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Research Article

AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH VARIABLE TYPE DEMAND RATE AND DIFFERENT SELLING PRICES

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ABSTRACT

Many goods undergo decay or deterioration over time which suffer from depletion by direct spoilage while stored. So decay or deterioration of these goods in stock is a very realistic feature and it is necessary to use this factor in inventory models. In this paper we have developed an order level inventory model for constant rate of deterioration. We have also considered a variable type of demand which behaves differently in the given time horizon. The demand rate is constant for a certain fixed time and then the demand varies linearly with time. This paper also deals with different selling prices in two different time periods. The objective of the model is to find the optimal on-hand inventory by considering the profit function.

KEYWORDS: Deterministic Inventory Model, Deterioration, Profit functions, Two different selling prices, Variable demand rate

INTRODUCTION

One of the most developed fields of Operations Research is inventory modeling. Inventory has been defined as an idle resource that possess economic value by Monks(1987). Keeping an inventory for future sales or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. Usually the demand rate is decided by the amount of the stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons motivate the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as shortages will fetch loss to the producers. On the other hand, deterioration is an important natural phenomenon and the consequent loss due to decay of items may be quite significant. Mainly when, physical goods are stocked for future use, in some items such as medicines, foodstuff, dairy items, volatile liquids, the process of deterioration is observed.

Resh et al., (1976) and Donaldson (1977) are the first researchers who considered an inventory model with a linear trend in demand. The time dependent demand patterns reported above are linear, that is, the demand increases continuously with time or decreases continuously along with the time. Dave and Patel (1981), Dutta and Pal (1992), considered time proportional demand. Goyal (1986) considered linear trend in demand. Hariga and Benkherouf (1994) considered exponential time varying demand for deteriorating items. Hill(1995) proposed a time dependent demand pattern by considering it as the combination of linearly time dependent and exponentially time dependent of demand in two successive time periods over the entire time horizon and termed as “ramp- type” time dependent demand pattern.. The works done by Roy(2008), Sabahno(2009), Mirzazadeh(2010), Gayen and Pal(2009) are some of the models for deteriorating items based on different realistic situations. Deterioration is defined as decay, spoilage, loss of utility of the product as defined by Shah and Shukla(2009) .

In the present paper demand rate is considered as constant to a fixed time and then it varies linearly with time. We have also considered different selling prices in two time periods since the demand decrease with time.. It is assumed that lead time is zero and shortages are not allowed. The objective of the model is to find the on-hand inventory by maximizing the profit function.

FUNDAMENTAL ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

- Demand rate is variable with respect to time.
- Single inventory will be used.
- Lead time is zero.
- Shortages are not allowed.
- Replenishment rate is infinite but size is finite
- Time horizon is finite.
- There is no repair of deteriorated items will occur during the cycle

Following notations are made for the given model:

I(t) = On hand inventory at time t

R(t) = Demand

θ=The constant deterioration rate where $0 \leq \theta \leq 1$

Io=Inventory at time t=0

s_1= Selling price per unit in (0,t1)

s_2=Selling price per unit in(t1,T)

c=Unit cost of the item per unit time

Q=On-hand inventory

H=Holding cost per unit item per unit time

r=Replenishment cost per replenishment which is a constant

T=Duration of a cycle

R_0=Initial demand rate

a=Rate of change of demand with respect to t.

DESCRIPTION OF THE MODEL

In this model we consider the rate of demand R(t) to be a constant up to a certain time t=t_1 and after which it varies linearly with time. If I(t) be the on hand inventory at time t≥0, then at time t+Δt, the on-hand inventory will be

$$I(t + \Delta t) = I(t) - R(t)\,\Delta t - \theta.\, I(t).\, \Delta t$$

Dividing by Δt on both sides,

$$\frac{I(t+\Delta t)}{\Delta t} = \frac{I(t)}{\Delta t} - \frac{R(t)\,\Delta t}{\Delta t} - \frac{\theta.I(t).\Delta t}{\Delta t}$$

$$\frac{I(t+\Delta t)}{\Delta t} - \frac{I(t)}{\Delta t} = - R(t) - \theta.\, I(t)$$

taking lim on both sides as Δt→ 0,

$$\lim_{\Delta t\to 0} = \left(\frac{I(t+\Delta t)}{\Delta t} - \frac{I(t)}{\Delta t}\right)$$

$$= \lim_{\Delta t\to 0}(-R(t) - \theta.I(t))$$

$$\frac{dI}{dt} = - R(t) - \theta.\, I(t) \qquad \to (1)$$

Define $R(t) = R_0 + a(t-t_1).H(t-t_1)$ → (2)

where $H(t-t_1) = 1$ for $t \geq t_1$

$= 0$ for $t \leq t_1$

Now equation (1) becomes,

$$\frac{dI}{dt} = -R_0 - \theta.I(t), \text{if } 0 \leq t \leq t_1 \qquad \to (3)$$

$$= -R_0 - a(t - t1) - \theta.I(t), \text{if } t_1 \leq t \leq T \qquad \to (4)$$

Take integral with respect to t in equation (3)

$$\int \frac{dI}{dt}dt = -\int (R(t) + \theta.\, I(t))\, dt$$

$$I(t) = -\frac{(R(t)+\theta.I(t))}{\theta} + c$$

$$I(t) = \left[\frac{-R_0}{\theta} - I(t)\right]e^{-\theta t} + c \qquad \to (*)$$

When I = Io and t = 0,

$$0 = \frac{-R_0}{\theta} - 0 + c$$

$$\frac{R_0}{\theta} = c$$

Now (*) becomes

$$Io = \left[\frac{-R_0}{\theta} - Io\right]e^{-\theta t} + \frac{R_0}{\theta}$$

$$I = \frac{-R_0}{\theta} + e^{-\theta t}\left[Io + \frac{R_0}{\theta}\right] \qquad \to (5)$$

Again using $I=I_1$ at $t = t_1$

$$I_1 = \frac{-R_0}{\theta} + e^{-\theta t}\left[Io + \frac{R_0}{\theta}\right]$$

$$I_1 = \frac{-R_0}{\theta} + e^{-\theta t}\left[\frac{Io\theta + R_0}{\theta}\right]$$

$$\theta = -R_0 + e^{-\theta t}[Io\theta + Ro]$$

Taking log on both sides,

$$\text{Log } I_1\theta = -\log Ro + (-\theta t_1) + \log(Io\,\theta + Ro)$$

$$\theta t_1 = -\log Ro - \text{Log } I_1\theta + \log(Io\,\theta + Ro)$$

$$= \log\left(\frac{(Io\,\theta + Ro)}{(I1\,\theta + Ro)}\right)$$

$$t_1 = -\frac{1}{\theta}\log\left(\frac{(I1\,\theta + Ro)}{(Io\,\theta + Ro)}\right) \qquad \to (6)$$

From equation (4),

$$\frac{dI}{dt} = -R_0 - a(t - t_1) - \theta.I(t), \text{if } t_1 \leq t \leq T$$

Taking integral with respect to t on both sides,

$$I = -\int Rodt - \int\left(a(t - t_1) + \theta.I(t)\right)dt$$

$$= -R_0 t - \left[\frac{a(t-t_1) + \theta.I(t)}{\theta}\right] + c$$

$= -R_0 t - \frac{a(t-t_1)}{\theta} - I(t) + c$

$\rightarrow (**)$

To find c applying initial condition, I=Io, t = T

Now (**)becomes,

$I = \frac{a(T-t_1)}{\theta}$ for $t_1 \le t \le T$

when $I = I_1$ at $t = t_1$ we have

$I_1 = \frac{a(T-t1)}{\theta}$

since $t1 = -\frac{1}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)$

$I_1 = \frac{a\left[T+\frac{1}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)\right]}{\theta}$

$= \frac{aT}{\theta} + \frac{\frac{a}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)}{\theta}$

$\theta I1 = aT + \frac{a}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)$

$\theta I1 - \frac{a}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) = aT$

Divide by a on both sides,

$\frac{\theta I_1}{a} - \frac{a}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) = T$

$\frac{\theta I_1}{a} + \frac{a}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) = T$

Now average on hand inventory is given by,

$Q = \int_0^T I dt = \int_0^{t_1} I dt + \int_{t_1}^T I dt$

Now substituting the corresponding values of I we have,

$Q = \frac{1}{\theta}\left[Io + \frac{R_0}{\theta}\right]\left(1 - e^{-\theta t}\right) - \frac{Rot1}{\theta} + \frac{a}{2\theta}(T - t_1)^2$

$= \frac{1}{\theta}\left[Io + \frac{R_0}{\theta}\right]\left(1 - e^{-\frac{t\theta}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)}\right) - \frac{Ro}{\theta}\left(-\frac{1}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)\right) + \left[\frac{a}{2\theta}\left(\frac{\theta I1}{a} - \frac{1}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)\right) + \frac{1}{\theta}\left[Io + \frac{R_0}{\theta}\right]\right]^2$

$= \frac{1}{\theta}\left[Io + \frac{R_0}{\theta}\right]\left(1 - \frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) + \frac{Ro}{\theta^2}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) + \frac{a}{2\theta} * \frac{\theta I1^2}{a^2}$

$= \frac{1}{\theta}\left[Io + \frac{R_0}{\theta}\right]\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)} + \frac{Ro}{\theta^2}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) + \frac{\theta I1^2}{2a}$

$= \frac{1}{Ro+Io\theta}(Io\theta + R_0)(I - I1) + \frac{Ro}{\theta^2}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) + \frac{\theta I1^2}{2a}$

$= \frac{Io\theta+Ro}{\theta} * \frac{Io-I1}{Ro+Io\theta} + \frac{Ro}{\theta^2}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right) + \frac{\theta I1^2}{2a}$ (#)

Integrating the first part of (#) we get

$\int_0^{t1} -\frac{R_0}{\theta} + e^{-\theta t}\left[Io + \frac{R_0}{\theta}\right] dt$

$= -\frac{R_0}{\theta} t \Big|_0^{t1} + \left[Io + \frac{R_0}{\theta}\right]\left(-\frac{e^{-\theta t}}{\theta}\right)_0^{t1}$

$= -\frac{Rot_1}{\theta} + \left[Io + \frac{R_0}{\theta}\right] * -\frac{1}{\theta}\left(e^{-\theta t} - 1\right)$

$= \frac{1}{\theta}\left(1 - e^{-\theta t}\right) * \left[Io + \frac{R_0}{\theta}\right]$

Integrating the second part of the above equation (#) we get

$\int_{t1}^{T} \frac{a}{\theta}(T - t)dt$

$= \frac{a}{\theta}\left[[T(t)]_{t_1}^{T} - \frac{t^2}{2}]_{t_1}^{T}\right]$

$= \frac{a}{\theta}\left[T(T - t_1) - \frac{1}{2}(T^2 - {t_1}^2)\right]$

$= \frac{a}{\theta}(T - t_1)\left[T - \frac{1}{2}(T - t_1)\right]$

$= \frac{a}{2\theta}(T - t_1)[2T - T - t_1]$

$= \frac{a}{2\theta}(T - t_1)^2$

$\therefore Q = \frac{1}{\theta}\left[Io + \frac{R_0}{\theta}\right] * \left[1 - e^{-\theta t}\right] - \frac{R_0 t_1}{\theta} + \frac{a}{2\theta}(T - t_1)^2$

$= \frac{Io - I_1}{\theta} + \frac{\theta I_1^2}{2\theta} + \frac{R_0}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)$

The profit function, ϕI_0

$\phi(I_0) = \frac{1}{T}\left[(s_1 + s_2 - c - r)I_0 - \frac{h}{T}\left(\frac{Io - I_1}{\theta} + \frac{\theta I_1^2}{2\theta} + \frac{R_0}{\theta}\log\left(\frac{(I1\,\theta+Ro)}{(Io\,\theta+Ro)}\right)\right)\right]$

Differentiate with respect to I_0 *on both sides,*

$\frac{d}{dI_0}(\phi(I_0)) = \frac{s_1+s_2-r}{T} - \frac{h}{T}\left[\frac{1}{\theta} - \frac{R_0}{\theta^2}\left(\frac{\theta}{R_0 + I_\theta}\right)\right]$

$= \frac{s_1+s_2-r}{T} - \frac{h}{T\,\theta} + \frac{hR_0}{T\theta(Io\,\theta+Ro)}$

$= \frac{\theta(s_1+s_2-r)}{T\theta} - \frac{h}{T\,\theta} + \frac{hR_0}{T\theta(Io\,\theta+Ro)}$

$= \frac{1}{T\theta}\left[\theta(s_1 + s_2 - r) - h + \frac{hR_0}{T\theta(Io\,\theta+Ro)}\right]$

The necessary condition for (I_0) *to attain maximum is* $\frac{d[(\phi(I_0)]}{dI_0} = 0$ *which gives,*

$I_0 = \frac{R_0(s_1+s_2-c-r)}{h-\theta(s_1+s_2-c-r)}$

Again differentiating with respect to I_0 both sides

$\frac{d^2}{dI_0^2}(\phi(I_0)) = \frac{1}{T\theta}\left(\frac{-hR_0(0+\theta)}{(Io\,\theta+Ro)^2}\right)$

$= \frac{1}{T\theta}\left(\frac{-hR_0\theta}{(Io\,\theta+Ro)^2}\right)$

$= -\frac{hR_0}{T(Io\,\theta+Ro)^2} \le 0$

It will give a global maximum for profit function $(\phi(I_0))$

CONCLUSION

The above model deals with an inventory model of variable demand rate , that is, demand rate is constant up to a time *t* after which the demand rate varies linearly with time. The paper also deals two different selling prices for two different time periods . During the period (0.t), the demand rate is maintained at a constant level but after that period the amount of inventory decreases continuously with time but the effect of deterioration is maintained throughout the cycle. Hence the inventory level decreases due to the combined effect of demand as well as deterioration. The model is solved by maximizing the profit function and on hand inventory is also found. The above model can also be studied under shortages, backlogging and backordering.

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