# ESTIMATION OF ELASTIC CRACK OPENING DISPLACEMENT FOR THIN ELBOWS WITH CIRCUMFERENTIAL THROUGH-WALL CRACKS 

Min-Kyu Kim<br>Sungkyunkwan University, Suwon, Republic of Korea

Han-Bum Surh<br>Korea Atomic Energy Research Institute Daejeon, Republic of Korea

Min-Gu Won<br>Sungkyunkwan University, Suwon, Republic of Korea

Nam-Su Huh<br>Seoul National University of<br>Science and Technology, Seoul, Republic of Korea

Moon-Ki Kim<br>Sungkyunkwan University, Suwon, Republic of Korea

## ABSTRACT

Leak-before-break (LBB) is an important concept that could confirm design and integrity evaluation of nuclear power plant piping. For the LBB analysis, the detective leakage rate should be calculated for a through-wall cracked pipes. For this calculation, the crack opening displacement (COD) calculation is essential

Recently, sodium faster reactor (SFR) which has thin-walled pipes with $R_{m} / t$ ranged 30-40 was introduced and then the investigation of these thin walled pipes and elbows has received great attention in the LBB evaluation. In this context, the threedimensional finite element (FE) analyses for thin elbows with circumferential crack under in-plane bending are carried out to investigate the elastic COD values.

Finally, the solution for elastic COD which can cover sufficiently thin elbow is successfully addressed.

## NOMENCLATURE

$a \quad$ Half crack length for circumferential through-wall crack
$E \quad$ Young's modulus
$M \quad$ Global bending moment
$R_{b} \quad$ Bend radius of pipe and elbow.
$R_{i} \quad$ Inner radius of pipe and elbow
$R_{m} \quad$ Mean radius of pipe and elbow
$R_{o} \quad$ Outer radius of pipe and elbow
$t \quad$ Wall thickness of pipe and elbow
$V \quad$ Shape factor for COD
$V_{s} \quad \mathrm{~V}$ of the straight pipe
$V_{e} \quad \mathrm{~V}$ of the elbow pipe
$\delta \quad$ COD
$\theta \quad$ Half crack angle
$v$ Poisson's ratio
$\phi \quad$ Angle from extrados to intrados of elbow
$\lambda \quad$ Dimensionless geometric parameter for elbow
$\delta_{e} \quad$ Elastic COD
$\sigma^{\infty} \quad$ Remote stress

## INTRODUCTION

An elbow, used in various industries, is one of the most important parts at the piping system. Because of geometrical shape, elbow has higher probability of defects occurrence than straight pipe. If the pipe is damaged, it can lead enormous economic losses in the industrial fields. Moreover, in the NPP(Nuclear Power Plant), flaws in the piping system can cause a leak of the radioactive chemicals. Therefore, it is necessary to apply proper assessment techniques to ensure the integrity of piping system at nuclear power plant[1].

LBB[2] is a major assessment technique for the piping system. LBB assessment is the concept that if detectable through wall crack length in the pipe is smaller than critical crack length, a large rupture accident can be prevented. Before the LBB introduced, the DEGB(Double Ended Guillotine Break), one of the extreme accident case at the NPP was prevented by additional structures like pipe whip restraint and jet impingement shield. However these structures over constrained the pipe and additional restraint stress could occur due to them. In order to
secure the safety of the NPP piping system, the LBB could be applied to the pipe. Moreover, as the elbow has more severe loading condition than the straight pipe, applying the LBB to the elbow can improve of the safety of the piping system. For applying the LBB to elbow pipe, it is needed to define the sensible leakage amount of the coolant. The leakage can be predicted from the $\operatorname{COD}$ (Crack Opening Displacement) value of the through wall crack[3].

Currently, the research on LBB in the case of the PWRs(Pressurized Water Reactors), forming a significant part of the Korea NPPs, have been performed frequently, and the technique has been well established. However, since it has different characteristics in terms of operating conditions, structural shape and materials compared to PWRs, the current LBB concept is hard to apply at the SFR(Sodium Fast Reactor)[4].

SFR is one of the fourth-generation (Gen IV) nuclear power plants, which uses the liquefied sodium as the primary coolant fluid. Because of their high level of safety, high fuel efficiency and reduced level of nuclear wastes, SFR is regarded as one of the most promising candidates of next generation NPP in Korea nowadays. SFR has special features as $500^{\circ} \mathrm{C}$ or higher operating temperature and a low operating pressure around the atmospheric pressure. Therefore, the piping system of the SFR has a very thin wall thickness compared to the existing commercial PWRs. The studies on thin walled pipe structure were not carried out and conventional studies limited in specific thickness pipes. Therefore, more research on the fracture characteristics of the thin walled pipe need to be performed[5].

In this paper, elastic COD in thin elbow pipe was examined. The elbow is selected because of its higher probability for occurrence of flaws. Considering elbow pipes with various thickness and circumferential cracks with various lengths, systematic three-dimensional finite element analyses were conducted. Based on the results of the analyses, fracture mechanics parameter prediction method was investigated. By quantifying the effect of the thin walled pipe, elastic fracture mechanics parameters prediction of the shape of closed form and tabulated form are presented. The results of this research are expected to be useful in the LBB application of high temperature reactors such as SFR.

## FINITE ELEMENT ANALYSIS

FE analyses are performed to estimate elastic COD for circumferential through-wall cracked elbows. The schematic geometry for analysis is expressed in Fig. 1. $R_{b}, R_{o}, R_{m}, R_{i}$ and $t$ denote the bend radius, outer radius, mean radius, inner radius and the wall thickness of the pipes, respectively. The half crack length, $a$, is determined by half crack angle, $\theta$. In this study, the dimensionless geometric parameter for elbows, $\lambda$ is used as denoted in equation (1).

$$
\begin{equation*}
\lambda=\frac{R_{b} / R_{m}}{R_{m} / t} \tag{1}
\end{equation*}
$$

In this paper, six values of $R_{m} / t$ which cover sufficiently thin walled pipes, five values of $\theta / \pi$ which cover practical range of crack length, and six values of $R_{b} / R_{m}$ are assumed as geometry parameters. Note that the in-plane bending has two modes depending on bending direction, closing and opening bending. The summary of the geometry parameters in linear elastic analysis for the circumferential through-wall cracked elbows are listed in Table 1.

The FE (Finite Element) analyses for examining the elastic COD are performed by using ABAQUS[6], one of the commercial multi-purpose FE analysis program.


Figure 1 Schematic geometry of elbows with circumferential through-wall cracks

Table 1 Summary of geometry parameters

| $R_{m} / t$ | $R_{b} / R_{m}$ | $\theta / \pi$ | Loading |
| :---: | :---: | :---: | :---: |
| $5,10,20$, | $2,3,4$, | $0.0625,0.125,0.25$, | In-plane |
| $30,40,50$ | 5,15 | $0.375,0.5,0.6$ | Bending |

The typical FE models of a circumferential cracked elbows are depicted at Fig. 2. To reduce necessary computational resources, quarter model was considered by using symmetry condition of the model. In order to avoid the hourglassing problem at the crack tip, causing a singularity, the 20 -node iso-parametric quadratic brick element with reduced integration(type C3D20R in ABAQUS element library) is utilized[6]. The number of entities are 2,396 and 12,639 as for elements and nodes,
respectively. The materials were assumed to be isotropic elastic with Young's modulus $E$ and Poisson's ratio $v$. The values of $E$ $=200 \mathrm{GPa}$ and $v=0.3$ were used as genetic steel materials in this research.

For the global bending, the nodes on the pipe end surface are constrained to a reference node located at center of the pipe by using multi-point constraint(MPC) option in ABAQUS. By using the MPC option, the researcher can easily apply the boundary and loading conditions to the connected slave nodes with the reference node. The corresponding bending moment is applied to the reference node. The symmetry conditions are applied each of the sliced surface except for the crack surface and a point at the opposite side of the crack is fixed as pin point to prevent translation rigid body motion. The COD is able to be extracted directly from nodal displacement at mid thickness point at the center of the crack[7].


Figure 2 Typical FE model of circumferential cracked elbow

## DETERMINATION OF CRACK OPENING/CLOSURE

Specific behaviors are observed on the circumferential cracked elbow subjected to in-plane bending. Intuitively, the circumferential crack located at intrados crack of elbow is opened under in-plane opening bending. On the other hand, if the circumferential crack is located at extrados crack of elbow, it seems that the crack is open under in-plane closing bending.

However, in some cases the crack does not open in this manner, and the analysis results are different from the intuitive results. The elastic stress distribution of uncracked elbow subjected inplane closing bending is shown in Fig. 3. As shown in right side of Fig. 3, the closing bending moment lead to the axial tension at the extrados and axial compression at the intrados. On the other hand, the net section of elbow is ovalized due to the opening bending moment. Because this ovalization stretch the intrados crack, it causes the compressive stress along the perpendicular direction to the net section plane due to Poisson's contraction. In the left side of Fig. 3, the gray region and colored region represent the stress state under axial compression and tension, respectively. If the crack is postulated in the gray region, the crack closure can occur due to ovalization-induced axial compression[8].

In order to examine the crack behavior in opening or closing modes, elastic FE analysis was performed for the circumferential cracked elbow at intrados crack and extrados crack. Elastic analysis results for cracked elbow subjected to in-plane bending are shown in the following Fig. 4. The crack open displacement along the crack length is denoted $\delta$ and maximum $\delta$ is $\delta_{\max }$. As a result, when the $R_{m} / t$ is small, crack closure does not occurred. In contrast, when the $R_{m} / t$ is large, crack closure occurs. This means, when the thick walled elbow can endure bending moments than the thin walled elbow. As the elbows in the SFR is thin model, the crack closure is not expected on inplane moment condition.


Figure 3 Elastic stress distribution of the elbow loading in-plane bending

(a) Small $R_{b} / R_{m}$

(b) Large $R_{b} / R_{m}$

Figure 4 Typical crack opening and closure behavior

## ESTIMATION OF ELASTIC COD

The elastic COD for circumferential through-wall cracked elbows can be expressed as follow $\mathrm{Eq}(2)[9]$ :

$$
\begin{equation*}
\delta_{e}=\frac{4 \sigma^{\infty} R_{m} \theta}{E} V \tag{2}
\end{equation*}
$$

Where $\delta_{e}$ is elastic COD from FE analysis, $R_{m}$ is mean radius, $\theta$ is half crack angle, $V$ is elastic COD shape factor and $\sigma^{\infty}$ is remote stress. The remote stresses corresponding to applied loads for the pipes were determined as

$$
\begin{equation*}
\sigma^{\infty}=\frac{M}{\pi R_{m}^{2} t} \tag{3}
\end{equation*}
$$

Therefore, the proposed solution is expected to estimate the elastic COD for thin-walled pipes with circumferential throughwall cracks. From equation (2), the value of $V$ for circumferential through-wall crack can calculated. Calculated shape factor equation for COD is represented as followed equation (3). The applicability range of the proposed equations is $R_{m} / t=5 \sim 50$ and $\theta / \pi<0.6$.

## RESULT

From the elastic FE analyses, each parameters are found. The values of $V$ are obtained as listed table 2 and $3 . V$ values are expressed equation (4). In order to focusing on the $V$ values, equation (2) is changed to equation (4).

$$
\begin{equation*}
V=\frac{\delta_{e}}{4 \sigma^{\infty} R_{m} \theta} \tag{4}
\end{equation*}
$$

$V_{e}$ is $V$ of the elbow pipe and $V_{s}$ is $V$ of the straight pipe.

The horizontal axis is $\lambda$ and the vertical axis is $V_{e} / V_{s}$. The graphs are organized by each of $R_{m} / t$ and $\theta / \pi$, respectively. Fig. 5 shows the variation of $V_{e} / V_{s}$ values according to geometric conditions. As depicted in Fig. 5(a), the value of $V_{e} / V_{s}$ goes close to 1 with increasing of the $\lambda$ value. And the $V_{e} / V_{s}$ goes to 1 at the fixed $\theta / \pi$ condition, as shown in Fig 5(b). Please note that even the $V_{e} / V_{s}$ results are converged to the 1 , there is less effect of the $R_{m} / t$ compared with the effect of the $\theta / \pi$. Since $V_{e} / V_{s}$ is close to 1 as the $\lambda$ increasing, it is shown that the $V_{e}$ of the elbow is converged to the $V_{s}$ of the straight pipe.


Figure $5 \quad$ Variation in $\mathrm{V}_{\mathrm{e}} / \mathrm{V}_{\mathrm{s}}$ along to the $\lambda$ of circumferential through wall cracked elbow

Table 2 Tabulated V values for the elbows with circumferential through-wall cracks at intrados under in-plane opening bending

| $R_{m} / t$ | $\theta_{l} / \theta_{2}$ | $R_{b} / R_{m}$ |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 4 | 5 | 15 |
| 5 | 0.125 | 0.595868 | 0.865899 | 0.976069 | 1.033129 | 1.172173 |
|  | 0.250 | 1.438065 | 1.526383 | 1.55507 | 1.57479 | 1.687185 |
|  | 0.375 | 2.913398 | 2.849858 | 2.772106 | 2.726647 | 2.729886 |
|  | 0.500 | 6.151493 | 5.754817 | 5.466853 | 5.285014 | 4.980041 |
|  | 0.600 | 11.87441 | 10.88161 | 10.26764 | 9.882278 | 9.091957 |


| 10 | 0.125 | CC | 0.162382 | 0.43384 | 0.625444 | 1.191499 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.250 | 0.829796 | 1.100385 | 1.287309 | 1.419696 | 1.938405 |
|  | 0.375 | 3.22666 | 3.326216 | 3.295592 | 3.262273 | 3.441369 |
|  | 0.500 | 8.920412 | 8.438693 | 7.918264 | 7.50106 | 6.649449 |
|  | 0.600 | 20.21501 | 17.89961 | 16.29746 | 15.15051 | 12.40499 |
| 20 | 0.125 | CC | CC | CC | CC | 1.004149 |
|  | 0.250 | CC | 0.081792 | 0.349899 | 0.612834 | 2.034295 |
|  | 0.375 | 3.270735 | 3.559595 | 3.589684 | 3.607738 | 4.212538 |
|  | 0.500 | 12.96709 | 12.78204 | 12.14565 | 11.44607 | 9.137446 |
|  | 0.600 | 36.91612 | 32.64717 | 29.52538 | 27.00538 | 18.23615 |
| 30 | 0.125 | CC | CC | CC | CC | 0.642404 |
|  | 0.250 | CC | CC | CC | CC | 1.775236 |
|  | 0.375 | 2.934107 | 3.421425 | 3.4724 | 3.502985 | 4.508207 |
|  | 0.500 | 15.7359 | 16.20997 | 15.61357 | 14.77554 | 11.20017 |
|  | 0.600 | 52.93728 | 46.935 | 42.67861 | 39.23781 | 24.13654 |
| 40 | 0.125 | CC | CC | CC | CC | 0.23162 |
|  | 0.250 | CC | CC | CC | CC | 1.344281 |
|  | 0.375 | 2.332826 | 3.091419 | 3.203974 | 3.247146 | 4.574683 |
|  | 0.500 | 17.70435 | 19.13827 | 18.68728 | 17.73904 | 13.10191 |
|  | 0.600 | 68.83418 | 61.18272 | 55.66096 | 51.39195 | 30.39381 |
| 50 | 0.125 | CC | CC | CC | CC | CC |
|  | 0.250 | CC | CC | CC | CC | 0.867892 |
|  | 0.375 | 1.549497 | 2.629187 | 2.844877 | 2.921732 | 4.579074 |
|  | 0.500 | 19.04022 | 21.74564 | 21.50392 | 20.49055 | 15.12619 |
|  | 0.600 | 84.83497 | 75.67744 | 68.68921 | 63.56037 | 37.62179 |

Table 3 Tabulated V values for the elbows with circumferential through-wall cracks at extrados under in-plane opening bending

| $R_{m} / t$ | $\theta_{1} / \theta_{2}$ | $R_{b} / R_{m}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 4 | 5 | 15 |
| 5 | 0.125 | 0.383555 | 0.600006 | 0.774823 | 0.895119 | 1.125835 |
|  | 0.250 | 0.940751 | 1.180308 | 1.366678 | 1.488479 | 1.680131 |
|  | 0.375 | 2.075892 | 2.288665 | 2.456814 | 2.563932 | 2.70909 |
|  | 0.500 | 4.274631 | 4.426686 | 4.551779 | 4.638074 | 4.794283 |
|  | 0.600 | 8.386547 | 8.295105 | 8.308168 | 8.36042 | 8.589026 |
| 10 | 0.125 | CC | 0.003764 | 0.258047 | 0.49913 | 1.193494 |
|  | 0.250 | 0.286365 | 0.671216 | 1.062794 | 1.380556 | 2.091007 |
|  | 0.375 | 2.11406 | 2.505343 | 2.868123 | 3.14408 | 3.637154 |
|  | 0.500 | 5.681709 | 5.92687 | 6.086009 | 6.21074 | 6.447299 |
|  | 0.600 | 12.96413 | 12.03582 | 11.53761 | 11.33689 | 11.28672 |
| 20 | 0.125 | CC | CC | CC | CC | 1.132782 |
|  | 0.250 | CC | CC | 0.287443 | 0.276073 | 2.628578 |
|  | 0.375 | 1.419625 | 1.974447 | 2.67785 | 3.294806 | 5.106198 |
|  | 0.500 | 7.142508 | 7.963953 | 8.428837 | 8.645482 | 9.087798 |
|  | 0.600 | 21.28836 | 19.44538 | 18.01048 | 17.06642 | 15.51117 |
| 30 | 0.125 | CC | 0.66871 | 0.91112 | CC | 0.832581 |
|  | 0.250 | CC | 1.98242 | 1.72171 | 1.1666 | 2.711041 |
|  | 0.375 | 0.419685 | 0.857334 | 1.767254 | 2.687389 | 6.112892 |
|  | 0.500 | 7.441294 | 8.920071 | 9.937018 | 10.43096 | 11.25217 |
|  | 0.600 | 28.74976 | 26.252 | 24.04734 | 22.47986 | 19.19199 |
| 40 | 0.125 | 0.15193 | CC | 0.86507 | CC | 0.394635 |
|  | 0.250 | CC | CC | 2.88866 | CC | 2.391417 |
|  | 0.375 | CC | CC | 0.494257 | 1.652868 | 6.737905 |
|  | 0.500 | 6.938346 | 9.076513 | 10.79954 | 11.69111 | 13.10191 |
|  | 0.600 | 35.5012 | 35.591 | 29.63252 | 27.44501 | 22.61708 |
| 50 | 0.125 | 0.34382 | CC | CC | CC | CC |
|  | 0.250 | CC | CC | CC | CC | 1.788572 |
|  | 0.375 | 1.578008 | 1.984596 | 1.00064 | 0.335894 | 7.073144 |
|  | 0.500 | 5.758551 | 8.53556 | 11.0913 | 12.52304 | 14.69853 |
|  | 0.600 | 41.48004 | 38.42753 | 34.80171 | 32.03585 | 25.85336 |

## CONCLUDING REMARKS

In this research, elastic COD of thin-walled elbows with through-wall cracks are evaluated through linear elastic FE analyses. The effects of geometric parameters on elastic COD are systematically investigated based on FE analyses results. While investigating the $V$ values, the behavior of circumferential crack at elbows were examined. As the results, the crack opening can be occurred even in the elbow at the thin walled pipe condition. Through comparing the $R_{b} / R_{m}-\lambda$ values, the validity of the result at this research is confirmed. By comparing $\theta / \pi-\lambda$ values, it was verified that $V_{e}$, the $V$ of elbow is converged to $V_{s}$, the $V$ of straight pipe. The results of this research are quite predictable with the suggested equation and tabulated $V$ values. Therefore, the result of this research is expected to be useful in the LBB application of high temperature reactors with thin walled piping systems.

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