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*Research Article*

# A Continuous Trust-Region-Type Method for Solving Nonlinear Semidefinite Complementarity Problem

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We propose a new method to solve nonlinear semidefinite complementarity problem by combining a continuous method and a trust-region-type method. At every iteration, we need to calculate a second-order cone subproblem. We show the well-definedness of the method. The global convergent result is established.

## 1. Introduction

This paper deals with the semidefinite complementarity problem (SDCP) with respect to a mapping $F : S \to S$, denoted by SDCP($F$), to find an $X \in S$ such that

$$(X, F(X)) \in S_+ \times S_+, \qquad \langle X, F(X)\rangle = 0, \tag{1}$$

where $S \subset R^{n\times n}$ is a set comprising those $X \in R^{n\times n}$ that are real symmetric. SDCP is the generalization of linear complementarity problems (LCPs) and semidefinite programs (SDPs) which has wide applications in engineering and economics [1]. The study of this problem can be dated back to the work of Shibata et al. [2]. Since then much attention has been attracted to SDCPs and various reformulations of SDCPs to minimization problem based on merit functions have been presented [3–6]. In general, there are two ways to derive the global convergence of an algorithm: trust-region methods and line search methods. The above methods proposed for solving SDCP($F$) are all based on a line search strategy; methods based on trust-region technique are relatively fewer. Despite having been studied by many researchers [7, 8], trust-region methods are robust, can be applied to ill-conditioned problems, and have strong global convergence properties. Therefore, different from the above methods, we propose a new algorithm based on trust-region method to solve SDCPs.

A function $F : S \to S$ is said to be monotone if

$$\langle X - Y, F(X) - F(Y)\rangle \geq 0 \tag{2}$$

for any $X, Y \in S$. An SDCP($F$) is called a monotone SDCP($F$) if the involved function is a monotone function. The Frobenius norm of a matrix $X$ is defined by

$$\|X\|_F := \|X\| := \sqrt{\langle X, X\rangle}. \tag{3}$$

Let $DF(X) : S \to S$ be a linear operator satisfying

$$\lim_{\Delta X \to 0} \frac{\|F(X + \Delta X) - F(X) - DF(X)\Delta X\|}{\|\Delta X\|} = 0; \tag{4}$$

then, $F$ is said to be Fréchet differentiable at $X$ and $DF(X)$ is the Fréchet derivative of $F$ at $X$. The function $F$ is said to be differentiable if it is differentiable at each $X \in S$ and to be continuously differentiable if also $DF(X)$ is continuous at each $X \in S$. In this paper, we suppose that $F : S \to S$ is a continuously differentiable monotone function.

Recently, there has been much interest in SDCP($F$). A few methods have been developed to solve this problem, such as interior point methods, merit function methods, and noninterior point continuation/smoothing methods [3–5].

Our new algorithm is based on the following smoothed Fischer-Burmeister function:

$$\Phi_\mu(X) = X + F(X) - \sqrt{X^2 + F(X)^2 + 2\mu^2 I}, \tag{5}$$

where $(\mu, X, Y) \in R \times S \times S$ and $I$ is the $n \times n$ identity matrix. This smoothing function was introduced by Kanzow [9] in the case of the NCP based on the Fischer-Burmeister function. Let

$$H_\mu(X) := \left\|\Phi_\mu(X)\right\|^2. \tag{6}$$

From Lemma 1 of [3], we know that if $\mu \to 0$, then

$$H_\mu(X) \longrightarrow H_0(X^*) := \left[X^* - \left[X^* - F(X^*)\right]_+, 0\right]^T, \tag{7}$$

where $[X^* - Y^*]_+$ denotes the orthogonal projection of $X^* - Y^*$ at $S_+$, whereas by Lemma 2.1 of [4],

$$H_0(X^*) = 0 \Longleftrightarrow X^* \text{ sloves SDCP } (F). \tag{8}$$

Thus, we can solve SDCP($F$) by using the following approach: reformulate SDCP($F$) as a system of nonsmooth equation $H_0(X) = 0$ and then approximate nonsmooth equations by parameterized smooth equations $H_\mu(X) = 0$; we solve the smooth equations at each iteration and make $\|H_\mu(X)\|$ decrease gradually by reducing the smoothing parameter $\mu$ to zero. In practice, however, it is usually impossible to solve the equation $H_\mu(X) = 0$ exactly for $\mu > 0$.

In this paper, we present a continuous and approximate method to solve SDCP($F$). At each iteration, the method solves a quadratic semidefinite program, which can be converted to a linear semidefinite program with a second-order cone constraint. A subproblem of this kind can be solved quite efficiently by using some recent software for semidefinite and second-order cone programs. The method is shown to be globally convergent under certain assumption.

The rest of this paper is organized as follows. Section 2 gives the algorithm and discusses the well-posedness for the algorithm; Section 3 analyzes the global convergence for the new algorithm; Section 4 presents the numerical results for the new algorithm; Section 5 concludes this paper.

## 2. The Algorithm

In this section, we will propose a smoothing trust-region-type algorithm for solving SDCP($F$) and prove that the proposed algorithm is well-defined.

Define

$$Q_{\mu_k}(\Delta X) := \left\|\Phi_{\mu_k}(X_k) + D\Phi_{\mu_k}(X_k)\Delta X\right\|^2. \tag{9}$$

We begin with a formal statement of the algorithm which is in the spirit of [10–12].

*Algorithm 1* (continuous trust-region-type method).

(S0) (Initialization) Choose $0 < \rho_1 < \rho_2 < 1$, $0 < \sigma_1 < 1 < \sigma_2$, $c_{\max} \geq c_{\min} > 0$, $c_0 \in [c_{\min}, c_{\max}]$, $X^0 \in S_+$, $\Gamma_0 := (1+\mu)\|\Phi_0(X^0)\|$, $\beta_0 := \|\Phi_0(X^0)\|$, $\kappa := \sqrt{2n}$, $\mu_0 := ((\varepsilon/(2\Gamma_0\kappa))\beta_0^2)^2$, and set $k := 0$.

(S1) Find the solution $\Delta X^k \in S$ of the subproblem

$$\min_{\Delta X \in S^{n\times n}} \frac{1}{2} c_k \langle \Delta X, \Delta X\rangle + Q_{\mu_k}(\Delta X) \quad \text{s.t. } X^k + \Delta X \geq 0. \tag{10}$$

If $\mu_k = 0$ and $\Delta X^k = 0$, then *STOP*.

(S2) Compute the ratio

$$r_k := \frac{H_{\mu_k}(X^k) - H_{\mu_k}(X^k + \Delta X^k)}{H_{\mu_k}(X^k) - Q_{\mu_k}(\Delta X^k)}. \tag{11}$$

If $r_k \geq \rho_1$, then the $k$th iteration is called successful, and we set $X^{k+1} := X^k + \Delta X^k$; otherwise, the $k$th iteration is called unsuccessful, and we set $X^{k+1} := X^k$.

(S3) If

$$\left\|\Phi_0(X^{k+1})\right\| \leq \max\left\{\eta\beta_k, \varepsilon^{-1}\left\|\Phi_0(X^{k+1}) - \Phi_{\mu_k}(X^{k+1})\right\|\right\}, \tag{12}$$

then set

$$\beta_{k+1} := \left\|\Phi_0(X^{k+1})\right\| \tag{13}$$

and choose $\mu_{k+1}$ such that

$$0 < \mu_{k+1} \leq \min\left\{\left(\frac{\mu}{2\Gamma_0\kappa}\beta_{k+1}^2\right), \frac{\mu_k}{4}\right\}; \tag{14}$$

otherwise, let $\beta_{k+1} := \beta_k$ and $\mu_{k+1} := \mu_k$.

(S4) Update $c_k$ as follows.

If $r_k < \rho_1$, set $c_{k+1} := \sigma_2 c_k$.
If $r_k \in [\rho_1, \rho_2)$, set $c_{k+1} := \text{mid}(c_{\min}, c_k, c_{\max})$.
If $r_k \geq \rho_2$, set $c_{k+1} := \text{mid}(c_{\min}, \sigma_1 c_k, c_{\max})$.

(S5) Set $k \leftarrow k+1$, and go to (S1).

To verify that Algorithm 1 is well-defined, we need the following properties of the smoothed Fischer-Burmeister function (5).

**Lemma 2** (see [1]). *Let $(\mu, X) \in R \times S^{n\times n}$ and $\Phi_\mu(X)$ be defined by* (5). *Then*

(i) *if $\mu > 0$, $\Phi_\mu(X)$ is continuously differentiable at any $X \in S^{n\times n}$;*

(ii) *for any $\mu_1, \mu_2 > 0$ and $X \in S^{n\times n}$, it follows that*

$$\left\|\Phi_{\mu_1}(X) - \Phi_{\mu_2}(X)\right\| \leq \sqrt{2n}\left|\mu_1 - \mu_2\right|. \tag{15}$$

**Lemma 3.** *Let $X^k$ be a given iterate and let $\Delta X^k$ be the solution of the corresponding subproblem* (10). *Then*

$$H_{\mu_k}(X^k) - Q_{\mu_k}(\Delta X^k) \geq \frac{1}{2} c_k \left\langle \Delta X^k, \Delta X^k\right\rangle. \tag{16}$$

*Proof.* Since $X^k \geq 0$, the symmetric matrix $\Delta X := 0$ is feasible for the subproblem (10). But $\Delta X^k$ is a solution of this subproblem, so we obtain

$$\frac{1}{2} c_k \left\langle \Delta X^k, \Delta X^k\right\rangle + Q_{\mu_k}(\Delta X^k) \leq Q_{\mu_k}(0) = H_{\mu_k}(X^k). \tag{17}$$

This proves our statement. □

The above lemma ensures that the denominator in the ratio $r_k$ is always nonnegative. Note that this implies that the sequence $\{H_{\mu_k}(X^k)\}$ is monotonically nonincreasing. We next show that this denominator is equal to zero if and only if the termination criterion in step (S1) is satisfied. Hence, step (S2) is visited only if the denominator is positive, so that Algorithm 1 is well-defined.

**Lemma 4.** *Let $X^k$ be a given iterate and $\Delta X^k$ the solution of the corresponding subproblem. Then $H_{\mu_k}(X^k) - Q_{\mu_k}(\Delta X^k) = 0$ if and only if $\Delta X^k = 0$.*

*Proof.* First assume that $\Delta X^k = 0$. Then $H_{\mu_k}(X^k) - Q_{\mu_k}(\Delta X^k) = 0$ since the definition of $Q_{\mu_k}$ implies $Q_{\mu_k}(0) = H_{\mu_k}(X^k)$. Conversely, let $H_{\mu_k}(X^k) - Q_{\mu_k}(\Delta X^k) = 0$. Lemma 3 then implies $0 = (1/2)c_k\langle \Delta X^k, \Delta X^k\rangle = (1/2)\|\Delta X^k\|^2$ and hence $\Delta X^k = 0$. □

Next we have to justify our termination criterion in step (S1). To this end, we will show that this criterion is satisfied if and only if the current iterate $X^k$ is a stationary point of $H_{\mu_k}(X)$.

Before we arrive at this result, we first take a closer look at subproblem. Let $X^k$ be a given iterate and let $\Delta X^k$ be the unique solution of this subproblem. Since this subproblem is a convex program with a strictly feasible set, this problem is equivalent to its KKT conditions. In other words, $\Delta X^k$ is a solution of subproblem if and only if there exist Lagrange multipliers $U^k \in S^{n\times n}$ such that the following KKT conditions hold:

$$\begin{gathered} c_k \Delta X^k + DQ_{\mu_k}\left(\Delta X^k\right) - U^k = 0, \\ X^k + \Delta X^k \succeq 0, \quad U^k \succeq 0, \\ \left\langle U^k, X^k + \Delta X^k \right\rangle = 0. \end{gathered} \tag{18}$$

Now, if $\Delta X^k = 0$ is the unique solution of this subproblem, then the system yields

$$\begin{gathered} DQ_{\mu_k}(0) - U^k = 0, \\ X^k \succeq 0, \quad U^k \succeq 0, \\ \left\langle U^k, X^k \right\rangle = 0. \end{gathered} \tag{19}$$

However, these conditions are nothing, but the KKT conditions for the following problem are

$$\min_{X\in S_+} \left\| \Phi_{\mu_k}(X) \right\|^2. \tag{20}$$

Summarizing these observations, we obtain the following result.

**Theorem 5.** *Let $\mu_k = 0$. If $\Delta X^k = 0$ is the (unique) solution of the subproblem for some $c_k > 0$, then $X^k$ is a stationary point of the original problem. Conversely, if $X^k$ is a stationary point of the original problem, then $\Delta X^k = 0$ is the unique solution of subproblem for every $c_k > 0$.*

*Proof.* The statements follow immediately from the preceding arguments. □

## 3. Convergence Analysis

Throughout this section, we assume that Algorithm 1 generates an infinite sequence $\{X^k\}$. Our aim is to establish a global convergence result for Algorithm 1. More precisely, we will show any accumulation point of $\{X^k\}$ is a stationary point of the original problem.

**Lemma 6.** *Let $\{X^k\}$ be a sequence generated by Algorithm 1, and let $\mu_k$ and $\{X^k\}_{k\in K}$ be subsequences converging to 0 and some matrix $X^*$, respectively, in such a way that $\{c_k\|\Delta X^k\|\}_{k\in K} \to 0$. Then $X^*$ is a stationary point of the original problem.*

*Proof.* First note that $X^*$ is symmetric positive semidefinite and hence feasible for original problem. Furthermore, since $c_k \geq c_{\min} > 0$ for all $k \in N$, the assumption $\{c_k\|\Delta X^k\|\}_{k\in K} \to 0$ implies $\{\|\Delta X^k\|\}_{k\in K} \to 0$. By continuity, we also have $DH_{\mu_k}(X^k) \to DH_0(X^*)$ as $k \to \infty$, $k \in K$. This together with system (18) implies that

$$\begin{gathered} U^k = c_k \Delta X^k + DQ_{\mu_k}\left(\Delta X^k\right), \\ DQ_0(0) =: U^* \end{gathered} \tag{21}$$

as $k \to \infty$, $k \in K$. Therefore, taking the limit $k \to \infty$ on the subsequence $K$ in the KKT condition (18), we obtain

$$\begin{gathered} DQ_0(0) - U^* = 0, \\ X^* \succeq 0, \quad U^* \succeq 0, \\ \left\langle U^*, X^* \right\rangle = 0. \end{gathered} \tag{22}$$

Hence we conclude that $X^*$ is a stationary point of the original problem. □

Another main step toward our global convergence result is contained in the following technical lemma.

**Lemma 7.** *Let $\{X^k\}$ be a sequence generated by Algorithm 1 and $\{X^k\}_{k\in K}$ a subsequence converging to some matrix $X^*$. If $X^*$ is not a stationary point, then one has $\limsup_{k\to\infty, k\in K} c_k < \infty$.*

*Proof.* Let $\overline{K} := \{k-1 \mid k \in K\}$. Then we have $\{X^{k+1}\}_{k\in\overline{K}} \to X^*$. We will show that $\limsup_{k\to\infty, k\in\overline{K}} c_{k+1} < \infty$. Assume the contrary. Then, if necessary, we may suppose without loss of generality that

$$\lim_{k\to\infty, k\in\overline{K}} c_{k+1} = \infty. \tag{23}$$

The updating rule in step (S3) then implies that none of the iterations $k \in \overline{K}$ with $k$ sufficiently large is successful since otherwise we would have $c_{k+1} \leq c_{\max}$ for all these $k \in \overline{K}$. Hence, we have

$$r_k < \rho_1 \tag{24}$$

and $X^k = X^{k+1}$ for all $k \in \overline{K}$ large enough. Since $\{X^{k+1}\}_{k\in\overline{K}} \rightarrow X^*$, this implies $\{X^k\}_{k\in\overline{K}} \rightarrow X^*$, too. Further, noticing that $c_{k+1} = \sigma_2 c_k$ for all unsuccessful iterations, we also have

$$\lim_{k\rightarrow\infty, k\in\overline{K}} c_k = \infty \tag{25}$$

because of (23). We now want to show that

$$r_k \longrightarrow 1 \quad \text{as } k \longrightarrow \infty,\ k \in \overline{K}, \tag{26}$$

which would then lead to the desired contradiction to (24). To this end, we first note that

$$\liminf_{k\rightarrow\infty, k\in\overline{K}} c_k \left\|\Delta X^k\right\| > 0. \tag{27}$$

In fact, if $c_k\|\Delta X^k\| \rightarrow 0$ on a subsequence, we would deduce from Lemma 6 that $X^*$ is a stationary point in contradiction to our assumption. Hence there is a constant $\gamma > 0$ such that

$$c_k \left\|\Delta X^k\right\| \geq \gamma, \quad k \in \overline{K}. \tag{28}$$

By Lemma 2, this implies

$$H_{\mu_k}\left(X^k\right) - Q_{\mu_k}\left(\Delta X^k\right) \geq \frac{1}{2} c_k \left\langle \Delta X^k, \Delta X^k \right\rangle \geq \frac{1}{2}\gamma \left\|\Delta X^k\right\| \tag{29}$$

for all $k \in \overline{K}$ sufficiently large.

We further note that $\{\|\Delta X^k\|\}_{k\in\overline{K}} \rightarrow 0$. Otherwise, it would follow from (25) that $c_k\|X^k\|^2 \rightarrow \infty$ on a suitable subsequence. This, in turn, would imply that the optimal value of the subproblem tends to infinity. However, this cannot be true since the feasible matrix $\Delta X^k := 0$ would give a smaller objective value. Hence we have $\{\|\Delta X^k\|\}_{k\in\overline{K}} \rightarrow 0$.

Taking this into account and using $\{X^k\}_{k\in\overline{K}} \rightarrow X^*$ and the fact that $F$ is continuously differentiable, we obtain through standard calculus arguments

$$\left|H_{\mu_k}\left(X^k\right) - Q_{\mu_k}\left(\Delta X^k\right)\right| = o\left(\left\|\Delta X^k\right\|\right) \quad \text{as } k \longrightarrow \infty,\ k \in \overline{K}. \tag{30}$$

Summarizing these observations, we get

$$\begin{aligned} \left|r_k - 1\right| &= \left|\frac{H_{\mu_k}(0) - H_{\mu_k}\left(X^k + \Delta X^k\right)}{H_{\mu_k}(0) - Q_{\mu_k}\left(\Delta X^k\right)} - 1\right| \\ &= \left|\frac{Q_{\mu_k}\left(\Delta X^k\right) - H_{\mu_k}\left(X^k + \Delta X^k\right)}{H_{\mu_k}(0) - Q_{\mu_k}\left(\Delta X^k\right)}\right| \\ &\leq \frac{o\left(\left\|\Delta X^k\right\|\right)}{(1/2)\gamma\left\|\Delta X^k\right\|} \longrightarrow 0 \end{aligned} \tag{31}$$

as $k \rightarrow \infty$, $k \in \overline{K}$. This contradiction to (24) completes the proof. □

As a direct consequence of this lemma, we obtain the following result.

**Lemma 8.** *Let $\{X^k\}$ be sequence generated by Algorithm 1. Then there are infinitely many successful iterations.*

*Proof.* If not, there would exist an index $k_0 \in N$ with $r_k < \rho_1$ and $X^k = X^{k_0}$ for all $k \geq k_0$. This implies $c_k \rightarrow \infty$ due to the updating rule in (S3). However, since $X^{k_0}$ is not a stationary point and $\{X^k\} \rightarrow X^{k_0}$, we get a contradiction to Lemma 6. □

We are now in the position to prove the main convergence result for Algorithm 1.

**Theorem 9.** *Let $\{X^k\}$ be a sequence generated by Algorithm 1. Then, any accumulation point of this sequence is a stationary point of the original problem.*

*Proof.* Let $X^*$ be an accumulation point and $\{X^k\}_{k\in K}$ subsequence converging to $X^*$. Since $X^k = X^{k+1}$ for all unsuccessful iterations $k$ and since there are infinitely many successful iterations by Lemma 7, we may assume without loss of generality that all iterations $k \in K$ are successful.

Assume that $X^*$ is not a solution. Lemma 6 then implies

$$\limsup_{k\rightarrow\infty, k\in K} c_k < \infty. \tag{32}$$

Hence there is a constant $\gamma > 0$ such that

$$c_k \geq \gamma, \quad k \in K. \tag{33}$$

Since each iteration $k \in K$ is successful, we also have $r_l \geq \rho_1$. Consequently, we obtain from Lemma 2

$$\begin{aligned} H_{\mu_k}\left(X^k\right) - H_{\mu_k}\left(X^k + \Delta X^k\right) &\geq \rho_1\left(H_{\mu_k}\left(X^k\right) - Q_{\mu_k}\left(\Delta X^k\right)\right) \\ &\geq \frac{1}{2}\rho_1 c_k \left\langle \Delta X^k, \Delta X^k\right\rangle \\ &\geq \frac{1}{2}\rho_1 c_{\min}\left\|\Delta X^k\right\|^2 \end{aligned} \tag{34}$$

for all $k \in K$. Since $\{H_{\mu_k}(X^k)\}$ is monotonically nonincreasing and bounded from below by, for example, $H_0(X^*)$, we have $H_{\mu_k}(X^k) - H_{m_k}(X^{k+1}) \rightarrow 0$ as $k \rightarrow \infty$. Therefore, we obtain $\{\Delta X^k\}_{k\in K} \rightarrow 0$ from (34). By (33), this also implies $\{c_k\|X^k\|\}_{k\in K} \rightarrow 0$. But then Lemma 6 shows that $X^*$ is a solution in contradiction to our assumption. This completes the proof. □

In the following, we will give one stronger global convergence result. Define the index set

$$\begin{aligned} \aleph := \ & \{0\} \cup \left\{k \mid \left\|\Phi\left(X^{k+1}\right)\right\| \right. \\ & \left. \leq \max\left\{\eta\beta_k, \mu^{-1}\left\|\Phi\left(X^{k+1}\right) - \Phi_{\varepsilon_k}\left(X^{k+1}\right)\right\|\right\}\right\} \\ = \ & \left\{k_0 = 0 < k_1 < k_2 < \cdots\right\}. \end{aligned} \tag{35}$$

**Lemma 10.** *If $F$ is a $P_0$ function, then the sequence $X^k$ generated by Algorithm 1 remains in the level set*

$$L_0 := \left\{X \in S \mid H(X) \le (1+\varepsilon)^2 H\left(X^0\right)\right\}. \quad (36)$$

*Proof.* Let $k$ be an arbitrary nonnegative integer, and let $k_j$ be the largest number in $\aleph$ such that $k_j \le k$, as $\aleph$ is as defined from (35). It is easy to deduce from step 3 of Algorithm 1 that

$$\begin{gathered} \mu_k = \mu_{k_j}, \quad \beta_k = \beta_{k_j}, \quad \text{as } k_j \le k < k_{j+1}, \\ \left\|\Phi_{\mu_k}\left(X^k\right)\right\| \le \left\|\Phi_{\mu_{k_j}}\left(X^k\right)\right\|, \quad \text{as } k_j \le k < k_{j+1}. \end{gathered} \quad (37)$$

Set

$$U_j := \left\{X \in S \mid \left\|\Phi_{\mu_k}\left(X^k\right)\right\| \le \left\|\Phi_{\mu_{k_j}}\left(X^k\right)\right\|\right\}. \quad (38)$$

As $k$ is an arbitrary integer and $x_k \in U_j$, it follows that $U_j \subseteq L_0$.

Next, by induction, we will prove

$$U_j \subseteq L_0, \quad \forall j \ge 0. \quad (39)$$

In view of Lemma 2, we deduce that $\forall X \in U_j$,

$$\begin{aligned} \|\Phi(X)\| &\le \left\|\Phi_{\mu_{k_j}}\left(X^k\right)\right\| + \kappa\sqrt{\mu_{k_j}} \\ &\le \left\|\Phi_{\mu_{k_j}}\left(X^{k_j}\right)\right\| + \kappa\sqrt{\mu_{k_j}} \\ &\le \left\|\Phi\left(X^{k_j}\right)\right\| + 2\kappa\sqrt{\mu_{k_j}} \\ &\le \beta_{k_j} + \kappa\sqrt{\mu_{k_j}}. \end{aligned} \quad (40)$$

If $j = 0$, then by (40) we have

$$\begin{aligned} \|\Phi(X)\| &\le \left\|\Phi\left(X^0\right)\right\| + 2\kappa\sqrt{\mu_0} \\ &\le (1+\varepsilon)\left\|\Phi\left(X^0\right)\right\|, \quad \forall X \in U_0. \end{aligned} \quad (41)$$

This proves $U_0 \in L_0$.

Suppose $U_{j-1} \subseteq L_0$ for some $j > 0$. Then, $x_{k_{j-1}} \in L_0$ and hence $\beta_{k_j-1} \le \Gamma_0$. Set

$$\begin{aligned} \aleph_1 &:= \left\{k \in \aleph \mid \eta\beta_{k-1} \ge \mu^{-1}\left\|\Phi\left(X^k\right) - \Phi_{\mu_{k-1}}\left(X^k\right)\right\|\right\}, \\ \aleph_2 &:= \left\{k \in \aleph \mid \eta\beta_{k-1} < \mu^{-1}\left\|\Phi\left(X^k\right) - \Phi_{\mu_{k-1}}\left(X^k\right)\right\|\right\}. \end{aligned} \quad (42)$$

It follows from step 3 of Algorithm 1 and Lemma 2 that

$$\beta_{k_j} \le \eta\beta_{k_j-1} = \eta\beta_{k_{j-1}}, \quad \text{if } k_j \in \aleph_1, \quad (43)$$

or

$$\begin{aligned} \beta_{k_j} \le \frac{\kappa}{\mu}\sqrt{\mu_{k_j-1}} = \frac{\kappa}{\mu}\sqrt{\mu_{k_{j-1}}} \le \frac{1}{2C_0}\beta^2_{k_j-1} \le \frac{1}{2}\beta^2_{k_{j-1}}, \\ \text{if } k_j \in \aleph_2. \end{aligned} \quad (44)$$

This implies that

$$\beta_{k_j} \le \delta_3\beta_{k_{j-1}}, \quad \forall k_j \in \aleph, \quad (45)$$

where $\delta_3 = \max\{1/2, \eta\}$. Moreover, we have

$$\mu_{k_j} \le \frac{1}{4}\mu_{k_j-1} = \frac{1}{4}\mu_{k_{j-1}}, \quad \forall k_j \in \aleph. \quad (46)$$

From (45) and (46), we deduce that, for $j > 0$,

$$\begin{aligned} &\beta_{k_j} \le \delta_3^j\beta_0 = \delta_3^j\left\|\Phi\left(X^0\right)\right\|, \\ &\mu_{k_j} \le \frac{1}{4^j}\mu_0 \le \frac{\varepsilon^2}{4^{j+1}(\Gamma_0\kappa)^2}\left\|\Phi\left(X^0\right)\right\|^4 \le \frac{\varepsilon^2}{4^{j+1}\kappa^2}\left\|\Phi\left(X^0\right)\right\|^2. \end{aligned} \quad (47)$$

Combining (47) with (40), we have

$$\begin{aligned} \|\Phi(X)\| &\le \delta_3^j\left\|\Phi\left(X^0\right)\right\| + \frac{\varepsilon}{2^j}\left\|\Phi\left(X^0\right)\right\| \\ &\le \delta_3^j(1+\varepsilon)\left\|\Phi\left(X^0\right)\right\|, \quad \forall X \in U_j, \end{aligned} \quad (48)$$

which implies $U_j \subseteq L_0$. Hence (39) is proved and Lemma 10 is valid. □

It follows from Fischer [13] that if $F$ is a $P_0$ function or, more generally, an $R_0$-function, then the level set $L_0$ as defined in Lemma 10 is compact. Lemma 3 shows that the sequence $\{H_{\mu_k}\}$ is monotonically decreasing and converges.

**Theorem 11.** *Assume that $F$ is a $P_0$ function. Let $\{X^k\}$ be a sequence generated by Algorithm 1. If there exists at least an accumulation point in the sequence $\{X^k\}$, then the index set $\aleph$ defined by (35) is infinite:*

$$\lim_{k\to\infty}\mu_k = 0, \qquad \lim_{k\to\infty}\Phi\left(X^k\right) = 0, \qquad \lim_{k\to\infty}\Phi_{\mu_k}\left(X^k\right) = 0. \quad (49)$$

*Proof.* We first prove that set $\aleph$ is infinite. By contradiction, assume that $\aleph$ is finite. Let $\bar{k}$ be the largest number in $\aleph$. Then for all $k \ge \bar{k}$, $\mu_k = \mu_{\bar{k}}$ and $\beta_k = \beta_{\bar{k}}$. Denote

$$\bar{\mu} := \mu_{\bar{k}}, \qquad \bar{\beta} := \beta_{\bar{k}}, \qquad \phi(X) := \Phi(X) - \Phi_{\bar{\mu}}(X). \quad (50)$$

Then $\forall k > \bar{k}$,

$$\left\|\Phi\left(X^k\right)\right\| > \max\left\{\eta\bar{\beta}, \mu^{-1}\|\phi(X)\|\right\}, \quad (51)$$

$$\Phi\left(X^k\right) = \phi\left(X^k\right) + \Phi_{\bar{\varepsilon}}\left(X^k\right). \quad (52)$$

From Theorem 9, it follows that there exists at least an accumulation point $\bar{X} \in L_0$ of $\{X^k\}$ such that

$$\nabla H_{\bar{\mu}}\left(\bar{X}\right) = 0. \quad (53)$$

Next, assume that subsequence $\{X_k\}_{k\in\widehat{\aleph}}$ converges to $\bar{X}$. In view of (53), we have $\{\Phi_{\bar{\mu}}(X^k)\}_{k\in\widehat{\aleph}} \to \Phi_{\bar{\mu}}(\bar{X}) = 0$ and hence there exists $\hat{k} \ge \bar{k}$ such that, for all $k \in \widehat{\aleph}$ with $k \ge \hat{k}$,

$$\left\|\Phi_{\bar{\mu}}\left(X^k\right)\right\| \le (1-\mu)\eta\bar{\beta}. \quad (54)$$

This together with (51) and (52) shows that, for all $k \in \widehat{\aleph}$ with $k \geq \widehat{k}$,

$$\begin{aligned}\left\|\Phi_{\overline{\mu}}\left(X^k\right)\right\| &\leq (1-\varepsilon)\left\|\Phi\left(X^k\right)\right\| \\ &\leq (1-\varepsilon)\left(\left\|\Phi_{\overline{\mu}}\left(X^k\right)\right\| + \left\|\phi\left(X^k\right)\right\|\right);\end{aligned} \quad (55)$$

that is,

$$\left\|\Phi_{\overline{\varepsilon}}\left(X^k\right)\right\| < \left(\varepsilon^{-1}-1\right)\left\|\phi\left(X^k\right)\right\|, \quad (56)$$

which means

$$\begin{aligned}\left\|\Phi\left(X^k\right)\right\| &\leq \left\|\Phi_{\overline{\varepsilon}}\left(X^k\right)\right\| + \left\|\phi\left(X^k\right)\right\| \\ &< \varepsilon^{-1}\left\|\phi\left(X^k\right)\right\|, \quad \text{as } k \in \widehat{\aleph},\ k \geq \widehat{k}.\end{aligned} \quad (57)$$

This contradicts (51). Hence the set $\aleph$ is infinite.

Next, $\{\mu_k\} \to 0$ follows immediately from the updating rule of $\mu_k$ and the fact that the set $\aleph$ is infinite. Moreover, by the proof of Lemma 10, we deduce

$$\left\|\Phi\left(X^k\right)\right\| \leq \delta_4^j (1+\varepsilon)\left\|\Phi\left(X^0\right)\right\|, \quad \text{as } k_j \leq k < k_{j+1}. \quad (58)$$

Because the set $\aleph$ is infinite, it follows from Lemma 10 and (58) that

$$\lim_{k\to\infty}\Phi\left(X^k\right) = 0, \qquad \lim_{k\to\infty}\Phi_{\mu_k}\left(X^k\right) = 0. \quad (59)$$

This completes the proof. □

As a consequence of the above theorem, we get the following global convergence result.

**Corollary 12.** *Assume that $F$ is a $P_0$ function. Let $\{X^k\}$ be a sequence generated by Algorithm 1. Then every accumulation point of the sequence $\{X^k\}$ is a solution of NCP(F).*

## 4. Numerical Experiments

*4.1. The Reformulation for Subproblem.* To test the numerical performance of Algorithm 1, we implemented the method in MATLAB (Version 7.0) using the SDPT3-Solver (Version 3.0) for the corresponding subproblems. First, we will give the reformulation of the subproblem. In order to solve nonlinear semidefinite programs of the form (1) by Algorithm 1, we have to be able to deal with a subproblem given by

$$\min_{\Delta X \in S^{n\times n}} \frac{1}{2}c_k \langle \Delta X, \Delta X\rangle + Q_{\mu_k}(\Delta X) \quad \text{s.t. } X^k + \Delta X \geq 0. \quad (60)$$

For this purpose, we would like to use the SDPT3-Solver (version 3.0) from [14]. This software is designed to solve linear semidefinite programs with cone constraints of the form

$$\begin{aligned}\min \quad & \sum_{j=1}^{n_s}\left\langle C_j^s, X_j^s\right\rangle + \sum_{i=1}^{n_q}(c_i^q)^T x_i^q + \left(c^l\right)x^l \\ \text{s.t.} \quad & \sum_{j=1}^{n_s}\left(A_j^s\right)^T \operatorname{svec}\left(X_j^s\right) + \sum_{i=1}^{n_q}(A_i^q)^T x_i^q + \left(A^l\right)^T x^l = b, \\ & X_j^s \in S_+^{s_j\times s_j}, \quad \forall j, x_i^q \in K_q^{q_i} \quad \forall i, x^l \in R_+^{n_l},\end{aligned} \quad (61)$$

where $X_j^s$, $X_j^s$ are symmetric matrices of dimension $s_j$; $c_i^q$, $x_i^q$ are vectors in $R^{q_i}$; $S_+^{s_j\times s_j}$ denotes the $s_j$-dimension positive semidefinite cone defined by $S_+^{s_j\times s_j} := \{X \in S^{s_j\times s_j} : X \geq 0\}$; $D_q^{q_i}$ denotes the $q_i$-dimensional second-order cone defined by $K_q^{q_i} := \{x = (x_1, x_{2:q_i}^T)^T \in R^{q_i} : x_1 \geq \|x_{2:q_i}\|\}$; $c^l$ and $x^l$ are vectors in $R^{n_l}$; $A_j^s$ are $\overline{s}_j \times m$ matrices with $\overline{s}_j = s_j(s_j+1)/2$; $A_i^q$ and $A^l$ are $q_i \times m$ and $l \times m$ matrices, respectively; and svec is the operator defined by $\operatorname{svec}(X) := (X(1,1), \sqrt{2}X(1,2), X(2,2), \sqrt{2}X(1,3), \sqrt{2}X(2,3), X(3,3), \ldots)^T \in R^{n(n+1)/2}$ for any symmetric matrix $X \in S^{n\times n}$.

We now want to rewrite the problem (60) in the form of (61). To this end, we need to make some reformulations, which will be described step by step in the following.

First, we drop the constant from the objective function without affecting the problem. Next, we introduce the auxiliary variable $S \in S^{n\times n}$ and set $X^k + \Delta X = S$. Because $\Delta X$ needs only to be symmetric and not to be positive semidefinite, we set $\Delta x = \operatorname{svec}(\Delta X)$ and write the problem in terms of $\Delta x \in R^{\overline{n}}$ with $\overline{n} := n(n+1)/2$. Then problem (60) is equivalent to

$$\begin{aligned}\min \quad & \left(\frac{1}{2}c_k + \left\|\operatorname{svec}\left(D\Phi_{\mu_k}\left(X^k\right)\right)\right\|^2\right)\|\Delta x\|^2 \\ & + 2\operatorname{svec}\left(\Phi_{\mu_k}\left(X^k\right)D\Phi_{\mu_k}\left(X^k\right)\right)^T\Delta x \\ \text{s.t.} \quad & \operatorname{svec}\left(X^k\right) + \Delta x = \operatorname{svec}(S), \\ & \Delta x \in R^{\overline{n}}, \quad s \geq 0.\end{aligned} \quad (62)$$

By introducing the second-order cone constraint $\|\Delta x\|t$, the above problem can be further rewritten as

$$\begin{aligned}\min \quad & \left(\frac{1}{2}c_k + \left\|\operatorname{svec}\left(D\Phi_{mu_k}\left(X^k\right)\right)\right\|^2\right)t^2 \\ & + 2\operatorname{svec}\left(\Phi_{\mu_k}\left(X^k\right)D\Phi_{\mu_k}\left(X^k\right)\right)^T\Delta x \\ \text{s.t.} \quad & \operatorname{svec}\left(X^k\right) + \Delta x = \operatorname{svec}(S), \\ & \|\Delta x\| \leq t, \quad \Delta x \in R^{\overline{n}}, \quad S \geq 0, \quad t \in R.\end{aligned} \quad (63)$$

Unfortunately, the term $t^2$ is not linear as required in (61). So we replace $t^2$ by the new variable $s \geq 0$ and add the constraint $t^2 \leq s$. But this constraint can be rewritten as the semidefinite constraint

$$\begin{pmatrix} s & t \\ t & 1 \end{pmatrix} \succeq 0. \quad (64)$$

Introducing once again an auxiliary variable, problem (62) and hence the original subproblem (60) are equivalent to

$$\begin{aligned} \min \quad & \left(\frac{1}{2}c_k + \left\|\operatorname{svec}\left(D\Phi_{\mu_k}\left(X^k\right)\right)\right\|^2\right)s \\ & + 2\operatorname{svec}\left(\Phi_{\mu_k}\left(X^k\right)D\Phi_{\mu_k}\left(X^k\right)\right)^T \Delta x \\ \text{s.t.} \quad & -\operatorname{svec}\left(X^k\right) = \Delta x - \operatorname{svec}(S), \quad \|\Delta x\| \leq t, \\ & \begin{pmatrix} s & t \\ t & 1 \end{pmatrix} - W = 0, \\ & \Delta x \in R^{\bar{n}}, \quad W \succeq 0, \quad S \succeq 0, \quad t \in R, \quad s \in R_+. \end{aligned} \quad (65)$$

We write the equality constraint

$$\begin{pmatrix} s & t \\ t & 1 \end{pmatrix} - W = 0, \quad (66)$$

in the svec-notation. Then we get

$$\begin{aligned} \min \quad & \left(\frac{1}{2}c_k + \left\|\operatorname{svec}\left(D\Phi_{\mu_k}\left(X^k\right)\right)\right\|^2\right)t \\ & + 2\operatorname{svec}\left(\Phi_{\mu_k}\left(X^k\right)D\Phi_{\mu_k}\left(X^k\right)\right)^T \Delta x \\ \text{s.t.} \quad & -\operatorname{svec}\left(X^k\right) = \Delta x - \operatorname{svec}(S), \quad \|\Delta x\| \leq t, \\ & \begin{pmatrix} s \\ \sqrt{t} \\ 1 \end{pmatrix} - \operatorname{svec}(W) = 0, \\ & \Delta x \in R^{\bar{n}}, \quad W \succeq 0, \quad S \succeq 0, \quad t \in R, \quad s \in R_+. \end{aligned} \quad (67)$$

We are now in a position to give the explicit correspondence between the parameters, variables, and input data in our last problem formulation (67) and those in the SDPT3 standard form. The problem parameters are given by

$$\begin{aligned} & n_s := 2, \qquad n_q := 1, \qquad s_1 := n, \\ & s_2 := 2, \qquad q_1 := 1 + \bar{n}, \qquad l := 1 + 2m. \end{aligned} \quad (68)$$

The variables are given by

$$\begin{aligned} & X_1^s := S \in S_+^{n\times n}, \qquad X_2^s := W \in S^{2\times 2}, \\ & x_1^q := \left(t, \Delta x^T\right)^T \in K_q^{1+\bar{n}}, \\ & x^l := \left(s, \xi^T, \omega^T\right)^T \in R^{1+2m}. \end{aligned} \quad (69)$$

The input datum in the objective function is given by

$$\begin{aligned} & C_1^s := 0 \in S^{n\times n}, \qquad C_2^s := 0 \in S^{2\times 2}, \\ & c_1^q := \left(0, \operatorname{svec}\left(Df\left(X^k\right)\right)^T\right)^T \in R^{1+\bar{n}}, \\ & c^l := \left(\frac{1}{2}c_k, \alpha_k e, 0\right) \in R^{1+2m} \end{aligned} \quad (70)$$

with

$$e = (1, \ldots, 1)^T \in R^m. \quad (71)$$

Finally, the matrices $A_1^s \in R^{\bar{n}\times(\bar{n}+3+m)}$, $A_2^s \in R^{3\times(\bar{n}+3+m)}$, $A_1^q \in R^{(1+\bar{n})\times(\bar{n}+3+m)}$, and $A^l \in R^{(1+m+m)\times(\bar{n}+3+m)}$ and the vector $b \in R^{\bar{n}+3+m}$ are given by

$$\begin{aligned} & A_1^s = \left(-I \;\; |0|\, 0\right), \qquad A_2^s = \left(0 \;\; |-I|\, 0\right), \\ & A_1^q = \left(\begin{array}{c|ccc|ccc} 0 & 0 & \sqrt{2} & 0 & 0 & \cdots & 0 \\ & 0 & 0 & 0 & & & \\ I & \vdots & \vdots & \vdots & -\operatorname{svec}(W) & \cdots & -\operatorname{svec}(w) \\ & 0 & 0 & 0 & & & \end{array}\right), \\ & A^l = \left(\begin{array}{c|ccc|ccc} 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & & 0 & & & I & \\ 0 & & 0 & & & -I & \end{array}\right), \\ & b = \left(-\operatorname{svec}\left(X^k\right)^T |0 \;\; 0 \;\; -1|\, g\left(X^k\right)^T\right)^T. \end{aligned} \quad (72)$$

This is the desired reformulation.

It may be worth mentioning that problem (60) can also be transformed as

$$\begin{aligned} \min \quad & \left(\frac{1}{2}c_k + \left\|\operatorname{svec}\left(D\Phi_{\mu_k}\left(X^k\right)\right)\right\|^2\right)t \\ & + 2\operatorname{svec}\left(\Phi_{\mu_k}\left(X^k\right)D\Phi_{\mu_k}\left(X^k\right)\right)^T \Delta x \\ \text{s.t.} \quad & -\operatorname{svec}\left(X^k\right) = \Delta x - \operatorname{svec}(S), \quad \|\Delta x\| \leq t, \\ & \begin{pmatrix} s & t \\ t & 1 \end{pmatrix} - W = 0, \\ & \Delta x \in R^{\bar{n}}, \quad W \succeq 0, \quad S \succeq 0, \quad t \in R, \quad s \in R_+. \end{aligned} \quad (73)$$

Since the constraint $\|\Delta x\|^2 \leq t$ is equivalent to

$$\begin{pmatrix} t & \Delta x^T \\ \Delta x & I \end{pmatrix}, \quad (74)$$

problem (73) can further be reformulated as a linear semidefinite program that involves a semidefinite cone constraint instead of a second-order cone constraint. However, such a semidefinite representation is much more expensive in terms of memory requirement. Therefore, we adopted the reformulation (73) in our numerical experiments.

Table 1: Numerical results.

| $n$ | $m$ | Algorithm 1 | | Algorithm 13 | |
|---|---|---|---|---|---|
| | | Average iteration | Average CPU | Average iteration | Average CPU |
| 5 | 2 | 5.4 | 0.03 | 15 | 0.04 |
| 10 | 5 | 13.54 | 2.22 | 20.57 | 3.88 |
| 10 | 10 | 12.88 | 2.25 | 19.27 | 4.00 |
| 15 | 5 | 15.97 | 1.02 | 55.92 | 10.23 |
| 15 | 10 | 20.74 | 1.05 | 34.27 | 15.17 |
| 20 | 5 | 30.11 | 10.22 | 100.68 | 25.55 |
| 20 | 10 | 37.35 | 11.05 | 157.39 | 30.11 |
| 25 | 5 | 41.99 | 24.54 | 217.22 | 44.67 |
| 25 | 10 | 47.02 | 30.22 | 348.04 | 61.22 |
| 25 | 15 | 45.11 | 27.66 | 477.93 | 110.84 |

*4.2. Numerical Results.* We present some numerical tests using Algorithm 1. All the codes are written in MATLAB 7.10. The tests are conducted on a DELL computer with Intel(R)Core(TM)i5-2400 processor (3.10 GHz) and 4.00 GB of memory on Windows 7.

Consider the following nonlinear semidefinite complementarity problem with $F(X) := XX-2X+I$. It is obvious that the solution set of SDCP$(F)$ is nonempty, since $X = I$ is its one solution. The parameters in the algorithm can be presented as follows: $\rho_1$ and $\rho_2$ can be randomly generated from $[0.1, 0.5]$ and $[0.6, 1]$, respectively; $\sigma_1$ and $\sigma_2$ are randomly generated from $[0.5, 1]$ and $[1.5, 2]$, respectively; $c_{\min}$ and $c_{\max}$ are randomly generated from $[0.01, 1]$ and $[500, 1000]$, respectively; $\mu$ is randomly chosen from $[10, 20]$; $c_0$ is randomly generated from $[c_{\min}, c_{\max}]$ and $X^0 := A^T A$, where $A \in R^{m\times n}$ with every entry being randomly generated from $[0, 1]$. The stopping criterion is set as $\|X^k\| \le 10^{-6}$ and $\mu_k \le 10^{-6}$.

For the purpose of comparison, we also solve this problem by the following descent algorithm based on the method proposed in [15].

*Algorithm 13* (decent direction method).

(S0) (Initialization) Choose $0 < \rho < 1$, $0 < \alpha < 1$, $X^0 \in S_+$, $\Gamma_0 := (1 + \mu)\|\Phi_0(X^0)\|$, $\beta_0 := \|\Phi_0(X^0)\|$, $\kappa := \sqrt{2n}$, $\mu_0 := ((\varepsilon/(2\Gamma_0\kappa))\beta_0^2)^2$, and set $k := 0$.

(S1) Find the solution $\Delta X^k \in S$ of the subproblem

$$\min_{\Delta X\in S^{n\times n}} \frac{1}{2}\langle \Delta X, \Delta X\rangle + Q_{\mu_k}(\Delta X) \quad \text{s.t. } X^k + \Delta X \ge 0. \tag{75}$$

If $\Delta X^k = 0$, then *STOP*.

(S2) Compute $\alpha_k = \max\{1, \alpha, \alpha^2, \ldots\}$ such that

$$H_{\mu_k}(X^k) \ge H_{\mu_k}(X^k + \alpha_k \Delta X^k) + \rho\alpha_k Q_{\mu_k}(\Delta X^k). \tag{76}$$

(S3) Let $x^{k+1} := X^k + \alpha_k \Delta X^k$. If

$$\left\|\Phi_0(X^{k+1})\right\| \le \max\left\{\eta\beta_k, \varepsilon^{-1}\left\|\Phi_0(X^{k+1}) - \Phi_{\mu_k}(X^{k+1})\right\|\right\}, \tag{77}$$

then set

$$\beta_{k+1} := \left\|\Phi_0(X^{k+1})\right\| \tag{78}$$

and choose $\mu_{k+1}$ such that

$$0 < \mu_{k+1} \le \min\left\{\left(\frac{\mu}{2\Gamma_0\kappa}\beta_{k+1}^2\right), \frac{\mu_k}{4}\right\}; \tag{79}$$

otherwise, let $\beta_{k+1} := \beta_k$ and $\mu_{k+1} := \mu_k$.

(S4) Set $k \leftarrow k + 1$, and go to (S1).

The above descent algorithm uses different NCP function from the one used in [15]. The parameters in this algorithm are set as follows: $\rho$ can be randomly generated from $[0.1, 0.5]$; $\alpha := 0.5$; the starting point $X^0$ and the stopping criteria are the same as Algorithm 1.

We now solve this problem 40 times by Algorithms 1 and 13, respectively, with the initial point $X^0$ being randomly generated as above. Table 1 lists the numerical results for the applications of Algorithms 1 and 13. The average number of iterations and the average computational time (CPU time) are reported in Table 1. The results generally show that our method is efficient in solving this problem.

## 5. Conclusion

In this paper, we propose a trust-region method to solve nonlinear semidefinite complementarity problem. The well-posedness of the new method is proved and the global convergence is also presented. The numerical comparisons with the descent algorithm show the efficiency of the proposed method. For further study, we will discuss the convergent rate of the algorithm.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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