The quantitative analysis of trade policy: a strategy in global competitive conflict

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Abstract: The quantitative analysis of trading wars is now an actual and modern problem although the first works on optimisation of the custom duties date back to the end of the 19th century. The problem of reducing the number of possible solutions to the conflict and precise identifying of the optimal strategies is important. Applying mathematics makes the problem more formal, allows removing political tension in search of optimum solutions to all participants of the trading conflict.

We consider that some state sells a product manufactured both in the country and abroad to other countries. The country-consumer of the product faces a choice of the import regulation in relation to the situation by means of the import duty size. A goal of such regulation may be the income of taxes and customs payments or the growth of the domestic manufacturer part in sales volumes. A goal of the importer which it achieves by the import volume variation is either profit, or a number of workplaces in the countries making the imported product.

We introduce three multicriteria problems which correspond to different purposes of economic agents. The Pareto set is constructed for each problem. The main attention is paid to the Pareto set reducing based on additional information on the preference relation of the economic agents. Several calculations are made and illustrative examples are given.

Keywords: quantitative analysis; optimal tariff; trade policy; risk management; multicriteria problem; Pareto set.

Biographical notes: V.D. Noghin graduated from Leningrad State University in 1971 and author of more than 120 papers and books related to multicriteria (vector) optimisation, multiple criteria decision-making, multiple criteria decision analysis and mathematical modelling.

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1   Introduction

Being the tool of trade policy protectionism, remains demanded in interstate relations, despite active actions of GATT and WTO. It is enough to recall almost 20 years' «banana war» between the USA and the EU in which the US banana suppliers from Latin America to the European markets have been met by huge import duties as the EU prefers exporters from Africa and Oceania. Only recently, WTO has agreed about some decrease in the duty, but not about full deliverance of the US import. In turn, the US Government has limited import of wines, cheeses, electrical devices and other goods from Europe. Past year, during crisis, the USA imposed anti-dumping duties concerning the Chinese tire covers, pipes for oil drilling rigs, a copper foil; per 2010 – concerning the Chinese manufactured gift packing. In February 2010, China in turn declared anti-dumping measures concerning the US chicken meat. The list of the goods Russia introduces restrictions to, makes up hundreds of names. Thus, the quantitative analysis of trading wars is an actual and modern problem though the first works on optimisation of the custom duties date back to the end of the 19th century. We may mention here the names of Mendeleyev (1950), Edgeworth (1908) and Bickerdike (0906). Publications on the topic proceed now (e.g., Zissimos, 2009; Bradford, 2006; Lahiri et al., 2002; Keen and Wildasin, 2004; Prasolov, 1999) with even greater activity.

Though negative and positive sides of protectionist decisions are known to everybody, the governments of countries keep on the commercial policy aiming at certain interests of their own. Thus, in «banana war» the lobby of companies Dole, Chiquita, Del Monte influenced the US Government while European countries were supporting economies of their former colonies in Africa, on Caribbean Islands and the countries of the Pacific region. The aim of import duty introduction may be an increase of number of workplaces, increase of competitiveness in a certain branch or getting of additional inputs in the country budget. Formally restriction on commerce can be expressed in import quotes, sanitary or other norms. After protect restriction introduction the country on which these restrictions are directed, tries to use the retaliation measures complicating the trading relations between the states and business inside the countries. Search of agreement between the clashing countries can continue for years. It distracts great material and human resources. Finally, usually there is some quantitative solution of the trading conflict expressed either by moderate duties, or reasonable quotas. However, the more versions of such solutions exist, the longer are the negotiations about acceptance of the solution. The problem of reducing the number of possible solution to the conflict and precise identifying of the optimal strategies is significant. Applying mathematics makes the problem more formal, allows removing political tension in search of optimum solutions to all participants of the trading conflict.

2   Multicriteria choice problem and the Edgeworth-Pareto principle

Let several economic agents operate and each of them has their criterion (goal function). Let us assume that every participant tends to maximise their criterion function. Moreover, we suppose that every participant is interested in a selection of 'mutually advantageous' (mutually acceptable) decisions within the set of feasible variants. First of all, it means

that the participants will disagree on such a situation when the benefit at least for one participant may be improved without any losses for the others. Such situation may be adequately described by means of a multicriteria choice model.

The multicriteria choice model $\langle X, f, \succ_X \rangle$ contains the following three objects: X is a set of feasible variants (solutions). Economic agents carry out a choice within it; $f = (f_1, ..., f_m)$, $m \geq 2$, is a numerical vector function (criterion) defined on X and taking on numerical values in arithmetic vector space R^m; i^{th} component of this criterion f_i represents the goal function of ith economic agent; $\succ_X$ is an asymmetric collective preference relation defined on X. Expression $x_1 \succ_X x_2$ for x_1, $x_2 \in X$ means, that the variant x_1 is more preferable than x_2 for all economic agents. In other words, the collective will choose the first and will not select the second variant from these two.

To solve the multicriteria choice problem means to point out a definite subset of X, which is called a set of chosen (selected) variants and is denoted by $C(X)$. In particular cases this set may consist of one element or even be empty.

The main difficulty of the multicriteria choice theory is that there is not existing 'unique true definition' for the set $C(X)$, relevant for arbitrary multicriteria problem. Any attempt to propose a strict formal definition for a set $C(X)$ is unproductive since for each collective of participants this set is unique. Its formulation, as a rule, depends on wide range of the most various conditions and circumstances which cannot be described mathematically. That is why it is more fruitful to propose some upper estimates for the unknown set $C(X)$ using some additional information on the collective preference relation (Noghin, 2005).

Let us introduce a set of feasible vectors $Y = f(X) \subset R^m$ and a set of selected vectors $C(Y) = F(C(X))$. We assume that a collective preference relation $\succ_Y$ (which is usually unknown in practice) is defined on Y and also $x_1 \succ_X x_2 \Leftrightarrow f(x_1) \succ_Y f(x_2)$ for all $x_1 \in \tilde{x}_1$, $x_2 \in \tilde{x}_2$; $\tilde{x}_1$, $\tilde{x}_2 \in \tilde{X}$, where $\tilde{X}$ is a set of equivalence classes generated by the equivalence relation $x_1 \sim x_2 \Leftrightarrow f(x_1) = f(x_2)$ on the set X.

In terms of vectors the multicriteria choice model $\langle Y, \succ_Y \rangle$ includes the set of feasible vectors Y and the collective preference relation $\succ_Y$ defined on Y. To solve such multicriteria choice problem means to find the set of chosen vectors $C(Y)$. Between both problems (in terms of variants and in terms of vectors) there is an obvious relationship which provides the way of rewriting any statement given in one language into another language.

Let us recall that a set of Pareto-optimal (effective) vectors and Pareto-optimal variants are defined by

$$P(Y) = \left\{ y^* \in Y \mid \text{there is no such } y \in Y \text{ that } y \geq y^* \right\}$$

$$P_f(X) = \left\{ x^* \in X \mid \text{there is no such } x \in X \text{ that } f(x) \geq f\left(x^*\right) \right\}$$

respectively. Here, $y' \geq y''$ means that any component of the first vector is greater or equal to the corresponding component of the second vector, and also $y' \neq y''$.

The following statement holds.

The Edgeworth-Pareto Principle (Noghin, 2005): Let some two 'reasonable' axioms (see Noghin, 2005) are satisfied. Then for any set of selected vectors $C(Y)$ the inclusion $C(Y) \subset P(Y)$ holds.

According to this principle the collective, accepting certain 'reasonable' axioms, should select the 'best' variants only within the Pareto set. It must be noted, that the Edgeworth-Pareto principle is true to transitive as well as to intransitive collective preference relations.

3   Mathematical model of the problem

The formal statement of the problem was considered in Prasolov (1999) with some different way of decision-making, however, in the given work the basic economic targets of the process participants remain the same. Let some state (e.g., the Russian Federation) sells (at its home-market) other countries a product manufactured both in the country and abroad. Owing to this commerce the country receives the income

$$S = txp + \tau qy, \tag{1}$$

where t is the value-added tax (now in Russia $t = 18\%$), x is manufacture volume of the given product in Russia, p is a market price of the product at home market in Russia, τ is the import duty (*ad valorem)* for the product import, q is the price of the product abroad, y is volume of import.

The importer (all importers from all countries) of the given product receives profit

$$D = y\big[p - (1+\tau)q\big]. \tag{2}$$

The country-consumer of the product faces a choice of the import regulation in relation to the situation by means of the import duty size (or some quota system which is not spoken about in the given work). A goal of such regulation may be the income of taxes and customs payments (1) or the growth of the domestic manufacturer part in sales volumes. A goal of the importer which it achieves by the import volume variation is either profit (2), or number of workplaces in the countries making the imported product which in this work is accepted proportional to the import volume.

Let us set three problems.

- *Problem A:* Find the Pareto set for two criterion functions S and D under inequalities $y \geq 0$, $\tau \geq 0$, $D \geq 0$.

  Internal manufacture is considered constant in this problem. The result of the Problem A solution determines possibilities of mutual strategy of the state and the importer. Pareto set specifies area of compromises in a trade policy and the choice within it allows establishing mutually advantageous strategy by negotiations (or, e.g., by the method of «tests and mistakes»).

- *Problem B:* Find the Pareto set for three criterion functions S, D and $Y = y$ under inequalities $y \geq 0$, $\tau \geq 0$, $D \geq 0$.

Internal manufacture *x* in this problem is also considered constant. The result of the Problem B defines possibilities of mutual strategy of the country-consumer, importing companies and the countries producing import, since the additional optimisation objective is the increase (or not reduction) of workplaces numbers. Such research becomes important if import to a consuming country is great enough to make the trade unions (or departments supervising export) of the manufacturing country, demand from the government the retaliation to import duties establishment. For example, restriction of chicken gammons deliveries from the USA to the Russian Federation («Bush legs») led to an essential pressure on integrated poultry farms in the state of Maryland in the USA and caused political reactions of the US Government. Whereas estimation of the Pareto set, could probably, allow finding the comprehensible conciliatory decision to the problem.

- *Problem C:* Find the Pareto set for three criterion functions *S*, *D* and *X* = *x* under inequalities $y \geq 0$, $\tau \geq 0$, $D \geq 0$.

   Let us consider, that internal manufacturing *x* in this problem is variable and conforms to function of the offer of a domestic production. In such statement, the problem analyses possibilities of protectionist behaviour with the objective to make own manufacturing more competitive or simply to increase the volume of own manufacturing that corresponds to care of the consuming country government about workplaces in the country.

In all offered problems, the price of the product for a foreign market is constant, therefore, it can be chosen as a money unit during all work: $q = 1$. In a similar way, in first two problems the volume of internal manufacture is constant and consequently it is possible to choose this value as a measure unit of import volumes and to make a consumption: $x = 1$.

   Besides, let us assume that the internal consumer spends a certain amount *M* from its own budget for given goods. Then the curve of demand is defined by an equation

$$p(x + y) = M. \tag{3}$$

As a result of these assumptions use for first two problems we receive three criterion functions

$$S = \frac{0.18M}{1 + y} + \tau y, \tag{4}$$

$$D = y\left(\frac{M}{1 + y} - 1 - \tau\right), \tag{5}$$

$$Y = y. \tag{6}$$

Criterion function (6) does not participate in Problem A. Here, variables are $y \geq 0$, $\tau \geq 0$ and *M* is a positive parameter (below in calculations and charts *M* is considered equal to 6, and in formulas the letter name of the parameter is left).

4   Problems solution

Let us start finding the Pareto set in *Problem A*. In accordance with function (5), an import profitableness condition $D \geq 0$ is expressed by an inequality $\tau \leq \dfrac{M}{1+y} - 1$ that corresponds to the area Ω in Figure 1.

Figure 1   Set (y, τ) for which trade is possible, or breaking-even import area (see online version for colours)
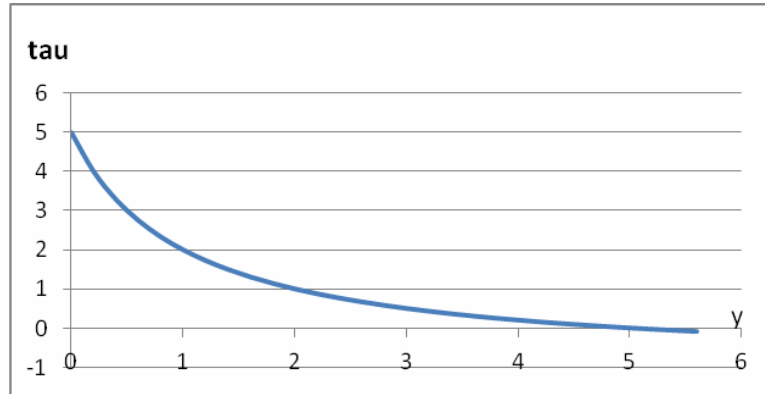
The equations of the function level lines (4) and (5) on a plane (y, τ) are as follows

$$\tau = \frac{1}{y}\left(S - \frac{0.18M}{1+y}\right), \tag{7}$$

$$\tau = \frac{M}{1+y} - \frac{D}{y} - 1, \tag{8}$$

and their graphs are given in the pictures below.

In Figure 2, the more is value S, the upper is the level line and in Figure 3 – on the contrary: the more is value D, the lower is the level line.

In the domain $\Omega \subset R^2$ the level lines with values S and D, depending on the chosen point, can be constructed for any point $(\hat{y}, \hat{\tau})$:
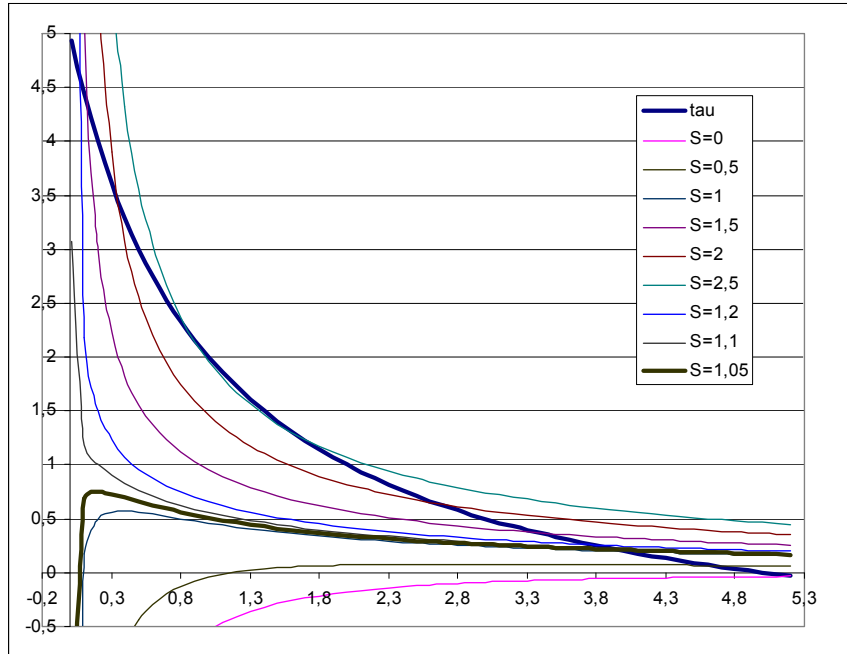
$$\hat{S} = \frac{0.18M}{1+\hat{y}} + \hat{\tau}\hat{y}, \quad \hat{D} = \hat{y}\left(\frac{M}{1+\hat{y}} - 1 - \hat{\tau}\right).$$

Substituting the received values of levels in (7) and (8), it is possible to deduce the equations of the level lines passing through $(\hat{y}, \hat{\tau})$:

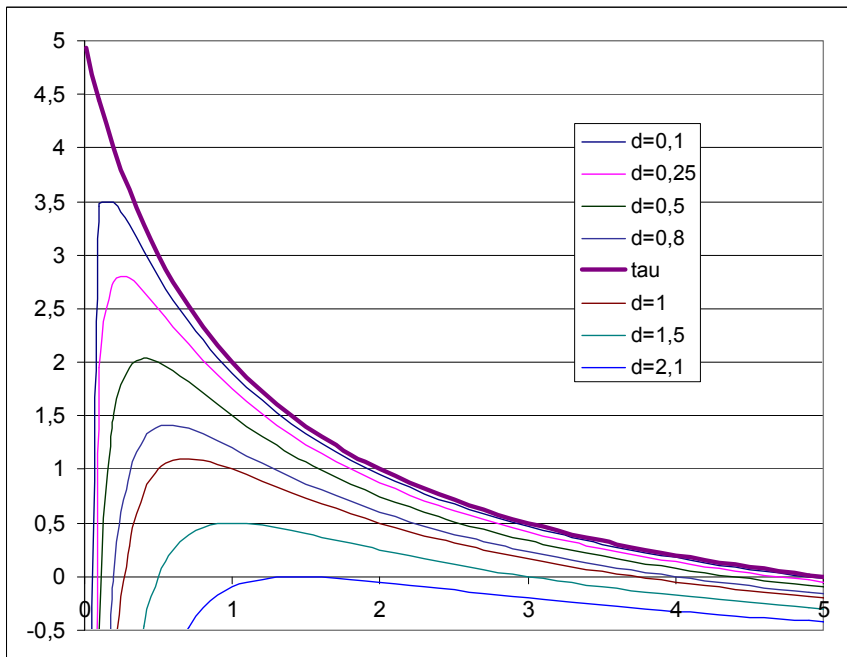$$\tau_S = \frac{1}{y}\left(\hat{\tau}\hat{y} + \frac{0.18M}{1+\hat{y}} - \frac{0.18M}{1+y}\right), \tag{9}$$

$$\tau_D = \frac{M}{1+y} - 1 - \frac{\hat{y}}{y}\left(\frac{M}{1+\hat{y}} - 1 - \hat{\tau}\right).. \tag{10}$$

Figure 2 Lines of the criterion function *S* levels (see online version for colours)

Note: The bold line, crossing the level lines, marks the area of feasible values in the problem.

Figure 3 Lines of criterion function *D* levels (see online version for colours)

If these lines are crossed in $(\hat{y}, \hat{\tau})$ at a non-zero angle there is a direction from $(\hat{y}, \hat{\tau})$ along which both criterion functions $S(y, \tau)$ and $D(y, \tau)$ increase. It means, such points cannot belong to Pareto set. Therefore, Problem A can be solved only in those points in which the level lines are tangent to each other. Using (9) and (10) let us record a condition of the contact:

$$\frac{d\tau_S}{dt}\bigg|_{\hat{y}} = \frac{d\tau_D}{dy}\bigg|_{\hat{y}},$$

or in another form

$$\frac{0.18M}{\hat{y}(1+\hat{y})^2} - \frac{\hat{\tau}}{\hat{y}} = \frac{1}{\hat{y}}\left(\frac{M}{1+\hat{y}} - 1 - \hat{\tau}\right) - \frac{M}{(1+\hat{y})^2}.$$

Reducing to common denominator, we receive $\hat{y} = \sqrt{0.82M} - 1$. For case $M = 6$ $\hat{y} \approx 1,218$. In this case $\hat{\tau}$ may be arbitrary for an interval $\left[0, \sqrt{\dfrac{M}{0.82}} - 1\right]$. In fact, we have proved the following theorem.

Statement 1: The set of locally Pareto-optimum solutions for finding of the compromise between a consuming country and an importer belongs to the set of points

$$\bar{y} = \sqrt{(1-t)xM/q} - x, \quad \tau \in \left[0, \sqrt{\frac{M}{(1-t)xq}} - 1\right]$$

on a plane $(\bar{y}, \tau)$. Obviously, $\hat{y} = \bar{y}$ for $x = 1$, $q = 1$, $t = 0.18$.

Numerical computer calculations show that the set specified in Statement 1 is the globally Pareto-optimum set.

Remark 1: According to a procedure stated in Prasolov (1999), an importer accepts the strategy

$$y^* = \arg\max D(y, \tau)$$

at fixed τ, and then the state determines a level τ^* corresponding to y^*. Designations and assumptions of the given work imply

$$D'_y = \frac{M}{1+y} - 1 - \tau - \frac{My}{(1+y)^2} = 0.$$

And, consequently, $y^* = \sqrt{\dfrac{M}{\tau^* + 1}} - 1$, or $\tau^* = \dfrac{M}{(1+y^*)^2} - 1$. In this case, the country's profit is reached on a curve

$$S^*(\tau) = 0.18\sqrt{M(\tau+1)} + \tau\left(\sqrt{\frac{M}{\tau+1}} - 1\right).$$

The greatest value of this profit appears for $\tau* \approx 2.29$, when $M = 6$, and in this case $y^* \approx 0.35$. After substituting these values in S and D $S^* \approx 0.6$; $D^* \approx 0.4$ are received. At the same time, Pareto set provides the way to accept a conciliatory decision choosing from an interval [0, 1.7]. Rough estimates show, that S grows from 0.5 up to 2.5, and D decreases from 2.1 up to 0 at $\hat{\tau}$ increasing. In particular, for $\hat{\tau} = 0.9$, $S = 1.6$; and $D = 1$ are obtained. Thus, the result of commercial policy choice with use of Pareto set gives much best result.

Problem B solution: Appearance of the third criterion function $Y = y$ representing interests of the import countries-manufacturers, changes construction of the Pareto set within $\Omega = \{(y, \tau): y \geq 0, \tau \geq 0, D \geq 0\}$ a little.

Let us consider three non-zero vectors on a plane (y, τ), starting at a point $(\hat{y}, \hat{\tau}) \in \Omega$:

$$grad S = \begin{pmatrix} \tau - \dfrac{0.18M}{(1+y)^2} \\ y \end{pmatrix}; \quad grad D = \begin{pmatrix} \dfrac{M}{(1+y)^2} - 1 - \tau \\ -y \end{pmatrix};$$

$$grad Y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

The ends of these vectors designate a triangle on a plane (y, τ). Angles between the vectors for the point $(\hat{y}, \hat{\tau}) \in \Omega$ are calculated easily:

$$\varphi(grad\ S,\ grad\ D) = arc\cos \frac{\dfrac{M\tau}{(1+y)^2} - \tau - \tau^2 - \dfrac{0.18M^2}{(1+y)^4} + (1+\tau)\dfrac{0.18M}{(1+y)^2} - y^2}{|\ grad\ S\ | \cdot |\ grad\ D\ |};$$

$$\varphi(grad\ S,\ grad\ Y) = arc\cos \frac{\tau - \dfrac{0.18M}{(1+y)^2}}{|\ grad\ S\ |},$$

$$\varphi(grad\ D,\ grad\ Y) = arc\cos \frac{\dfrac{M}{(1+y)^2} - (1-\tau)}{|\ grad\ D\ |},$$

Statement 2[1]: If the following equality holds

$$\varphi(grad\ S,\ grad\ D)\big|_{(\hat{y},\hat{\tau})} + \varphi(grad\ S,\ grad\ Y)\big|_{(\hat{y},\hat{\tau})} + \varphi(grad\ D,\ grad\ Y)\big|_{(\hat{y},\hat{\tau})} = 2\pi,$$

then the point $(\hat{y}, \hat{\tau}) \in int\ \Omega$ belongs to the locally Pareto-optimum set.

Proof: Points *A, B* and *C* correspond to end points of vectors *grad S, grad D, grad Y*. It is clear from Figure 4, that there exist such directions from the point $(\hat{y}, \hat{\tau})$ that values of all three criterion functions can be increased. While position of the points in Figure 5 does not presuppose it. Thus, the Pareto point in local sense is conformed to a case when the point $(\hat{y}, \hat{\tau})$ is inside triangle *ABC*. In translation into «angular language» it conforms to the equality in the theorem condition.

Figure 4   The point $(\hat{y}, \hat{\tau})$ is outside the triangle
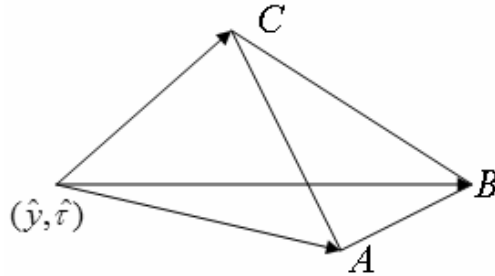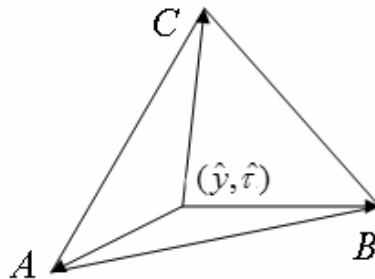

Figure 5   The point $(\hat{y}, \hat{\tau})$ is inside the triangle

Using only the definition of Pareto-optimality, O. Baskov showed that in Problem B the Pareto set looks like

$$\left\{ (y, \tau) : y \geq \sqrt{(1-t)xM / q} - x,\ \tau \geq 0,\ D \geq 0 \right\}.$$

Let us pass to problem C. As before, the basic criterion functions are defined by expressions (1) and (2), but equality (3) now includes variable *x*, this is manufacturing of a considered product inside the country, and it is defined by supply function which is considered linear here for simplicity of conclusions. In practice to form a full problem in sense of economic interpretations, it is necessary to involve statistical methods of this function identification. So, let the supply function look like

$$p = ax + b,$$

where apparently $a > 0$, $b > 0$. Substituting $x = (p - b) / a$ found from last equality in (3), it is possible to find the following equation for the internal price

$$p = \frac{b - ay}{2} + \sqrt{\frac{(b - ay)^2}{4} + aM}.$$

Here, one of two roots of the appearing quadratic equation is chosen because of $p > 0$.

As before, let us reduce the quantity of parameters, entering special measure units of money amount and sales volumes: $q = 1$ and $a = 1$. Then variable value of the price at

home market and variable quantity of product units produced in the country are defined by formulas

$$p = \frac{b-y}{2} + \sqrt{\frac{(b-y)^2}{4} + M}, \; x = -\frac{b+y}{2} + \sqrt{\frac{(b-y)^2}{4} + M}.$$

Let us substitute the obtained expressions for the price and the volume of own manufacture in criterion functions (1) to (2):

$$S = txp + \tau y = \tau y + t\left[M - \frac{y}{2}\left(b - y + \sqrt{(b-y)^{2+4M}}\right)\right],$$

$$D = \frac{y}{2}\left[b - y + \sqrt{(b-y)^2 + 4M} - 2(1+\tau)\right].$$

Let us calculate appropriate derivatives:

$$\frac{\partial S}{\partial y} = \tau + \frac{t}{2}\left[2y - b - \sqrt{(b-y)^2 + 4M} - \frac{y(y-b)}{\sqrt{(b-y)^2 + 4M}}\right], \frac{\partial S}{\partial \tau} = y;$$

$$\frac{\partial D}{\partial y} = \frac{1}{2}\left[b - 2y + \sqrt{(b-y)^2 + 4M} + \frac{y(y-b)}{\sqrt{(b-y)^2 + 4M}} - 2(1+\tau)\right], \frac{\partial D}{\partial \tau} = -y;$$

$$\frac{\partial X}{\partial y} = \frac{1}{2}\left(\frac{y-b}{\sqrt{(b-y)^2 + 4M}} - 1\right), \frac{\partial X}{\partial \tau} = 0.$$

Further analysis requires some information on parameter b which is present at the internal supply function of a product. If value b was less than 1 it would mean, that it was possible to produce enough amount of the product in the country without any problems. As the price outside the country for a product unit equals 1, than for $b < 1$ manufacturing is possible with modern technologies and absence of deficiency of production factors only. It makes no sense to increase production if $b > 4$. Therefore, it is considered that $1 < b < 4$.

As $\frac{\partial S}{\partial \tau} = y$ and $\frac{\partial D}{\partial \tau} = -y,$ it is necessary to estimate first of all the Pareto set for two criterion functions S and D. In points of this set gradients should be collinear vectors, i.e.,

$$grad \; S = \lambda grad \; D.$$

The last will hold at solutions of the following equation with regard to y:

$$2y - b - \sqrt{(b-y)^2 + 4M} - \frac{y(y-b)}{\sqrt{(b-y)^2 + 4M}} + \frac{2}{1+t} = 0.$$

Here, b, M, t are parameters. For $M = 6$ and $t = 0.18$ it is possible to establish by means of the computer program (EXCEL MS), that the given equation has the unique positive solution and it approximately linearly depends on parameter b:

$$\breve{y} \approx 0.3355b + 1.633.$$

On a plane (y, τ) condition $D > 0$ is equivalent to an inequality

$$\tau < \frac{b-y}{2} + \sqrt{\frac{(b-y)^2}{4} + M} - 1 = \breve{\tau}.$$

Since $\dfrac{\partial X}{\partial y} < 0,$ addition of the third criterion function $X = x$ leads to an estimation of the Pareto set in the form of a curvilinear trapeze $\{0 \le y \le \breve{y},\ 0 \le \tau \le \breve{\tau}\}$ depending on parameters $b, M, t.$

5   Reducing of the Pareto set

As a rule, the Pareto set is quite wide, and the selection within it is difficult. For this reason, the so-called Pareto set reducing problem arises (Noghin, 2008). It should be understood that the reducing of the Pareto set may be justified only on the basis of some additional information on the preferences. Many authors replace this information with some heuristic considerations or certain 'plausible' assumptions in order to narrow down the feasible set, and to facilitate decision-making problem by the same token.

A characteristic feature of heuristic methods is the impossibility to describe the class of problems for which this heuristic method guarantees the desired result. From this perspective, the axiomatic approach can be considered as more valid since the one strictly separates the class of problems for which it is intended, from all the others, where the use of the axiomatic approach does not guarantee a desired result.

Some authors assume that the choice should be carried out the team-members themselves after analysing of the whole Pareto set submitted to them (or some substantial part of the set mentioned). Indeed, if there is only a small number of Pareto-optimal elements (best of all – two), the choice can be made by comparison of these options and analysis of their strengths and weaknesses.

However, even in the case of two Pareto-optimal elements the choice may be complex if, e.g., the number of participants (and, accordingly, criteria) is large. When the Pareto set is fairly wide, the direct analysis of Pareto-optimal variants is difficult and a formalised procedure should be available for successful choice.

In Noghin (2008), the conventional classification of different approaches to the Pareto set reduction is proposed. One of them is the axiomatic approach, which has been developed since the early 80s by the first author of this paper. According to this approach, we have to accept four definite axioms which characterise the collective's behaviour in decision-making process as 'reasonable' (see Noghin, 2005).

Let us consider any two vectors $y' = (y_1', ..., y_m')$ and $y'' = (y_1'', ..., y_m'')$ belonging to the Pareto set. By definition of the Pareto set, there must exist two non-empty sets $A, B \subset I = \{1, 2, ..., m\}$ such as

$$y_i' - y_i'' = w_i > 0 \quad \forall i \in A;\ y_j'' - y_j' = w_j > 0 \quad \forall j \in B;$$

$$y_s' = y_s'', \ \forall s \in I \setminus (A \cup B).$$

Here, the first vector is greater than the second in components of A, while the second one exceeds the first in components of B. Other components of the two vectors are equal. If

the economic participants discard one of the two vectors (e.g., the second), this means that the first vector is more preferable to the second, i.e., $y' \succ y''$.

The relation $y' \succ y''$ is a 'quantum of information' about the preference relation of the economic participants in the decision process, which shows the willingness of members of group B to accept the loss of no more than w_j, while members of group A at the same time expect to receive allowances in an amount not less than w_i. This 'quantum of information' indicates a definite 'discrimination' of members of group B and some 'privileges' for members of group A. The given information reduces the Pareto set in one vector y''. But due to acceptance of some axioms, the reduction may be much greater. Namely, we have the following result.

Theorem 1: (Noghin, 2005). Let some four 'reasonable' axioms are accepted. The 'quantum of information' is available. Then for any set of selected variants $C(X)$ the inclusions $C(X) \subset P_g(X) \subset P_f(X)$ are valid, where the 'new' vector criterion g has the following components

$$f_i \text{ for all } i \in I \setminus B; \; g_{ij} = w_j f_i + w_1 f_j \text{ for all } i \in A, j \in B.$$

In accordance with Theorem 1, the new Pareto set $P_g(X)$ represents a more precise upper estimation for the unknown set $C(X)$ than the initial Pareto set. We should choose the 'best' variants just in $P_g(X)$.

Let us return to Problem A. It follows from (4) to (5) that for a fixed y the goal functions S and D are linear, their slopes are numbers with the opposite signs. Therefore, if in accordance with Proposition 1 to fix $y = \bar{y}$, then there appear two linear functions of the variable τ with slopes equal to $\bar{y}$ and $-\bar{y}$, respectively.

Let us assume that after negotiations the state, having some means of pressure to the importer, forces it to agree to an assignment in an amount not exceeding amount $w_D > 0$ in D, while the state hopes to increase the value of criterion S at least in $w_S > 0$. This is a certain 'quantum of information'. Due to Theorem 1, it is necessary to form a new bi-criterion problem with objective functions S and $w_D S + w_S D$, and after that to find the corresponding new Pareto set. In the case $w_D = w_S$, the slope of $w_D S + w_S D$ is equal to 0. The Pareto set with respect to criteria S and Const will coincide with the set of maximum points for S on the interval specified in Proposition 1, i.e., with the point

$$\bar{y} = \sqrt{(1-t)xM/q} - x, \; \bar{\tau} = \sqrt{\frac{M}{(1-t)xq}} - 1.$$

The same result is obtained in the case of $w_D > w_S$, since the slope of $w_D S + w_S D$ will have the same sign as that of the function S. If $w_D > w_S$, then the new Pareto set will coincide with the initial one. In the last case, this 'quantum of information' does not allow to narrow down the Pareto set.

Similarly, it is possible to consider the symmetric situation when the importer has means of pressure to the state, and it agrees to accept the loss of no more than w_S, and thus has 'no objection' against the importer obtaining the gain of at least w_D. In this case, the new bi-criterion problem with the functions $w_D S + w_S D$ and D appears. If $w_D \leq w_S$ then the Pareto set consists of a single point $\bar{y} = \sqrt{(1-t)xM/q} - x$, $\bar{\tau} = 0$, whereas the Pareto set reducing does not occur for $w_D > w_S$.

Let us consider Problem *B* with three criteria *S*, *D* and *Y*. Theoretically, there may be two cases

1   the 'quantum of information' concerns two criteria

2   the 'quantum of information' concerns three criteria.

For example, in case (2) the state may have an instrument of pressure, forcing the importer make certain concessions in the criterion *D*; while the state itself expects to get some increase in values of the criteria *S* and *Y*. Having some 'quantum of information', we can consider this kind of information to reduce the Pareto set by using Theorem 1.

Now let, e.g., the state and the importer be ready to concede by criterion *Y* no more than one unit to increase not less than 0.3 in criteria *S* and *D*, i.e., $w_S = w_D = 0.3$, $w_y = 1$. Then, as O. Baskov showed, the corresponding Pareto set is described as follows

$$\left\{ (\tau, y) \,\Big|\, \sqrt{\frac{(1-t)Mx}{q}} - x \le y \le \sqrt{\frac{(1-t)Mx}{q-0.6}} - x,\, \tau \ge 0,\, D \ge 0 \right\}$$

Let us consider Problem B. As it has been specified earlier, here the Pareto set is a curvilinear trapeze $\{(y,\ \tau)\,|\,0 \le y \le \bar{y},\ 0 \le \tau \le \bar{\tau}\}$, where $\bar{y} \approx 0.3355b + 1.633$ and

$$\bar{\tau} = \frac{b-y}{2} + \sqrt{\frac{(b-y)^2}{4} - M} - 1.$$

Let the state can compel the importer to lose in criterion *D* (the profit of the importer) not exceeding $w_D > 0$ units in order to increase the value of criterion *S* (tax revenues in the budget) not less than $w_S > 0$ units. According to Theorem 1, to construct the new Pareto set it is necessary to deal with a problem with three criteria *S*, $w_D S + w_S D$, and *x* (assuming that the development of its own industry and, in particular, quantity of workplaces have less priority, than budgetary receipts). In opposite case, when the state is ready to make a concession in w_S, and the importer thus receives an additive at least in w_D units, there is a multicriteria problem with three criteria $w_D S + w_S D$, *D*, and *x*.

One more variant turns out, when the state is ready to lose a quantity of w_x units in internal manufacture in order to increase the value of *S* not less than w_S. Then there appear the following three goal functions *S*, *D*, and $w_x S + w_S x$. If the sate is concerned with workplaces and the development of its own manufacture, then criteria functions *x*, then *D*, $w_x S + w_S x$, and *x* should be considered.

Besides, in Problem C (as in Problem B) situations when concessions (prize) occur on any pair of criteria from available three *S*, *D* and *x* are theoretically possible.

Let us choose the following as an illustrative example with concrete numerical characteristics of Problem C. Let $M = 6$; $t = 0{,}18$; $b = 3$; $w_x = 1$; $w_S = 2$. Then

$$S = \tau y + 0.18\left[6 - \frac{y}{2}\left(3 - y + \sqrt{(3-y)^2 + 24} \right) \right],$$

$$D = \frac{y}{2}\left[3 - y + \sqrt{(3-y)^2 + 24} - 2(1+\tau) \right].$$

$$G = \left\{ \tau y + 0.18\left[6 - \frac{y}{2}\left(3 - y + \sqrt{(3-y)^2 + 24} \right) \right] \right\} + 2\left\{ -\frac{3+y}{2} + \sqrt{\frac{(3-y)^2}{4} + 6} \right\}.$$

To construct the Pareto set it is necessary to apply Statement 2. As a result following set is obtained

$$\left\{ y \in [1.0525; 2.6395], \tau \in \left[0, \frac{1}{2}\left(1 - y + \sqrt{(3-y)^2 + 24}\right)\right]\right\}.$$

Obviously, this set is essentially narrower than the initial one.

6   Conclusions

The paper deals with the problem of determining the import duties for a product that is also produced within the country. There are several economic agents: the customer government, the government of the importing country, the importers and manufacturers in the country. All of them have their own interests, which are formally expressed in the form of objective functions (profits or income), so this problem is multi-objective. The compromise domain (Pareto set) is constructed for three types of multicriteria problems, three options for the interests of various economic agents. Using additional assumptions on the behaviour of domestic consumers and producers, the Pareto set is considered for different purposes of economic agents. Several calculations were made and illustrative examples were considered in the given paper.

Particular attention is paid to the Pareto set reducing based on additional information about the preferences of economic agents.

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Notes

1   This statement also means, that, the cone, built by three vectors-gradients, contains the origin of coordinates as the internal point.