The Tradeoff of the Commons Under Stochastic Use

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Abstract

We develop a model of scarce renewable resources to study the commons problem. Our model formulation differs from the existing literature in that the use of the commons is assumed to be stochastic in nature. One example is the microwave spectrum for mobile and wireless communications. We investigate three mechanisms of resource allocation: free usage, the exclusive franchise, and a regulated monopoly. We show that the welfare tradeoff among them depends on the commons' characteristics and usage patterns. In particular, we find that property rights are not always the best solution. We then make three extensions that apply to spectrum allocations.

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1 Introduction

The problem of allocating exhaustible public resources has been extensively studied in the literature, initiated by Gordon (1954) and Scott (1955). Its organizational form known as the commons is generally considered a tragedy (Hardin, 1968), because it lacks a mechanism to prevent selfish overuse. Instead, property rights are generally believed to offer an efficiency solution. Renewable public resource, however, brings about a different set of perspectives in that

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the objective of an imaginative welfare-maximizing social planner is to maximize usage, and more precisely to maximize the value of the usage, while at the same time preventing overuse. Using a scheduling mechanism, McAfee and Miller (2012) first showed that a commons can be more efficient than property-rights-based solutions, and there exists a tradeoff between the two types of organizational forms. In their model the resources are excludable and indivisible. This means that not only a fixed number of customers may be accommodated at any one time, but the identity of these customers also has to be fixed during the time of resource use.

In this paper, we extend their conclusions to an alternative situation where a renewable resource is subject to use in a stochastic manner. On the surface, this seemingly small modification may appear trivial, but, in fact, introduces several important differences that have profound implications for seeking efficient allocation mechanisms. First, in our model the resource is excludable but divisible in that its usage is multiplexed into its designed capacity. The resource is excludable because its quality can degrade as the number of users increases. But it is divisible because the actual number of users and, more importantly, the identity of its users can vary from time to time.

In a commons scenario, this suggests that scheduling is neither critical nor practical. It is not critical because, unlike the McAfee and Miller (2012) model which is concerned with loss from coordination failure, here the downside of a service denial in the absence of scheduling does not incur much cost. This is because each demand is treated equally by the resource capacity regardless of the current status of the demand, and it will have equal access probability in the future. Scheduling is not practical, since usage is stochastically session-based such that the overhead cost for establishing every single session would be too great. Without scheduling, the commons essentially becomes a self-regulating system where equilibrium can be reached automatically among users when the resource gets too crowded. Some people may prefer leaving while others choose to stay, accepting the current service quality. The social welfare of this equilibrium varies randomly from time to time however, and in particular is certainly not maximized. In other words that the resource is allocated to those who value it most is clearly a zero probability event. On the contrary, allocative efficiency is certainly assured under property-rights solutions, albeit at an additional cost. It is precisely this tradeoff that lies at the heart of our analysis.

Our second difference from McAfee and Miller (2012) is that we assume that service quality is expected to be better under property-rights solutions than under the commons form. Technically our model allows for more people accessing the resource capacity, which is also called overbooking in network engineering.[1] In general, overbooking can be accommodated provided the probability

[1]McAfee & Miller (2012) is concerned about the loss from not using the resources. In our model. this problem

of service denial, or the blocking probability in technical terms, is managed within an acceptable level. For example, a short delay for free Internet access usually causes no concern at public Wi-Fi hotspots.[2] Since we endogenize pricing in our model, one would expect that paying customers naturally have a lower level of tolerance for blocking than those who have free use of the Internet.

Our model description probably applies most appropriately to the case of wireless spectrum. Rapid growth in wireless communications has increased the pressure for more spectrums to support more users, more uses and more capacity. To alleviate that pressure, major regulatory changes were introduced in some countries regarding spectrum allocation. Today some radio spectrums are indeed offered free of charge in many countries, for example Wi-Fi applications, but other spectrums are still allocated by defined property rights, such as via spectrum auctions.[3] We develop an analytical framework to provide a likely theoretical explanation as to why that might be the case by looking at the network characteristics of spectrum bands.

We establish three base models to analyze welfare implications of these mechanisms in regulating spectrum usage: a commons model, a model of an unregulated monopoly and a model of regulation. As expected, the regulation model is socially better off than the monopoly model, and the tradeoff between the commons and the regulation model hinges upon two major factors among other things. First, if the fixed cost of implementing a pricing mechanism is large, free use dominates the regulation model. Second, if the maximum acceptable blocking probability is large, free use dominates the regulation model. In other words, property-rights solutions only matter when people care a lot about the service quality or when the costs of implementing such solutions are relatively low.

We also make three extensions to the base models. We analyze the tradeoff between free use and the regulation model as a function of the cell-site coverage size, and find that the tradeoff condition that favors free use under the macro-cell architecture must also hold under the micro-cell condition. This means that free use generally favors spectrum bands of shorter reach, such as Wi-Fi and Bluetooth. The second extension concerns mixed service where the free-use traffic is mixed with the traffic of paying customers who enjoy priority with guaranteed quality of service. This may be viewed as a primitive version of the cognitive radio or the software-defined radio that is currently being actively developed. We find that the mixed-service model,

does not exist as we consider the case of overbooking. Telecommunication networks are usually designed for peak-time usage, and overbooking is commonly used when congestion is expected.

[2]When bits are transmitted in sessions under the TCP/IP protocol, blocking manifests in the form of longer delay.

[3]For a comprehensive review of spectrum auctions, see McAfee and McMillan (1996).

where a certain amount of free use is accommodated, always dominates the case where only paying customers have access. The third extension concerns channel bonding, which is another important technology feature of the cognitive radio that is currently under development. We investigate whether social welfare is enhanced when spectrum bands are combined together to deliver service as opposed to being used separately under the current regulatory regime. We show that this is always the case as channel bonding exhibits a kind of economy of scale property. This is because the efficiency gain from statistical multiplexing is likely to increase as the spectrum width increases.

These results point to some important implications for the spectrum policy debate. Our results generally support a phased approach to opening up more spectrums, perhaps starting with spectrum bands with shorter-range coverage, such as Wi-Fi and Bluetooth. Our results also support tiered services where prioritized paying traffic and free-use traffic can be multiplexed. This is essentially the most important feature currently being developed under cognitive radio or software-defined radio technologies.

The spectrum policy literature is quickly expanding, especially tangential to the economics profession. Noam (2012) provides a historical review of the evolution of economists' views on radio spectrum. Starting with Coase (1959), economists have favored property-rights solutions with regard to spectrum allocation. In more recent years, a significant debate has emerged over whether the government should make the spectrum open and free. Benkler (1998), Noam (1998) and Benkler (2002) advocate for an open spectrum based on the prediction that technology advancement will end the scarcity situation in terms of airwaves. Even in the event of scarcity, the spectrum should remain open but it should be priced. Minervini (2013) analyzes spectrum management deregulation reforms within the theoretical framework of transition economics, and shows how Anglo-Saxon and European countries have been implementing gradual reforms while Central America has opted for a fast transition to market mechanisms.

Hazlett (1998), Brennan (1998), and Cave and Webb (2004) argue that spectrum is still scarce, and they believe that a regime of open but priced access would impose prohibitive transaction costs. Hazlett (2008) advocates for abolishing the control of the Federal Communications Commission, thus permitting any wireless operations to exist within an owner's frequency space. Hazlett and Munoz (2009) investigate the relationship between spectrum policy and efficiency in the output market in a cross-country study of 28 mobile telephone markets. We compliment this strand of literature by pointing out the insight that not all frequency bands are created equal, and that a socially optimal policy depends on the network characteristics of spectrum bands. A better policy would be to take a hybrid approach where a combination of paying and free traffic is accommodated at the same time.

There has also been a separate small literature in recent years at the intersection of economics and telecommunications engineering field, which is devoted to studying implications of new radio technologies, such as cognitive radio or software-defined radio, on spectrum management policies. See for example Bastidas and Stine (2013) and Yuguchi (2012). Our second and third extensions to the base models essentially try to present a simplified primitive version of cognitive radio, and thus also contribute to this strand of literature as well.

The rest of the paper is organized as follows. We start by introducing the three base mechanisms: the commons, monopoly and regulation, detailing the tradeoffs among them. Section 3 uses the wireless spectrum example to investigate the three extensions to the base models. Concluding remarks and discussions of some policy implications are in Section 4.

2 Base Model

When people start to use the commons, their usage is stochastic in nature. This means that they are not using the resource all the time, and that there is a certain degree of statistical multiplexing gain. This type of stochastic usage can be analyzed with many kinds of sophisticated traffic models. But our use of a simple Bernoulli distribution to characterize usage preserves the economics implications of the tradeoff among the different resource allocation mechanisms without getting into the traffic engineering complexities. Formally let a person's usage y_i, $i = 1, 2 \ldots, N$, be an i.i.d. Bernoulli random variable taking on the value of 1 with probability q or 0 with probability $1 - q$. 1 means the user is actively using the services and 0 otherwise.

Suppose a commons has a capacity of M with $M < N$. Then a blocking probability exists when the commons supports n people for $M < n \leq N$. People can tolerate some level of blocking when the blocking probability is small, but they will simply give up when the blocking is too large. One may refer to this blocking probability as a quality of service index. Generally in telecommunications networks the blocking probability is very small. For example, in a telephone system, the blocking probability is usually less than 1%. This means $nq < M$, since the blocking probability is less than 0.5; i.e., the average aggregate usage must be less than the designed capacity.

When there are n persons using the service, the summation of n Bernoulli random variables, $Y^n = \sum_{i=1}^{n} y_i$, can be approximated by a normal distribution function according to the Central Limit Theorem. Then the blocking probability α, that is the probability of $Y^n \geq M$, is regulated by the following,

$$\alpha = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{M-\mu}{\sigma}} e^{-\frac{t^2}{2}} dt \tag{1}$$

where

$$\mu = E(Y^n) = E(\sum_{i=1}^{n} y_i) = \sum_{i=1}^{n} E(y_i) = nq,$$

$$\sigma = \sqrt{V(Y^n)} = \sqrt{V(\sum_{i=1}^{n} y_i)} = \sqrt{\sum_{i=1}^{n} V(y_i)} = \sqrt{\sum_{i=1}^{n} q(1-q)} = \sqrt{nq(1-q)}.$$

Letting $G(.) = F(.)^{-1}$ denote the inverse function of the norm distribution function $F(.)$, of which its value at $1-\alpha$ can be found in a standard normal distribution table, we can solve for n as

$$n = \frac{M}{q} + \frac{1}{2q}[G(1-\alpha)\sqrt{4M(1-q) + G(1-\alpha)^2(1-q)^2} + G^2(1-\alpha)(1-q)]. \tag{2}$$

Note the second part of the right hand side of (2) is always positive. This equation determines a one-to-one relationship between α and n. Apparently the larger the n, the smaller the upper bound of the integration set, and thus the larger the α. However, people cannot tolerate too large a blocking probability, as the commons resources would be of too poor quality to be of meaningful use. Let us suppose α^f is the maximum blocking probability that people can tolerate under free use. For simplicity, let us further suppose that α^f is valued such that the solution to n is an integer number.[4] Since people are selfish in usage, the commons would be congested to the largest extent possible. Therefore $n(\alpha^f)$ defines the equilibrium number of people in use of the services.

People's utility of using the commons can be specified by introducing a profile vector, $X = [x_1, x_2, \ldots, x_N]$, with its elements forming a decreasing series. That is, $x_i > x_j$ whenever $i < j$, $\forall i, j$. The ith person's utility from using the commons is simply specified as x_i. The higher this value, the more utility this person derives from the usage of the commons. Since $n(\alpha^f) \leq N$, how these $n(\alpha^f)$ people are selected from the entire population becomes an issue, depending on the resource allocation mechanism under use. We investigate three scenarios in the rest of this section:

- Commons - The allocation of the commons resources is random and everyone has an equal probability of use regardless of his utility value.

[4]To be rigorous, a floor function needs to be applied to n to arrive at the actual number of people the commons is able to be support.

- Monopoly - A monopolist charges a profit maximizing price p^m, and those people whose usage utility x_i exceeds p^m become customers.
- Regulation - A regulated firm charges a price p^r such that it makes zero profit. Those people whose usage utility x_i exceeding p^r become customers.

Case 1: Commons

Under the commons mechanism, everyone has an equal opportunity to use the commons even though their usage utilities are different. Mathematically, define a new set $S = \{s_1, s_2, \ldots, s_l\}$, where $l = C^N_{n(\alpha^f)}$, consisting of all subsets of X whose cardinality is $n(\alpha^f)$. Then each s_i is a possible group of people using the commons, and they all have an equal probability $1/l$ of using it. Define z_i as the summation of all elements in s_i, $i = 1, 2, \ldots, l$. The total social welfare in this case is a random variable depending on which s_i is selected. However, its expectation, which we denote as $W^f(\alpha^f)$, should be well behaved as follows:

$$W^f(\alpha^f) = \frac{1}{l}\sum_{i=1}^{l} z_i = \frac{1}{l}\sum_{i=1}^{l} \frac{n(\alpha^f)l}{N} x_i = \frac{n(\alpha^f)}{N}\sum_{i=1}^{N} x_i \tag{3}$$

Case 2: Monopoly

Now consider the case where the usage is regulated by a pricing mechanism operated by a monopolist. Suppose the introduction of this mechanism incurs an overall fixed cost, K, plus a variable cost, v, for serving each customer. The objective function is then:

$$\begin{gathered} \max_{\alpha^m} (p^m - v)n - K \\ s.t. \text{(i) } \alpha^m \text{ and } n \text{ satisfy (2)}, \\ \text{(ii) } \alpha^m \leq \alpha^f \\ \text{(iii) } x_i \geq p^m,\ \forall i < n \end{gathered} \tag{4}$$

Let us denote the solution to (4) as α^m and the resulting number of customers as $n(\alpha^m)$. Cancelling out the cash payment from customers to the monopolist, the total social welfare $W^m(\alpha^m)$ is deterministic in this case:

$$W^m(\alpha^m) = \sum_{i=1}^{n(\alpha^m)} x_i - [vn(\alpha^m) + K] \tag{5}$$

Case 3: Regulation

Under the regulation paradigm, the operator of the resource is allowed to make zero profit. Therefore α^r, defined as the maximum acceptable blocking probability under regulation, is derived from

$$\begin{aligned} &\max_{\alpha^r} \alpha \\ s.t.(\text{i}) \ &\alpha \text{ and } n \text{ satisfy (1)}, \\ &(\text{ii}) \ \alpha^r \leq \alpha^f \\ &(\text{iii}) \ x_i \geq p^r, \forall i < n \\ &(\text{iv}) \ (p^r - v)n - K = 0 \end{aligned} \tag{6}$$

Let us denote the solution to (6) as α^r and the resulting number of customers as $n(\alpha^r)$. The total social welfare $W^r(\alpha^r)$ is also deterministic:

$$W^r(\alpha^r) = \sum_{i=1}^{n(\alpha^r)} x_i - p^r n(\alpha^r) \tag{7}$$

Our next task is to compare $W^f(\alpha^f)$, $W^m(\alpha^m)$ and $W^r(\alpha^r)$ and to derive the conditions under which one social welfare outcome is larger than another.[5] To simplify our analysis, we need to impose some kind of structure on X. Without loss of generality, let's assume it is an equally distanced series. That is $x_i = A - (i-1)\epsilon \ i = 2, \cdots, N$ and $x_N > 0$, where ϵ is a small positive number. This means A is sufficiently large and ϵ is such that all potential users have a positive usage utility. With this specification, the social welfare under free use in equation (3) can be written more explicitly as:

$$W^f(\alpha^f) = n(\alpha^f)(A - \frac{N-1}{2}\epsilon) \tag{8}$$

i.e., the social welfare relies on an average person's willingness to pay.

For the monopoly and regulation cases, under our settings, it can be derived that

$$W^m(\alpha^m) = \frac{(3A - 3v + \epsilon)(A - v + \epsilon)}{8\epsilon} - K \tag{9}$$

and

$$W^r(\alpha^r) = \frac{2(A-v)[A - v + \epsilon + \sqrt{(A-v+\epsilon)^2 - 4\epsilon K}]}{8\epsilon} - K. \tag{10}$$

Detailed derivations of equations (8), (9), and (10) can be found in the Appendix. Next we compare the cases of regulation and monopoly and give the following result:

Proposition 1 *With respect to the comparison between monopoly and regulation, $n(\alpha^r) > n(\alpha^m)$ and $W^r(\alpha^r) > W^m(\alpha^m)$ with $\alpha^r > \alpha^m$, i.e., regulation is generally more efficient than monopoly.*

[5] We assume $\alpha^f > \alpha^r$ and $\alpha^f > \alpha^m$ for discussion simplicity in the rest of the paper.

This is after all not surprising that the social welfare outcome under an unregulated monopoly is generally worse off than it is under regulation. Regulation also tends to include a larger percentage of the population, but at the cost of providing a lesser degree of service quality.

Next note that $M[A - \frac{N-1}{2}\epsilon] \leq w^f(\alpha^f) \leq N[A - \frac{N-1}{2}\epsilon]$ since $M \leq n(\alpha^f) \leq N$. Depending on the exact value of α^f, it is obvious to derive the following result:

Proposition 2 *When the fixed cost, K, is sufficiently small, regulation can be more socially efficient than free use. When the fixed cost is sufficiently large, free use can be more socially efficient than regulation. If it is neither sufficiently small nor sufficiently large, there exists a threshold blocking probability α^{f*} such that for $\alpha^f < \alpha^{f*}$, regulation is more socially efficient than free use, while for $\alpha^f > \alpha^{f*}$, the opposite is true.*

What Proposition 2 says is that the condition under which free use dominates regulation is driven by two things. First, the fixed cost in implementing the pricing mechanism is important. If the cost is small, regulation can be more efficiently implemented and, thus, can be the better mechanism for allocating resources. If it is too large, then society is better off with the commons form, albeit the quality of service suffers. Second, the maximal acceptable blocking probability α^f is also important in that a large α^f tilts the comparison between free use and regulation more towards the former, while a small α^f tilts it towards the latter.

We can find real examples to validate Proposition 2. For example, cellular networks, which mostly consist of voice traffic and are sensible to delay, are usually built out of licensed spectrum. Wi-Fi networks, which support predominantly data traffic and are thus robust to delay, use free spectrum.

3 Extensions

In this section, we use the spectrum example to explore several extensions to our base models for practical applications, focusing on comparisons between the commons and the regulation.

3.1 Frequency Reuse

Spectrum bands have certain properties that dictate the cell size a base station can serve. Typically given the same transmission power, the higher the spectrum band, the shorter the distance a signal can travel and with less penetration power through trees and buildings. In general, wireless technologies may be categorized into two types of cell architectures, macro-cells whose cell radius can span several miles, such as any one of these 3G and 4G wireless technologies, and micro-cells that span only up to a few hundred feet, such as Wi-Fi. A micro-cell architecture can cover the same area as a macro-cell can with the help of frequency reuse, i.e., by assigning different spectrum bands to adjacent cells. We explore how cell size impacts the economics of stochastic commons.

Suppose in our original setup there are N number of potential customers uniformly distributed over a geographic area that can be adequately served by one macro-cell with capacity M. A fixed cost, K_1, is incurred if any pricing mechanism is introduced. Alternatively, this market can be served with θ number of micro-cells, $\theta > 1$, distributed uniformly across the terrain, with each cell having the same capacity, M, via frequency reuse by avoiding assigning the same spectrum bands to adjacent cells. Similarly a fixed cost, K_2, per each cell is incurred for these micro-cells, and we assume $K_2 = K_1$, if a pricing mechanism is introduced. Assume that the variable cost of serving each customer under any pricing mechanism, v, is the same for both macro- and micro-cells.

The tradeoff between the free use model and the regulation model under the macro-cell architecture is essentially already shown in section 2. The trade-off between the two spectrum allocation mechanisms under the micro-cell architecture critically depends on the number of customers supported by each micro-cell, which is equal to $\min\{n(\alpha^f), N/\theta\}$ or $\min\{n(\alpha^r), N/\theta\}$, depending on whether the commons model or regulation is adopted. If this maximum takes on the first value, it corresponds to the case of capacity-limited in cellular design. If the maximum takes on the second value, it corresponds to the case of range-limited in cellar design. We derive the condition under which the social welfare associated with free use dominates regulation. The goal here is to show that the condition that favors free use under the micro-cell architecture is easier and more relaxed to meet than under the macro-cell architecture. Since $n(\alpha^r) \leq n(\alpha^f)$, we have the following cases:

Case 1: $N/\theta \leq n(\alpha^r) \leq n(\alpha^f)$. In this case, all customers are served. The corresponding social welfare for regulation is

$$W^r = \sum_{i=1}^{N} x_i - \theta K_2 - vN = N(A - \frac{N-1}{2}\epsilon) - \theta K_2 - vN. \quad (11)$$

The corresponding social welfare for free use is

$$W^f = \sum_{i=1}^{N} x_i = N(A - \frac{N-1}{2}\epsilon). \quad (12)$$

Clearly, in this case, we favor free use. There is no tradeoff between free use and regulation, as in this case the population is so sparsely dispersed that it would be too costly to bring pricing to every customer.

Case 2: $n(\alpha^r) \leq N/\theta \leq n(\alpha^f)$. In this case, not all customers are served under regulation, while with free use for micro-cells all customers are served. Considering the variable serving cost v and the fixed cost θK_2, we again favor free use, as in the previous case.

Case 3: $n(\alpha^r) \leq n(\alpha^f) \leq N/\theta$. In this case, not all customers are served under both regulation and free use. We have $n(\alpha^f)$ (resp. $n(\alpha^r)$) out of N/θ consumers chosen under free use (resp. regulation). Under regulation, $\theta n(\alpha^r)$ number of customers in total are served and the customers with the highest usage utility are chosen. Then the corresponding social welfare for regulation under the micro-cell is

$$W_{mi}^r = \Sigma_{i=1}^{\theta n(\alpha^r)} x_i - \theta n(\alpha^r) v - \theta K_2. \tag{13}$$

The corresponding social welfare under free use for the micro-cell is

$$W_{mi}^f = \sum_{i=1}^{\theta n(\alpha^f)} x_i = \theta n(\alpha^f)(A - \frac{N-1}{2}\epsilon). \tag{14}$$

Denote the social welfare under the macro-cell for regulation as $W_{ma}^r = \sum_{i=1}^{n(\alpha^r)} x_i - vn(\alpha^r) - K_1$ and the social welfare under the macro-cell for free use as $W_{ma}^f = n(\alpha^f)(A - \frac{N-1}{2}\epsilon)$. We then have:

Proposition 3 *If free use is favored under the macro-cell, it must be the case that free use is also favored under the micro-cell, i.e., the condition for favoring free use is easier to meet under the micro-cell architecture.*

What Proposition 3 indicates is that the commons form is more suited for spectrum bands specified for micro-cell architectures. Practical examples corroborate our findings. For example, the free spectrum allocated so far, such as Wi-Fi and Bluetooth, are all micro-cell technologies.

3.2 Mixed Service

Suppose the network technology enables two kinds of service, one with a guaranteed maximum level of blocking probability for paying customers, and one with no quality of service guarantee but available to all users for free. A two-tier queuing model is usually used to model this kind of traffic pattern. But without getting into the complexities of a traffic model, we can alternatively think of an approximating scenario where the capacity, M, is carved into two parts, M^* exclusively reserved under regulation and $M - M^*$ free capacity, used respectively by N^* number of paying customers and $N - N^*$ number of free users. Obviously N^* represents the number of customers with the highest values in the utility profile $X = [x_1, x_2, \ldots, x_N]$, while all the rest in the utility profile randomly share the remaining capacity, $M - M^*$, for free. We investigate the optimal capacity for M^* from the perspective of maximizing social welfare, and compare its result with the pure free use model and the pure regulation model. We derive conditions under which one model generates high social welfare than the other.

For simplicity, we assume α^r is exogenously given and $\alpha^r \leq \alpha^f$. For M^* for paid capacity, the number of users under regulation, according to equation (2), is

$$N^* = \frac{M^*}{q} + \frac{1}{2q}[G(1-\alpha^r)\sqrt{4M^*(1-q) + G^2(1-\alpha^r)(1-q)^2} + G^2(1-\alpha)(1-q)], \tag{15}$$

which means, given α^r, N^* is a function of M^*. From the above equation, we have

$$\frac{dN^*}{dM^*} = \frac{1}{q} + \frac{1-q}{q}\frac{G(1-\alpha^r)}{\sqrt{4M^*(1-q)} + G^2(1-\alpha^r)(1-q)^2} > 0. \tag{16}$$

For the part of free capacity, we have $n(\alpha^f)$ people using the service, where

$$n(\alpha^f) = \frac{M-M^*}{q} + \frac{1}{2q}[G(1-\alpha^f)\sqrt{4(M-M^*)(1-q) + G^2(1-\alpha^f)(1-q)^2} + G^2(1-\alpha^f)(1-q)],$$

from which we have

$$\frac{dn(\alpha^f)}{dM^*} = -\frac{1}{q} - \frac{1-q}{q}\frac{G(1-\alpha^f)}{\sqrt{4(M-M^*)(1-q)} + G^2(1-\alpha^f)(1-q)^2} < 0. \tag{17}$$

Now the social welfare for the customers under regulation is

$$W^r(M^*) = \sum_{i=1}^{N^*} x_i - [vN^* + K] = (A - \frac{(N^*-1)\epsilon}{2})N^* - vN^* - K.$$

The corresponding welfare for those enjoying the free capacity is

$$W^f = \frac{n(\alpha^f)}{N-N^*}\sum_{i=N^*+1}^{N} x_i = n(\alpha^f)(A - \frac{(N^*+N-1)\epsilon}{2}).$$

The total social welfare becomes

$$W^{MS} = (A - \frac{(N^*-1)\epsilon}{2})N^* - vN^* - K + n(\alpha^f)(A - \frac{(N^*+N-1)\epsilon}{2}). \tag{18}$$

The first order condition of W^{MS} with respect to M^* yields

$$\frac{dW^{MS}}{dM^*} = (A - N^*\epsilon + \frac{\epsilon}{2} - v - \frac{n(\alpha^f)\epsilon}{2})\frac{dN^*}{dM^*} + (A - \frac{N^*+N-1}{2}\epsilon)\frac{dn(\alpha^f)}{dM^*}. \tag{19}$$

When $\frac{dW^{MS}}{dM^*} = 0$, the social welfare is maximized. Denote the maximized W^{MS} as $W^{MS}(M^*)$. We first compare the social welfare between pure regulation and mixed service.

Proposition 4 *Mixed service is always no worse than pure regulation in terms of maximizing social welfare.*

The above proposition indicates that mixed service is somewhat better than pure regulation.[6] Next, we give a comparison between mixed service and free use. First we show:

Lemma 1 *When $M^* \leq M - M^*$ with $\alpha^r < \alpha^f$, we have*

$$\frac{dN^*}{dM^*} > |\frac{dn(\alpha^f)}{dM^*}|.$$

This lemma implies that an increase in the paid capacity will cause the number of paying customers to increase more than the number of people using the free capacity when the paid capacity is less than half of the total capacity. With the implication of this result, we have:

Proposition 5 *When v is sufficiently small in the case of mixed service, more capacity should be allocated to regulation than to free use.*

The intuition behind the result is straightforward. Regulation can assure resources to be allocated to consumers with high usage utility, which can in turn increase the realized social welfare. However, this benefit is only valid when v is sufficiently small. When v becomes large, the benefit from high usage utility users can be quickly exhausted. Proposition 5 then indicates that it is generally welfare enhancing to have the free service as a piggyback feature on top of the licensed paying service, which is exactly what the cognitive radio is trying to achieve.

The fixed cost, K, plays a similar role in determining whether regulation should be employed or not. From free use to regulation, there is a jump in the social welfare by means of a fixed cost. Partial regulation is more favored only when K is sufficiently small. In this case, as we have stated earlier, if v is sufficiently small, more capacity should be allocated to regulation. Otherwise, if v is not that small, we prefer less capacity for regulation. The extreme case occurs when v is sufficiently large, in which case no regulation should be resorted to at all.

3.3 Channel Bonding

In most countries, spectrum maps are usually the result of cumulative license allocations over many years. These are typically done in a fragmented manner and, thus, in hindsight may not be optimized from the resource utilization perspective. One of the important features of the cognitive radio under development is its capability to dynamically aggregate bits and pieces of unused licensed spectrum bands for free data transmission. In this section we show that such

[6]However, the same argument in the proof of proposition 4 cannot be used to show that mixed service is always better than pure free use, since there is a jump in the social welfare by means of a fixed cost K and a variable cost v in the mixed service model.

aggregation has an inherent advantage that is somewhat similar to the concept of economy of scale, and thus could be welfare enhancing compared to a fragmented licensed use under rigid regulations.

Suppose in our original setup there are δ unused licensed bands each with capacity M such that they can be aggregated into δM, $\delta > 1$, via channel bonding for unlicensed usage. Suppose further that the original M capacity can support n number of customers. We examine the expanding rate of customers that can be served when the capacity is aggregated to δM.

Proposition 6 *When the capacity is aggregated from M to δM, the number of customers supported is expanded from n to βn with $\beta > \delta$.*

The implication of Proposition 6 is that if we combine these unused licensed spectrum bands into one unit for aggregate unlicensed use, then the number of customers that can be supported will increase more than linearly. The converse implication is that if we split a band into several equal units, the number of customers supported in each unit is going to decrease more than linearly. This is essentially the pitfall of licensing small pieces of spectrum bands in an ad hoc manner, which unfortunately has been the common practice in spectrum allocation in the past. On the other hand channel bonding has its inherent advantages in that economy of scale in spectrum utilization can be expected. This is because the statistical multiplexing gain is likely to increase as the spectrum width increases in a way similar to economy of scale. Channel bonding is an important feature of the cognitive radio currently under active development. From a policy point of view, our result indicates that it is merited not only because the technology enables the mining of unused or underused resources, but also because of its economy of scale property that was previously hampered by ad hoc and fragmented legacy allocations that were done over many years in the past.

4 Concluding Remarks

We build a model to study resource allocation under stochastic usage for three mechanisms: commons, unregulated monopoly, and regulation. Unlike the usual "curse of the commons" prediction in the literature, we find that property rights are not always the right solution to maximize social welfare as first proposed by McAfee and Miller (2012). There is an inherent tradeoff between these organizational forms driven by the resource characteristics. In the wireless spectrum example, we identify two factors as important. If the fixed cost of implementing a pricing mechanism is large, free use dominates property right solutions. Second, if the maximal acceptable blocking probability is large, in other words, if people's usage behavior is not particularly susceptible to occasional service degradation, the commons form appears to be a better solution than the property-rights solutions.

We then investigate three extensions to our base models that are quite relevant to wireless technology development. First we look at frequency reuse as it applies to micro-cell or macro-cell architectures. Spectrum bands that are suitable for more intensive frequency reuse are generally more suitable for the commons model. This generally applies to spectrum bands tailored for micro-cell wireless architectures, such as Wi-Fi and Bluetooth. Channel bonding and mixing traffic of different quality of service levels, which are the two important features enabled by the cognitive radio or software-defined radio technologies, are generally welfare enhancing.

Our analysis points to the important policy implication that the right answer to the debate over spectrum policy between the property-rights camp and the free spectrum camp may lie somewhere in-between. Some spectrums are inherently more suitable for the commons organizational form while others are better reserved for licensed use. The tradeoff between them is driven by network and cost characteristics that are likely to change over time as the technology advances. Therefore we believe the reform towards a more open spectrum policy may need to take a phased approach. One needs to take into account these detailed network characteristics to study spectrum policy issues.

In terms of future research directions, we mostly did not consider competition in this paper, which clearly applies to cellular service. There are at least four major cellular carriers in the US. Whether our result is robust to different market structures is still unclear. Another area to explore is to look at how the role of asymmetric information plays in our regulation model. Asymmetric information is traditionally an important issue in regulatory studies, albeit the cost structures of wireless technologies are becoming quite well understood these days.

Appendix

Derivations of (8), (9) and (10):

Equation (8): Notice that $W^f(\alpha^f) = \frac{n(\alpha^f)}{N}\sum_{i=1}^N x_i$, where $\sum_{i=1}^N x_i = N\frac{A+A-(N-1)\epsilon}{2} = N(A - \frac{N-1}{2}\epsilon)$ with $x_i = A-(i-1)\epsilon$. Thus

$$W^f(\alpha^f) = n(\alpha^f)(A - \frac{N-1}{2}\epsilon).$$

Equation (9): For the monopoly case, the first order condition of the maximization problem yields $A+\epsilon-2\epsilon n(\alpha^m) = v$, so $n(\alpha^m) = \frac{A+\epsilon-v}{2\epsilon}$. Then we have $p^m = A-(n(\alpha^m)-1)\epsilon = \frac{A+\epsilon-v}{2}$. Thus,

$$\begin{aligned} W^m(\alpha^m) &= \sum_{i=1}^{n(\alpha^m)} x_i - [vn(\alpha^m)+K] \\ &= \frac{A+A-\frac{A-v-\epsilon}{2\epsilon}\epsilon}{2}.\frac{A+\epsilon-v}{2\epsilon} - \frac{v(A+\epsilon-v)}{2\epsilon} - K \\ &= \frac{(3A-3v+\epsilon)(A+\epsilon-v)}{8\epsilon} - K. \end{aligned}$$

Equation (10): For the regulation case, $n(\alpha^r)$ solves

$$\frac{K}{n(\alpha^r)} + v = A-(n(\alpha^r)-1)\epsilon,$$

which is equivalent to

$$\epsilon n(\alpha^r)^2 - (A-v+\epsilon)n(\alpha^r) + K = 0.$$

Solving the above quadratic equation we get

$$n(\alpha^r) = \frac{A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\varepsilon K}}{2\epsilon}.$$

Then we have

$$\begin{aligned} W^r(\alpha^r) &= \sum_{i=1}^{n(\alpha^r)} x_i - [vn(\alpha^r)+K] \\ &= \frac{[A+A-(n(\alpha^r)-1)\epsilon]n(\alpha^r)}{2} - vn(\alpha^r) - K \\ &= \frac{-\epsilon n(\alpha^r)^2 + (A+\epsilon-v)n(\alpha^r) + (A-v)n(\alpha^r)}{2} - K. \end{aligned}$$

Note that from the above quadratic equation, we have

$$-\epsilon n(\alpha^r)^2 + (A+\epsilon-v)n(\alpha^r) = K.$$

Thus

$$W^r(\alpha^r) = \frac{(A-v)n(\alpha^r)+K}{2} - K.$$

With $n(\alpha^r)$ inserted, we have

$$W^r(\alpha^r) = \frac{2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]}{8\epsilon} - K.$$

Proof of Proposition 1:

First we prove $n(\alpha^r) > n(\alpha^m)$. To see this, note that

$$n(\alpha^r) - n(\alpha^m) = \frac{\sqrt{(A-v+\epsilon)^2 - 4\varepsilon K}}{2\epsilon} > 0.$$

By the one-to-one relationship between α and n, we immediately know that $n(\alpha^r) > n(\alpha^m)$ as claimed.

Next we prove $W^r(\alpha^r) > W^m(\alpha^m)$. Note that

$$W^r(\alpha^r) - W^m(\alpha^m) = \frac{2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]}{8\epsilon} - \frac{(3A-3v+\epsilon)(A+\epsilon-v)}{8\epsilon}.$$

To determine its sign, we only need to compare $2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]$ and $(3A-3v+\epsilon)(A+\epsilon-v)$. Note that

$$\begin{aligned}
2(\quad A \quad & -v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}] - (3A-3v+\epsilon)(A+\epsilon-v) \\
&= 4\epsilon K + 2(A-v)^2 + 2\epsilon(A-v) + 2(A-v)\sqrt{(A-v+\epsilon)^2-4\epsilon K} - 3(A-v)^2 + 4\epsilon(A-v) + \epsilon^2 \\
&= 4\epsilon K + 2(A-v)\sqrt{(A-v+\epsilon)^2-4\epsilon K} - [(A-v)^2 + 2\epsilon(A-v) + \epsilon^2] \\
&= 4\epsilon K + 2(A-v)\sqrt{(A-v+\epsilon)^2-4\epsilon K} - (A-v-\epsilon)^2 \\
&= 2(A-v)\sqrt{(A-v+\epsilon)^2-4\epsilon K} - ((A-v-\epsilon)^2 - 4\epsilon K) \\
&= \sqrt{(A-v+\epsilon)^2-4\epsilon K}(\sqrt{4(A-v)^2} - \sqrt{(A-v+\epsilon)^2-4\epsilon K}).
\end{aligned}$$

Now since

$$\begin{aligned}
4(A-v)^2 \quad &- \quad (A-v+\epsilon)^2 + 4\epsilon K \\
&= 4\epsilon K - [(A-v-\epsilon)^2 - 4(A-v)^2] \\
&= 4\epsilon K + (3A-3v+\epsilon)(A-v-\epsilon) \\
&> 0
\end{aligned}$$

by noting that $n(\alpha^f) - 1 = \frac{A-v-\epsilon}{2\varepsilon} > 0$. Thus we have $W^r(\alpha^r) > W^m(\alpha^m)$.

Proof of Proposition 2:

Note that $W^r(\alpha^r) = \frac{2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]}{8\epsilon} - K$ is a decreasing function of K while $W^f(\alpha^f) = n(\alpha^f)(A - \frac{N-1}{2}\epsilon)$ depends only on $n(\alpha^f)$ with $M \leq n(\alpha^f) \leq N$. Thus

- When $\frac{2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]}{8\epsilon} - K < M[A - \frac{N-1}{2}\epsilon]$, free use is more socially efficient.
- When $\frac{2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]}{8\epsilon} - K > N[A - \frac{N-1}{2}\epsilon]$, regulation is more socially efficient.
- When $M[A - \frac{N-1}{2}\epsilon] < \frac{2(A-v)[A-v+\epsilon+\sqrt{(A-v+\epsilon)^2-4\epsilon K}]}{8\epsilon} - K < N[A - \frac{N-1}{2}\epsilon]$, there exists an α^{f*} such that when $\alpha^f < \alpha^{f*}$, regulation is more efficient; otherwise free use is more efficient.

Proof of Proposition 3:

Since free use is more favored under macro-cell, we have

$$W^r_{ma} = \sum_{i=1}^{n(\alpha^r)} x_i - vn(\alpha^r) - K_1 < n(\alpha^f)(A - \frac{N-1}{2}\epsilon) = W^f_{ma}$$

Now we show $W^f_{mi} > W^r_{mi}$. To see this, note that $\sum_{i=1}^{\theta n(\alpha^f)} x_i < \theta \sum_{i=1}^{\theta n(\alpha^f)} x_i$, and thus

$$\sum_{i=1}^{\theta n(\alpha^f)} x_i - \theta n(\alpha^r)v - \theta K_2 < \theta \sum_{i=1}^{n(\alpha^f)} x_i - \theta n(\alpha^r)v - \theta K_1$$

by noting that $K_2 = K_1$ and x_i is decreasing.

Proof of Proposition 4:

Note that is the maximized value of W^{MS} with respect to M^* and the welfare for pure regulation is

$$W^{PR} = \sum_{i=1}^{N(M)} x_i - [vN(M) + K] = (A - \frac{(N(M)-1)\epsilon}{2})N^r - vN(M) - K,$$

which is a special case of W^{MS} with $N^* = N(M)$ and $n(\alpha^f) = 0$ and does not necessarily maximize W^{MS}. Thus mixed service (if available) is somewhat better than pure regulation.

Proof of Lemma 1:

Note that

$$\frac{dN^*}{dM^*} - |\frac{dn(\alpha^f)}{dM^*}| = \frac{1-q}{q}[\frac{G(1-\alpha^r)}{\sqrt{4M^*(1-q) + G^2(1-\alpha^r)(1-q)^2}} - \frac{G(1-\alpha^f)}{\sqrt{4(M-M^*)(1-q) + G^2(1-\alpha^f)(1-q)^2}}].$$

Now we show the above equation is positive whenever $M^* \le M - M^*$. To see this, let $B = \frac{G(1-\alpha^r)(1-q)}{\sqrt{4M^*(1-q)+G^2(1-\alpha^r)(1-q)^2}}$, and $C = \frac{G(1-\alpha^f)(1-q)}{\sqrt{4(M-M^*)(1-q)+G^2(1-\alpha^f)(1-q)^2}}$. Then

$$\frac{1}{B} = \sqrt{(1-q)^2 + \frac{4M^*(1-q)}{G^2(1-\alpha^r)}},$$

and

$$\frac{1}{C} = \sqrt{(1-q)^2 + \frac{4(M-M^*)(1-q)}{G^2(1-\alpha^f)}}.$$

Now since $\alpha^r < \alpha^f$, we have $G(1-\alpha^r) > G(1-\alpha^f)$. Thus we $M^* \le M - M^*$, we have $1/B < 1/C$ and thus $B > C$, which indicates that $\frac{dN^*}{dM^*} - |\frac{dn(\alpha^f)}{dM^*}| > 0$.

Proof of Proposition 5:

By Lemma 1, when $M^* < M - M^*$, $\frac{dN^*}{dM^*} > |\frac{dn'}{dM^*}|$. Thus with this restriction, if

$$A - N^*\epsilon + \frac{\epsilon}{2} - v - \frac{n(\alpha^f)\epsilon}{2} > A - \frac{N^* + N - 1}{2}\epsilon,$$

we have $dW^{MS}/dM^* > 0$ for $M^* \le M - M^*$. The above equation can be simplified to

$$\frac{N\epsilon}{2} > \frac{(N^* + n(\alpha^f))\epsilon}{2} + v,$$

which holds when v is small enough. In this case, increasing M^* will increase social welfare whenever $M^* \le N - M^*$. Thus the optimal capacity allocated to regulation does not happen whenever $M^* \le N/2$, which indicates that more capacity should be allocated to regulation in mixed service when v is sufficiently small.

Proof of Proposition 6:

Suppose the number of customers served by unused license spectrum is expanded to βn when the capacity is aggregated to δM for free use. When the capacity is M, we have

$$\alpha^r = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{M-nq}{\sqrt{nq(1-q)}}} e^{-\frac{t^2}{2}} dt.$$

Similarly when the capacity is δM, we have

$$\alpha^f = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\delta M-(\beta n)q}{\sqrt{(\beta n)q(1-q)}}} e^{-\frac{t^2}{2}} dt.$$

But since $\alpha^r < \alpha^f$, comparing the above two equations, we have

$$\frac{M - nq}{\sqrt{nq(1-q)}} > \frac{\delta M - (n\beta)q}{\sqrt{(n\beta)q(1-q)}}.$$

Next we show $\beta > \delta$ by way of contradiction. Suppose not, $1 < \beta \le \delta$, then we have

$$\sqrt{\beta} > \frac{\delta M - \beta nq}{M - nq} \ge \frac{\beta M - \beta nq}{M - nq} = \beta,$$

which cannot be true when $\beta > 1$. Thus $\beta > \delta$.

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