Optimal Sizing of a Series PHEV: Comparison between Convex Optimization and Particle Swarm Optimization

Mitra Pourabdollah[1], **Emilia Silvas**[2], **Nikolce Murgovski**[1], **Maarten Steinbuch**[2], **Bo Egardt**[1]

[1] *Chalmers University of Technology, Gothenburg, Sweden, (e-mail: {mitrap,nikolce.murgovski,bo.egardt}@chalmers.se),*
[2] *Eindhoven University of Technology, Eindhoven, The Netherlands, (e-mail: e.silvas,m.steinbuch@tue.nl)*

Abstract: Building a plug-in hybrid electric vehicle that has a low fuel consumption at low hybridization cost requires detailed design optimization studies. This paper investigates optimization of a PHEV with a series powertrain configuration, where plant and control parameters are found concurrently. In this work two often used methods are implemented to find optimal energy management with component sizes. In the first method, the optimal energy management is found simultaneously with the optimal design of the vehicle by using convex optimization to minimize the sum of operational and component costs over a given driving cycle. To find the integer variable, i.e., engine on-off, dynamic programming and heuristics are used. In the second method, particle swarm optimization is used to find the optimal component sizing, together with dynamic programming to find the optimal energy management. The results show that both methods converge to the same optimal design, achieving a 10.4% fuel reduction from the initial powertrain design. Additionally, it is highlighted that the usage of each of the method poses challenges, such as computational time (where convex optimization outperforms particle swarm optimization by a factor of 20) and the tuning effort for the particle swarm optimization and the ability to handle integer variables of convex optimization.

Keywords: Plug-in Hybrid Electric Vehicles, Optimal Sizing, Energy Management, Convex Optimization, Dynamic Programming, Particle Swarm Optimization.

1. INTRODUCTION

Hybrid electric vehicles (HEV) have an electric propulsion system, in addition to a conventional combustion engine. HEVs can reduce the fuel consumption by downsizing the engine, recovering braking energy, eliminating engine idling, and having extra power control freedom by the two power sources. Plug-in hybrid electric vehicles (PHEV) are the next generation of hybrid electric vehicles that have the ability to store energy from the electrical grid, using large capacity batteries. PHEVs may drive short trips entirely on stored electrical energy, thus decreasing the vehicle's dependency on petroleum. However, the main challenge with PHEVs is their high cost which motivates the study of optimal design for these vehicles.

The total cost of ownership of a PHEV depends directly on the size of the powertrain components. Moreover, energy management, which is the control strategy that determines the power split between engine and additional energy source at every time instant, affects the design. To exclude its influence on component sizing, the control strategy should ideally be part of the optimal design process (Sundström et al. [2008]). Hence, the problem of optimizing the total vehicle cost should be approached by optimization of both energy management and component sizes. The general procedure of sizing PHEVs is performed by optimizing total cost of vehicle ownership for a set of driving cycles. To reflect lifetime driving and charging behaviors of a driver, long driving cycles are needed to represent different driving situations.

Since the problem of sizing and control of a PHEV is nonlinear, mixed integer, and has several states, this poses significant challenges on the algorithms used to solve it. Researchers have used exhaustive search, i.e., the evaluation of the cost function for different combinations of the design variables, to roughly estimate the shape of the cost function and choose a design. This is both sub-optimal and time inefficient, which lead to the usage of optimization-based algorithms. Among these, evolutionary algorithms, such as particle swarm optimization (PSO), genetic algorithms or simulated annealing, have shown good results. To calculate the fuel consumption some of these methods use rule-based control strategies e.g.,Gao et al. [2007]. Besides the heuristic algorithms, dynamic programming (DP) has been extensively used together with evolutionary algorithms, e.g., by Li et al. [2012], Ebbesen et al. [2012], Ravey et al. [2012].

Evolutionary algorithms require parameter tuning, large computation times and have no proof of global optimality (Silvas et al. [2014]). As an alternative to evolutionary algorithms, the problem can be reformulated as a convex optimization problem as shown by Murgovski et al. [2011], Egardt et al. [2014], Pourabdollah et al. [2013]. Both component sizes and the complete *control trajectory* of the continuous variables can be included as optimization variables. Convex problems have a unique optimum and can be solved fast and reliably. However, the drawback with convex optimization is that integer variables, e.g., engine on-off or gear selection, cannot be included in the problem and therefore, should be given as an input a-priori to the optimization problem (Murgovski et al. [2011]). The value can be found either by heuristics e.g., as in Pourabdollah et al. [2013], or by iterative strategies as in [Murgovski et al., 2014], where the results are compared with DP.

In this paper, convex optimization is used and compared with a particle swarm optimization for finding optimal design of a series PHEV. For the convex optimization method, engine on-off is found by two methods. First, the engine on-off decision for convex optimization is found based on a simple rule. As an alternative, convex optimization and dynamic programming are combined in an iterative manner to update the engine on-off decisions, and the iterations are performed until the cost converges. Particle swarm optimization (PSO) also finds the optimal design in a nested way with DP. This method, used by Ebbesen et al. [2012], Nuesch et al. [2012], Silvas et al. [2014], searches for the powertrain component sizes using PSO and for any fixed powertrain sizes, the optimal energy management is computed by DP.

The rest of this paper is organized as follows. An overall picture of the optimization problem, the driving cycle and the model of the powertrain and its components are presented in Section 2. A brief explanation of the optimization methods, convex optimization, dynamic programming, and particle swarm optimization are provided in Section 3. Illustrative results from the study are shown in Section 4. Finally conclusions are drawn in Section 5.

2. PROBLEM FORMULATION AND MODELING

In this section, the problem formulation and modeling details are introduced. The studied PHEV, depicted in Fig. 1, is a series powertrain, where only the electric motor (EM) is mechanically linked to the drivetrain and can propel the wheels.

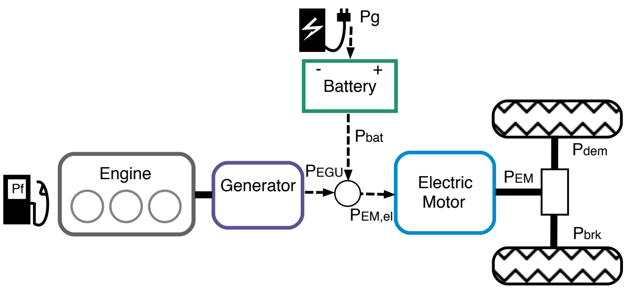

Fig. 1. Series PHEV powertrain configuration (solid lines: mechanical link, and dashed lines: electrical links).

The problem is formulated to find design and control variables that minimize an objective function under the presence of constraints. Here, the objective function is a weighted sum of operational costs over the driving cycle, J_{op}, and component costs, J_{comp}. The operational cost includes the consumed fossil fuel and electrical energy, and the components cost is the sum of the costs of battery and engine-generator unit (EGU). The problem can be stated as:

$$\min_{u,s} J = J_{op}(u,s) + J_{comp}(s) \tag{1}$$

subject to:

$$x(t+1) = f(x(t), u(t), s)$$
$$x \in X, u \in U, s \in S,$$

where x is the state variable vector, e.g., the battery state of charge; and s is the scaling factor for components, i.e., battery and EGU. The size of the electric motor is directly decided by the power demand from the driving cycle, and hence is not included in the problem. The control input variable u is defined as:

$$u = [u_d \; u_c], \tag{2}$$

where u_c includes the continuous control inputs of length N, e.g., engine or motor torque, and u_d includes the integer control inputs of size N, e.g., gear or engine on-off, and N is the number of time samples of the driving cycle. These variables are explained in more detail at the end of this section. The constraints in the problem are in the form of powertrain and component models, which are introduced in Section 2.2.

2.1 *Driving cycle and performance requirements*

In the optimization problem, we try to find the optimal component sizes and the energy management variables over a given driving cycle. Therefore, designing a vehicle requires knowledge about the lifetime driving of the vehicle. However, since it is impossible to predict the precise driving cycle and computational resources are limited, we use a long driving cycle that represents real-life driving [Kullingsjö et al., 2012].

The long driving cycle used in the optimization can reflect real-life driving, but might not include extreme situations that require high performance. Acceleration requirement is considered as an important vehicle attribute by many drivers, and is hence added in the constraints. Acceleration as a function of speed on a flat road is used to make a so called performance cycle, which is then appended to the driving cycle. The performance cycle includes speeds from zero to maximum speed, increasing according to the desired accelerations as explained by Pourabdollah et al. [2013]. The driving cycle used in the simulations is shown in Section 4. We assume that the battery has the possibility to be charged with constant power from the grid at charging occasions overnight, when the car is parked for 8 hours.

2.2 *Modeling*

In this section the models of the powertrain and its components are presented. The same models are used both for DP and convex optimization. A part of the problem in (1) fulfills convexity requirements, as it will be shown in the following. A convex function satisfies $f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y)$ for all $x, y \in \mathcal{R}^n$ with $0 \le \lambda \le 1$ [Boyd and Vandenberghe, 2009]. Powertrain components that are being described by quasi-static functions are approximated with nonlinear convex functions, together with some necessary variable changes. The accuracy of these approximations is acceptable and discussed in detail by Murgovski et al. [2011].

Powertrain: Starting from the driving cycle that is fully described by the velocity $v(t)$ and acceleration $a(t)$ (and, in this particular case, a zero road slope) at discrete time instants, the required traction force, F_t, and power demand, P_{dem}, are calculated from the driving cycle as

$$F_t(t) = \frac{c_d A_f \rho v(t)^2}{2} + m_{tot} g c_r + m_{tot} a(t), \tag{3}$$
$$P_{dem}(t) = F_t(t) v(t),$$

Table 1. Vehicle parameters

parameter	value
Baseline mass	m=1600 kg
Glider mass	m_g=1280 kg
Frontal area	A_f=2.37 m^2
Rolling resistance	c_r=0.009
Aerodynamic drag coefficient	c_d=0.33
Air density	ρ= 1.293 kg/m^3
Wheel radius	r_w=0.3 m
Ratio of the final gear	r_{fg}=4.2
EM reduction gear	r_{EM}=2

where c_d, A_f, m_{tot}, ρ, g, c_r, and a are air drag coefficient, frontal area, total vehicle mass, air density, gravitational acceleration, rolling resistance coefficient, and acceleration, respectively, with values given in Table 1. The total vehicle mass, m_{tot}, is the sum of the masses of the glider, battery, and EGU.

The powertrain model is described by mechanical power balance equation, given as

$$P_{dem}(t) + P_{brk}(t) = P_{EM}(t), \tag{4}$$

where P_{brk} and P_{EM} are the power dissipated at friction brakes and EM mechanical power. The electrical power balance equations during driving times, $\mathcal{T}_{drive}$, and charging times, $\mathcal{T}_{charge}$, are given as

$$P_{EM,el}(t) + P_{aux}(t) = P_{EGU}(t) + P_{b,int}(t) - P_{b,loss}(t), t \in \mathcal{T}_{drive}$$
$$P_{b,int}(t) - P_{b,loss}(t) = -P_g(t)\eta_g, k \in \mathcal{T}_{charge}, \tag{5}$$

where $P_{EM,el}$, P_{EGU}, $P_{b,int}$, $P_{b,loss}$, P_g, and η_g are the EM electrical power, the EGU power, the battery internal power (before the losses), the battery loss power, grid power, and grid efficiency. The electrical power used by auxiliary devices, P_{aux}, is estimated by a constant value of 500 W. For simplicity, the losses in the power electronics are included in the EM losses and the rotational inertia (including the inertia of the wheels, the differential, the EM and the EGU) is neglected in the models.

Battery: The battery consists of s_b identical cells, each modeled as a constant open circuit voltage V_{oc} in series with a constant internal resistance, R. The power losses and the stored energy of the battery, E_b, are calculated as

$$P_{b,loss}(t) = \frac{RP_{b,int}(t)^2}{s_b V_{oc}^2}, \tag{6}$$

$$E_b(t+1) = E_b(t) - h(t)\, P_{b,int}(t). \tag{7}$$

The battery internal power is positive when discharging. During the available parking periods the vehicle is charged with constant current and power. Then, without loss of generality, the charging energy can be modeled as if entering the battery in one extra long sample at the parking occasions. The battery cells have capacity of $Q = 159Wh$, power of $880W$, and mass of $m_{b,1}$, giving the total pack mass as $m_b = m_{b,1}s_b$.

Electric Machine: The losses of the EM and power electronics are gathered in a power loss map, $P_{EM,loss}$, where the losses are measured at steady-state for different torque-speed combinations. At each time instant on the driving cycle, the EM angular speed, ω_{EM}, is calculated in advance as

$$\omega_{EM}(t) = r_{EM}v(t)\frac{r_{fg}}{r_w}, \tag{8}$$

where r_w, r_{fg}, and r_{EM} are the wheel radius, ratio of the final gear (differential), and EM reduction gear respectively. The power losses are approximated by a second-order polynomial in torque as

$$P_{EM,loss}(t) = a_1(t)P_{EM}(t)^2 + a_2'(t)P_{EM}(t) + a_3(t), \tag{9}$$

$$P_{EM,el}(t) = P_{EM}(t) + P_{EM,loss}(t), \tag{10}$$

$$= a_1(t)P_{EM}(t)^2 + a_2(t)P_{EM}(t) + a_3(t), \tag{11}$$

where the coefficients $a_1 \geq 0$, $a_2 = a_2' + 1$ and a_3 are functions of EM speed, ω_{EM}, and hence time. The values are calculated from data using least squares method for a number of grid points of ω_{EM}. For the speed values not belonging to the grid nodes, the coefficients are obtained by linear interpolation (Murgovski et al. [2011]).

Engine-generator unit: The fuel power, $P_{f,base}$, of a baseline EGU is a function of the generator power and is approximated with a second-order polynomial in P_{EGU} as

$$P_{f,base}(P_{EGU,base}) = b_1 P_{EGU,base}^2 + b_2 P_{EGU,base} + b_3 \tag{12}$$

where $b_j \geq 0$; $j \in \{0, 1, 2\}$ [Murgovski et al., 2011]. The model of the scaled EGU is obtained by applying linear relation to the fuel and electric power of the baseline EGU, i.e. $P_f = s_{EGU}P_{f,base}$ and $P_{EGU} = s_{EGU}P_{EGU,base}$. The fuel power of the scaled EGU then becomes

$$P_f(t) = b_1\frac{P_{EGU}^2(t)}{s_{EGU}} + b_2 P_{EGU}(t) + e_{on}(t)b_3 s_{EGU}, \tag{13}$$

where e_{on} is a binary signal that is introduced to remove the idling losses $b_3 s_{EGU}$, when the EGU is off. To preserve the problem convexity, e_{on} is decided prior to the optimization.

The EGU mass scales linearly with the mass of the baseline EGU with 100 kW power , i.e. $m_{EGU} = m_{EGU,1}s_{EGU}$, where $m_{EGU,1}$ is a linear weight coefficient.

2.3 Problem Formulation

The problem introduced briefly in (1) is now given in more detail. The operational cost J_{op} includes the consumed fossil fuel and electrical energy as

$$J_{op} = \sum_{t=1}^{N} w_f P_f(t)h(t) + w_e P_g(t)h(t), \tag{14}$$

where $w_f = \frac{\rho_f}{\rho_{LHV}}$, $w_e = \frac{\rho_{el}}{1000 \cdot 3600}$, and ρ_{LHV} is the lower heating value of gasoline. The fuel power, P_f, and charger power, P_g, are converted to an equivalent cost in EUR using energy prices ρ_f for gasoline and ρ_{el} for electricity. The sampling interval $h(t)$ is equal to 1 s at the driving instances and at charging instances is equal to the entire charging period [Pourabdollah et al., 2013].

The component cost, $J_{comp}(s)$, is the sum of the costs of battery and EGU. The remaining cost of the vehicle is independent of sizing and can be therefore excluded from the problem. The components cost is calculated as the depreciation over the driving cycle, i.e., the proportion of the components cost given by the ratio between the length of the cycle, d, and the lifetime driving distance of the vehicle. If payment is equally divided in vehicle lifetime with yearly interest rate of $p_c = 5\%$, the components cost is given by

$$J_{comp} = \frac{d}{d_y y_v}\left(1 + p_c\frac{y_v + 1}{2}\right)(cost_b + cost_{EGU}), \tag{15}$$

where y_v is the vehicle lifetime, and d_y is the average traveled distance of the vehicle in one year. For each component, the cost model is a linear function

$$cost_b = cost_{j,0} + cost_{j,1}\, s_j \,\forall\, j \in \{b, EGU\}, \tag{16}$$

where $cost_{j,0}$ is the initial cost and $cost_{j,1}$ is the linear cost coefficients.

The state variable, x, in (1) is the energy in the battery given in (7). The decision variables firstly include, s, the dimensionless component sizes for battery and EGU. The continuous input variables, u_c, which are related to the energy management, are determined for every time instant. These variables are EGU power, $P_{EGU}(t)$, battery internal power, $P_{b,int}(t)$, battery state of energy, $E_b(t)$, grid power, $P_g(t)$, and braking power, $P_{brk}(t)$. The integer input variable, u_d, consists of the binary engine on-off variable, $e_{on}(t)$.

As mentioned earlier, the total vehicle mass in (3) is computed as the sum of the masses of the glider, m_0, battery, and EGU,

$$m_{tot} = m_0 + m_{b,1}s_b + m_{EGU,1}s_{EGU}. \tag{17}$$

Here, we distinguish the concept of a baseline vehicle that has predefined values for the scaling factors s_b and s_{EGU}. The demanded power of the baseline vehicle is hereafter denoted by $P_{dem,base}$.

3. OPTIMIZATION METHODS

The optimization methods used in this paper, namely dynamic programming, convex optimization and particle swarm optimization are explained in this section.

3.1 Dynamic programing

Dynamic programming is a method to solve optimal control problems based on the Bellman's principle of optimality (Bellman [1957]). The dynamic programming algorithm proceeds backward in time. The problem starts with a final time cost, which is assumed to be zero for PHEVs, because, unlike HEVs, there is no constraint on the final battery state of charge. At each time instant, DP finds the optimal energy management and engine on-off that minimizes a total cost. DP has been widely used in automotive applications to find the optimal energy management which minimizes fuel consumption, since it can handle nonlinear constraints and integer variables (Lin et al. [2003], Hofman et al. [2012]). The main drawback of DP is its computational time which increases exponentially with the number of states and number of components sizes. Therefore, in order to find the optimal design of vehicles, DP is often used with other optimization methods.

3.2 Convex optimization

Convex optimization is also used to solve the problem of finding the optimal design and energy management [Egardt et al., 2014, Murgovski et al., 2011, Pourabdollah et al., 2013]. The powertrain and component models, in addition to the cost and weight models, are formulated as convex to solve a convex problem. Modeling of the powertrain and its components to guarantee the convexity is the main step of the optimization method. Once the problem is defined as a convex optimization problem, it can be solved in a relatively short time, using efficient solvers. Integer variables such as engine on-off cannot be included in the problem and hence need to be decided a-priori to the convex optimization to preserve convexity. The constraints in the convex problem include the physical limits of the components in addition to the powertrain and component models, introduced in Section 2.2. The complete problem formulation is given as

$$\min w_f \sum_{t=1}^{N} P_f(t)h(t) + w_e \sum_{t=1}^{N} P_g(t)h(t) + J_{comp} \tag{18a}$$

variables: $P_{EGU}, P_{b,int}, E_b, P_g, P_{brk}, s_b, s_{EGU}$

subject to:

$$P_{dem}(t) + P_{brk}(t) = P_{EM}(t) \tag{18b}$$

$$P_{EM,el}(t) + P_{aux}(t) = P_{EGU}(t) + P_{b,int}(t) - P_{b,loss}(t) \tag{18c}$$

$$P_g(t)\eta_g = -P_{b,int}(t) + P_{b,loss}(t), t \in \mathcal{T}_{charge} \tag{18d}$$

$$P_{b,loss}(t) \geq \frac{RP_{b,int}(t)^2}{s_b V_{oc}^2} \tag{18e}$$

$$P_{EM,el}(t) \geq a_1(t)P_{EM}(t)^2 + a_2(t)P_{EM}(t) + a_3(t) \tag{18f}$$

$$E_b(t+1) = E_b(t) - h(t)P_{b,int}(t) \tag{18g}$$

$$P_f(t) = b_1 \frac{P_{EGU}(t)^2}{s_{EGU}} + b_2 P_{EGU}(t) + e_{on}(t) b_3 s_{EGU} \tag{18h}$$

$$E_b(t) \in s_b[E_{b,min}, E_{b,max}]$$

$$P_{b,int}(t) \in s_b[P_{b,int,min}, P_{b,int,max}]$$

$$P_g(t) \in [0, P_{g,max}(t)]$$

$$P_{EGU}(t) \in s_{EGU}[0, P_{EGU,max,base}]$$

for $t \in \mathcal{T}_{drive}$ unless is defined otherwise.

In order to guarantee the problem convexity, the equality signs in (18e) and (18f) are relaxed to inequalities. The optimal result will satisfy equality because otherwise energy is wasted which is not optimal. Moreover, the number of cells, s_b, is relaxed to a real value. The relaxation will introduce a rounding error that has a small influence on the optimal result [Egardt et al., 2014, Murgovski et al., 2011]. To solve the problem, a tool, CVX [Grant and Boyd, 2010], is used to automatically translate it to a second order cone form, required by a publicly available solver, e.g. Sedumi [Strum, 2011].

Integer variables in convex optimization problem As mentioned earlier, integer variables cannot be included in the convex optimization problem. Therefore the decision on engine on-off is found a-priori to the problem. The problem has been studied by different researchers. Elbert et al. [2014] derived the globally optimal engine on-off analytically, whereas Murgovski et al. [2014] used a heuristic method based on pontryagin's maximum principle. However, these approaches do not include sizing of the engine.

Heuristics have been used also by Pourabdollah et al. [2013] to find the integer variables in the problem of component sizing. The engine is turned on if at a certain time instant the power demand is higher than a power threshold, P_{on}, and is turned off otherwise. The optimization is then repeated over different values of P_{on} to find the threshold that minimizes the cost. An alternative way to find engine on-off decision is by using convex optimization and DP methods alternately. In order to do this, the optimization starts with DP with initial component sizes. DP finds the optimal energy management including engine on-off for the given component sizes. The engine on-off decision is then given to the convex optimization to find the new component sizes. This iteration is continued until the cost and component sizes converge. The procedure is illustrated in the right part of in Fig. 2.

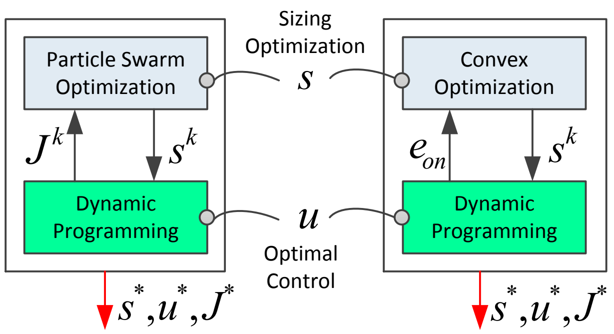

Fig. 2. Bi-level optimization frameworks compared for component sizing and control: (Method 1/right) the proposed combined convex optimization and DP, and (Method 2/left) the combination of Particle Swarm Optimization and DP.

3.3 Particle swarm optimization

Particle swarm optimization (PSO) is a stochastic search method inspired by the coordinated motion of animals living in groups, originally developed by Kennedy and Eberhart [1995]. Using a population of candidate solutions, called *particles*, in the design space, called *swarm*, PSO optimizes the problem by improving the candidate solution with regard to a given quality measure. To begin with, the method initializes the swarm randomly in the design space, with each particle having assigned a position, χ, and a velocity, υ. Then, the objective function is evaluated for each particle and a global best is determined, G. Iteratively the velocity, υ_i, and position, χ_i, of each particle are changing towards their own best, B_i, and globally best location by

$$\begin{aligned} \upsilon_i^{k+1} &= \phi^k \upsilon_i^k + \alpha_1[\beta_{1,i}(B_i - \chi_i^k)] + \alpha_2[\beta_{2,i}(G - \chi_i^k)], \\ \chi_i^{k+1} &= \chi_i^k + \upsilon_i^{k+1}, \end{aligned} \tag{19}$$

with χ_i^k and υ_i^k the current position and velocity of particle i in generation number k, and ϕ the particle inertia which causes a certain momentum of the particles. Parameters $\beta_{1,2} \in [0, 1]$ are uniformly distributed random values and $\alpha_{1,2}$ are acceleration constants.

In nested combinations with other algorithms, PSO has proven as a good candidate in design of HEVs, as shown by Williamson et al. [2005], Gao et al. [2007], Sundström [2009], Nuesch et al. [2012], Silvas et al. [2014]. Motivated by these studies, we will combine PSO with DP, as depicted in Fig. 2 and use this bi-level design method as a benchmark comparison.

4. RESULTS

In this section, the results of the simultaneous optimization of energy management and component sizing are given. The optimization is performed over near 3 hours/176 km long real life driving cycle, followed by a performance cycle. There are also 4 occasions where the car has the possibility to charge the battery with constant grid power for 8 hours. As mentioned, the size of the EM is decided by the maximum power demand of the driving cycle, which is equal to 108.5 kW.

In the first method, convex optimization is used to find the optimal design. A simple rule is used to find the engine on-off based on the baseline power demand required by the vehicle when following the driving cycle, $P_{dem,base}(t)$. At every time instant, if the power demand is higher than a power threshold, P_{on}, the engine is turned on and is turned off otherwise. The convex optimization problem is iterated over several values of P_{on} to find the best result. The result of battery size and total cost for different values of P_{on} are shown in Fig. 3. The optimal power threshold to turn the engine on is equal to 16kW which gives total cost as €20.81, battery size equal to 8.07 kWh ($s_b = 50.64$) and EGU size of 66.87 kW ($s_{EGU} = 0.69$).

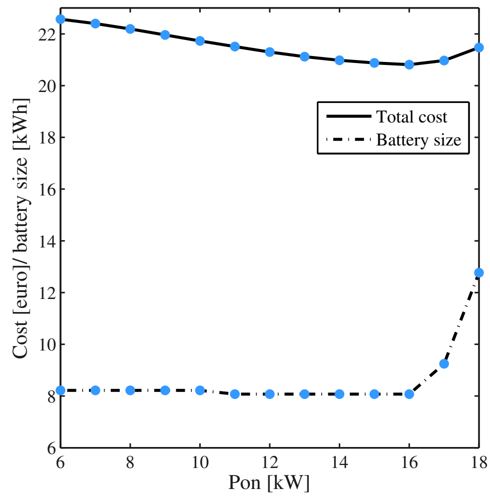

Fig. 3. Number of battery cells and the total cost obtained by convex optimization using different power threshold to turn engine on.

The convex optimization is then combined with DP in an iterative manner to optimize the engine on-off as shown in Fig.2. As mentioned, the iteration starts with DP, with initial component sizes. These initial inputs can have some impact on the final results ending in a local optimum. For example, if the initial battery size is much larger than the optimal value, in the first iteration of DP, the vehicle is propelled most of the time by the cheap energy from the battery and the engine is turned off in order to reduce the losses. Giving this engine on-off to the convex optimization, the battery may stay oversized since the convex optimization is not able to alter the engine on-off decision. In this way, the iterative optimization never gets a chance to converge to a smaller battery size. To avoid this problem, a small battery with $s_b = 13$, or size equal to 2 kWh, is chosen as the initial value.

The procedure of using convex optimization and DP is continued until the cost and component sizes converge. In Fig. 4 the results of 12 iterations are given. As we can see, the optimal sizes of the engine-generator unit and the battery converge after the first iteration. The cost however, decreases in the next iteration, where DP finds the optimal energy management for the given sizes. The total cost decreases from €22.93 in the first iteration to €20.54 in the last. The optimal battery size is equal to 8.2 kWh ($s_b = 51.42$) and EGU size is equal to 66.17 kW ($s_{EGU} = 0.66$). The driving cycle used in the optimization together with the result of the optimal state of charge from the last iteration are shown in Fig. 5.

In the second method, and for comparison, we solve the same sizing and control problem using PSO for finding component

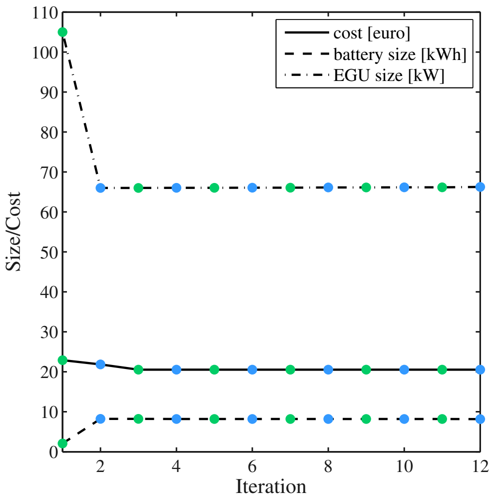

Fig. 4. Number of battery cells, EGU size, and the total cost over 12 iteration of DP (green) and convex optimization (Blue), starting with small battery size.

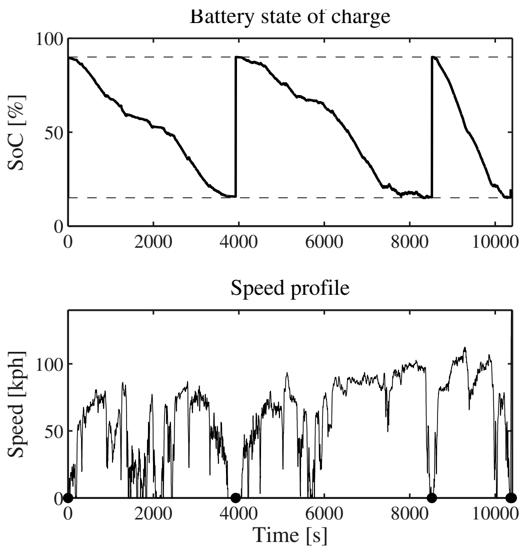

Fig. 5. The optimal SoC (top) and the speed profile of the driving cycle (bottom). The charging occasions are denoted by black dots.

sizing and DP for finding the optimal control as shown in Fig.2. The PSO algorithm is simulated for different particles number, $p \in [5, 40]$, and generations number, $g \in [5, 10]$. It is required to have at least 30 particles for the algorithm to reach the global optimum (see Fig. 6), or 10 particles to reach a 'close to' optimum value (within 0.5% of the global optimum), while 8 generations are required in average for the algorithm to converge to a solution. The optimal battery size is equal to 8.2 kWh ($s_b = 51.44$), EGU size is equal to 66.17 kW ($s_{EGU} = 0.66$) and the total cost is 20.54 euros.

4.1 Computational Efficiency

As shown in the results, both convex optimization/DP and PSO/DP methods find the same optimized solution. However, the computational time of the two methods differ. For the 167 minutes long driving cycle, the evaluation of each DP simulation takes 250 seconds and each convex optimization

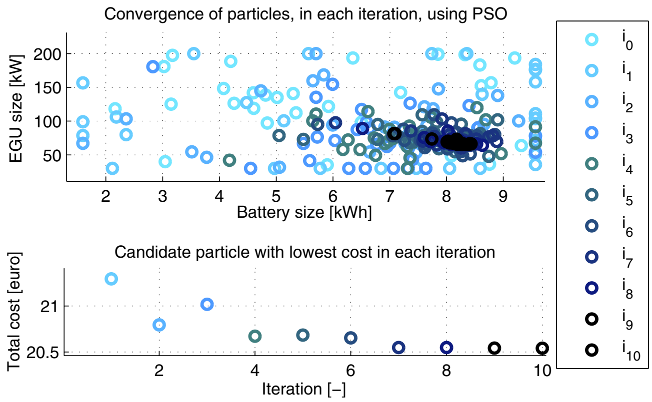

Fig. 6. The optimal sizing, using the PSO method, results in an design with $s_b = 51.44$ and $s_{EGU} = 0.66$, that gives a total cost of 20.54 euros.

problem takes around 120 seconds of calculations[1]. Using convex optimization together with heuristics to find e_{on} takes $13 \cdot 120$ seconds or 26 minutes. Computation of the iterative method of convex optimization/DP takes $6 \cdot (250+120)$ seconds or 37 minutes.

In the second method presented in Fig. 6, several particles are computed at each iteration. With each DP evaluation taking 150 seconds, the total computation time is 150 sec $\cdot 30$ particles $\cdot 10$ generations $= 45000$ seconds ($\simeq$ 12hours and 30 min). The results show the significant benefit of using convex optimization to optimally size a series PHEV.

5. CONCLUSIONS

In this paper, we address the problem of finding the optimal powertrain component sizing and control algorithm of a PHEV, with respect to hybridization and fuel costs, using a real-life measured driving cycle. To solve this problem, we use convex optimization together with heuristics or dynamic programming to find the engine on-off decisions. The results are compared with another existing method, i.e., PSO/DP. As shown by results, the convex optimization based method proves to be faster alternative to current methods that can find the global unique solution. Using simple rules to find the engine on-off gives results close to optimal, whereas DP can improve the result with a very high accuracy, with a relatively short time compared to an evolutionary based algorithm, PSO. Moreover, extra states, such as battery state of health or engine thermal state, or design variables, such as electric motor scaling, can simply be added to convex optimization problem without much increase in computational burden. However, the computational time of DP or evolutionary methods explodes exponentially by increasing the number of states or design parameters.

In this work we have investigated a series configuration, on one driving cycle. Future work can include more studies, including different configurations, larger set of driving cycles, different battery technologies, and pricing scenarios. Although both methods used in this study have converged to the same results, there is no general proof that the global minimum is reached. Further studies is needed for a theoretical basis to proof the convergence of the algorithms used.

[1] Simulations for method 1 were performed on a PC with Intel core 2 processor, at 2.67 GHz and 8 GB memory, and for method 2 on a similar PC, with Intel i7 processor at 2.2 GHz and 8 GB of memory.

6. ACKNOWLEDGMENT

This project is financially supported by the Swedish Hybrid Center.