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ON DESIGN AND PERFORMANCE OF INTERNAL EPITROCHOIDAL PUMPS

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ABSTRACT

In this study an analysis of rotor profiles geometry in internal epitrochoidal pumps and a systematic study of their performance as a function of the geometrical parameters are introduced. Internal pumps are used in various fields, like automotive, alimentary, or medical-scientific. This machines consist of two rotors: generally the inner rotor has epitrochoidal profile and the profile of the outer rotor is determined as conjugate to the inner one. The rotor geometry and the final performance of the whole machine has been studied by many researchers, using different methods. In this paper the construction of the profiles is performed by the theory of gearing, in order to calculate performance indexes. In particular the pump type with epitrochoidal outer rotor has been considered. Even if it is impossible to establish an optimal profile valid for every application, the analysis of the results obtained allows to choose the design parameters, in order to optimize the shape of the rotors for any particular application.

INTRODUCTION

Internal lobe pumps, usually called gerotors, have been widely studied in the last decades. These machines are constituted by a cylindrical housing and two rotors rotating at different angular velocities with a fixed ratio. The inner rotor transmits the motion to the outer one by a number of contact points equal to the number of lobes of the inner rotor plus one. The chambers in which the fluid is elaborated are enclosed by the same contact points, so that their volume varies during rotary motion. In order to separate the high and low pressure zones, a common solution consists in realizing suitable inlet and discharge ports in the housing plates (see Figure 1).

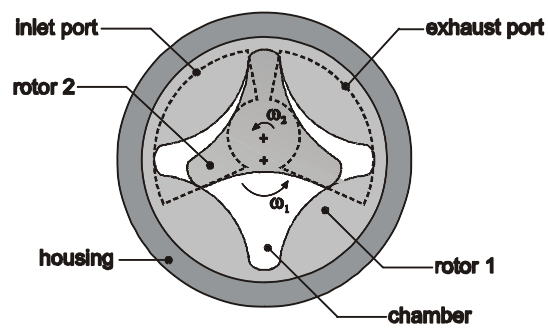


Figure 1. Example of epitrochoidal lobe pump

The rotors geometry has been studied by many researchers, using different methods for the profiles calculation [1-7]. The procedure for rotor profile generation isn't univocally defined: usually the second rotor profile is obtained as conjugate of a given first rotor geometry. Considering the epitrochoidal geometry (see Figure 1), the profile of rotor-1 is constituted in the contact zone by an arc of a circle. As a result, the profile of the second rotor is the conjugated to the first one, and it is obtained as an envelope of an epitrochoid. Another approach consists in considering the profile of the second rotor (rotor-2 in Figure 1) as an envelope of an epitrochoid, and the profile of the first rotor (rotor-1 in Figure 1) is obtained as its conjugate.

According to these approaches, Colbourne [1] and Beard [2] studied the effects of geometrical parameters on the profiles curvature and displacement of epitrochoidal gerotors; Shung [3] proposed a deeper analysis of parameters dependency for the curvature, compression ratio and geometry of conjugate profiles, presenting general equations describing the geometry of these machines.

Litvin [4] successfully applied the theory of gearing [5], in order to calculate the analytical formulas of the conjugate profile, obtaining the profile equations in a particularly synthetic form. The paper by Vecchiato [6], improved the geometrical analisyis determining possible transmission errors due to machining tolerances or wear.

Another approach, less general but more direct, has been proposed by Mimmi [7], starting from the geometry of the external rotor and based on simple geometrical relations.

Once the rotors profiles are obtained, it is possible to calculate performance and operational indexes of the machine [1, 2, 7]. In particular, the most important performance indexes related to cycloidal pump are the specific flow rate and the flow rate irregularity, while for the evaluation of the contact mechanism between the rotors, the pressure angle, the specific slipping and the curvature in the contact points are considered.

In this paper the parametric formulations of rotors geometry is explicitly given in a synthetic form, based on the theory of gearing [4, 5]. Furthermore, a parametric classification of gerotors is proposed, based on three geometrical parameters suitably chosen. In this way the performance indexes can be obtained for every geometric configuration. In particular, the possibility of a pump with epitrochoidal outer rotor has been analyzed, showing some advantages in terms of performance, but also some counter-indications from the operational point of view.

NOMENCLATURE

- a epitrochoid generating radius;
- e eccentricity ($e = r_2 - r_1$);
- I flow rate irregularity;
- K r_1/e ratio;
- $\mathbf{M}$ coordinate transformation matrix;
- n_1 rotor-1 lobe number;
- n_2 rotor-2 lobe number ($n_2 = n_1 - 1$);
- r_1 rotor-1 pitch radius;
- r_2 rotor-2 pitch radius ($r_2 = r_1 \cdot n_2 / n_1$);
- q instantaneous flow rate;
- R specific flow rate;
- $\mathbf{r}$ profile position vector;
- s axial chamber depth;
- u slipping velocity between rotors;
- α_1 ρ/e ratio;
- α_2 a/e ratio;
- γ non-dimensional curvature in contact points;
- δ pressure angle;
- Γ profile curve;
- ρ rotor-1 lobe radius (see Fig.3);
- σ rotors specific slipping;
- ϕ polar reference angle;
- ψ motion angle;
- $1,2$ conjugate, epitrochoidal rotor.

ROTORS PROFILE GEOMETRY

Referring to Figure 2, the equation of the epitrochoid Γ_{epi} which generates the rotor-2 can be obtained as the curve described by point C, attached to the pitch circumference of radius r_1, at a distance a from its center O_1.

In order to avoid the presence of singular points, the profile of rotor-2 is constituted by the envelope of the epitrochoidal profile, given by a circumference of radius ρ. It is important to underline that, if the internal envelope is considered (see Figure 2a), the epitrochoidal rotor-2 results the inner one, while on the contrary, considering the external envelope (see Figure 2b), the inner rotor results to be the conjugate profile.

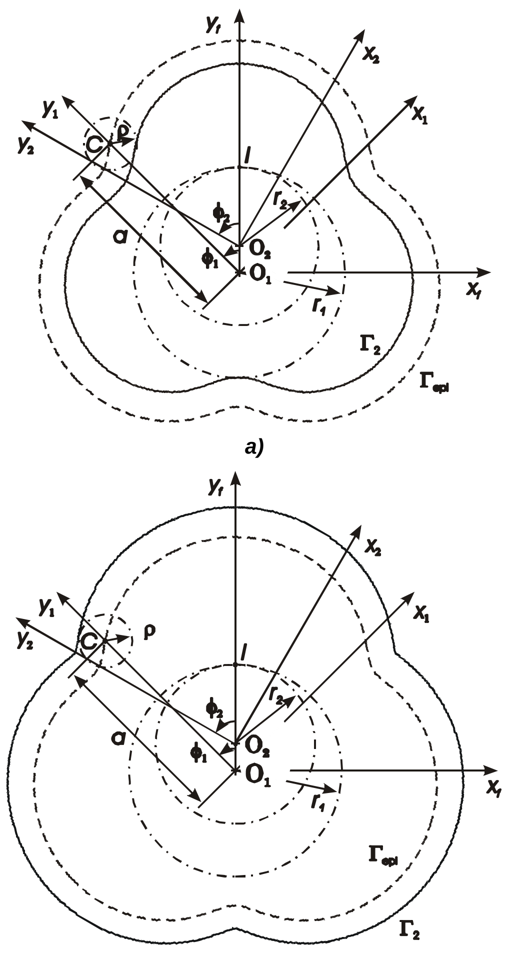


Figure 2. Generation of epitrotrochoidal profile Γ_2: *a)* internal envelope, *b)* external envelope

In order to calculate the envelope profile Γ_2, it is sufficient to consider the equation of the epitrochoid Γ_{epi} and the contribute given by its normal vector. As a result we obtain Eqs. 1-4:

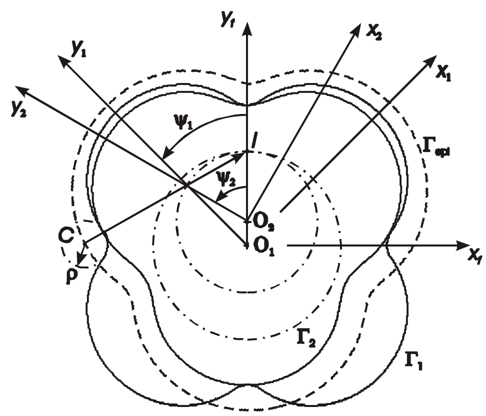
$$\mathbf{r}_2(\phi_2)=\begin{bmatrix} x_2(\phi_2) \\ y_2(\phi_2) \\ 0 \\ 1 \end{bmatrix}=\begin{bmatrix} A(\phi_2)\sin\left(\frac{\phi_2}{K}\right)+B(\phi_2)\sin\phi_2 \\ A(\phi_2)\cos\left(\frac{\phi_2}{K}\right)+B(\phi_2)\cos\phi_2 \\ 0 \\ 1 \end{bmatrix} \quad (1)$$

$$A(\phi_2)=a\cdot\left(1+\frac{\rho}{K\cdot C(\phi_2)}\right) \quad (2)$$

$$B(\phi_2)=e\cdot\left(-1-\frac{\rho}{C(\phi_2)}\right) \quad (3)$$

$$C(\phi_2)=\sqrt{\frac{a^2}{K^2}+e^2-2\frac{ea}{K}\cos\left(\phi_2-\frac{\phi_2}{K}\right)} \quad (4)$$

The profile of Γ_1 (see Figure 3), defined by vector $\mathbf{r}_1$, is determined as conjugated to rotor-2: the theory of gearing [5] has been used.



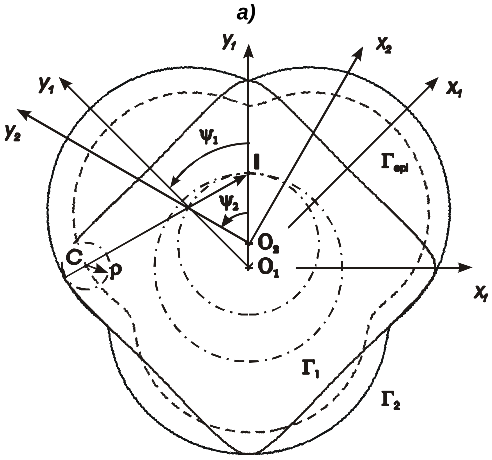


Figure 3. Generation of conjugate profile Γ_1: *a*) internal envelope, *b)* external envelope

Referring to Figure 3 and considering the coordinate transformation matrix $\mathbf{M}_{12}$, the rotor-1 profile, in both cases *a)* and *b)*, can be obtained by the system of equations constituted by the equation of meshing and the coordinate transformation:

$$\begin{cases} f(\phi_2,\psi_2)=0 \\ \mathbf{r}_1=\mathbf{M}_{12}\cdot\mathbf{r}_2 \end{cases} \quad (5)$$

In accordance with [4], the equation of meshing yields two solutions (see Eqs. 6-7):

$$\psi_2=\phi_2+n\cdot\pi \quad (6)$$

$$\psi_2=\phi_2+(2n+1)\cdot\pi-2\cdot(\theta+\phi_1) \quad (7)$$

$$\theta=\arctan\left(\frac{\sin\phi_1}{\lambda-\cos\phi_1}\right) \quad (8)$$

Finally, the rotor-1 profile $\mathbf{r}_1$ equation results:

$$\mathbf{r}_1=\begin{bmatrix} A(\phi_2)\sin\left(\frac{\phi_2}{K}-\frac{\psi_2}{K}\right)+B(\phi_2)\sin\left(\phi_2-\frac{\psi_2}{K}\right)+e\cdot\sin\psi_1 \\ A(\phi_2)\cos\left(\frac{\phi_2}{K}-\frac{\psi_2}{K}\right)+B(\phi_2)\cos\left(\phi_2-\frac{\psi_2}{K}\right)+e\cdot\cos\psi_1 \\ 0 \\ 1 \end{bmatrix} \quad (9)$$

Two different expressions of $\mathbf{r}_1$ are obtained, using the solutions in Eqs. 6-8. The conjugate profile Γ_1 is the combination of the two expressions, as a function of the motion angle ψ. The expression of $\mathbf{r}_1$ relating to the solution in Eq. 6, that results applicable for the contact points, is:

$$\mathbf{r}_1(\psi_1)=\begin{bmatrix} x_1(\psi_1) \\ y_1(\psi_1) \\ 0 \\ 1 \end{bmatrix}=\begin{bmatrix} B(\psi_1)\sin(\psi_1)+e\cdot\sin\psi_1 \\ A(\psi_1)+B(\psi_1)\cos(\psi_1)+e\cdot\cos\psi_1 \\ 0 \\ 1 \end{bmatrix} \quad (10)$$

Figure 4 shows the pitch circles, the generating epitrochoid and the profiles obtained for a series of geometrical parameters that will be described in the next paragraph.

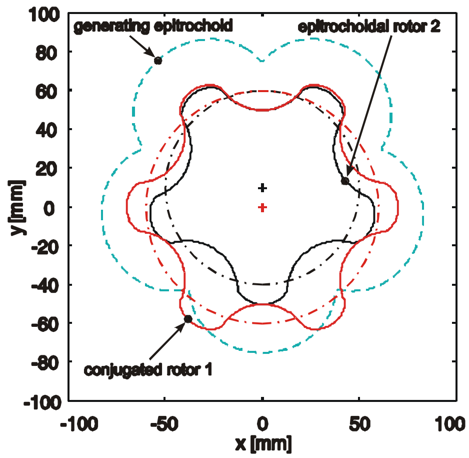


Figure 4. Pitch circles, generating epitrochoid and rotor profiles. Geometrical parameters: e=10 [mm], n_2=5, α_1=2.5, α_2=7.5

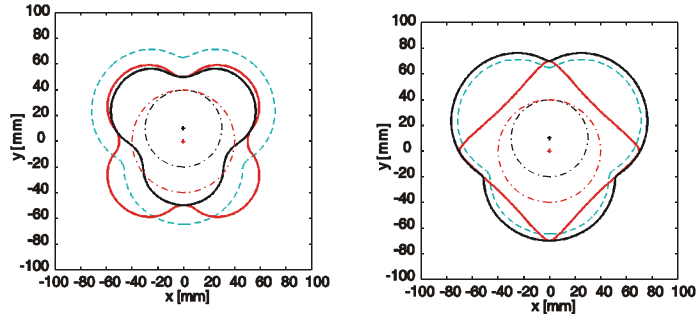
DESIGN PARAMETERS

The geometry of rotor profiles of the epitrochoidal pumps is determined on the basis of three non-dimensional design parameters. In this work, the considered parameters are:

- n_2 rotor-2 (epitrochoidal profile) lobe number;
- α_1 ρ/e ratio;
- α_2 a/e ratio;

The parameter α_1 has been considered negative when we take into account the envelope external to the generating epitrochoid (see Figure 3b), and consequently the epitrochoidal rotor is the outer one. In fact, in this case we can obtain the profiles by considering negative values of ρ in Eqs. 2-4. The parameter α_1 has limitations due to the curvature of the epitrochoid: the value of ρ can not be greater than radius of curvature of the epitrochoid [4]. On the contrary, the parameter α_2 is limited in its minimum value in order to prevent undercutting. In any case, α_2 must be higher than n_1 in order to avoid loops in the epitrochoidal profile. In Figures 5-7 some example families of profiles obtained for various parameter values are shown.

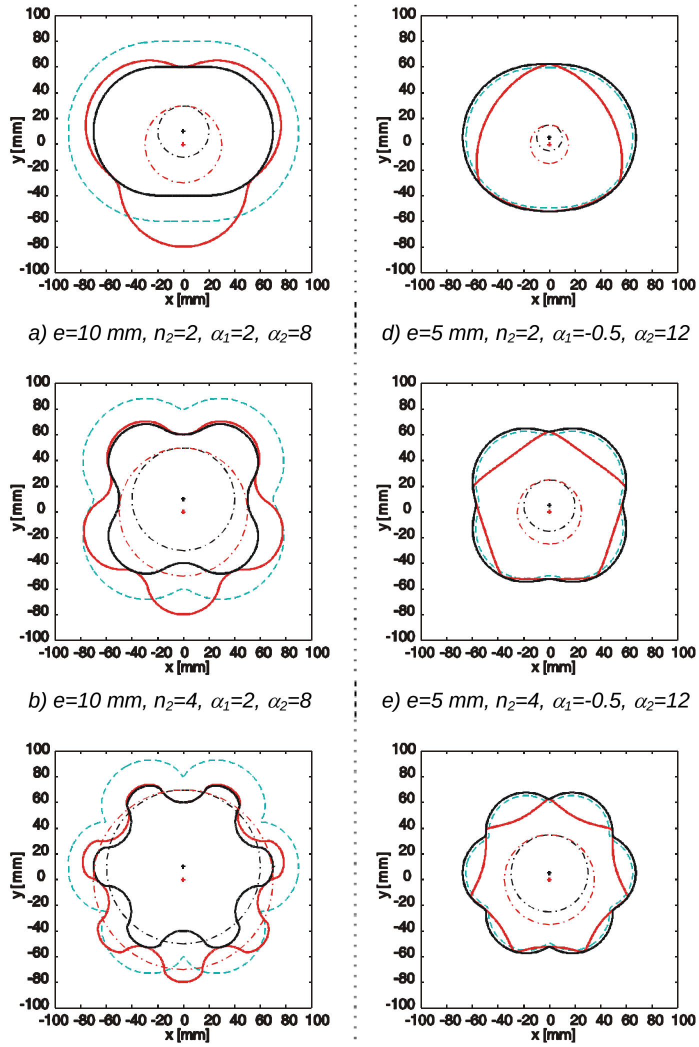
Variation of parameter α_1 (=ρ/e)



a) e=10 mm, n_2=3, α_1=1.5, α_2=6.5 a) e=10 mm, n_2=3, α_1=-0.5, α_2=6.5

Figure 5. Rotor profiles varying design parameter α_1

Variation of lobe number n_2 α_1>0 (internal envelope) | **Variation of lobe number n_2 α_1<0 (external envelope)**



a) e=10 mm, n_2=2, α_1=2, α_2=8
d) e=5 mm, n_2=2, α_1=-0.5, α_2=12
b) e=10 mm, n_2=4, α_1=2, α_2=8
e) e=5 mm, n_2=4, α_1=-0.5, α_2=12
c) e=10 mm, n_2=6, α_1=2, α_2=8
f) e=5 mm, n_2=6, α_1=-0.5, α_2=12

Figure 6. Rotor profiles varying design parameter n_2

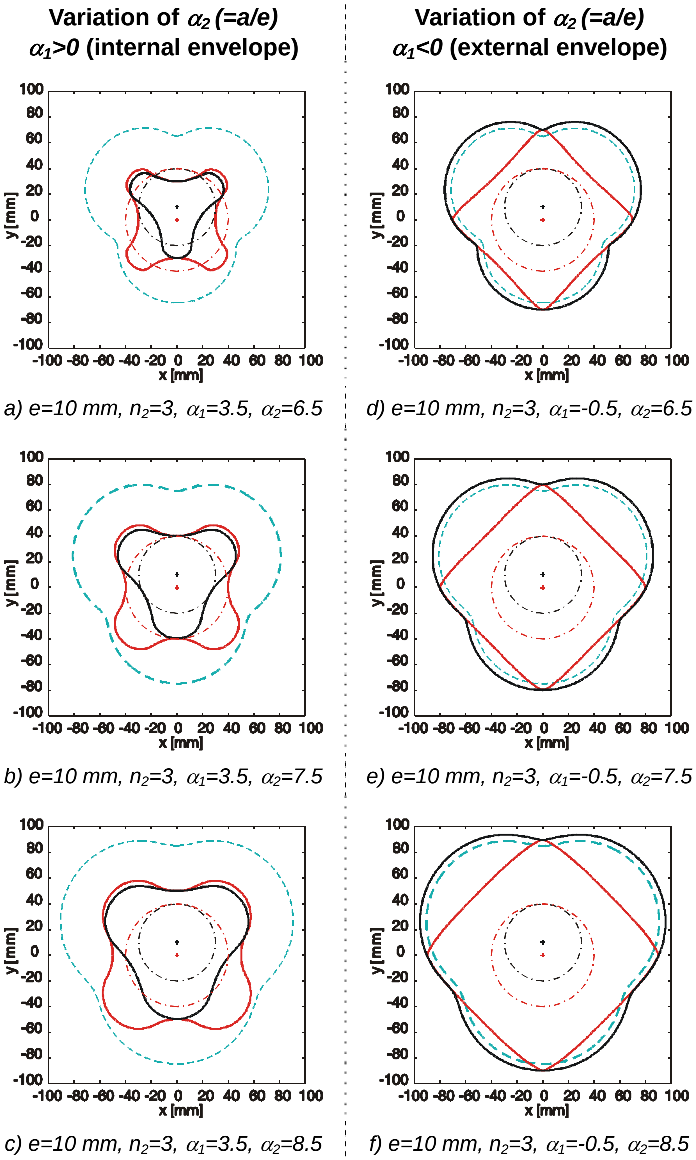


Figure 7. Rotor profiles varying design parameter α_2

PERFORMANCE AND OPERATIONAL INDEXES

FLOW RATE INDEXES

The specific flow rate R is the ratio between the volume of fluid conveyed in one revolution of the inner rotor and the smallest cylindrical volume occupied by the rotor assembly. This volume results as the product between A_{max}, the area of the smallest circular section in which the rotors can be inscribed, and the axial depth s:

$$R = \frac{q_{med} \cdot 2\pi}{A_{\max} \cdot s} \qquad (11)$$

where q_{med} is the mean flow rate.

Referring to Figure 8, if A_i and A_{i+1} are two consecutive contact points, $d\phi_1$ and $d\phi_2$ the infinitesimal rotations of rotor-1 and rotor-2, and ω_1 and ω_2 their angular velocities, the volume variation of the i-th chamber, delimited by A_i and A_{i+1}, is provided by two contributes, one from rotor-1 and another from rotor-2, equal to:

$$dV_{1i}=s\cdot d\phi_1\cdot((O_1A_i)^2-(O_1A_{i+1})^2)/2 \qquad (12)$$

$$dV_{2i}=s\cdot d\phi_2\cdot((O_2A_i)^2-(O_2A_{i+1})^2)/2 \qquad (13)$$

The variations have opposite sign and consequently the total infinitesimal variation of i-th chamber volume results $dV_i=dV_{1i}-dV_{2i}$.

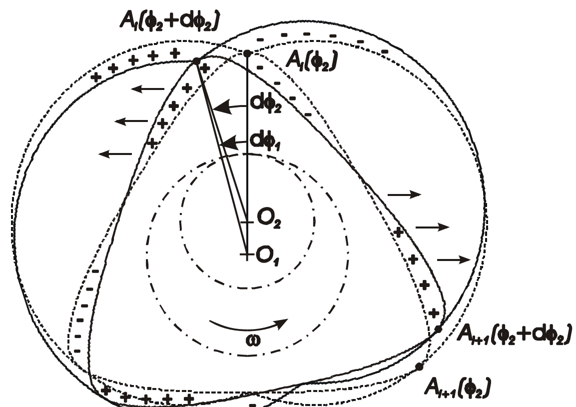


Figure 8. Volume variation of chambers, anticlockwise rotation

The i-th chamber instantaneous flow rate is the time derivative of dV_i:

$$q_i(\phi_2)=\frac{1}{2}\cdot s\cdot\omega_1\cdot\left(\left(\overline{O_1A_i}^2-\overline{O_1A_{i+1}}^2\right)-\left(\overline{O_2A_i}^2-\overline{O_2A_{i+1}}^2\right)\cdot\frac{r_1}{r_2}\right) \qquad (14)$$

where ω_1 is the angular speed of rotor-1.

Under the hypothesis of volumetric efficiency equal to one and of an incompressible fluid, the expression of the pump instantaneous flow rate can be obtained as the sum of the contributes of the various chambers producing delivery:

$$q_{ist}(\phi_2)=\sum q_i^+(\phi_2) \qquad (15)$$

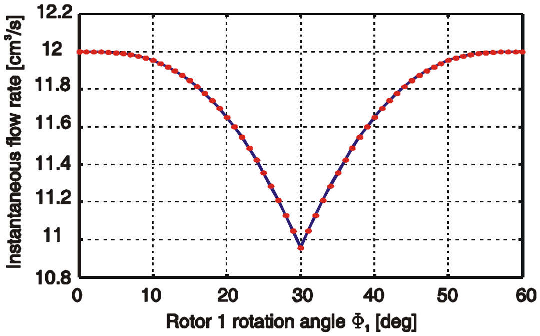


Figure 9. Instantaneous flow rate for the pump in Figure 4, ω_1=10 rad/s

The mean flow rate and the flow rate irregularity can be calculated by Eqs.14-15:

$$q_{med} = \frac{\int_0^{2\pi/n_2} q_{ist}(\phi_2)\cdot d\phi_2}{2\pi/n_2} \quad (16)$$

$$I = \frac{\max(q_{ist}(\phi_2)) - \min(q_{ist}(\phi_2))}{q_{med}} \quad (17)$$

OPERATIONAL INDEXES

Regarding the mechanical operating way of the pump, the following indexes has been considered:

- δ, pressure angle;
- σ_1 and σ_2, rotor specific slippings;
- γ_1 and γ_2, rotor non-dimensional curvatures in correspondence to the contact points.

During the rotation, the minimum pressure angle must be sufficiently reduced, in order to achieve the correct driving of the rotors: in Figure 10 the plot of the most favorable pressure angle between the rotors, in correspondence to the contact points for the pump in Figure 4, is shown.

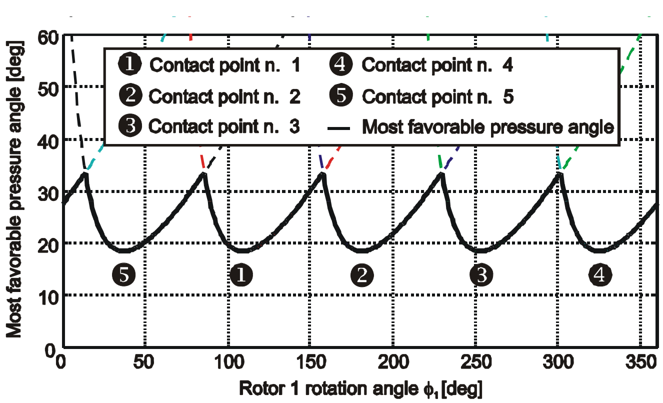


Figure 10. Most favorable pressure angle for the pump in Figure 4

The specific slipping may be related to the wear of the rotors. If u_1 and u_2 are the slipping velocities between the rotors, the specific slipping can be determined by the following equations:

$$\sigma_1 = \left|\frac{u_2 - u_1}{u_1}\right| \quad (18)$$

$$\sigma_2 = \left|\frac{u_2 - u_1}{u_2}\right| \quad (19)$$

Finally, the curvature is related to the contact pressure between the profiles: its value has been adimensionalized by the eccentricity e. The curvature of the conjugate rotor in the contact points is constant, equal to $1/\rho$.

PARAMETRIC PERFORMANCE ANALYSIS

The performance indexes have been analyzed for different geometrical configurations. The results obtained have shown the following relationships between the design parameters and the performance:

- an increase of the value of α_1 generally produces a greater specific flow rate;
- if n_2 is even, the consequence of an increase of the absolute value of α_1 is a greater flow rate irregularity;
- if n_2 is odd, higher α_1 values generally produce a lower flow rate irregularity;
- if n_2 is even, lower α_2 values increase the specific flow rate and the flow rate irregularity;
- if n_2 is odd, lower α_2 values increase the specific flow rate and reduce the flow rate irregularity;
- lower n_2 values in general reduce the specific flow rate and increase the flow rate irregularity;
- the pumps optimized for flow rate irregularity characterized by a even number n_2 of lobes of the epitrochoidal rotor-2 show a flow rate irregularity always lower in comparison with the optimized pumps with an odd foregoing or succeeding lobe number n_2.

Many of these results are confirmed by literature [1, 7]. The analysis has then been focused on the parameter α_1. In particular the possibility of negative values of α_1 has been considered: in this case, the epitrochoidal rotor is the external envelope of the generating epitrochoid. The specific flow rate is evaluated referring to a complete rotation of the inner rotor, that is usually the driving one. If α_1 is negative, the rotor-2 is the outer one: consequently, the inner rotor angular speed is lower than the outer one (see Figures 5b, 6b, 7b), differently from the classical configuration (see Figures 5a, 6a, 7a). If we consider the rotation of the epitrochoidal rotor-2, the specific flow rate would be reduced by the ratio r_2/r_1.

Accordingly, the achieved results show a significant raise of the specific flow rate when α_1 becomes negative, without a remarkable variation of the trend concerning the flow rate irregularity. Apart from the discontinuity in correspondence of the leap, the specific floe rate continues to increase with α_1 increasing.

In Figure 11 are shown two diagrams obtained for the performance indexes by varying the parameter α_1: the arrows indicate better performance indexes values.

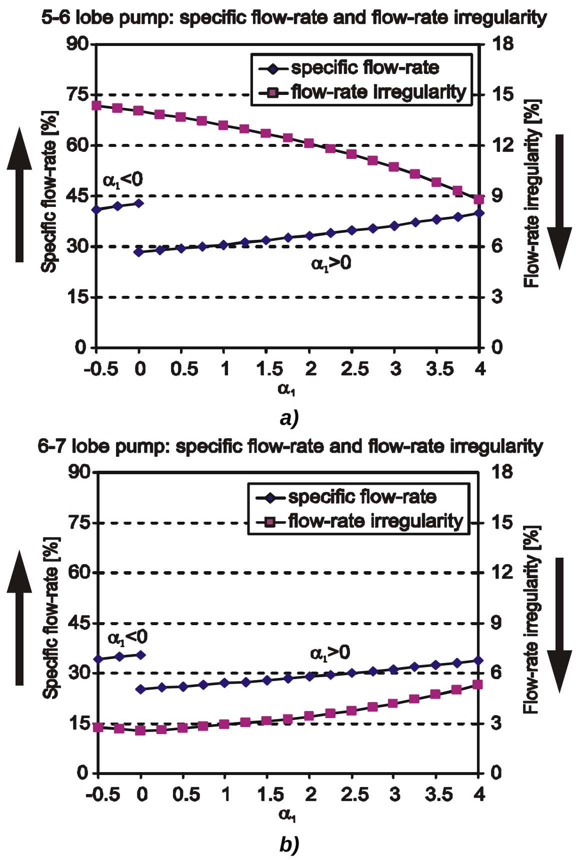


Figure 11. Specific flow rate and flow rate irregularity varying α_1: *a)* n_2 odd, *b)* n_2 even

It can be stated that the highlighted trend, with sudden increase of the specific flow rate without the variation of the irregularity around $\alpha_1=0$, is common for any value of the design parameters α_2 and n_2, and it corresponds to the passage from the configuration of Figure 3a to the configuration of Figure 3b. However the pressure angle, for α_1 near zero, generally significantly increases, as can be observed in Figure 12, due to the growth of the rotor curvature in contact points. Besides, another consequence of the low curvature radius is that the specific slipping of the conjugate rotor increases too.

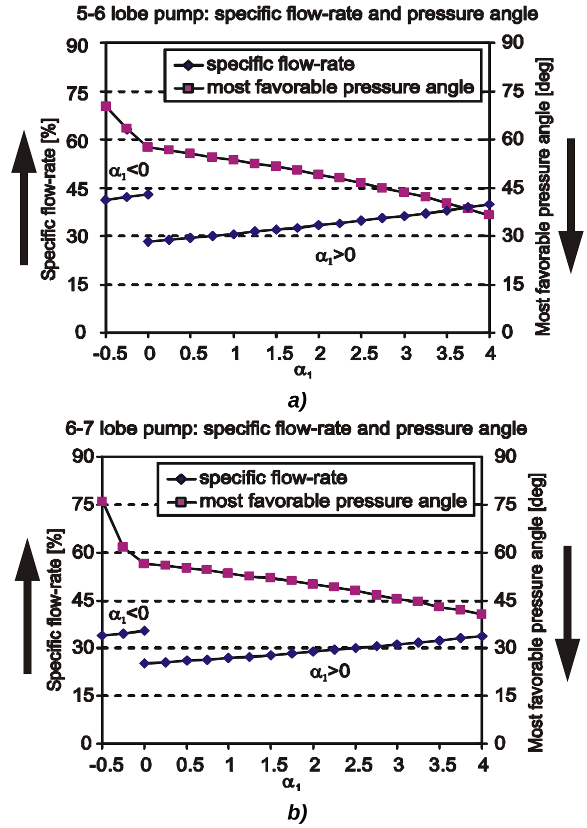


Figure 12. Specific flow rate and most favourable pressure angle varying α_1: *a)* n_2 odd, *b)* n_2 even

As a result, the pumps with external epitrochoidal profile require an external synchronizing system to allow the motion transmission between the rotors, and to avoid severe wear conditions.

CONCLUSIONS

In this study we have analyzed the geometry of epitrochoidal lobe pump and their performance as a function of design parameters.

Once the method for the calculation of profiles has been chosen, three geometrical parameters have been selected in order to set the shape of profiles. In particular, applying the used approach, besides the common pumps, the pump type with epitrochoidal outer rotor can be considered too, by the variation of one design parameter.

Once the profiles are obtained, various indexes have been calculated in order to evaluate the performance and operational conditions on the base of the design parameters.

In particular, the configuration characterized by the outer epitrochoidal rotor permits to obtain an increase in the specific flow rate without a consequent reduction of irregularity performance. However, this configuration has two important counter-indications: in fact, the presence of high values of the pressure angle imposes to use an external synchronizing system, and the wear conditions can be critical, due to the high values of the specific slipping between the rotors,.

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