several approaches have been proposed for automatic object segmentation. Some of the main challenges include the extraction of object boundaries or regions from images with noise and intensity inhomogeneity. Other factors such as weak object edges, low resolution, and complex geometries can also affect the accuracy and efficiency of the shape extraction process.

Active contours provide an effective framework for object segmentation as they can easily adapt to shape variations. Various types of information can also be incorporated to regularize the smoothness and shape of the contour. Active contour models usually take the form of image gradient based approaches [1] and region based approaches [2, 3]. As conventional edge based methods are driven by external energies derived from local image information, they are often affected by local minima such as image noise, and have difficulties dealing with weak object edges. Region based methods make use of regional statistics such as means and variances to derive the external energies or forces, and are thus more robust to noise interference. However, as region based models [2, 3] are often based on the assumption that image objects consist of distinct regional statistics, they cannot deal with intensity inhomogeneity in images. Various groups have also incorporated shape prior information into their models. Many of these techniques are based on statistical assumptions, i.e. the training shapes are constrained to a Gaussian distribution. This can easily restrict the range of applications as real world objects can often take on complex shape variations.

In this paper, we propose a new variational level set model for efficient segmentation of images. The proposed method uses an image based energy derived from the global interaction of image gradient vectors [4] to attract the active contour towards object boundaries. This image based energy greatly improves the performance of the active contour in handling weak edges, image noise and arbitrary cross-boundary initializations. We also incorporate statistical shape prior information into the variational segmentation model using nonparametric shape density distribution [5, 6]. By using kernel density estimation (KDE) to the space of shapes, the shape prior can model arbitrary shape distributions, and can be applied to segment various shapes from occluded or noisy images.

2 PROPOSED METHOD

The proposed variational model consists of an image attraction force which propagate contours towards object boundaries, and a global shape force which draws the model towards similar shapes represented in the training set. In this section, we formulate the variational segmentation model using Bayesian inference. Therefore, the segmentation of an image I can be considered as minimizing the following energy functional:

$$E(\phi) = -\log(p(I|\phi)) - \log(p(\phi)) = E_{\text{image}}(\phi) + \alpha E_{\text{shape}}(\phi) \tag{1}$$

where $E_{\text{image}}(\phi)$ represents the image based term, $E_{\text{shape}}(\phi)$ represents the shape prior and α is a constant weighting term.

We recently proposed a new image attraction force based on hypothesized gradient vector interactions [4] for contour evolution. Here, we formulate the image attraction force in a variational framework so that statistical prior information can be conveniently incoporated into the model. The proposed image based energy functional takes the following form:

$$E_{\text{image}}(\phi) = \nu \int_{\Omega} g(\mathbf{x})|\nabla H_{\epsilon}(\phi)|d\mathbf{x} + \int_{\Omega} G(\mathbf{x})H_{\epsilon}(\phi)d\mathbf{x} \tag{2}$$

where ν is a constant parameter, $g(\mathbf{x}) = 1/|1 + \nabla I|$, and H_{ϵ} is the regularized Heaviside function [2]. $G(\mathbf{x})$ represents the gradient vector interaction field given as:

$$G(\mathbf{x}) = \int_{\Omega} \frac{\hat{\mathbf{r}}_{\mathbf{x}\mathbf{x}'}}{r^k_{\mathbf{x}\mathbf{x}'}} \cdot \nabla I(\mathbf{x})d\mathbf{x} \tag{3}$$

where $\hat{\mathbf{r}}_{\mathbf{x}\mathbf{x}'}$ is the unit vector from pixel location $\mathbf{x}$ to $\mathbf{x}'$ and $r_{\mathbf{x}\mathbf{x}'}$ is the distance between the pixels. k is a constant which is set to the dimension of the image data (i.e. $k = 2$ for 2D image). $G(\mathbf{x})$ can be computed efficiently as a vector convolution using fast Fourier transform (FFT), and some effects caused by spurious edges can be removed by not considering pixels with edge magnitude smaller or greater than a certain percentage of the maximum magnitude. The first term in (2) induces the segmentation model to favour minimal length and smooths the contour, while the second term attracts the active contour towards image object boundaries.

The gradient vector interaction field $G(\mathbf{x})$ utilizes image pixels or voxels across the whole image domain, and thus gives a global representation of the geometric configuration. This provides the active contour with a high invariancy to initializations and a large attraction range. It also increases the robustness of the active contour against image noise and weak edges.

The shape based energy functional is defined using a shape distance measure [5] as:

$$E_{\text{shape}}(\phi) = D^2(\phi, \phi_i) = \int_{\Omega} (H(\phi(\mathbf{x} + \mu_{\phi}) - H(\phi_i))^2 d\mathbf{x} \tag{4}$$

where $\{\phi_i\}_{i=1\dots N}$ is a set of training shapes, and μ_ϕ is the center of gravity of the shape ϕ. The shape distance provides a dissimilarity measure which is invariant to translation of the shape ϕ. Intrinsic alignments with respect to scale and rotation can also be incorporated in the model [5]. Here, we use the nonparametric technique of kernel density estimation (KDE) to model the statistical shape distribution:

$$p(\phi) \propto \frac{1}{N}\sum_{i=1}^{N} \exp\left(-\frac{1}{2\sigma^2} D^2(\phi, \phi_i)\right) \tag{5}$$

where σ is the kernel width, and is set to the mean nearest-neighbor distance.

The minimization of the energy functional in (1) generates a segmentation model which simultaneously attracts the active contour towards image object boundaries and similar shapes in the training set. The gradient descent with respect to the shape ϕ can be derived using calculus of variation as:

$$\frac{\partial \phi}{\partial t} = -\frac{\delta E_{\text{image}}(\phi)}{\delta \phi} - \alpha \frac{\delta E_{\text{shape}}(\phi)}{\delta \phi} \tag{6}$$

The gradient flow of the image based energy is given as:

$$\frac{\delta E_{\text{image}}(\phi)}{\delta \phi} = \nu g(\mathbf{x}) \nabla \cdot \left(\frac{\nabla \phi(\mathbf{x})}{|\phi(\mathbf{x})|}\right) \delta_\epsilon(\phi(\mathbf{x})) - G(\mathbf{x}) \delta_\epsilon(\phi(\mathbf{x})) \tag{7}$$

where δ_ϵ is the regularized version of the Dirac delta function, i.e. the derivative of H_ϵ, and the gradient flow of the shape based energy is defined as:

$$\frac{\delta E_{\text{shape}}(\phi)}{\delta \phi} = \frac{\sum_i w_i \frac{\partial D^2(\phi,\phi_i)}{\partial \phi}}{2\sigma^2 \sum_i w_i} \quad \text{where } w_i = \exp\Big(-\frac{1}{2\sigma^2} D^2(\phi, \phi_i)\Big) \tag{8}$$

The shape derivative with respect to ϕ is given as:

$$\begin{aligned}\frac{\delta D^2(\phi, \phi_i)}{\delta \phi} &= 2\delta_\epsilon(\phi(\mathbf{x}))\Big((H_\epsilon(\phi(\mathbf{x})) - H_\epsilon(\phi_i(\mathbf{x} - \mu_\phi)) + \frac{(\mathbf{x} - \mu_\phi)^T}{\int\limits_\Omega H_\epsilon(\phi) d\mathbf{x}} \\ &\quad \times \int (H_\epsilon(\phi(\mathbf{x}') - H_\epsilon(\phi_i(\mathbf{x}' - \mu_\phi)\, \delta_\epsilon(\phi(\mathbf{x}')) \nabla \phi(\mathbf{x}')\, d\mathbf{x}'\Big)\end{aligned} \tag{9}$$

3 RESULTS

In this section, we show that the proposed method can be applied efficiently for image object segmentation. Figure 1 shows the segmentation of multiple annular-like objects from an image with 70% noise, occlusions and intensity variation. The training set consist of 20 images with annular-like objects of varying shapes. It is shown that the proposed active contour with shape prior can extract the shapes from the occluded and noisy image efficiently.

Figure 2 depicts the segmentation of carotid from computed tomography (CT) images. In this example, 20 training shapes are manually generated to model the shape distribution. Note that the image data consist of various structures such as adjacent vessels and bones, and image regions representing the carotid often contain diffused edges and intensity inhomogeneity. Therefore, careful initializations are often required for purely image based segmentation model to delineate the shape of the structure. As shown in the figure, the proposed active contour with image and shape based energy can be applied to segment the carotid structure efficiently.

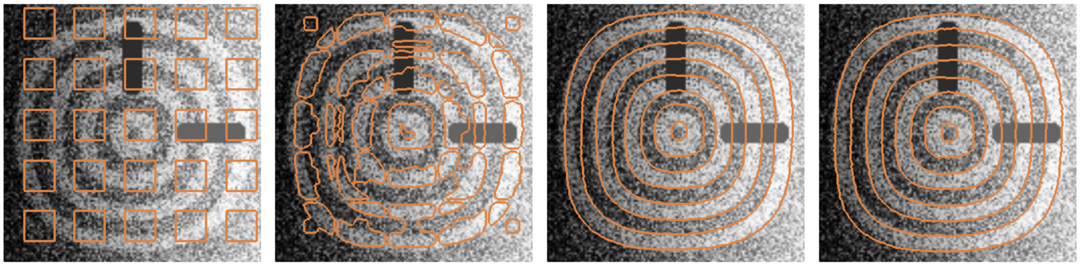

Figure 1: Segmentation of annular-like shapes using the proposed method.

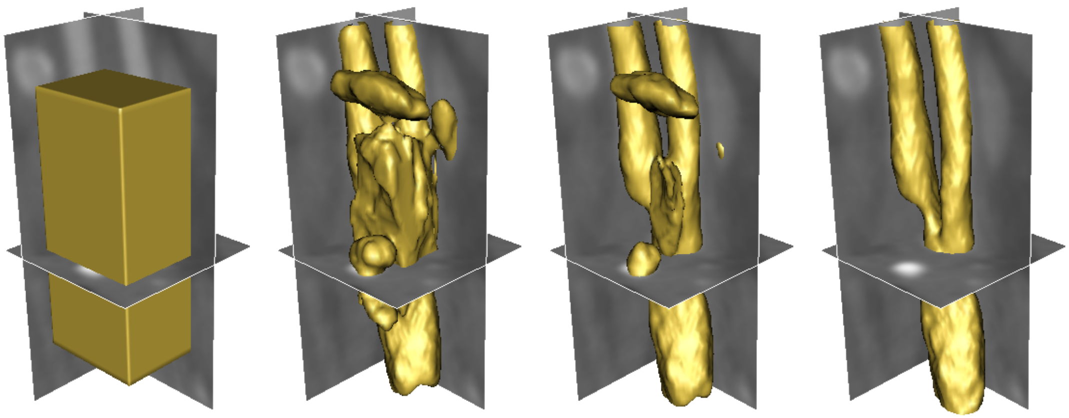

Figure 2: Segmentation of carotid from CT image using the proposed method.

4 CONCLUSIONS

We have presented a new level set segmentation model with statistical shape prior using a variational approach. The image based energy is derived from the global interaction of gradient vectors. The active contour model is thus more robust to image noise and weak edges, and has a strong initialization invariancy. By using kernel density estimation, the incorporated shape prior can model arbitrary shape distributions. The proposed model can thus be applied to segment complex shapes from images of various modalities efficiently.

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