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ANALYSIS OF A VIBRATING-BEAM-BASED MICROMIXER

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ABSTRACT

The mixing of two or more streams in microscale devices is a slowly molecular diffusion process due to the unique laminar flows, and some ‘turbulence’ based mixing technologies which are effective in macroscales become hard to implement in such small dimensions. The chaotic advection based mixing, depending on the stretching and folding of interface, has been proved to be effective for low Reynolds numbers (Re) and is a very promising technology for micro mixing. We propose a new mixing concept based on a vibrating micro-beam in microfluidic channels to generate chaotic advection to achieve an efficient mixing. The simplicity of the proposed mixer design makes microfabrication process easy for practical applications.

The feasibility of the concept is evaluated computationally and moving mesh technique (ALE) is utilized to trace the beam movement. The simulation shows that the mixing quality is determined by parameters such as flow velocities, amplitudes and frequencies of vibrating beam. The Reynolds number (*Re*) is less than 2.0, Pelect number (*Pe*) ranges from 5 to 1000, and Strohal number (*St*) 0.3 to 3.0. It was found that vortex type of flows were generated in microchannel due to the interaction between beam and channel wall. The mixing efficiency with this design is well improved comparing with the flows without beam vibration.

INTRODUCTION

Micromixer is one of the important components in microfluidic systems for applications like biochemistry analysis, drug delivery and sequencing or synthesis of nucleic acids where the mixing of reagents like small molecules, large macromolecules and particles become critical. But the unique features in microscale flows such as low Reynolds number, less turbulence, limited available space make the a rapid and uniform mixing of relevant reagents to become a difficult task. The microscale mixing is mainly achieved by molecular diffusion, resulting in an extremely slow process and long micro channel for complete mixing [1, 2].

Generally, micromixers can be categorized as passive micromixers and active micromixers[3]. Passive micromixers do not require external energy and the mixing is typically accomplished by driving fluids through channels with delicate, fixed geometries to produce the chaotic advection [4]. Active micromixers use the disturbance generated by an external field for the mixing process. Different external disturbances are used for the operation of active micromixers, like vibration [5], pressure field disturbance [6], electro kinetic disturbance [7], ultrasonic disturbance [8] and magnetic hydrodynamic disturbance [9]. The faster and on-demand mixing within a short distance makes active mixers attractive to many applications. Commercial CFD software CFD-ACE [10], FLUENT [11, 12] have been used to predict the performance of micromixers. The remarkable qualitative agreements between simulation and experiment have been found. Moreover, the simulations also provide insight into the complexity of microscale fluid mechanics.

A new concept using a vibrating micro-beam to disturb the interface in a micro-channel is proposed to enhance mixing. The presumption is that the vortex and rolling-up of interface can be formed in the microchannel flow to promote mixing by increasing the contact area between streams. The performance of micromixer with different parameters, as vibrating velocity, frequency, fluid velocities, diffusivity and channel height, were analyzed from the simulations with COMSOL [13].

The principle of vibrating-beam-based micromixer provides the following performance characteristics and advantages: a) rapid mixing in a micro-scale channel can be achieved with the disturbance of the vibrating beam; b) vibrating can be easily activated and controlled by external sources such as electrostatic, magnetic and PZT etc; c) the mixer can be used for a wide variety of fluids with different electrochemical characteristics; the proposed method can achieve an efficient local mixing region which can also be used for analytic detection in biological immunoassays through generating swirling patterns in the fluid and thereby enhance the transport of the reagents to the reaction surface [14].

DESIGN AND METHODOLOGY

The basic geometries of micromixer include a straight micro-channel (400μmx50μm) and a micro-beam (50μmx2μm) located at the middle as shown in Figure 1. Two fluid streams with different concentrations (c0=1 and c0=0) flow into the upper and lower half of channel.

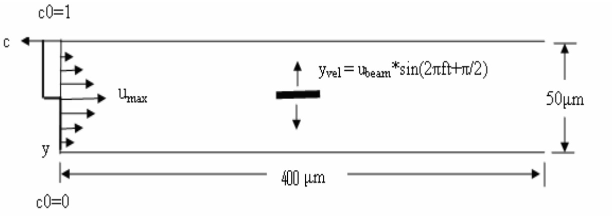


Fig. 1: Schematic geometry and operating parameters

The micro-beam vibrates periodically in the vertical direction (y-direction) and no moving exists in horizontal direction (x-direction). The beam vibration velocity in y-direction follows a sinusoidal equation:

$$u_y = u_{beam}\sin(2\pi ft + \pi/2) \tag{1}$$

Where u_{beam} is the beam vibrating velocity amplitude (m/s) and f is vibrating frequency in Hz. Figure 2 represents a typical vertical vibrating velocity with time progress.

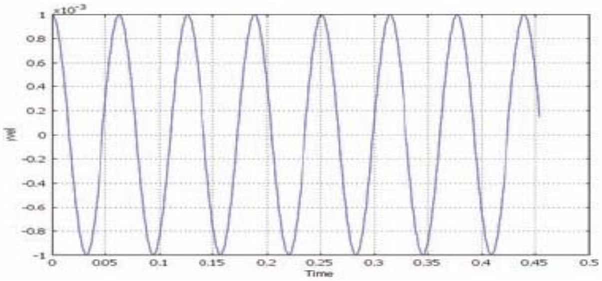


Fig. 2: Vertical velocity, u_y of the micro beam

It is assumed that the flow is fully developed prior to entering the channel, which has a typical parabolic velocity profile as follows:

$$u = u_{\max} * (1-(y-h_c/2)^2)/(h_c/2)^2 \tag{2}$$

Where h_c is height of the microchannel.

The liquid in microchannel is assumed to be an incompressible Newtonian liquid. The momentum and mass balances are described by Navier - Stokes equation and continuity equation as

$$\rho\frac{\partial \bar{u}}{\partial t} + \rho(\bar{u}\cdot\nabla)\bar{u} = \nabla\cdot\left[-P\bar{I} + \eta\left(\nabla\bar{u} + \left(\nabla\bar{u}\right)^T\right)\right] + \bar{F} \tag{3}$$

$$\nabla\bar{u} = 0 \tag{4}$$

Where $\bar{u}$ is for the velocity vector, ρ for the density, P for the pressure, η for the dynamic viscosity of fluid.

The species transport in the channels is described by the diffusion–convection (CD) equation as:

$$\frac{\partial c}{\partial t} + \nabla\cdot(-D\nabla c) = R - \bar{u}\nabla c \tag{5}$$

Where c and D represent the concentration and diffusivity of the species, respectively.

The moving mesh-Arbitrary Lagrangian Eulerian (ALE), method was implemented to capture the vibration of beam [13]. The finite element method is utilized in simulation.

RESULTS AND DISCUSSION

The dimensionless parameters such as Reynolds number (*Re*), Peclet number (*Pe*), and Strohal number (*St*) should be paid special attention for analyzing simulation results of micro mixer. The Reynolds (*Re*) number is defined as:

$$\mathrm{Re} = u_{\max}D_h/\gamma \tag{6}$$

The $u_{\max}$ is the centre velocity of parabolic velocity profile. D_h and γ are the hydraulic diameter and the dynamic viscosity of fluid, respectively. *Re* represents the ratio between momentum and viscous frictions. The Reynolds numbers in this mixer are well below the transient value of channel flow (2200) which leads to laminar flows. The Peclet number (*Pe*) is defined as:

$$Pe = u_{\max}L/D \tag{7}$$

Where L is the mixing path, which is on the same order of the height of channel in this study and D is the diffusivity of the fluids. Peclet number indicates the ratio of mass transport due to convection and diffusion. A high *Pe* means a convection-dominant flow. The Strohal number (*St*) is defined as

$$St = fD_h/u_{\max} \tag{8}$$

which represents the ratio of residence time of a species in the channel to the time period of disturbance in an active micromixer. The *f* is the vibrating frequency of micro-beam. A high *St* is related to a highly disturbed flow.

We choose a common method to evaluate the quality of mixing by using mixing index [12]:

$$s = \sqrt{\sum (c - \bar{c})^2 / (N - 1)} \qquad (9)$$

Where c and $\bar{c}$ represent the local and average concentrations, which can be extracted from the concentration profile at exit and *N* is the number of sampling. The zero value of mixing index means a perfect mixing and a unit value refers to little mixing. Good mixing is illustrated by a low mixing index value.

The effects of five important parameters including beam vibrating amplitude and frequency, fluid velocity and diffusivity, and channel dimension on the performance of vibrated-beam-based micromixer were analyzed. The beam vibrating amplitudes, u_{beam}, varies from 0.1 to 1.5mm/s, the vibrating frequencies, *f*, from 12 Hz to 48Hz, the fluid flow velocities, u_{max}, from 0.1 mm/s to 2.0 mm/s, and the diffusivity of fluid, *D*, from 10^{-10} to 10^{-8} m^2/s, and microchannel height from 50 μm to 200 μm. The corresponding *Re* varies from 0.01 to 0.2, *Pe* from 5 to 500 and *St* from 0.8 to 16 respectively, as shown in Fig. 3.

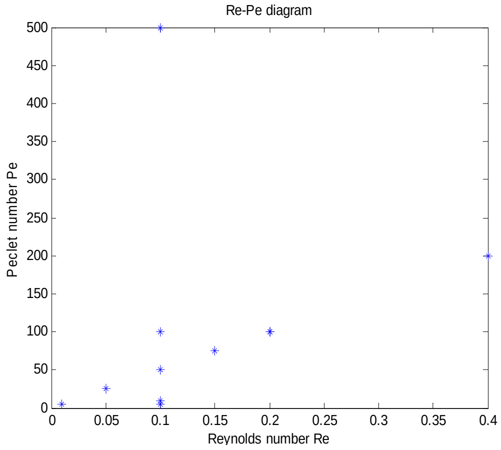


Fig. 3: The *Pe-Re* diagram for micro-beam mixer

The default parameters in our research were set as: u_{beam}=1.0mm/s, *f*=16 Hz, *D*=10^{-9}m^2/s and u_{max}=1.0mm/s; and viscosity of fluid is similar to that of water (10^{-6}m^2/s). All numerical results was checked with mesh dependence study.

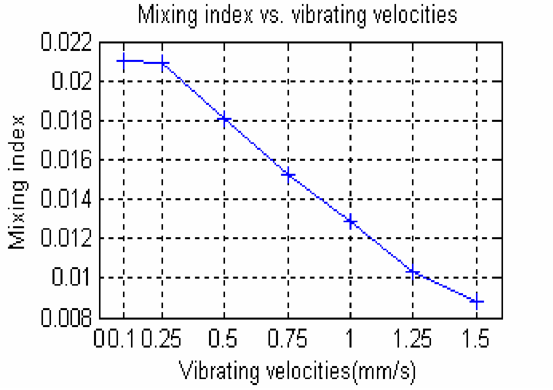
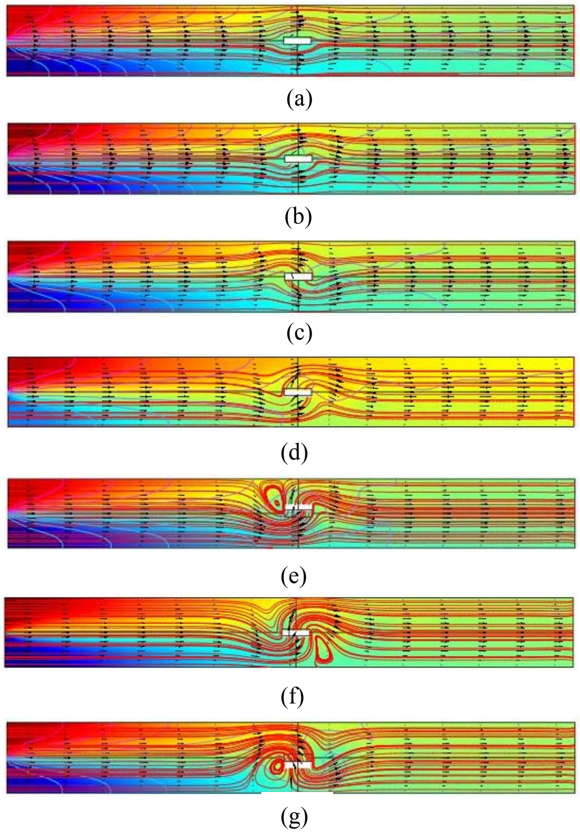


Fig. 4: Mixing index vs. vibrating velocities



(a)

(b)

(c)

(d)

(e)

(f)

(g)

Fig. 5: Flow fields vs. different vibrating velocities (mm/s) (a) 0.1, (b) 0.25, (c) 0.5, (d) 0.75, (e) 1.0, (f) 1.25, (g) 1.5

Effect of vibrating velocity amplitudes

Seven different beam velocity amplitudes including 0.1, 0.25, 0.50, 0.75, 1.0, 1.25 and 1.5mm/s are chosen to clarify their effect while maintaining the constant *Re* (=0.1), *Pe* (=50) and *St* (=1.6). The frequency, diffusivity and beam velocity are default values. The results of mixing indexes are shown in Figure 4. The mixing quality increases almost linearly from 0.011 to 0.003 when the vibrating amplitudes increase from 0.25 to 1.5mm/s. There is no significant improvement when amplitude changes from 0.1 to 0.25 mm/s. The careful inspection on the flow fields in Figure 6 shows that vortex can be developed gradually around the edges of the plate depending on the location of the beam.

In the Figure 5, the fluid streamline, velocity and concentration are represented by red lines, blue and red colors (red: c0=1 and blue: c0=0), and black arrows.

Effect of vibrating frequency

The effect of frequency on the mixing is studied by changing the frequency from 12Hz to 48Hz, which results in the change of *St* from1.2 to 4.8 correspondingly, with the constant *Re* (=0.1) and *Pe* (=50). The results shown in Figure 6 indicate that the mixing effect actually decreases with the frequency's increasing. The sharp increase in mixing index is found between 12Hz and 24Hz, but slow between 24and 48Hz. This seems to be contrary to our intuition. The corresponding flow fields are reported in Figure 7. It is anticipated that the higher vibrating frequency will result in stronger disturbance to flow near the vibrating beam. However, as seen in the Figure 7, the displacement of the beam is also decreased which consequentially lead to the diminishment of vortex, which is the key for a high quality mixing in microchannel. In our case, the displacement of the beam *l* can be described as

$$l = \frac{u_{beam}}{2\pi f}\sin(2\pi f t) \qquad (10)$$

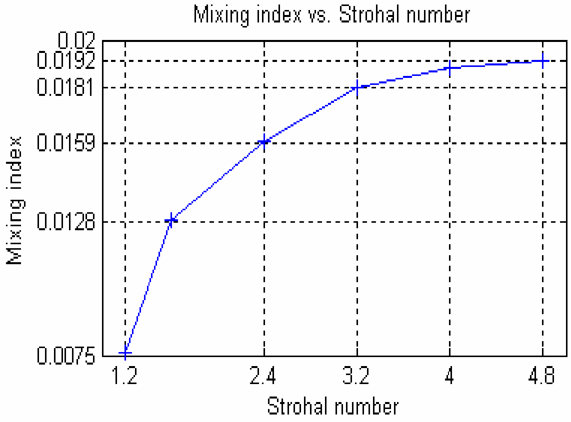


Fig. 6 : Mixing index vs. Strohal number

Increasing the frequency will reduce the displacement assuming constant vibrating velocity amplitude. Therefore, the moving distance of beam is the critical parameter in beam-beased micromixer.

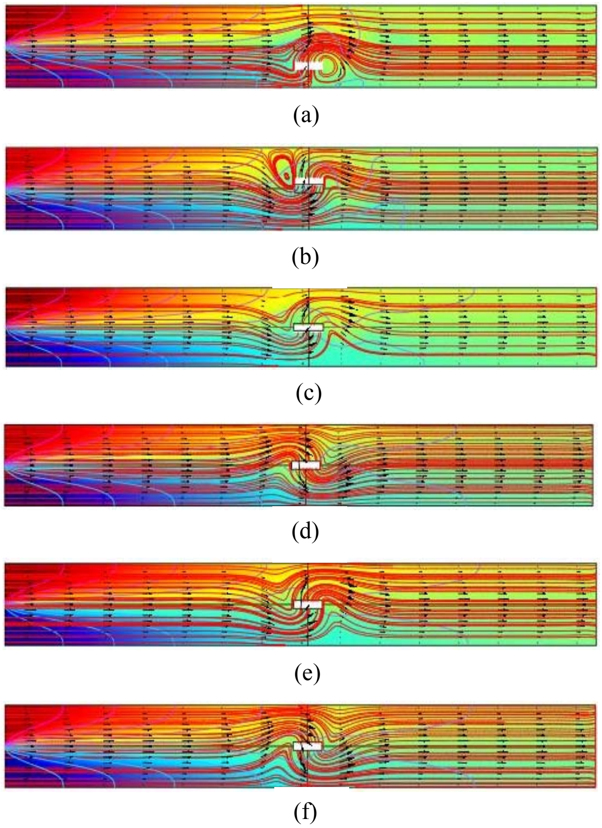


Fig. 7: Flow fields vs. different vibrating frequencies at (a)12Hz, (b)16Hz, (c)24Hz, (d)32Hz, (e)40Hz, (f)48Hz

Here we give a rough explanation of the observed phenomena. We first integrate the equation (10) over a vibration period [0, t] based on the definition of kinetic energy. The result is described in the following equation:

$$KE = \frac{u_{beam}^2}{4}\left[t + \frac{1}{4\pi f}\sin\left(4\pi f t\right)\right] \qquad (11)$$

From this equation, we can see the kinetic energy can be divided into two parts: one is a linear increase part which is related to average velocity and the second part is a periodic part which can be considered as the result of beam vibration. A high frequency lead to the decrease of second part and consequently the decreases of the total kinetic energy since the vibrating amplitudes, u_{beam}, are the same for these cases.

Effect of flow velocities

The flow velocity determines the residence time of the fluid in the microchannel. Therefore, it is more favorable if the fluid flows through the micro-channel at lower velocity assuming both beam vibrating velocity and frequency are fixed. The flow velocities are increased from 0.1mm/s to 2.0mm/s, which lead to increase of Reynolds number from 0.01 to 0.20 and the reduction of Strohal number from 16 to 0.8. Figure 8 and Figure 9 illustrate the mixing index and flow fields at different flow velocities. When the values of *Re* are in the range of 0.01 and 0.05, the calculated mixing index have extremely small values (less than 10^{-4}), which means good mixing. But with the increasing of the Reynolds number from 0.1 to 0.2, the mixing index increases rapidly from 0.0013 to 0.048, which indicates that the lower quality mixing at higher fluid flow velocity.

As shown in Figure 9, when *Re*=0.01, a high chaotic flow field (large vortex) is formed for a good mixing and small mixing index. But the vortex become smaller and smaller and eventually disappears when the flow velocity increase from 0.01 to 1 mm/s, and therefore leads to a poor quality mixing. It seems that the mixing is hardly promoted at all when the flow rate is large. This is because when the fluid velocity is fast, the vortex and roll up of streams is hardly formed, which results in very short interaction between streams of different concentrations. To achieve a good mixing, a stronger vibration is needed in this situation.

It is obviously that the vortex formation in the micro-channel depends on both the flow velocities and vibrating amplitudes. The lower fluid flow velocity with higher vibrating amplitude gives the better mixing quality in Figure 10, where a summary of their relationship with mixing index is presented.

Effect of diffusivity

The diffusivity plays another important role in the effectiveness of the vibrating beam-based micromixer. The Peclet number are increased from 5 to 500 when the diffusivity decreases from 10^{-8} to10^{-10} m^2/s and the mixing index decreases from 0.203 to 10^{-5}. In this study, Reynolds number and Strohal number are same as those of Figure 5 (b). In Figure 12, the contours of concentration are represented by black lines. For a very low diffusivity, i.e. 10^{-10} m^2/s, the interface of different concentrations is disturbed periodically, but no immediate mixing is achieved and a long

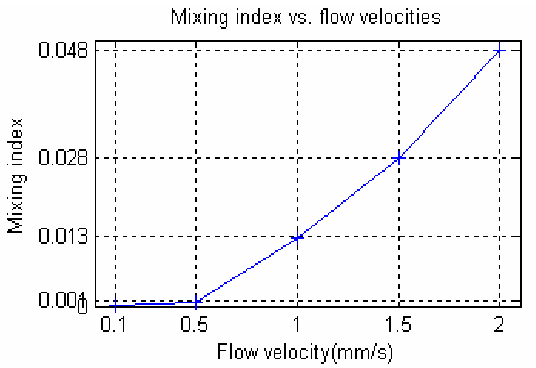


Fig. 8: Mixing index vs. flow

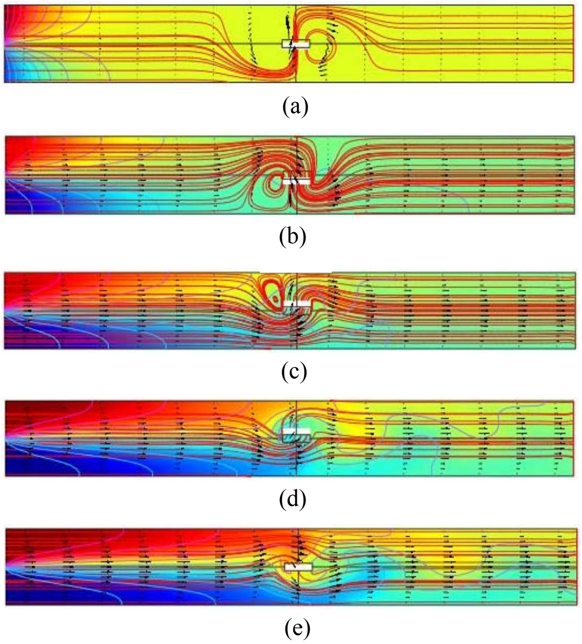

(a)
(b)
(c)
(d)
(e)

Fig. 9: Flow fields vs. flow velocities (a) 0.1mm/s, (b) 0.5mm/s, (c) 1.0mm/s, (d) 1.5mm/s, (e) 2.0mm/s

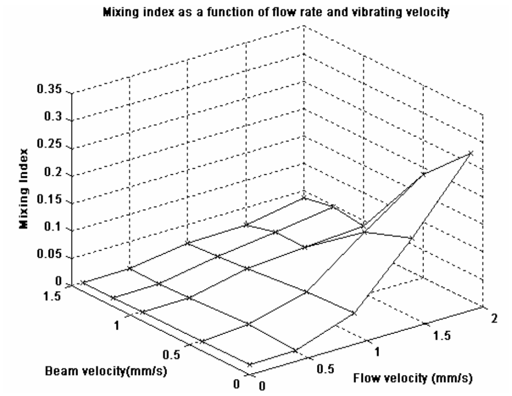


Fig. 10: The mixing index as a function of flow velocity and beam vibration amplitude

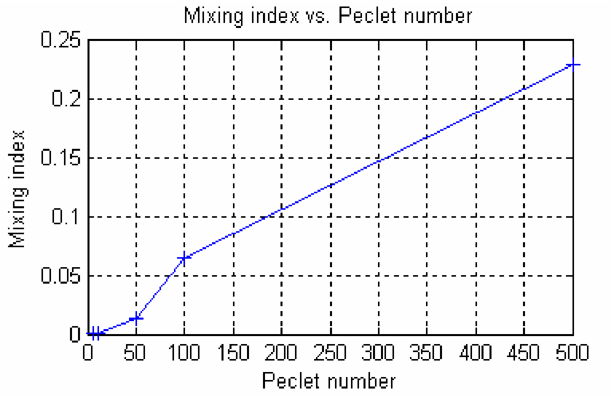


Fig. 11: Mixing index vs. Peclet number

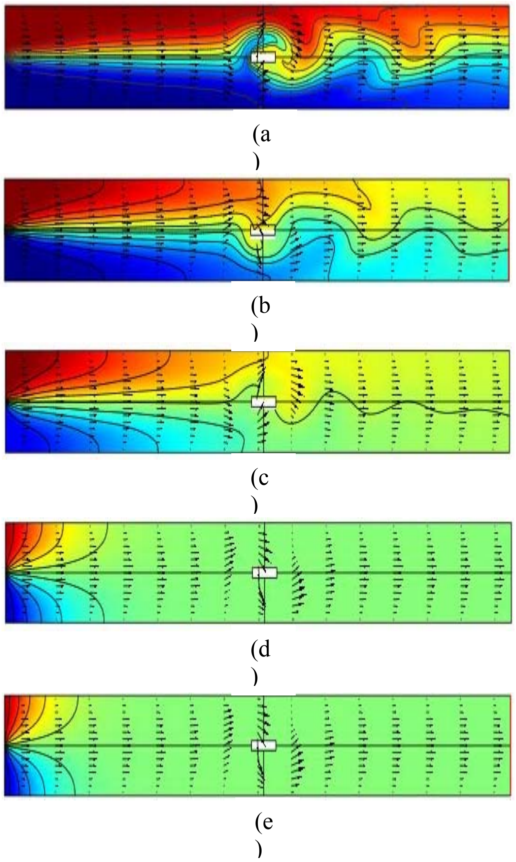


Fig. 12: Concentration fields vs. fluid diffusivity (a)10^{-10}m^2/s, (b) $5\text{x}10^{-10}$m^2/s, (c) 10^{-9}m^2/s, (d)$5\text{x}10^{-9}$m^2/s, (e) 10^{-8}m^2/s.

Channel length is required for a good quality mixing. The good mixing happens for a specific combination of vibrating amplitude, flow velocity and diffusivity. However, an array of vibrating beams should be able to achieve a rapid mixing in a short distance and the investigation is underway to develop an array vibrating beam micromixer.

Effect of channels' height

The interaction between vibrating beam and microchannel results in the vortex which is responsible for the folding and rolling-up of streams and further a good mixing. Here we simulate the effect of channel height on the effectiveness of vibrating-beam micromixer. In this case, the heights of channel are set as 50, 100 and 200 microns. The other parameters are set as default. With the fixed vibrating parameters and flow velocity, the mixing index increases with the increase of height of channel, as shown in Figure 13. Again, vortex is formed for microchannel of height of 50 microns. The corresponding mixing quality of floe fields can be seen in this figure and Figure 14. With the increasing of the channel height, the mixing quality decreases sharply. As we point out earlier, the close interaction between beam and channel wall is the key for the good mixing.

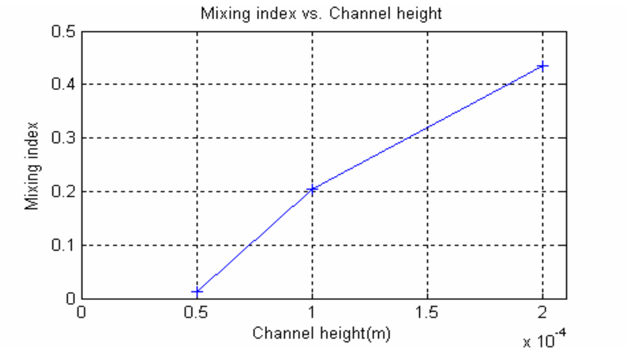


Fig. 13: Mixing index vs. channel heights

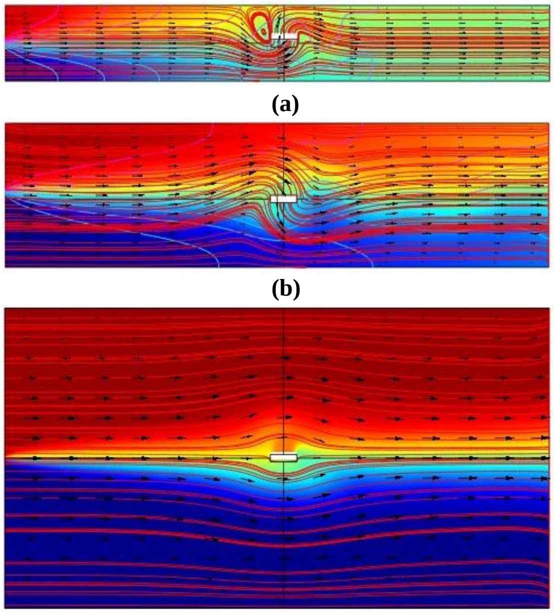


Fig. 14: Flow fields vs. different vibrating velocities

CONCLUSION

In this paper, a new concept of micro-beam mixer for microfluidic mixing have been presented, a preliminary study on the micromixer based on vibrating-beam is performed computationally. In summary, operating parameters including vibrating velocity, frequency, flow velocity, diffusivity and channels' height are systematically studied to address their effects on mixing. The importance of *Re*, *Pe* and *St* are shown in the analysis. The vibrating beam disturbs the flow fields, the vortex and the rolling up of the interface can be formed in the micro-channel flow to promote mixing by increasing the contact area. The results convince us that the vibrating-beam-based mixing is a feasible technology for a quick mixing in micro-scale. The simulations provide guidance to the design of single beam micromixer, and further investigation on array of beams micromixer is underway.

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REFERENCES

[1] Nguyen, N., Wu, Z., 2005, "Micromixers-a review", Journal of Micromechanics and Microengineering, **15**, pp. 1-16.

[2] Stroock, A., Dertinger, S., Ajdari, A., Mezic, I., Stone, H., Whitesides, G., 2002, "Chaotic mixer for microchannels", Science, **295**, pp. 647-651.

[3] Hessel, V., Lowe, H., Schonfeld, F., 2005, "Micromixers—a review on passive and active mixing principles", Chemical engineering science, **60**, pp. 2479-2501.

[4] Maeng, J., Yoo, K., Song, S., 2006, "Modeling for fluid mixing in passive micromixers using the vortex index". Journal of Korean Physical Society, **48**, pp. 902-907.

[5] Ito, Y., Komori, S., 2006, "A vibration technique for promoting liquid mixing and reaction in a microchannel". AIChE Journal, **52**, pp. 3011-3017.

[6] Glasgow, I., Aubry, N., 2003, "Enhancement of microfluidic mixing using time pulsing", Lab on a chip, **3**, pp 114-20.

[7] Oddy, M.H., Santigo J.G., Mikkelsen J.C., 2001, "Electrokinetic instability micromixing", Analytical Chemistry, **73**, pp. 5822-5832.

[8] Yang, Z., Matsumoto, S., Goto, H., Matsumoto, M., Maeda, R., 1999, "Ultrasonic micromixer for microfluidic systems", Sensors and Actuator A: Physical, **93**, pp. 266-272.

[9] Bau, H.H., Zhong, J., Yi, M., 2001, "A minute magneto hydrodynamic (MHD) mixer", Sensors and Actuator B: Chemical, **79**, pp. 207-215.

[10] Lu, L.-H., Ryu, K.S., Liu, C., 2002, "A magnetic microstirrer and array for microfluidic mixing", Journal of Microelectromechanical system, **11**, pp. 462-469.

[11] Wong, S.H., Ward, M., C.L., Wharton, C.W., 2004, "Micro T-mixer as a rapid mixing micromixer", Sensors and Actuators B: Chemical, **100**, pp. 359-379.

[12] Bergman, R., Efremov, A., Woehl,P., 2006, "Numerical mixing analysis of a vaned circular micromixer", Proceedings of ASME ICNMM, pp. 1-5.

[13] http://www.comsol.com

[14] Myszka, D., G., 1998, "Survey of the 1998 optical biosensor literature", Journal of Molecular Recognition, **12**, pp. 390–408.