

# Bundling decisions in procurement auction with sequential tasks\*

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## Abstract

This paper investigates the principal's bundling decision in procurement auction in the context of a project consisting of two sequential tasks, where task externality exists and information arrives sequentially. We show that although increasing in the number of bidders in the market of the second task always tilts the principal's choice towards unbundling, increasing in the number of consortium that can perform both tasks tilts the principal's choice towards unbundling if and only if the externality is positive.

*Keywords:* Auction, Bundling, Design-Bid-Build, Design-Build, Procurement, Public-Private Partnerships

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# 1 Introduction

In the emergence of a highly specialized society, contracts were the guiding and organizing vehicle for optimal division and control of tasks. For a typical project consisting of closely related multiple phases, whether the owner contracts with single or separate entities for different phases represents a critical component of procurement strategy. For example, recent project delivery methods has witnessed a swift away from design-bid-build (D-B-B) towards design-build (D-B)<sup>1</sup>. As an illustration, the Design-Build Institute of America (DBIA) has reported that the number of D-B projects accounted for more than 30% of the construction in the U. S. in 2001 as compared to 5% in 1985 (Beard et al. (2001); Tulacz (2002)).

A trend in the provision of infrastructure services towards Public-Private Partnerships (PPP)<sup>2</sup> has been documented as opposed to conventional short-term contracts. PPPs are now extensively used across Europe, Canada, the US and a number of developing countries. Estimations show that between 1984 and 2002, 82% of all water projects and 92% of all transport projects were PPPs (Oppenheimer and MacGregor (2004)). Furthermore, 30% of all services provided by the larger EU governments are delivered through PPPs (Torres and Pina (2001)). Traditionally employed for transportation, energy and water system, PPPs have recently penetrated into IT services, accommodation, leisure facilities, prisons, military training, waste management, schools and hospitals<sup>3</sup>.

Although labeled with different titles, these contracting methods share some common features. First, auctions are pervasively used while selecting the contractors (see, e.g., McAfee and McMillan (1986); Laffont and Tirole

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<sup>1</sup>In D-B-B, separate entities are responsible for each the design and construction of a project. In D-B, however, design and construction aspects are contracted with a single entity known as the design-builder.

<sup>2</sup>PPP is characterized by long term contracts between a public sector authority and a private party, in which tasks of designing, building and operating are bundled together to form a special purpose vehicle.

<sup>3</sup>See Martimort and Iossa (2008) for more detailed examples on PPP.

(1987)). This suggests that there is information asymmetry between the project principal and agents. In the absence of asymmetric information, the principal can always do better by selecting the most efficient contractor and using take-it-or-leave-it (TIOLI) offers without occurring the cost of organizing an auction. Second, activities in the preceding task have an impact on the project quality or operating cost of the succeeding task. Third, bidders can acquire further information on their valuation of the exact size of the project, the quality attributes of the infrastructure, the quantity and prices of inputs, the technology available between the different tasks. This sequential arrival of information leads to different information structure under task bundling and unbundling. For instance, estimations about the building cost in the D-B framework is much less accurate than that in the D-B-B framework, while operators in a conventional approach have more accurate estimation about the operating costs than that in PPPs.

This paper aims at investigating the optimal choice between bundling versus unbundling. A project consists of two sequential tasks, task 1 and task 2, namely, designing and building as in the debate of project delivery methods or building and operating as in the scenario of PPPs. Any cost reducing activity is non-observable and non-contractible, which typically raises moral hazard problem. Assume that there exists task externality, i.e., activity in task 1 has an impact on the operation cost of task 2. Furthermore, information regarding agents' cost type for task 2 arrives only in period 2. In period 1, agents only know their cost type for task 1.

To minimize the expected total payment to the agents, the principal can choose between two regimes, bundling or unbundling. Under the bundling regime, the prospective consortium is selected to perform both tasks through competitive bidding for an incentive contract. Under the unbundling regime, the contractor for the two tasks are selected via two sequential auctions. The auction we consider is the first-price sealed-bid auction.

In procurement, some tasks can only be performed by only a few firms while others by many. Assume  $N_1$  firms can perform task 1 and  $N_2$  firm can

perform task 2, with  $N_1 < N_2$ . For example, in the construction industry, the number of firms who can build is larger than that of firms who can design. In the bundled auction, a designer and a builder have to form a consortium before participating in the auction. As a result, the number of consortiums is equal to  $N_1$ . We define  $N_2$  the competitiveness in the market of task 2 and  $N_1$  the competitiveness in the market of joint-tasks.

There are two crucial differences that determine the relative advantage and disadvantage of the two procurement regimes. The first one is the “externality internalization”. In the auction organized in period 1, agents have private information on the cost of task 1 under both regimes. Hence, the winner earns information rent, which increases with the share of the cost of task 1 borne by this firm. As a result, there is a trade-off between providing incentives and reducing the winner’s information rent in the auctions. In the presence of positive task externality, a higher cost reduction effort in task 1 leads to a lower operation cost for task 2. Hence, bundling serves as a device for internalizing task externality and mitigating the agency problem of task 1. For negative externalities, internalization is bad because the principal has to let the winner bear a larger share of the cost of task 1 to induce the same level of effort in the first period, leaving too much information rent to the agent. Consequently, whether “externality internalization” tilts the principal’s choice towards bundling depends on whether the externality is positive or negative.

The second difference between the two regimes is “sequential information”. The assumption that agents can only observe their cost of task 2 in period 2 has two effects. Unlike in unbundling, in which the most efficient agent for task 2 is chosen, the consortium chosen in the bundled auction only has the average cost of performing task 2. Thus, there is an efficiency loss of bundling in period 2. Second, unlike in bundling, agents have private information and hence information rent should be given to the winner while auctioning task 2 in the unbundled regime. As competition in task 2 increases, the efficiency loss of bundling becomes larger and the informa-

tion rent given to the agent in unbundling becomes smaller. This makes unbundling more attractive. This feature of sequential information tilts the principal's choice towards unbundling when competition in the second period is large.

The paper shows how the optimal incentive scheme differs under the two procurement regimes. More importantly, the difference in the incentive scheme determines how other factors, such as competitiveness in the joint-task market and uncertainties involved the two periods, affect the principal's decision on bundling.

For positive externalities, the winner is rewarded a smaller power of incentive in the first period under bundling than under unbundling. This is because externality internalization already gives him some first-period incentive. Consider an increase of the competitiveness in the joint-task market. This implies that the winner's information rent in the auction in the first period will be reduced in both regimes. The reduced information rent, as we shown in the paper, is increasing with the power of incentive and hence smaller under bundling. Consequently, unbundling becomes relatively more favorable.

With risk-averse agents, an increase in the uncertainty in the first period is costly for the principal since he has to pay the winner a higher risk premium. The increased risk premium is increasing in the winner's first-period power of incentive and hence smaller under bundling. Therefore, bundling becomes relatively more favorable. The above results are reversed for negative externalities.

Now consider the power of incentive in the second period. Consider the case of positive externality, while a high power of the second-period incentive provides incentives only in the second period under unbundling, it provides incentives in both the second and the first period under bundling. As a result, when the externality is positive enough, the power of the second-period incentive is higher under bundling. Using the same logic as above, an increase in the uncertainty in the second period tilts the principal's choice

toward unbundling.

Other than infrastructure procurement, our model applies to other programs, such as scientific research where task 1 and 2 can be recognized as basic research and applied R&D activities. Our results are consistent with empirical evidences that both integration and separation exist in practice. For instance, in the German Research Center for Artificial Intelligence, the same scientists carry out both basic research and applied R&D and product transfer (Wahlster (2002)). On the other hand, a successful software development process is characterized by the separation of R&D activities and production activities (Royce (2002)).<sup>4</sup>

Our paper belongs to the literature on task separation and integration in presence of agency problem. Earlier work by Holmstrom and Milgrom (1990, 1991) shows that tasks should be bundled (unbundled) if they are complements (substitutes). Similar results can be found in the literature on optimal ownership structures in the PPPs. For example, Martimort and Pouyet (2008) studies the PPP problem in the context of moral hazard and find that whether bundling is preferable only depends on the sign of the externality between two tasks (see also Hart (2003), Bennetta and Iossa (2006), Iossa and Martimort (2008), Hoppe and Schmitz(2008), Chen and Chiu(2010)). However, all these papers assume that there is only one agent in each period. Instead, we consider a more common situation where there are several potential agents and an auction is organized for selecting contractors and information arrives sequentially. Especially, considering auction allows us to discuss how the competitiveness in the market affect the principal's bundling decision.

We borrow extensively from McAfee and McMillan (1986), which studies the problem of bidding for incentive contracts. Laffont and Tirole (1987) also studies the problem of auctioning incentive contract. However, none of these scenarios have been related to the bundling choice.

Another literature related to our paper is multi-object auctions. A semi-

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<sup>4</sup>See in Patrick W. Schmitz (2005) for more examples.

nal paper by Palfrey (1983) considers the question of bundling versus separate auctions. The paper shows that the seller always prefers the bundling auction when there are two bidders, but he tends to prefer separate auctions as the number of bidders becomes larger. Similar results can be found in Chakraborty (1999). Those papers assume bidders know their individual information when the auction takes place. Jeitschko and Wolfstetter (2002) and Grimm (2007) consider sequential auctions with the same timing of information revelation as in ours. However, all these papers consider auctioning multiple objects instead of multiple tasks.

This paper is organized as follows. Section 2 presents the basic framework. Section 3 solves the problem and identifies the conditions that favor bundling (or unbundling). Section 4 discusses the situation of risk-averse agents. We conclude in Section 5.

## 2 The Model

A principal wants to procure a project consisting of two sequential tasks. To facilitate expression, we focus on the case of D-B-B vs. D-B and throughout the paper refer the two tasks as designing and building.<sup>5</sup> The project gives a fixed benefit that is so large that the principal always want to implement it. As a result, the principal's objective is to minimize the total implementation costs that he needs to pay to the agents. Our major concern is whether the principal contracts with single or separate agents for the two tasks.

Assume  $N_1$  firms can design and  $N_2$  firm can build, with  $N_1 < N_2$ .<sup>6</sup> In an unbundled auction, the designers and builders are separate entities. However, in the bundled auction, a designer and a builder have to form a consortium before participating in the auction. This is common in procurement auctions where bidders are required to prove their ability to carry out all the tasks.

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<sup>5</sup>Of course, one can also view the two tasks as building and operating as in conventional contracting vs. PPPs.

<sup>6</sup>The case  $N_1 > N_2$  will be discussed later and we show that our main insights still stand.

As a result, the number of consortium is equal to  $N_1$ . We call  $N_2$  the competitiveness in the building market and  $N_1$  the competitiveness in the market of the joint-task. All the agents and the principal are risk neutral.

In period 1, the agent exerts effort to complete the task of designing and the cost is

$$c_1 = \theta_1^{n_1} - e_1, \quad (1)$$

where  $\theta_1^{n_1}$  denotes the cost type of agent  $n_1$ , who has been selected to design, and  $e_1$  denotes his cost-reducing effort.  $\theta_1^{n_1}$  is assumed to be i.i.d. with c.d.f  $F_1(\theta_1^{n_1})$  and p.d.f.  $f_1(\theta_1^{n_1})$  on  $[\underline{\theta}_1, \bar{\theta}_1]$

In period 2, the agent exerts effort to complete the task of building and the cost is

$$c_2 = \theta_2^{n_2} - e_2 - \delta e_1, \quad (2)$$

where  $\theta_2^{n_2}$  denotes the cost type of agent  $n_2$ , who has been selected to build,  $e_2$  denotes his cost-reducing effort, and  $\delta e_1$  captures the externality from the preceding quality-improving activity. Assume  $\theta_2^{n_2}$  is i.i.d. with c.d.f.  $F_2(\theta_2^{n_2})$  and continuous p.d.f.  $f_2(\theta_2^{n_2})$  on  $[\underline{\theta}_2, \bar{\theta}_2]$ . Moreover,  $\theta_2$ s and  $\theta_1$ s are independent. As in the standard literature,  $F_i(\theta_i)$  satisfies the monotone hazard rate property:  $\frac{F_i(\theta_i)}{f_i(\theta_i)}$  is increasing.

Exerting effort  $e_i$  costs  $\psi(e_i)$ , with  $\psi'(e_i) > 0$ ,  $\psi''(e_i) > 0$ ,  $\psi'''(e_i) \geq 0$  and  $\psi'(0) = 0$ . In the regime of task bundling, these disutility functions are additive, that is to say, the effort cost is  $\psi(e_1) + \psi(e_2)$ .

Following the literatures, we assume that  $\theta_1$  and  $\theta_2$  are the private information of agents  $n_1$  and  $n_2$ , respectively and that  $e_i$  is neither observable nor contractible, while  $c_i$  is observable and contractible.

The sign of  $\delta$  determines the sign of the externality between the two tasks. Positive  $\delta$  means positive externality, while negative  $\delta$  means negative externality. Negative externality happens, for example, when agents make some innovations in the designing technology that reduce the designing cost. Such innovations may require agents to learn new job processes and hence increases the building cost (See Martimort and Pouyet (2008)). Positive



externality between the two tasks are well documented in the second-sourcing literature.<sup>7</sup>

Assume that agent  $n_1$  observes  $\theta_1^{n_1}$  privately in period 1.  $\theta_2^{n_2}$  can only be observed by agent  $n_2$  in period 2. In the DBB vs. DB case, the estimation of the building cost requires certain information, such as the exact size of the project, the quality attributes of the infrastructure, the quantity and prices of inputs, and the available technology at the time of construction, which is available only at the end of the design period. We hence reasonably assume that the cost of building can only be observed after completion of the design.

We focus on mechanisms that are commonly used in practice: bundling or unbundling. In bundling, the principal auctions the two tasks to one single firm; in unbundling, the two tasks are auctioned separately. The auction format we consider is the first-price sealed-bid auction: The one with the lowest bid wins the auction and is rewarded the contract.

The timing is as follows<sup>8</sup>:

- 1, The principal chooses the regimes between bundling and unbundling.

**Bundling**

- 2, Then designers and builders form design-builders. A design-builder only knows the cost parameter of design at this time.

- 3, The principal organizes an auction for the bundled tasks. Bidders bid

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<sup>7</sup>For example, Laffont and Tirole (1988) assume the agent can, in the first period, make monetary investments that lower the second period cost:  $c_1 = \theta_1^{n_1} - e_1 + d(i)$  and  $c_2 = \theta_2^{n_2} - e_2 - i$ . Although  $e_1$  has no direct effect on  $c_1$ , it affects  $c_2$  indirectly through investment  $i$ . Indeed, keeping  $c_1$  unchanged, an increase in  $e_1$  leads to an increase in  $i$ , which ultimately reduces  $c_2$ : a positive externality between  $e_1$  and  $c_2$ . Using their method of modelling positive externality will generate the same prediction as in our model, as we will argue later.

<sup>8</sup>Under unbundling, the principal is potentially better off if he can ask all the builders to pay a fixed payment in the first period to get participation permits in the auction that will be organized in the second period. By doing so, he extracts all the information rent of the builders. But this requires i) the principal can commit to an auction that is to be organized long time later; ii) firms are not cash-constrained. Those requirements are strong and hence such a possibility may not be feasible.

and the winner wins the contract.

4, The winner exerts efforts in design period and then the costs in the first period are realized.

5, A design-builder becomes privately informed about his cost parameter of build and then exert effort in the build period.

6, Build costs are realized and the contract is executed.

### **Unbundling**

2, Designers become privately informed. The principal organizes an auction for designing. All designers bid and the winner wins a designing contract.

3, The winning designer exerts effort in the design period. Costs in the first period are realized and the designing contract is executed.

4, Builders become privately informed. The principal organizes an auction for building. All builders bid and the winner wins a building contract.

5, Build costs are realized and the building contract is executed.

**Contracts.** Following McAfee and McMillan(1986), we only consider the linear contract, the most commonly used contract, in this paper<sup>9</sup>. In case of unbundling, the contract for task  $i$  is

$$t(b_{n_i}, c_1) = b_{n_i} + \alpha_i c_i,$$

where  $b$  is the bid of the winner and  $c$  is the realized cost.<sup>10</sup> In addition to the winner's bid, the principal also pays a share of the realized cost: If  $\alpha = 0$ , the contract is a fixed price contract; if  $\alpha = 1$ , the contract is a cost-plus contract. The share  $1 - \alpha$  is called the power of incentive. Throughout the paper, we will impose the condition  $\alpha_i \leq 1$ . Otherwise, the agent's net payoff would increase with the realized cost, and thus he would always inflate the cost.

Notice that the contracts are short term in the sense that the payment of the winner in the first period does not depend on the realized cost in the second period. Although doing is beneficial for the principal, it may not be

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<sup>9</sup>The optimal contract is considered in appendix and it will not change our results.

<sup>10</sup>McAfee and McMillan (1986) argues that a more general form  $t(b, c) = F + \alpha_1 c + \alpha_2 b$  can be reduced to the form we used in the paper.

feasible when payments cannot be delayed because of the well known limited commitment of local government<sup>11</sup>.

Under bundling, the contract is

$$t(b_{n_1}, c_1, c_2) = b_{n_1} + \alpha_1 c_1 + \alpha_2 c_2.$$

## 2.1 Complete information benchmark

Suppose the efforts  $e_1$  and  $e_2$  as well as the private information  $\theta_1$  and  $\theta_2$  can be observed and contracted. Then the principal implements the first best outcome. That is, he chooses unbundling and selects the agent with the lowest cost type  $\theta_i$  to perform the task  $i$ . The first best efforts  $e_1^*$  and  $e_2^*$  are determined by equalling the marginal cost and the marginal benefit of each effort:  $\psi'(e_1^*) = 1 + \delta$  and  $\psi'(e_2^*) = 1$ .

## 3 The bundling decision

In this section, we first solve the principal's problem of unbundling and bundling separately, and then compare the expected total payment under the two regimes.

### 3.1 Unbundling

**Agents' optimization.** In period 1, once selected, the agent with cost parameter  $\theta_1$  has expected utility

$$\begin{aligned} \pi_1(\theta_1) &= b_1 + \alpha_1 c_1 - c_1 - \psi(e_1) \\ &= b_1 - (1 - \alpha_1)\theta_1 + (1 - \alpha_1)e_1 - \psi(e_1). \end{aligned} \quad (3)$$

Maximizing over  $e_1$  gives  $\psi'(e_1) = 1 - \alpha_1$  and hence  $e_1 = \psi'^{-1}(1 - \alpha_1)$ . Thus, the principal's choice of the share ratio  $\alpha$  determines the agent's choice

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<sup>11</sup>See Laffont and Tirole (1993, chapter 8) and Martimort and Pouyet (2008).

of cost reducing activity. The larger the share of costs paid by the principal, the smaller the effort expended to lower costs.

The contract is awarded by means of a first-price, sealed-bid auction. Knowing the form of the contract they will be awarded if successful, the potential agents choose their bids  $b_{n_1}$ . We consider the symmetric solution and let  $b_1(\theta_1)$  be each agent's bidding strategy. Builder  $n_1$ 's ex ante expected utility is

$$E\pi_1 = [1 - F(b_1^{-1}(b_{n_1}))]^{N_1-1}\pi_1(\theta_1), \quad (4)$$

and agent chooses  $b_{n_1}$  to maximize his expected utility. In equilibrium, we have  $b_{n_1} = b_1(\theta_1^{n_1})$ . Substituting this into the first-order condition from the above equation yields

$$\frac{b_1'(\theta_1)}{\pi_1(\theta_1)} = \frac{(N_1 - 1)f_1(\theta_1)}{1 - F_1(\theta_1)}. \quad (5)$$

Combining (3) and (5), we have

$$\pi_1'(\theta_1) = \frac{(N_1 - 1)f_1(\theta_1)}{1 - F_1(\theta_1)}\pi_1(\theta_1) - (1 - \alpha_1).$$

Solving the differential equation, we obtain

$$\pi_1(\theta_1) = (1 - F_1(\theta_1))^{-(N_1-1)} \left( K + (1 - \alpha_1) \int_{\theta_1}^{\bar{\theta}_1} (1 - F_1(s))^{N_1-1} ds \right), \quad (6)$$

where  $K$  is some constant. Since  $\pi_1(\bar{\theta}_1) = 0$ , we have  $K = 0$ . The term  $\pi_1(\theta_1)$  is also known as information rent, representing the benefit the winner enjoys from his information advantage.

Notice that  $\pi_1(\theta_1)$  is increasing with the power of incentive  $1 - \alpha_1$ . From the point of view of agents, that the principal bears a share of  $\alpha_1$  of the realized cost is equivalent to say that every one has cost type  $(1 - \alpha_1)\theta_1$  and the principal bears no realized cost. A lower power of incentive means everyone's cost type has a more concentrate distribution and hence the information rent

is smaller. Extremely, with  $1 - \alpha_1$  close to 0, everyone has almost the same cost type and hence earns a rent close to zero.

Combining with (3), we obtain the bidding strategy:

$$b_1(\theta_1) = (1 - \alpha_1) \left( (1 - F_1(\theta_1))^{-(N_1-1)} \int_{\theta_1}^{\bar{\theta}_1} (1 - F_1(s))^{N_1-1} ds + \theta_1 \right) + (\psi(e_1) - (1 - \alpha_1)e_1). \quad (7)$$

The first term of the bidding strategy, reflecting the adverse selection part, is decreasing in  $\alpha_1$ : bidders will bid more aggressively if the share of the cost paid by the principal  $\alpha$  is larger. The second one, reflecting the benefit of that the agent can get from cost reducing activity, is fully extracted by the principal.

Using the same technique, we know that  $e_2 = \psi'^{-1}(1 - \alpha_2)$  and that the bidder's expected utility  $\pi_2$  and bidding strategy  $b$  in the second period are

$$\pi_2(\theta_2) = (1 - \alpha_2) (1 - F_2(\theta_2))^{-(N_2-1)} \int_{\theta_2}^{\bar{\theta}_2} (1 - F_2(s))^{N_2-1} ds, \quad (8)$$

$$b_2(\theta_2) = (1 - \alpha_2) \left( (1 - F_2(\theta_2))^{-(N_2-1)} \int_{\theta_2}^{\bar{\theta}_2} (1 - F_2(s))^{N_2-1} ds + \theta_2 \right) + (\psi(e_2) - (1 - \alpha_2)e_2) - \delta(1 - \alpha_2)e_1. \quad (9)$$

The additional part  $\delta(1 - \alpha_2)e_1$  stems from the fact that the effort in the first period has an externality on the cost in the second period and every bidder knows that. As a result, they will reduce their bids by this fixed part so that the one with cost parameter  $\underline{\theta}_2$  still earns zero utility.

**Optimal linear contract.** If the agent with cost parameter  $\theta_i$  wins, the expected payment of the principal is  $b_i(\theta_i) + \alpha_i c_i$ . On average, the total payment of the principal in period  $i$  is

$$\tau_i = N_i \int_{\underline{\theta}_i}^{\bar{\theta}_i} (b_i(\theta_i) + \alpha_i c_i) (1 - F_i(\theta_i))^{(N_i-1)} f_i(\theta_i) d\theta_i.$$

The principal will choose  $\alpha_1$  and  $\alpha_2$  to minimize  $\tau^u = \tau_1 + \tau_2$ .

**Lemma 1** *The total payment in unbundling is given by the following expression*

$$\begin{aligned} \tau^u = & \sum_{i=1}^2 E\theta_{i \min} + \sum_{i=1}^2 (1 - \alpha_i) E \frac{F_i}{f_i} (\theta_{i \min}) \\ & - \{(1 + \delta) e_1 - \psi(e_1) + e_2 - \psi(e_1)\}. \end{aligned} \quad (10)$$

where  $\theta_{i \min} = \min(\theta_i^1 \dots \theta_i^{N_i})$ .

**Proof.** See Appendix. ■

The total payment consists of three parts. The first part is the expected exogenous cost of the winner. The second is the expected information rent of the winner. Because agents have private information, this part should be strictly positive. The third part is the benefit from cost-reducing activities. The competition between bidders enables the principal to extract all of it. F.O.Cs give

$$0 = \frac{\alpha_1^u + \delta}{\psi''(\psi'^{-1}(1 - \alpha_1^u))} - E \frac{F_1}{f_1} (\theta_{1 \min}), \quad (11)$$

$$0 = \frac{\alpha_2^u}{\psi''(\psi'^{-1}(1 - \alpha_2^u))} - E \frac{F_2}{f_2} (\theta_{2 \min}). \quad (12)$$

We call the first terms of the RHS of the above equations the moral hazard effects and the second terms the information rent effects<sup>12</sup>. To understand the moral hazard effect, notice that the social return of agents' activity  $e_1$  is  $(1 + \delta) e_1 - \psi(e_1)$ . Differentiating the social return with respect to  $\alpha_1$ , we obtain the first term of the RHS of (11). Since this part is uniform for all the agents regardless of their cost types, the principal can extract all of it. The information rent effects comes from the fact that, all other things being equal, the winner's information rent is smaller if the principal pays a larger share of the realized cost. Hence, we have a trade-off between providing incentive, which requires  $\alpha$  to be small, and the information rent reduction, which requires  $\alpha$  to be large.

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<sup>12</sup>It is called bidding-competition effects in McAfee and McMillian(1986).

One can also see that the optimal  $\alpha_2^u$  is independent of the choice of  $\alpha_1^u$ . This is because we assume effort  $e_1$  in the first period affects the cost in the second in a deterministic and additive way. The additive assumption ensures that more effort in the first period does not affect the marginal returns of effort in the second period. The deterministic assumption ensures that the information asymmetric condition will not be changed by the effort in the first period. Relaxing any one of the two assumptions makes  $\alpha_2^u$  dependent on  $\alpha_1^u$ .

We summarize our findings in the following proposition:

**Proposition 1** *Under unbundling, the optimal power of incentives are determined by (11) and (12) and the optimal efforts in both periods are downward distorted:  $e_1^u < e_1^*$  and  $e_2^u < e_2^*$ .*

**Proof.** Immediately from (11), (12) and that  $\psi'(e_1^*) = 1 + \delta$  and  $\psi'(e_2^*) = 1$ . ■

## 3.2 Bundling

**Agents' optimization.** The winner with type  $\theta_1$  has expected utility

$$\pi_1(\theta_1) = t(b_1, c_1, Ec_2) - c_1 - Ec_2 - \psi(e_1) - \psi(e_2),$$

where  $c_2 = E\theta_2 - e_2 - \delta e_1$ , since the agents have no information on  $\theta_2$  at the time of auction. Notice that once the contract is given, the choice of  $e_1$  and  $e_2$  does not depend on  $\theta_1$  or  $\theta_2$ . The F.O.Cs give  $e_2 = \psi'^{-1}(1 - \alpha_2)$  and  $e_1 = \psi'^{-1}(1 - \alpha_1 + \delta(1 - \alpha_2))$ . Using the same technique as in the above section, we know that the bidder's expected utility is still given by (5). The total payment is

$$\begin{aligned} \tau^b &= E\theta_{1\min} + E\theta_2 + (1 - \alpha_1) E \frac{F_1}{f_1}(\theta_{1\min}) \\ &\quad + \psi(e_1) - (1 + \delta)e_1 + \psi(e_2) - e_2. \end{aligned} \tag{13}$$

Minimizing over  $\alpha_1$  and  $\alpha_2$  gives

$$0 = \frac{\alpha_1^b + \delta\alpha_2^b}{\psi''(\psi'^{-1}(1 - \alpha_1^b + \delta(1 - \alpha_2^b)))} - E\frac{F_1}{f_1}(\theta_{1\min}). \quad (14)$$

$$\begin{aligned} 0 &= \frac{\alpha_2^b}{\psi''(\psi'^{-1}(1 - \alpha_2^b))} + \frac{(\alpha_1^b + \delta\alpha_2^b)\delta}{\psi''(\psi'^{-1}(1 - \alpha_1^b + \delta(1 - \alpha_2^b)))} \\ &= \frac{\alpha_2^b}{\psi''(\psi'^{-1}(1 - \alpha_2^b))} + \delta E\frac{F_1}{f_1}(\theta_{1\min}). \end{aligned} \quad (15)$$

**Proposition 2** *As compared to unbundling, 1) the power of incentive is smaller(larger) in the first period, if the externality is positive(negative):*

*$1 - \alpha_1^b \leq 1 - \alpha_1^u$  iff  $\delta \geq 0$ ;*

*2) the power of incentive is larger(smaller) in the second period if the externality is not too negative(negative enough):  $1 - \alpha_2^b \geq 1 - \alpha_2^u$  iff  $\delta \geq \widehat{\delta}$ , where  $\widehat{\delta} = -\frac{E\frac{F_2}{f_2}(\theta_{2\min})}{E\frac{F_1}{f_1}(\theta_{1\min})} < 0$ ;*

*3) i) the optimal efforts in the first period under the two regimes are equal and downward distorted:  $e_1^u = e_1^b < e_1^*$ ;*

*ii) the optimal effort in the second period is larger(smaller) if the externality is not too negative(negative enough):  $e_2^b \geq e_2^u$  iff  $\delta \geq \widehat{\delta}$ ;*

*iii) the optimal effort in the second period is upward(downward) distorted if the externality is positive(negative):  $e_2^b \geq e_2^*$  iff  $\delta \geq 0$ .*

**Proof.** See appendix. ■

To understand 1), notice that, given the same power of incentive under the two regimes in the first period, effort in the first period is higher in bundling because of internalization of the externality between the two tasks, if the externality is positive. Consider a marginal increase in the power of incentive:  $d(1 - \alpha_1)$ . The marginal benefit, which is the resulting increased efficiency from the increased effort in the first period, is smaller in bundling because of two reasons. First, the increased effort is smaller under bundling, since  $de_1 = \frac{1}{\psi''(e_1)}d(1 - \alpha_1)$  is decreasing in  $e_1$ . Second, for a marginal increase in effort  $e_1$ , the resulting increased marginal efficiency is smaller in bundling,



since the efficiency  $(1 + \delta) e_1 - \psi(e_1)$  is concave in  $e_1$ . On the other hand, the marginal cost, which is the resulting increased information rent, is the same under the two regimes. In optimum, the principal chooses the power of incentive so that the marginal benefit and the marginal cost are equal. Consequently, the optimal power of incentive is smaller under bundling. The result is reversed for negative externality.

The intuition for 2) is as follows. In unbundling, because of the trade off between providing incentive and reducing the winner's information rent, the principal has to pay a strictly positive share of  $c_2$ , which is independent of the externality  $\delta$ . In bundling, agents are still uninformed and hence there is no information rent on their cost in the second period at the auctioning time. If there is no externality (i.e.,  $\delta = 0$ ), it is optimal to let the agent bear the whole share of  $c_2$  (i.e.,  $1 - \alpha_2 = 1$ ). In the presence of positive externality, rather than just letting the agent bear the whole share of  $c_2$ , the principal prefers to slightly increase  $1 - \alpha_2$  by an amount  $d(1 - \alpha_2)$ . By doing so, the efficiency is just diminished by a second-order term  $\psi''(e_2)(d(1 - \alpha_2))^2$ , since  $1 - \alpha_2 = 1$  is the level of power of incentive that maximize the second period efficiency. Instead, the principal can then decrease the power of incentive in the first period  $1 - \alpha_1$  by  $\delta d(1 - \alpha_2)$  without losing incentive in the first period. The reduced  $1 - \alpha_1$  saves the expected information rent left to the agent in the first period to the first order  $\delta E \frac{E_1}{f_1}(\theta_{1 \min}) d(1 - \alpha_2)$ . In optimum, the principal gives super-powered incentive in the second period (i.e.,  $1 - \alpha_2 > 1$ ). For negative externality, the result is reversed and the principal would bear a strictly positive share of  $c_2$ , which is increasing with the extent of externality. If the extent is not too large, the share in bundling is still smaller than that in unbundling. The argument also explains 3) *ii*) and *iii*), since effort in the second period is solely determined by the power of incentive in the second period under both regimes.

This proposition also indicates that agent's effort in the first period is the same under both bundling and unbundling. Although the agent in the first period does not internalize the externality between two tasks in unbundling,

the principal can correct this by letting him bear a larger share of  $c_1$ . (*More explanation needed*).

### 3.3 Bundling or Unbundling?

The principal will choose unbundling iff  $\tau^u \leq \tau^b$ . Combining (10) and (13), we have

$$\begin{aligned} \tau^u - \tau^b = & \underbrace{\{E\theta_{2\min} - E\theta_2\}}_{-} + \underbrace{\{(\alpha_1^b - \alpha_1^u) E \frac{F_1}{f_1} (\theta_{1\min})\}}_{+/-} + \underbrace{\{(1 - \alpha_2^u) E \frac{F_2}{f_2} (\theta_{2\min})\}}_{+} \\ & - \underbrace{\{e_2^u - \psi(e_2^u) - (e_2^b - \psi(e_2^b))\}}_{+/-} \end{aligned} \quad (16)$$

The RHS of the above equation consists of three parts representing three differences between bundling and unbundling. The first difference is allocation efficiency. Unbundling enables the principal to allocate the contract to the agents with lowest  $\theta_2$  while bundling cannot, since agents have no information on  $\theta_2$  at the time of auction. The second difference is the information rent given to agents. The principal has to pay the winner information rent for the agents' private information  $\theta_1$  under both bundling and unbundling. The difference arises in this part because the principal pays a different share of  $c_1$  under the two regimes. Moreover, the principal needs to pay information rent on  $\theta_2$  under unbundling while he does not need to under bundling since agents are uninformed on  $\theta_2$  at the time of auction. The third difference is the benefit generated by the agent's cost-reducing activity. By lemma 2, agent exerts the same effort in the first period under the two regimes. Thus, the difference in this part vanishes. Efforts in the second period, however, are different.

From (16), we see that the expected total payment under unbundling can be either larger or smaller than bundling, depending on parameters. The following proposition tells us how the externality and the competitiveness in markets affect the relative attractiveness of bundling and unbundling.

**Proposition 3** 1) *If the externality is positive(negative), increasing the extent of externality tilts the principal's choice towards bundling(unbundling).*

2) *Increasing the competitiveness in the building market tilts the principal's choice towards unbundling.*

3) *If the externality is positive(negative), increasing the competitiveness in the market of the joint-task tilts the principal's choice towards unbundling(bundling).*

**Proof.** See Appendix. ■

Proposition 3 1) is nothing new and proved by Bennett and Iossa (2006) and Martimort and Pouyet (2008). The latter two points are new here. Proposition 3 2) is straightforward. As the competitiveness in the building market becomes stronger, the principal's payoff does not change in bundling while it increases in unbundling. The logic for Proposition 3 3) is as follows. Increasing  $N_1$  benefits both bundling and unbundling in two ways. First, it increases allocation efficiency, i.e.,  $\theta_{1\min}$  is decreasing in the sense of first-order stochastic dominance. However, there is no difference between bundling and unbundling in terms of this benefit and therefore it would not change the principal's bundling decision. Second, it decreases the winner's information rent,  $(1 - \alpha_1) E \frac{F_1}{f_1}(\theta_{1\min})$ , which is increasing if the share of the cost of designing borne by the winner,  $(1 - \alpha_1)$ , increases. As we have argued before, this share is smaller in bundling if the externality is positive. As a result, bundling benefits less from the decrease in information rent and hence becomes relatively less attractive.<sup>13</sup> By the same logic, we see the result will be reversed if the externality is negative.

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<sup>13</sup>The key for this result is that, comparing to unbundling, the power of incentive in the first period is lower under bundling. The result also holds using Laffont and Tirole (1988)'s method of modelling positive externality. Under unbundling, agent will choose  $i = 0$  and the power of incentive in the first period is chosen to balance the standard trade off between providing incentive in the cost reducing activity and information rent reduction. Under bundling, however, a higher power of incentive in the first period has an additional cost: it reduces agent's incentive to invest in  $i$ . As a result, the principal will provide a lower power of incentive under bundling.

**Remark** If  $N_2 < N_1$ , the number of bidders in the bundling auction is  $N_2$ .

Obviously, the principal's payoff is independent on  $N_1$  under bundling and strictly increasing in  $N_1$  under unbundling. Moreover, it's easy to get that  $\frac{\partial \tau^u - \tau^b}{\partial N_2} = \frac{\partial}{\partial N_2} \{ (E\theta_{2\min}^{N_2} - E\theta_{1\min}^{N_2}) + (1 - \alpha_2^u) E \frac{F_2}{f_2} (\theta_{2\min}^{N_2}) - (1 - \alpha_1^b) E \frac{F_1}{f_1} (\theta_{1\min}^{N_2}) \}$ , where  $\theta_{1\min}^{N_2} = \min(\theta_1^1, \dots, \theta_1^{N_2})$  and  $\theta_{2\min}^{N_2} = \min(\theta_2^1, \dots, \theta_2^{N_2})$ . Calculation gives  $\frac{\partial^2 \tau^u - \tau^b}{\partial N_2 \partial \delta} = \frac{\partial}{\partial N_2} E \frac{F_1}{f_1} (\theta_{1\min}^{N_2}) \frac{\partial \alpha_1^b}{\partial \delta} < 0$ . Assuming  $\theta_2$ s and  $\theta_1$ s are i.i.d, we have  $\frac{\partial \tau^u - \tau^b}{\partial N_2} < 0$  for small  $\delta$  and  $\frac{\partial \tau^u - \tau^b}{\partial N_2} > 0$  for large  $\delta$ . Large positive(negative) externality determines that the principal is more likely to choose unbundling if  $N_2$  is larger(smaller).

**Welfare.** One interesting problem is whether the principal bundles too much or too little from the point of view of social welfare. To answer this question, assume that the social welfare is equal to the sum of the utility of the principal and the agents:  $W^b = -\tau^b + E\pi_1^b(\theta_{1\min})$  and  $W^u = -\tau^u + E\pi_1^u(\theta_{1\min}) + E\pi_2^u(\theta_{2\min})$ . The information rent that the principal has to give to the winner is just a wealth redistribution and does not affect the total social surplus. Using this criteria, one can easily see

$$\begin{aligned} W^u - W^b &= -(\tau^u - \tau^b) + E\pi_1^u(\theta_{1\min}) + E\pi_2^u(\theta_{2\min}) - E\pi_1^b(\theta_{1\min}) \\ &= -(\tau^u - \tau^b) + (\alpha_1^b - \alpha_1^u) E \frac{F_1}{f_1} (\theta_{1\min}) + (1 - \alpha_2^u) E \frac{F_2}{f_2} (\theta_{2\min}). \end{aligned}$$

Clearly, if the externality is positive, we have  $W^u - W^b > -(\tau^u - \tau^b)$ : if the principal chooses unbundling, then unbundling is desirable from the point view of social welfare. However, the reverse does not necessarily hold, meaning that there are some cases where the unbundling is socially desirable while the principal chooses bundling. For negative externality, it is ambiguous whether  $W^u - W^b$  is larger than  $-(\tau^u - \tau^b)$ . Nevertheless, we have the following proposition:

**Proposition 4** *There exists  $\tilde{\delta} < 0$ , such that the principal bundles too much if  $\delta > \tilde{\delta}$  and too little if  $\delta < \tilde{\delta}$ , from the point view of social welfare.*

**Proof.** See Appendix. ■

## 4 Risk-aversion

In this section, we assume agents are risk averse and check whether our result in proposition 3 still holds. Moreover, we also want to ask how the agents' risk-aversion as well as the uncertainties involved in the two periods affect the principal's bundling decision.

To get an analytic solution and makes our presentation clear, we make the following specific assumptions: 1) Agents has CARA utility function  $U(x) = \frac{1-e^{-rx}}{r}$ ; 2) the effort cost function is quadratic  $\psi(e) = \frac{1}{2}e^2$ ; 3) both  $\theta_2$  and  $\theta_1$  are distributed exponentially on  $[0, +\infty)$ ,  $F_2(\theta_2) = 1 - e^{-\theta_2}$  and  $F_1(\theta_1) = 1 - e^{-\theta_1}$ .

Since agents are risk-averse, it is necessary to add some noisy term on  $c_1$  and  $c_2$ :  $c_1 = \theta_1^{n_1} - e_1 + \varepsilon_1$  and  $c_2 = \theta_2^{n_2} - e_2 - \delta e_1 + \varepsilon_2$ . Assume  $\varepsilon_1$  and  $\varepsilon_2$  are normally distributed with zero mean and variance  $\sigma_1^2$  and  $\sigma_2^2$ .

**Unbundling.** Let's start from period 2. The profit of the winner, whose cost parameter is  $\theta_2$ , is

$$\pi_2(\theta_2) = b_2 - (1 - \alpha_2)(\theta_2 - e_2 - e_1\delta) - \frac{1}{2}e_2^2 - (1 - \alpha_2)\varepsilon_2$$

The certainty-equivalence of his expected utility is then,

$$\pi_2^E(\theta_2) = b_2 - (1 - \alpha_2)(\theta_2 - e_2 - e_1\delta) - \frac{1}{2}e_2^2 - \frac{1}{2}r(1 - \alpha_2)^2\sigma_2^2 \quad (17)$$

It can easily be shown that each successful agent will choose an effort  $e_2 = 1 - \alpha_2$ . We consider the symmetric solution and let  $b_2(\theta_2)$  be each agent's bidding strategy. Builder  $n_2$ 's ex ante expected utility is

$$EAU = [1 - F_2(b_2^{-1}(b_{n_2}))]U(\pi_{2n_2}^E),$$

and agent  $n_2$  chooses  $b_{n_2}$  to maximize his expected utility. In equilibrium, we have  $b_{n_2} = b(\theta_2^{n_2})$ . Substituting this into the first-order condition from the above equation yields

$$\frac{U'(\pi_2^E(\theta_2))}{U(\pi_2^E(\theta_2))} b_2'(\theta_2) = \frac{(N_2 - 1)f_2(\theta_2)}{1 - F_2(\theta_2)}. \quad (18)$$

**Lemma 2** *The selected agent's certainty-equivalence is*

$$\pi_2^E(\theta_2) = \frac{1}{r} \log \left( 1 + \frac{r(1 - \alpha_2)}{N_2 - 1} \right), \quad (19)$$

*and his expected utility is*

$$U(\pi_2^E(\theta_2)) = \frac{(1 - \alpha_2)}{r(1 - \alpha_2) + N_2 - 1}. \quad (20)$$

**Proof.** See appendix. ■

Let  $T_2(\theta_2)$  be the expected payment by the principal when the builder with  $\theta_2$  makes the lowest bid. Using (17) and the expression of  $c_2$ , we have

$$\begin{aligned} T_2(\theta_2) &= E_{\varepsilon_2}(b_2(\theta_2) + \alpha_2 c_2) \\ &= \theta_2 + \pi_2^E(\theta_2) - e_1 \delta + \frac{1}{2} r (1 - \alpha_2)^2 \sigma_2^2 - \frac{1 - \alpha_2^2}{2}. \end{aligned}$$

The principal's expected payment  $\tau_2 = ET_2(\theta_{2\min})$  is given by the following expression

$$\begin{aligned} \tau_2 &= \left\{ E\theta_{2\min} + \frac{1}{r} \log \left( 1 + \frac{r(1 - \alpha_2)}{N_2 - 1} \right) \right. \\ &\quad \left. - \frac{1 - \alpha_2^2}{2} - e_1 \delta \right\} + \frac{1}{2} r (1 - \alpha_2)^2 \sigma_2^2. \end{aligned} \quad (21)$$

The terms in the curly braces are the same as in the risk neutral case. The last term is the risk-premium that the principal needs to pay to the agent.

Using the same technique, one can easily get  $e_1 = 1 - \alpha_1$  and the selected agent's certainty-equivalence is

$$\pi_1^E(\theta_1) = \frac{1}{r} \log \left( 1 + \frac{r(1 - \alpha_1)}{N_1 - 1} \right). \quad (22)$$

The expected payment of the principal in the first period is

$$\begin{aligned}\tau_1 &= E\theta_{1\min} + \frac{1}{r} \log \left( 1 + \frac{r(1 - \alpha_1)}{N_1 - 1} \right) \\ &\quad + \frac{1}{2} r (1 - \alpha_1)^2 \sigma_1^2 - \frac{1 - \alpha_1^2}{2}.\end{aligned}\quad (23)$$

The principal will choose  $\alpha_1$  and  $\alpha_2$  to minimize  $\tau^u = \tau_1 + \tau_2$ . The F.O.Cs give

$$0 = \alpha_1^u + \delta - \frac{1}{N_2 - 1 + r(1 - \alpha_1^u)} - r(1 - \alpha_1^u)\sigma_1^2, \quad (24)$$

$$0 = \alpha_2^u - \frac{1}{N_2 - 1 + r(1 - \alpha_2^u)} - r(1 - \alpha_2^u)\sigma_2^2. \quad (25)$$

**Bundling.** After being selected, the agent will internalize the externality between two tasks. The corresponding efforts he will exert are  $e_1 = 1 - \alpha_1 + \delta(1 - \alpha_2)$  and  $e_2 = 1 - \alpha_2$ . The same technique as in the unbundling case immediately gives

$$\begin{aligned}\tau^b &= \{[E\theta_{1\min} + E\theta_2] + \frac{1}{r} \log \left( 1 + \frac{r(1 - \alpha_1)}{N_1 - 1} \right) \\ &\quad - [(1 + \delta)e_1 - \frac{e_1^2}{2} + e_2 - \frac{e_2^2}{2}]\} \\ &\quad + \left\{ \frac{1}{2} r (1 - \alpha_1)^2 \sigma_1^2 + \frac{1}{2} r (1 - \alpha_2)^2 \sigma_2^2 \right. \\ &\quad \left. + (1 - \alpha_2) E\theta_2 - \frac{\log(1 + r(1 - \alpha_2))}{r} \right\}.\end{aligned}\quad (26)$$

The terms in the first curly braces are the same as in the risk neutral case. The terms in the second curly braces are the risk premiums that the principal needs to pay to the agents. Notice that the agent bear some share of three risks:  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\theta_2$ .

F.O.Cs give

$$0 = \alpha_1^b + \delta\alpha_2^b - \frac{1}{N_1 - 1 + r(1 - \alpha_1^b)} - r(1 - \alpha_1^b)\sigma_1^2, \quad (27)$$

$$0 = \delta(\alpha_1^b + \delta\alpha_2^b) + \alpha_2^b - r(1 - \alpha_2^b)\sigma_2^2 - E\theta_2 + \frac{1}{1 + r(1 - \alpha_2^b)}. \quad (28)$$

**Lemma 3** *i)  $\alpha_1^b \geq \alpha_1^u$  iff  $\delta \geq 0$ ; ii)  $\alpha_2^b < \alpha_2^u$  iff  $\delta \geq \hat{\delta}$ , where*

$$\hat{\delta} = -\frac{\frac{1}{N_2-1+r(1-\alpha_2^u)} + r(\alpha_2^b - \alpha_2^u)\sigma_2^2 - E\theta_2 + \frac{1}{1+r(1-\alpha_2^b)}}{\frac{1}{N_1-1+r(1-\alpha_1^b)} + r(1-\alpha_1^b)\sigma_1^2}. \quad (29)$$

**Proof.** See appendix. ■

The lemma replicates the result in the case of risk neutral. Unlike in risk neutral, however,  $\hat{\delta}$  is not necessarily negative. To see why, it is sufficient to see that  $\alpha_2^b > \alpha_2^u$ , i.e., the power of incentive in unbundling is higher, could happen when no externality exists:  $\delta = 0$ . In this case, (28) and (25) implies  $\alpha_2^b > \alpha_2^u$  iff

$$E\theta_2 - \frac{1}{1+r(1-\alpha_2^b)} > \frac{1}{N_2-1+r(1-\alpha_2^u)}. \quad (30)$$

Increasing  $\alpha_2$  generates a benefit in bundling that never happens in unbundling: it reduces agent's exposure to the risk  $\theta_2$ . On the other hand, it also generates a benefit in unbundling that never happens in bundling: it reduces agent's information rent. If the marginal gain from reducing risk premium in bundling is larger than that from reducing information rent in unbundling, the principal will bear a larger share of  $c_2$  in bundling. If (30) holds, principal gives higher power of incentive to the agent in bundling only if  $\delta$  is positive enough, i.e.,  $\hat{\delta} > 0$ . Notice (30) is more likely to happen if  $N_2$  is large.

**Proposition 5** *The results in proposition 1 still holds with risk-averse agents. Moreover, we have the following two new results:*

- i) Increasing  $\sigma_1^2$  tilts the principal's choice towards bundling(unbundling), if  $\delta > 0$  ( $\delta < 0$ ).*
- ii) Increasing  $\sigma_2^2$  tilts the principal's choice towards unbundling(bundling), if  $\delta > \hat{\delta}$  ( $\delta < \hat{\delta}$ ).*

Besides all the results and insights with risk-neutral agents, the above proposition gives clear idea about how the change in the uncertainties involved in the two periods affect the principal's bundling decision. Increasing uncertainty in the first period makes the principal worse off under both



bundling and unbundling, since he has to pay a higher risk premium to the agent. If the externality is positive  $\delta > 0$ , agent bear a larger share of the risk  $\varepsilon_1$  and therefore the principal suffers more in unbundling. Consequently, bundling becomes relatively more attractive. The result is reversed if the externality is negative. Similarly, principal becomes worse off as the uncertainty in the second period increases. If the externality is large enough,  $\delta > \widehat{\delta}$ , agent bears a smaller share of the risk  $\varepsilon_2$  and therefore the principal suffers less in unbundling. Consequently, bundling becomes relatively less attractive. The result is reversed if externality is small,  $\delta < \widehat{\delta}$ .

What remains unsolved is how a change in agent's risk-aversion attitude affects the principal's bundling decision. From (19), (21), (23), (22) and (26) and the envelope theorem, we have

$$\begin{aligned} \tau^u - \tau^b = & \underbrace{\frac{\partial}{\partial r} \left( \frac{\partial E\pi_1^E}{\partial r} \Big|_{\alpha=\alpha^u} - \frac{\partial E\pi_1^E}{\partial r} \Big|_{\alpha=\alpha^b} \right)}_{+/-} + \underbrace{\frac{\partial}{\partial r} E\pi_2^E}_{-} + \underbrace{\frac{1}{2} \left( (1 - \alpha_1^u)^2 - (1 - \alpha_1^b)^2 \right) \sigma_1^2}_{+/-} \\ & + \underbrace{\frac{1}{2} \left( (1 - \alpha_2^u)^2 - (1 - \alpha_2^b)^2 \right) \sigma_2^2}_{+/-} - \underbrace{\frac{\partial}{\partial r} \frac{-\log(1 + r(1 - \alpha_2))}{r}}_{+} \end{aligned} \quad (31)$$

Five differences of the total payment between the regimes arises due to a marginal increase in the degree of risk-aversion. First, it reduces the certainty equivalence given to the agent in the first period under both regimes. If externality is positive(negative), the principal gives the winner a higher(lower) certainty equivalence and hence benefit more(less) in unbundling. Consequently, this term is negative(positive). Second, it reduces the certainty equivalence given to the winner in the second period under unbundling. Third, it increases the risk premium caused by  $\varepsilon_1$ . For positive(negative) externality, the principal let the agent bear a higher(lower) share of  $\varepsilon_1$  and hence suffers more(less) in unbundling. Therefore, this term is positive(negative). Fourth, it increases the risk premium caused by  $\varepsilon_2$ . For  $\delta > \widehat{\delta}$  ( $\delta < \widehat{\delta}$ ), the principal let the agent bear a lower(higher) share of  $\varepsilon_2$  and hence suffers less(more) in unbundling. Thus, this term is negative(positive). Last, it increases the risk-premium caused by  $\theta_2$  under bundling.

Although whether principal is more likely to choose unbundling when agents become more risk-averse is ambiguous, the following observations from (31) give some idea about the direction:

- Observation** i) If  $\sigma_1^2 = 0$ , then the principal is more likely to choose unbundling as the agents becomes more risk-averse for positive externality.
- ii) For  $\delta > \max(0, \widehat{\delta})$ , the principal is more likely to choose unbundling(bundling) as the agents becomes more risk-averse if:  $\sigma_1^2$  is small (large);  $\sigma_2^2$  is large (small).
- iii) For  $\delta < \min(0, \widehat{\delta})$ , the principal is more likely to choose unbundling(bundling) as the agents becomes more risk-averse if:  $\sigma_1^2$  is large(small);  $\sigma_2^2$  is small (large).

## 5 Concluding Remarks

This paper combines the literature on bundling tasks with the literature of auctioning incentive contracts. Externality between the two tasks play important role of determining how other factors, such as competition on the market of joint task, uncertainties involved in the two tasks, agents' risk aversion attitude, affect the principal's bundling decision.

There are many future works remaining undone. First, we implicitly assume that, once bundling, the winner will build by himself in any circumstance, even when his realized cost turns out to be very high. It would be interesting to relax this assumption and explore whether the principal can do better by allowing the winner to subcontract with other bidders in the second period. Subcontracting is a common phenomenon in many procurement situations (see Kamien (1989), Gale, Hausch, and Stegeman (2000) and Grimm (2007)). In construction industry, the design-led design-build is a method in which the designer is responsible for both the design and construction, while he often subcontracts with on-site personnel.

Our conjecture is that bundling with subcontracting always dominates bundling without subcontracting. Consider the principal providing the same bundled contract in both cases. Competition between bidders enable the principal to extract all the surplus except for the information rent which is determined only by agents' private information distribution and the number of bidders in the first period. Since these two factors are the same under both bundling with and without subcontracting, the information rent should be the same. Thus, comparing which case is better for the principal is equivalent to compare which case generates higher surplus. Bundling with subcontracting can do no worse than bundling without subcontracting since the winner can always choose to build by himself. But the principal and the auctioneer, i.e., the winner in the first period, have different objectives. Inefficiency still arises in the second period in the bundling method even subcontracting is allowed. Thus, it remains unknown whether bundling with subcontracting is better than unbundling.

Second, our paper treats all agents in a symmetric way. Yet, it is well documented in the second sourcing literature that the effort in the first task increases the incumbent's cost advantage over the outsiders(see, Anton and Yao(1987), Riordan and Sappington (1989)). Consequently, the problem becomes auction with endogenous asymmetric bidders. Under this asymmetric treatment, unbundling provides incentive for the incumbent in the first period: he can work hard to increase the chance of getting the contract in second period auction.

Third, we have assumed that the agents have no information about their cost of building. It is natural to ask what is the principal's bundling decision if agents are also fully informed about their cost of building. Going to this direction, we can build a bridge between the incentive theory and multiproduct auctions.

Last, although we assume the activity in the first period is cost reduction activity, the model also applies to the case in which agent exerts quality-improving effort, as in the literature on PPP. As a result, we can then discuss

the optimal combination of ownership and the bundling decision.

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## 6 Appendix

### Proof of lemma 1

**Proof.** Substitute the expression  $b_i(\theta_i)$ ,  $c_i$  into  $\tau_1$  and  $\tau_2$ , and noticing the p.d.f of  $\theta_{i \min}$  is  $N_i(1 - F_i(\theta_i))^{(N_i-1)} f_i(\theta_i)$ , we have

$$\begin{aligned} \tau^u &= \sum_{i=1}^2 E\theta_{i \min} + (1 - \alpha_i) N_i \int_{\underline{\theta}_i}^{\bar{\theta}_i} \int_{\underline{\theta}_i}^{\bar{\theta}_i} (1 - F_i(s))^{N_i-1} ds f_i(\theta_i) d\theta_i \\ &\quad + \psi(e_1) - (1 + \delta)e_1 + \psi(e_1) - e_2. \end{aligned}$$

Notice that

$$\begin{aligned} &N_i \int_{\underline{\theta}_i}^{\bar{\theta}_i} \int_{\underline{\theta}_i}^{\bar{\theta}_i} (1 - F_i(s))^{N_i-1} ds f_i(\theta_i) d\theta_i \\ &= N_i \int_{\underline{\theta}_2}^{\theta_2} F_i(\theta_i) (1 - F_i(\theta_i))^{N_i-1} d\theta_i \\ &= E \frac{F_i}{f_i}(\theta_{i \min}). \end{aligned}$$

■

### Proof of Proposition 2

**Proof.** 1) is immediate by comparing (12) and (15).

By (11) and (14), we have

$$\frac{\alpha_1^u + \delta}{\psi''(\psi'^{-1}(1 - \alpha_1^u))} = \frac{\alpha_1^b + \delta\alpha_2^b}{\psi''(\psi'^{-1}(1 - \alpha_1^b + \delta(1 - \alpha_2^b)))}. \quad (32)$$

Denote  $h(x) = \frac{x}{\psi''(\psi'^{-1}(1 + \delta - x))}$ . Then, the above equation writes

$$h(\alpha_1^u + \delta) = h(\alpha_1^b + \delta\alpha_2^b).$$

Because  $\psi''$  and  $\psi'^{-1}$  are increasing functions,  $h(x)$  is monotonously increasing. Thus, we have

$$\alpha_1^u + \delta = \alpha_1^b + \delta\alpha_2^b. \quad (33)$$



(33) gives  $\alpha_1^b = \alpha_1^u + \delta(1 - \alpha_2^b)$ . Since  $(1 - \alpha_2^b) > 0$ , we have 2).

Notice  $e_1^u = \psi'^{-1}(1 - \alpha_1^u)$  and  $e_1^b = \psi'^{-1}(1 - \alpha_1^b + \delta(1 - \alpha_2^b))$  and hence (32) and (33) implies 3) i). ■

### Proof of Proposition 3

**Proof.** We only need to prove  $\frac{\partial(\tau^u - \tau^b)}{\partial\delta} > 0$ ;  $\frac{\partial(\tau^u - \tau^b)}{\partial N_2} < 0$ ; and  $\frac{\partial(\tau^u - \tau^b)}{\partial N_1} \leq 0 \Leftrightarrow \delta \geq 0$ .

Using envelope theorem, we have .

$$\begin{aligned} \frac{\partial(\tau^u - \tau^b)}{\partial\delta} &= -e_1^u + e_1^b - (\psi'(e_1^b) - (1 + \delta)) \frac{\partial e_1^b}{\partial\delta} \\ &= \alpha_1^b + \delta\alpha_2^b \\ &= \psi''(e_1^b) E \frac{F_1}{f_1}(\theta_{1\min}) > 0. \end{aligned}$$

The second equality is due to the fact that  $e_1^u = e_1^b$  and  $\psi'(e_1^b) = 1 - \alpha_1^b + \delta(1 - \alpha_2^b)$ . The last equality is due to (14).

$$\frac{\partial(\tau^u - \tau^b)}{\partial N_2} = \frac{\partial E\theta_{2\min}}{\partial N_2} + (1 - \alpha_2^u) \frac{\partial}{\partial N_2} E \frac{F_2}{f_2}(\theta_{2\min}) < 0.$$

The inequality is due to the fact that  $E\theta_{2\min}$  is decreasing in  $N_2$ ;  $\theta_{2\min}^{N_2+1} \stackrel{FOS}{\preceq}$   $\theta_{2\min}^{N_2}$ ,  $\frac{F_2}{f_2}$  is an increasing function. and hence  $\frac{\partial}{\partial N_2} E \frac{F_2}{f_2}(\theta_{2\min}) < 0$ .

$$\frac{\partial(\tau^u - \tau^b)}{\partial N_1} = (\alpha_1^b - \alpha_1^u) \frac{\partial}{\partial N_1} E \frac{F_1}{f_1}(\theta_{1\min}).$$

Since  $\theta_{1\min}^{N_1+1} \stackrel{FOS}{\preceq} \theta_{1\min}^{N_1}$  and  $\frac{F_1}{f_1}$  is increasing, we have  $\frac{\partial}{\partial N_1} E \frac{F_1}{f_1}(\theta_{1\min}) < 0$ . Thus,  $\frac{\partial(\tau^u - \tau^b)}{\partial N_1} \leq 0 \Leftrightarrow \alpha_1^b \geq \alpha_1^u \Leftrightarrow \delta \geq 0$ , by lemma 2. ■

### Proof of lemma 2

**Proof.** Using the technique of McAfee and McMillan (86), we solve the differential equation (18) and obtain

$$\begin{aligned}
U(\pi_2^E(\theta_2)) &= [1 - F_2(\theta_2)]^{1-N_2} e^{r(1-\alpha)\theta_2} (1-\alpha) \int_{\theta_2}^{\bar{\theta}_2} [1 - F_2(x)]^{N_2-1} e^{-r(1-\alpha)x} dx \\
&= \frac{1}{r} \{1 - [1 - F_2(\theta_2)]^{1-N_2} e^{r(1-\alpha)\theta_2} \int_{\theta_2}^{\bar{\theta}_2} (N_2 - 1) [1 - F_2(x)]^{N_2-2} e^{-r(1-\alpha)x} f_2(x) dx\},
\end{aligned}$$

where the second equality is obtained by integrating by parts. On the other hand, we have  $U(\pi_2(\theta_2)) = \frac{1 - e^{-r\pi_2(\theta_2)}}{r}$  by definition. Hence,

$$e^{-r\pi_2^E(\theta_2)} = [1 - F_2(\theta_2)]^{1-N_2} e^{r(1-\alpha)\theta_2} \int_{\theta_2}^{\bar{\theta}_2} (N_2 - 1) [1 - F_2(x)]^{N_2-2} e^{-r(1-\alpha)x} f_2(x) dx,$$

which gives

$$\pi_2^E(\theta_2) = \frac{N_2 - 1}{r} \log[1 - F_2(\theta_2)] - (1-\alpha)\theta_2 - \frac{1}{r} \log\left\{ \int_{\theta_2}^{\bar{\theta}_2} (N_2 - 1) [1 - F_2(x)]^{N_2-2} e^{-r(1-\alpha)x} f_2(x) dx \right\}.$$

Using our specific formula that  $F_2(\theta_2) = 1 - e^{-\theta_2}$ , we get our result immediately. ■

### Proof of lemma 3

**Proof.** Define  $h(x) = x - \frac{1}{N_1 - 1 + r(1-x)} - r(1-x)\sigma_1^2$ . Combining (24) and (27), we have

$$h(\alpha_1^b) - h(\alpha_1^u) = (1 - \alpha_2^b) \delta. \quad (34)$$

The s.o.c implies  $h'(x) > 0$ . Hence, (34) implies i).

From (27) we have  $\alpha_1^b + \delta\alpha_2^b = \frac{1}{N_1 - 1 + r(1-\alpha_1^b)} + r(1 - \alpha_1^b)\sigma_1^2$ . Substitute it into (28) and combining with (25), we obtain ii). ■

### Proof of Proposition 5

**Proof.** We need to prove  $\frac{\partial \tau^u - \tau^b}{\partial \delta} > 0$ ,  $\frac{\partial \tau^u - \tau^b}{\partial N_2} < 0$ ,  $\frac{\partial \tau^u - \tau^b}{\partial N_1} \leq 0 \Leftrightarrow \delta \geq 0$ ,  $\frac{\partial \tau^u - \tau^b}{\partial \sigma_1^2} \geq 0 \Leftrightarrow \delta \geq 0$ ,  $\frac{\partial \tau^u - \tau^b}{\partial \sigma_2^2} \leq 0 \Leftrightarrow \delta \geq \hat{\delta}$ .

From (23), (21) and (26) and the envelope theorem, we get

$$\begin{aligned}\frac{\partial \tau^u - \tau^b}{\partial \delta} &= e_1^b - e_1^u + \alpha_1^b + \delta \alpha_2^b \\ &= \alpha_1^u + \delta \\ &= \frac{1}{N_2 - 1 + r(1 - \alpha_1^u)} + r(1 - \alpha_1^u) \sigma_1^2 > 0.\end{aligned}$$

$$\frac{\partial \tau^u - \tau^b}{\partial N_2} = \frac{\partial}{\partial N_2} E\theta_{2\min} + \frac{\partial}{\partial N_2} \frac{1}{r} \log \left( 1 + \frac{r(1 - \alpha_2)}{N_2 - 1} \right) < 0.$$

$$\begin{aligned}\frac{\partial \tau^u - \tau^b}{\partial N_1} &= \frac{\partial}{\partial N_1} \frac{1}{r} \left( \log \left( 1 + \frac{r(1 - \alpha_1^u)}{N_1 - 1} \right) - \log \left( 1 + \frac{r(1 - \alpha_1^b)}{N_1 - 1} \right) \right) \\ &= \frac{\partial}{\partial N_1} \frac{1}{r} \log \left( \frac{N_1 - 1 + r(1 - \alpha_1^u)}{N_1 - 1 + r(1 - \alpha_1^b)} \right) \\ &= \frac{\partial}{\partial N_1} \frac{1}{r} \log \left( 1 + \frac{r(\alpha_1^b - \alpha_1^u)}{N_1 - 1 + r(1 - \alpha_1^b)} \right),\end{aligned}$$

which is negative iff  $\alpha_1^b \geq \alpha_1^u \Leftrightarrow \delta \geq 0$ .

$$\frac{\partial \tau^u - \tau^b}{\partial \sigma_1^2} = \frac{r}{2} \left( (1 - \alpha_1^u)^2 - (1 - \alpha_1^b)^2 \right),$$

which is positive iff  $\alpha_1^b \geq \alpha_1^u \Leftrightarrow \delta \geq 0$ .

$$\frac{\partial \tau^u - \tau^b}{\partial \sigma_2^2} = \frac{r}{2} \left( (1 - \alpha_2^u)^2 - (1 - \alpha_2^b)^2 \right),$$

which is negative iff  $\alpha_2^b < \alpha_2^u \Leftrightarrow \delta \geq \hat{\delta}$ . ■