

# Permission to exist\*

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## Abstract

We provide a new model that generates persistent performance differences amongst seemingly similar enterprises. Our model provides a mechanism whereby efficient incumbent rivals can give permission for an inefficient firm to exist in the presence of efficient entrants. We demonstrate that, in a repeated game, an efficient incumbent has a unilateral incentive to establish a relational contract that softens price competition to either strengthen the inefficient firm in a war of attrition that emerges post-entry or reduce the value to the inefficient firm of selling its position to entrants. The paper provides conditions under which that equilibrium exists and derives a number of empirical predictions as implications of the model. It is demonstrated that performance differences are likely to be associated with stability in the identity of firms in the market.

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# 1 Introduction

Perhaps one of the most developed empirical literatures in strategic management is that detailing the strong evidence of persistent performance differences (PPDs) amongst seemingly similar enterprises. [Bartelsman and Doms \(2000\)](#) summarised the literature in the 20th century while [Syverson \(2011\)](#) documented that literature in the 21st. The differences are significant, there are no identified common drivers (although [Gibbons and Henderson, 2013](#), offer an important claim for a managerial element). In opposition to a long-standing prediction from industrial economic theory, while impacted by competition, PPDs are not eliminated by it ([Syverson, 2004](#)). Our goal in this paper is to provide a new perspective in theoretically approaching this set of empirical puzzles as well as a model illustrating that new perspective.

To date, theories designed to account for PPDs have focused on initial starting conditions where differences between firms exist aiming to explain why competitive processes have not eliminated those differences. The canonical case is of competing firms in a market having different levels of productivity where the least efficient firms should be vulnerable to one or both of two forces: displacement by *expansion* from the efficient firms and/or displacement by entry from efficient firms (i.e., *selection*). As [Syverson \(2011\)](#) shows, initial differences between firms can persist in markets where (a) firms have natural limits on their size (say, due to diseconomies of scale) that prevent expansion and (b) entry barriers are high. Both conditions are necessary to allow differences to persist. If (a) did not hold, then efficient firms could expand and drive out inefficient ones while if (b) did not hold, then new firms (or new plants) could be created that mimic the processes of efficient firms.

To date, the theoretical literature has provided reasons why either (a) or (b) hold. For instance, the literature on learning and dynamic capabilities demonstrates that firms may be unable to expand to displace less efficient firms and may be bound by issues of history ([Gibbons and Henderson, 2013](#)). By contrast, the resource based theory of the firm, emphasises resources that incumbent firms hold that cannot be replicated or acquired by entrants ([Wernerfelt, 1984](#); [Barney, 1991](#)).

Here we want to take an approach that is complementary to existing theories but that imposes the additional burden that (a) and (b) do not hold; specifically, a situation where there are no barriers to expansion or barriers to entry preventing selection from working to displace inefficient firms with efficient ones. These assumptions mean that there are no fundamental constraints to efficient practices and resources being deployed

in the industry.<sup>1</sup> However, that means that in order for an inefficient firm to continue to operate, someone (and indeed, the ‘right’ someone) must give that firm *permission to exist*.<sup>2</sup>

Who can give an inefficient firm permission to exist? The inefficient firm itself has the incentive but permission requires the actions of someone external. Consumers in the market have the ability to allow an inefficient firm to continue operating but, given our assumption that neither (a) nor (b) hold, they have no incentive to do that as they would benefit, in terms of lower prices, from being supplied only by efficient firms. Another option is to consider the supply-side, but suppliers have no more incentive than consumers to deal with inefficient firms in an otherwise flexible vertical chain. That leaves the inefficient firm’s rivals as the only viable candidates participating in the market<sup>3</sup> to give it permission to exist.

A rich literature in antitrust economics (going back to Adam Smith) notes that rival firms have every interest to assist in preserving weaker competitors over stronger ones. Thus, from an incentive perspective, it is not difficult to imagine rivals wanting to give an inefficient firm permission to exist. But do they have the ability to give effect to that permission?<sup>4</sup>

In this paper, we provide a model where it is demonstrated that an efficient rival in a market may be able to give an inefficient firm permission to exist when neither (a) nor (b) hold (and, in fact, because of it). To sketch the mechanism, suppose that, initially, there is both an inefficient and an efficient firm in a market and that each engages in price competition (specifically, differentiated Bertrand competition) in selling to consumers. There is an infinite pool of potential entrants who can operate at the same productivity as the efficient firm. If interactions were not repeated, then

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<sup>1</sup>Makadok (1998) provides an empirical analysis of the money mutual fund industry that nominally satisfies these conditions and finds that there are significant early mover advantages. Gans and Quiggin (2003) demonstrate that in the absence of entry barriers a size distribution of firms can emerge when entrepreneurial ability is heterogeneous. Powell (2013) provides a model that emphasises the interaction between the internal organisation of the firm – specifically, its ability to establish relational performance contracts – and the competitive rents a firm earns in the market.

<sup>2</sup>The perspective offered here is inspired by work in coalitional game theory where agents agree to various trade relationships. A constraint on agreements is a notion of stability whereby agents must agree to participate in trade both as individuals and as groups relative to alternative arrangements that sub-groups might agree to. For more details see MacDonald and Ryall (2004).

<sup>3</sup>Of course, it is possible that institutions or governments can give firms permission to exist, but we leave that possibility aside here to be consistent with the path taken in the current strategic management literature.

<sup>4</sup>Excluding, of course, obvious violations of antitrust law and other mechanisms such as tacit collusion.

the inefficient firm would be displaced by an efficient entrant due to the effects of price competition; that is, an efficient firm could enter with the same product as the inefficient one and drive its sales to zero, causing exit. However, if interactions were repeated, then the efficient incumbent could establish the following relational contract with the inefficient firm: “I will maintain my price in competition with you sufficiently high so that, should you face entry, you have a stronger incentive to outlast an efficient entrant in the resulting war of attrition.” The idea is that the efficient firm softens price competition to such an extent that the inefficient firm earns higher profits than an efficient entrant would earn in long-run competition with the efficient incumbent. As wars of attrition involve losses, that will deter entry by efficient entrants and leave firms of different efficiencies in the market.

This describes the mechanism but, in what follows, there is more work to be done to establish this as an equilibrium outcome. First, softening competition is potentially costly to the efficient incumbent so we need to show that, despite these costs, an incumbent still has an incentive to implement the relational contract. Second, if an entrant could co-exist with an inefficient firm, then the “offer” from the efficient incumbent could not be made, so we need to find conditions under which the market is a natural oligopoly (or in our case, a natural duopoly).<sup>5</sup> Third, clearly, an explicit offer by one firm to another would violate antitrust laws, so we must show that the offer can be sustained as an equilibrium in an implicit or tacit form. Finally, to provide a complete theory, we need to characterise incentives to become more or less efficient in the first place so as to demonstrate that differences between firms can arise endogenously rather than as a given initial condition.

The paper proceeds as follows. In Section 2, we setup our baseline model involving consumers on the Hotelling line where firms can either be located at one end or the other. There, we describe the operation of wars of attrition for a place in the market that occurs when more than two firms enter the market. We also assume, following [Byford and Gans \(2014\)](#), that entrants can avoid that war and enter by acquiring an incumbent firm. We analyse the negotiation that arises in that case. In Section 3, we confine attention to strategies that are independent of history to demonstrate that there exists an equilibrium of that game whereby an inefficient firm is replaced by an efficient entrant as per the usual intuition for this market structure. Section 4 presents the main proposition of the paper and establishes how the efficient incumbent can

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<sup>5</sup>A natural oligopoly arises where a market can only accommodate a certain number of firms relative to demand when those firms have fixed costs or exogenous or endogenous ([Sutton, 1991](#)) sunk costs.

use a relational contract (of the form described above) to allow the inefficient firm to persist in the market. We provide conditions under which this relational contract is an equilibrium of our baseline model. We then conduct comparative statics on firm performance and provide empirical implications arising from the model. Section 5 then considers extensions including additional firms, relational contracts with predation and other forms of competition. A final section concludes.

## 2 A duopoly market

To provide the simplest expression for our theory, we assume that the market is a natural duopoly. This is a market where fixed costs are such that, in relation to demand, at most two firms can profitably operate in the market at any one time. This setting is of interest as it is a special case of the equilibrium outcome identified by [Sutton \(1991\)](#) in his analysis of endogenous sunk costs. Here we assume that fixed costs are exogenous but are such that the natural duopoly structure arises. Because of this feature, entry into the market will only be successful if it (a) occurs via acquisition of an incumbent or (b) results in an incumbent exiting the market. Below, we will explore the importance of this assumption on market structure.

### 2.1 Market structure

To fix ideas, we consider spatial price competition on a Hotelling line of unit length. Consumers in the market are a unit mass uniformly distributed along the line. We assume that each consumer demands a single unit of the good in each period. Consumers value the good at an amount  $v$  and face a transport cost of  $t > 0$  per unit travelled. In order to ensure that the market is covered, it is assumed that  $v \geq 5t$ .

At the outset, there are two firms present in the market: an efficient incumbent, firm 1, and an inefficient incumbent, firm 2. To be sure, while our starting point here is that there are intrinsic firm differences, following our main result in Section 4 below, we will endogenise efficiency making firms symmetric at the outset, and demonstrate that, in equilibrium, differences can emerge and, are sustained.

Firm 1 is located at the left end of the line and faces a constant marginal cost,  $c_1$ , that is normalised to 0. Firm 2, the inefficient incumbent, is located at the right end of the line and faces a constant marginal cost of  $c_2 \in (0, t)$ . All active firms also incur a common fixed cost of  $C > 0$  in each period, and discount the future by the common

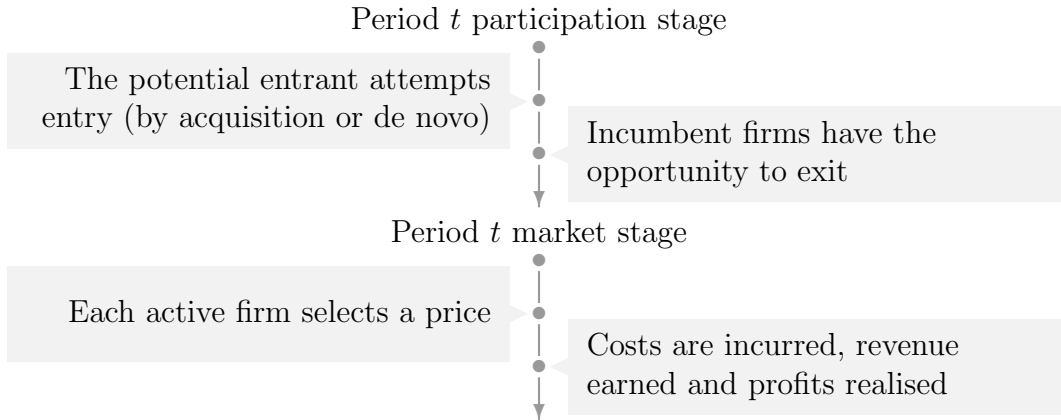


Figure 1: Timing

discount factor  $\delta \in (0, 1)$ .<sup>6</sup>

The market is open to entry by firms outside of the market. Initially, firm 3 exists as a potential entrant. Should firm 3 ever enter the market, a new firm, firm 4, emerges as a potential entrant and so on. Thus, the pool of potential entrants is infinite. We assume that entrants have access to the same efficient technology and practices employed by firm 1 and as such  $c_i = 0$  for  $i \in \{3, 4, \dots\}$ .

## 2.2 Timing

We model competition in the natural duopoly market as an infinite-horizon dynamic game. The timing of the game is illustrated in Figure 1.

Each period of the game begins with the *participation stage*. In this stage, the potential entrant chooses between three alternatives: (1) enter the market de novo, (2) attempt entry by acquisition, or (3) wait for the next period.

De novo entry requires a firm to construct a new set of production facilities in the market. This costs the entrant an amount  $K > 0$ . We assume that the entrant must locate its new facilities at either end of the line.<sup>7</sup>

<sup>6</sup>While we model firm differences here as being with regard to their marginal production costs, the model could accommodate other differences. For instance, firm 1 may have higher product quality than firm 2 with  $v_1 > v_2$  for all consumers. The outcomes here would be identical to the case where  $c_1 < c_2$ .

<sup>7</sup>For example, we might assume that planning restrictions prevent a firm from locating in a residential area, and that the available sites are on the edge of town. An immediate consequence of this assumption is that de novo entry, if it occurs, must result in the entrant collocating with one or other of the incumbent firms.

If instead, the firm attempts entry by acquisition, it must nominate the incumbent it wishes to acquire, and negotiate a fee for the purchase of the incumbent's assets. We do not explicitly model the bargaining process in this paper. Rather, we assume that if the value of the incumbent's assets to the entrant is greater than the value of these assets to their current owners, a transaction will occur for a fee  $F$  that is acceptable to both parties. In addition to the fee paid to the owners of the incumbent firm, the entrant also incurs a transaction cost  $k > 0$ . The cost  $k$  can be understood as the cost of conducting due diligence, replacing outdated technology and restructuring the firm.

If neither of these options is attractive to the potential entrant, then it may choose to do nothing in the current period. Note that exercising this option does not prevent the firm from attempting entry in a future period.

After the potential entrant has selected its action, the incumbent firms have the opportunity to exit the market. We assume that exit is costless (i.e., a firm can always declare bankruptcy) and irrevocable.

The participation stage is followed by the *market stage*. In the market stage the firms that are active in the market simultaneously select their prices. Consumers then make their purchases and profits are realised.

## 2.3 Wars of attrition

It is useful to distinguish between two *phases* of the game, depending on whether there are two or three firms active in the market. We say that the game is in a *duopoly phase* if at most one firm is present at each end of the line. During a duopoly phase, competition between firms is subdued because consumers prefer to purchase from nearby firms. In contrast, the game is said to be in a *war of attrition* if two or more firms are collocated at one end of the line. During a war of attrition, competition is more intense as consumers choose between the products of collocated firms on the basis of price alone.

Given this, we state a key assumption for our analysis.

$$C \in \left( c_2 \frac{3t - c_2}{4t}, \frac{t}{2} \right). \quad (1)$$

First, for the equilibrium in the market to be a natural duopoly, the fixed costs of operating in the market cannot be too low. Here, the lower bound on  $C$  is the profit an efficient entrant earns if it collocates with firm 2 and sets a price that is (slightly less than) firm 2's marginal cost. Second, if the fixed costs of operating in the market

are too high, the market will become a natural monopoly rather than duopoly. Thus, we assume an upper bound on  $C$  so that the market supports two firms.<sup>8</sup>

A war of attrition arises naturally in our setting. In each period, each firm chooses whether to remain in the market or not for another period. If the firms remain, they compete on price at an intensity dictated by their location (and, as we will see below, potentially, the history of other incumbents' pricing).

The willingness of a firm to endure a war of attrition (and remain in the market) depends both on the losses the firm endures during the war and the profits it receives if it wins. Suppose that firm  $i$  is collocated with one or more rivals. If the last of firm  $i$ 's rivals withdraws after  $\tau$  periods of conflict, firm  $i$  remains in the market and no further entry occurs, then firm  $i$ 's profit will be,

$$\Pi_i = \frac{1 - \delta^{\tau+1}}{1 - \delta} \pi_{i,W} + \frac{\delta^{\tau+1}}{1 - \delta} \pi_{i,D},$$

where  $\pi_{i,W} < 0$  represents the loss firm  $i$  incurs in each period of the war of attrition, and  $\pi_{i,D} > 0$  represents firm  $i$ 's profit in each period of the subsequent duopoly phase. Rearranging, we see that firm  $i$  earns a (weakly) positive profit if,

$$\theta_i = \frac{\pi_{i,D}}{-\pi_{i,W}} \geq \frac{1}{\delta^{\tau+1}} - 1. \quad (2)$$

In words,  $\theta_i$  represents the value to firm  $i$  of winning the war of attrition, normalised by the magnitude of the losses it incurs during the war. Given that the RHS of (2) is positive and increasing in  $\tau$ , it follows that firm  $i$ 's capacity to profitably endure a war of attrition is increasing in  $\theta_i$ . In the standard treatment of the war of attrition,  $\theta_i$  would be regarded as the firm's "type".

Wars of attrition typically possess multiple subgame perfect equilibria (SPEs). As is summarised by [Levin \(2004\)](#), there are typically two pure strategy equilibria where one or the other firm exits immediately and also a mixed strategy equilibrium where each exits the market with some probability. That mixed strategy equilibrium has the property that exit is delayed but can have some counter-intuitive comparative statics; for instance, the 'weak' firm may have a lower probability of exit than the 'strong' firm. For this reason, researchers have focused on alternative equilibrium concepts (for instance, the introduction of trembles and some incomplete information) to generate outcomes that are more intuitive (see [Myatt, 1999](#)).

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<sup>8</sup>The derivation of these bounds can be found in section 3.



In order to parallel the intuition that has emerged in the war of attrition literature, we will look for SPEs with the property that high types defeat low types, and identical types have an equal probability of victory. These properties are the analogue of monotone symmetric strategies in a war of attrition with unknown types (e.g., Bishop, Canning and Maynard Smith, 1978; Riley, 1980; Kreps and Wilson, 1982).

**Definition 1.** *The strategy of firm  $i$  is monotone in a war of attrition if, whenever firm  $i$  is collocated with another firm  $j$ , firm  $i$ 's participation stage action is:*

- (a) *Remain in the market if  $\theta_i > \theta_j$ .*
- (b) *Exit with probability  $\rho \in (0, 1)$ , where  $\rho$  leaves firm  $j$  indifferent between exiting and remaining in the market for one more period, if  $\theta_i = \theta_j$ .*
- (c) *Exit the market if  $\theta_i < \theta_j$ .*

*If more than one firm is collocated with firm  $i$ , then apply the rule with respect to the collocated firm with the highest type.*

It can be readily seen that if the strategy of each firm is monotone in the war of attrition, except for the special case where firms are symmetric, in equilibrium, there will be no war of attrition and the ‘stronger’ firm will be present in the market. That said, we can imagine extensions for future work that enrich the study of the war of attrition phase to incorporate asymmetric information or other equilibrium types that have been studied in the literature.

That said, it is useful to emphasise the key differences between the war of attrition, as it exists in our model, and the standard treatments. First, the war of attrition is simply one phase in which our dynamic game can find itself. Moreover, the game does not end when a winner emerges. Rather, the game can enter and exit the war of attrition phase any number of times as firms enter and exit the market. Second, in our game, the firm’s “type” in a war of attrition is endogenous. The profits a firm earns in each phase of the game ( $\pi_{i,D}$  and  $\pi_{i,W}$ ) depend on the actions firms take in the market stages. In section 4 we show how firms can strategically manipulate these values, effectively choosing a firm’s type.

### 3 Competing independent of history

We now turn to analyse the model in detail beginning with a benchmark case where the actions that firms take in markets are not dependent on history. This is an appropriate

starting point because, we will show, it captures the standard intuition as to why competitive processes lead to inefficient firms being replaced by efficient ones in markets. In the next section, we examine the equilibria that arise when actions can be history dependent. However, the equilibrium explored here remains an equilibrium in the broader context.

The way we do this is focusing, in this section, on situations where firms play only stationary strategies. Formally, a stationary strategy is a strategy in which firms condition their actions on the state of the game. That is, the subgame perfect equilibrium (SPE) in which the actions taken by firms depend on the number, locations and marginal costs of the firms present in the market, but not on the history of the game. The stationary SPE serves as the baseline for our analysis. It also functions as the threat point for the equilibria developed in Section 4.

The first step in characterising the stationary SPE is to establish the actions that firms take in the market stage of each period.

**Lemma 1.** *In each market stages of a stationary SPE, each firm plays its best response to the prices chosen by its rivals.*

*Proof.* Actions in the market stage have no effect on the state of the game. It follows that a firm's only concern when selecting its price is to maximise its profit in the current period.  $\square$

Lemma 1 states that, in a stationary SPE, the prices selected in any period will constitute a mutual best response. In other words, prices and profits in any given market stage will be exactly the Nash equilibrium prices and profits of a one-shot market-stage game.

Given this, we can now state those equilibrium prices. Suppose that firm  $i$  is alone at one end of the line, and that firm  $j$  has the lowest price of all firms located at the opposite end. The indifferent consumer is located a distance,

$$x^* = \frac{p_j - p_i + t}{2t},$$

from firm  $i$ . It follows that the profit firm  $i$  receives if it selects the price  $p_i$  can be written,

$$\pi_i(p_i, p_j) = (p_i - c_i) \frac{p_j - p_i + t}{2t} - C.$$

Firm  $i$ 's best response function takes the familiar form,

$$R_i(p_j) = \frac{p_j + t + c_i}{2}.$$

Note that  $R_i$  is increasing in  $p_j$  indicating that prices are strategic complements.

If firms 1 and 2 are the only firms present in the market, in a stationary SPE, firms set the prices  $p_1 = t + c_2/3$  and  $p_2 = t + 2c_2/3$ , and earn the corresponding profits,

$$\pi_1 = \frac{1}{2t} \left( t + \frac{c_2}{3} \right)^2 - C \quad \text{and} \quad \pi_2 = \frac{1}{2t} \left( t - \frac{c_2}{3} \right)^2 - C.$$

This outcome remains unchanged if firm 1 is replaced by an efficient entrant. If, instead, firm 1 competes against an efficient entrant, say firm 3, the firms will set the prices  $p_1 = p_3 = t$ , and earn profits of  $\pi_1 = \pi_3 = t/2 - C$ .

Now consider market-stage actions during a war of attrition in which a third firm, say firm 3, collocates with an incumbent. Because consumers distinguish between the collocated firms on the basis of price alone, competition between the firms is pure Bertrand. This means that if firm 3 collocates with firm 1, both firms will price at marginal cost  $p_1 = p_3 = 0$ , and make a loss equal to the fixed cost  $C$ . If firm 3 collocates with firm 2 we assume that firm 2 prices at marginal cost, and that firm 3 undercuts firm 2 by an arbitrarily small amount. In this case, firm 2 makes a loss of  $C$  and firm 3's loss is,

$$\pi_3(c_2, R_1(c_2)) = c_2 \frac{3t - c_2}{4t} - C < 0,$$

where the inequality follows from the assumed lower bound on  $C$ .

The interesting case to consider is where both types of entry (by acquisition or de novo) are viable. This will occur so long as,

$$\max\{k, K\} < \frac{1}{1 - \delta} \left( \frac{t}{2} - C \right).$$

In words, the cost of entering the market is strictly less than the discounted sum of the profits a firm earns as a member of an efficient duopoly. These are the profits an entrant expects if it displaces the inefficient incumbent.

We can now complete the characterisation of the stationary SPE. We begin by establishing that an entrant will only ever target the inefficient incumbent.

**Lemma 2.** *In a stationary SPE in which each firm's strategy is monotone in a war of attrition (definition 1), an entrant will neither attempt to purchase an efficient incumbent, nor enter de novo and collocate with an efficient incumbent.*

*Proof.* From Lemma 1 it follows that two (or more) collocated efficient firms incur identical losses during a war of attrition and expect identical duopoly profits in the event

of victory. This means that collocated efficient firms have identical types. Definition 1 then requires that each collocated firm exits with a probability that leaves its rival indifferent between exiting and remaining in the market for one more period. The payoff to exiting is zero, which means that expected payoff to remaining in the market must also be zero. Therefore, the expected return to entering and collocating with an efficient incumbent is  $-K < 0$ . Similarly, an entrant will not purchase an efficient incumbent as the entrant will be no more profitable than the incumbent post acquisition, and the entrant will incur a transaction cost  $k > 0$ .  $\square$

We can now demonstrate that an SPE exists where the inefficient incumbent is always immediately replaced by an efficient entrant using the least cost path.

**Proposition 1.** *In a stationary SPE in which each firm's strategy is monotone in a war of attrition (definition 1),*

- (a) *If  $k \leq K$  then firm 3 acquires firm 2 for a fee  $F \in [0, K - k]$ .*
- (b) *If  $K < k$  then firm 3 enters de novo, collocating with firm 2, and firm 2 exits immediately.*

*In either case, no further entry occurs.*

*Proof.* For (b),  $K < k$  implies that it is cheaper for a firm to construct a new presence in the market, than to restructure an incumbent firm. If firm 3 enters de novo and collocates with firm 2 then,

$$\theta_3 = \frac{\frac{t}{2} - C}{C - c_2 \frac{3t - c_2}{4t}} > \frac{\frac{1}{2t} \left(t - \frac{c_2}{3}\right)^2 - C}{C} = \theta_2.$$

Because firm 2 has a lower type, it exits the market immediately. It follows that de novo entry is profitable for firm 3. Note that firm 2 cannot do better by remaining in the market as it incurs a loss of  $C$  in each period of a war of attrition. Similarly, firm 3 has no incentive to exit as firm 2 exits with certainty.

For (a) note that the possibility of de novo entry remains as an outside option for firm 3. De novo entry costs firm 3 an amount  $K$  and reduces firm 2's profit to zero. It follows that firm 3 will acquire firm 2 for a fee  $F \in [0, K - k]$  that lies between these outside options. This interval exists if  $k \leq K$ .  $\square$

The SPE described in Proposition 1 conforms to the conventional wisdom on the role that entry and competition play in eliminating inefficient firms. When a firm lags behind its competitors, it becomes an attractive target for acquisition by an entrant that can utilise the firm’s position in the market more profitably than the current owners. In the absence of a sale, an inefficient incumbent is vulnerable to competition from an efficient de novo entrant that can price the incumbent firm out of the market.

## 4 Strategic intervention by the efficient incumbent

We are now in a position to introduce the primary conceptual innovation in this paper: That for there to be persistent differences in firm performance, some set of agents must provide inefficient firms with permission to exist. The previous section confirmed that, in the standard intuition, consumers would not provide that permission to an inefficient incumbent. Here, we examine whether the efficient incumbent has the ability and the incentive to provide that permission thereby exposing a new mechanism where firm differences may persist. Operationally, what the efficient incumbent is trying to do is to modify the inefficient incumbent’s type so that it will prevail in a war of attrition with an efficient entrant.

In the analysis of the previous section, the efficient incumbent was a passive player. Firm 1 did nothing to influence the price of firm 2 in the acquisition market, nor the type of firm 2 in a war of attrition. Yet firm 1 has a strong incentive to intervene. If an efficient entrant replaces firm 2, firm 1’s profit falls from  $\frac{1}{2t} (t + \frac{c_2}{3})^2 - C$  to  $t/2 - C$ .

In this section, we show how the efficient incumbent can prevent entry or acquisition by unilaterally reducing competition in the market and, in the process, directing a stream of profits to its inefficient rival. This action has two effects. First, the increased profits enjoyed by firm 2 increases the value of the firm to its owners. This, in turn, raises the price of firm 2 in the acquisition market.<sup>9</sup> At the same time, the stream of profits increases the prize firm 2 receives on winning a war of attrition, and hence, it increases firm 2’s type in any such war.

### 4.1 The relational contracting equilibrium

To begin with, we characterise a *relational contracting equilibrium*. The relational contracting equilibrium is an SPE of the game in which the *unilateral conduct* of firm 1,

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<sup>9</sup>An effect noted in Byford and Gans (2014).

reducing the intensity of competition during the duopoly phase, is sufficient to prevent entry. The term ‘relational contracting’ comes from the notion that firm 1 uses the value of its ongoing relationship or interaction with firm 2 to make credible a promise to act in a favourable manner in the market stage of the game.

**Definition 2.** *A relational contracting equilibrium is an SPE of the game in which:*

- (a) *All firms employ strategies that are monotone in a war of attrition.*
- (b) *In a duopoly phase in which firms 1 and 2 are active in the market, firm 1 sets the price,*

$$p_1^R = \max \left\{ \frac{3t + c_2}{2}, 2\sqrt{\frac{C}{2} \cdot \frac{(2t - c_2)(t - c_2)}{C - c_2 \frac{3t - c_2}{4t}}} - t + c_2 + \varepsilon \right\}, \quad (3)$$

*for some arbitrarily small  $\varepsilon > 0$ , and firm 2 plays its best response to  $p_1^R$ .*

- (c) *During a war of attrition, each firm plays its best response to the prices chosen by its rivals.*
- (d) *If firm 1 sets a price of less than  $p_1^R$  during a duopoly phase, or if firm 2 exits the market, the game immediately and permanently reverts to the stationary equilibrium outlined in Proposition 1.*

Definition 2 has a straightforward interpretation. First, in order to be consistent with the stationary equilibrium developed in Proposition 1, condition (a) requires all firms to employ strategies that are monotone in a war of attrition. This consistency is important as the stationary equilibrium also functions as the threat point in the relational contracting equilibrium.

The distinguishing feature of the relational contracting equilibrium is the equilibrium path duopoly-phase behaviour, characterised in condition (b). Firm 1 unilaterally raises its price above its best response. Firm 1’s price  $p_1^R$  is selected to maximise its equilibrium path profit subject to the requirement that both types of entry are rendered not viable. The derivations of the values in (3) can be found in the proof of Proposition 2 below.

Note that under condition (b), firm 2 always plays its best response to firm 1’s price. One useful feature of this as a condition of the SPE is that neither firm is colluding and

so there are no antitrust implications from firm 1's strategic behaviour.<sup>10</sup>

Should entry occur, condition (c) states that firms will set prices as they would in the stationary equilibrium. Setting higher prices would be counterproductive as it would reduce the losses incurred by the entrant. We consider the possibility of an incumbent adopting more aggressive pricing below.

Finally, condition (d) states that any deviation by firms 1 or 2 results in immediate reversion to the stationary equilibrium. This means that any failure by firm 1 to deliver the required profits to firm 2 results in firm 2 exiting the market in the following period. Moreover, condition (d) ensures that an entrant will not profit from firm 1's accommodating behaviour towards firm 2, and hence that the entrant's type in a war of attrition is not affected.

**Proposition 2.** *For sufficiently high  $\delta \in (0, 1)$ , a relational contracting equilibrium exists if and only if,*

$$p_1^R < \frac{1}{2} \left( \sqrt{t^2 + 6tc_2 + c_2^2} + 3t + c_2 \right). \quad (4)$$

*Proof.* The proof proceeds in three steps.

*Step 1: Firm 2's type is higher than an entrant's.* Given that firm 2 plays its best response to  $p_1^R$ , firm 2's profit in each period of a duopoly phase is,

$$\pi_2(R_2(p_1^R), p_1^R) = \frac{1}{2t} \left( \frac{p_1^R + t - c_2}{2} \right)^2 - C.$$

Profits during a war of attrition, and firm 3's profit if it succeeds in replacing firm 2, are as in the stationary SPE described in Proposition 1. It follows that firm 2 will have a higher type than an entrant if,

$$\theta_2 = \frac{\frac{1}{2t} \left( \frac{p_1^R + t - c_2}{2} \right)^2 - C}{C} > \frac{\frac{t}{2} - C}{C - c_2 \frac{3t - c_2}{4t}} = \theta_3.$$

Solving for  $p_1^R$  yields the inequality,

$$p_1^R > 2 \sqrt{\frac{C}{2} \cdot \frac{(2t - c_2)(t - c_2)}{C - c_2 \frac{3t - c_2}{4t}}} - t + c_2,$$

---

<sup>10</sup>While it is true that the equilibrium involves both firms charging prices in excess of "competitive" levels, this is the result of *unilateral conduct* by firm 1. Firm 2's response is always to maximise its profits in the current period. Moreover, because the unilateral conduct is not to the detriment of a competitor, it is unlikely to raise regulatory flags.

which is satisfied by Definition 2.

*Step 2: An entrant will be unable to acquire firm 2.* With the threat of de novo entry not credible, firm 3 can only acquire firm 2 if firm 3's post-entry profit, net of the transaction cost, is greater than firm 2's profit. Note that by definition 2,

$$p_1^R \geq \frac{3t + c_2}{2}.$$

Thus, in each period of a duopoly phase, firm 2 earns,

$$\pi_2(R_2(p_1^R), p_1^R) = \frac{1}{2t} \left( \frac{p_1^R + t - c_2}{2} \right)^2 - C \geq \frac{1}{2t} \left( \frac{5t - c_2}{4} \right)^2 - C.$$

Given that  $c_2 \in (0, t)$ ,

$$\begin{aligned} \frac{1}{2t} \left( \frac{5t - c_2}{4} \right)^2 - C &> \frac{t}{2} - C \\ &> \frac{t}{2} - C - (1 - \delta)k, \end{aligned}$$

which is firm 3's post-entry profits, net of the amortised transaction cost.

*Step 3: For sufficiently high  $\delta \in (0, 1)$ , firm 1 will not deviate if  $p_1^R$  satisfies (4).* Firm 1 earns a profit of,

$$\pi_1(p_1^R, R_2(p_1^R)) = p_1^R \frac{3t - p_1^R + c_2}{4t} - C,$$

in each period of a duopoly phase. Because  $p_1^R > R_1(R_2(p_1^R))$ , firm 1 can increase its profit in the current period by lowering its price. The cost of deviating is reversion to the stationary equilibrium in the following period. Given that firm 2 will be immediately replaced by an efficient entrant in the stationary equilibrium, firm 1's profits in all future periods will be  $t/2 - C$ . It follows that if firms are sufficiently patient, firm 1 will not deviate so long as,

$$p_1^R \frac{3t - p_1^R + c_2}{4t} - C > \frac{t}{2} - C.$$

Solving for  $p_1^R$  yields (4). □

Proposition 2 establishes the necessary and sufficient conditions for a relational contracting equilibrium to exist for some discount factor. It confirms that strategic behaviour by firm 1 can alter the long-term competitive outcome in the industry by deterring the entry of efficient firms. The precise level of the critical discount factor



will depend on the magnitude of firm 1's equilibrium path profits, relative to its profits in the stationary equilibrium.

Before turning to consider the comparative statics and empirical implications of Proposition 2, it is useful to note that the equilibrium can exist even in situations where firm 1 would, in the stationary equilibrium with entry barriers, have a sufficient efficiency advantage to drive firm 2 out of the market. Specifically, if  $C$  is relatively large, lying in the non-empty interval,

$$C \in \left( \frac{1}{2t} \left( t - \frac{c_2}{3} \right)^2, \frac{t}{2} \right),$$

then firm 2 makes a loss in each period of a stationary equilibrium. If subsequent entry is not possible, then firm 1 is happy to see firm 2 exit the market as this leaves it with a monopoly, albeit with a single location. If, entry is possible then firm 1 cannot preserve this monopoly and is, therefore, better off protecting firm 2.<sup>11</sup> This demonstrates that even when condition (a) in the introduction does not hold (that is, expansion by existing firms can drive inefficient firms out of the market), the relational contracting equilibrium can exist with firms with different levels of efficiency persisting.

## 4.2 Firm performance

We now turn to consider the comparative statics of the relational contracting equilibrium in order to better understand the empirical implications of our theory. In a relational contracting equilibrium, prices along the equilibrium path, and the resultant profits, depend both on firm 2's relative inefficiency ( $c_2$ ) and the fixed cost of operating in the market ( $C$ ).<sup>12</sup> From definition 2 it follows that firm 1's duopoly phase price  $p_1^R$  is the piecewise maximum of two functions,

$$p_{\max} = \frac{3t + c_2}{2}, \tag{5}$$

and for arbitrarily small  $\varepsilon > 0$ ,

$$p_{\text{war}} = 2\sqrt{\frac{C}{2} \cdot \frac{(2t - c_2)(t - c_2)}{C - c_2 \frac{3t - c_2}{4t}}} - t + c_2 + \varepsilon. \tag{6}$$

---

<sup>11</sup>If  $C$  lies below this level then firm 2 is profitable in the stationary equilibrium and cannot be displaced unless firm 1 reverts to predatory pricing as explained in Section 5 below.

<sup>12</sup>The travel cost  $t$  also appears in the definition of  $p_1^R$ , however  $t$  is only significant in its magnitude relative to  $c_2$  and  $C$ . Within the model, the effects of an increase in  $t$  are equivalent to proportional decreases in  $c_2$  and  $C$ .

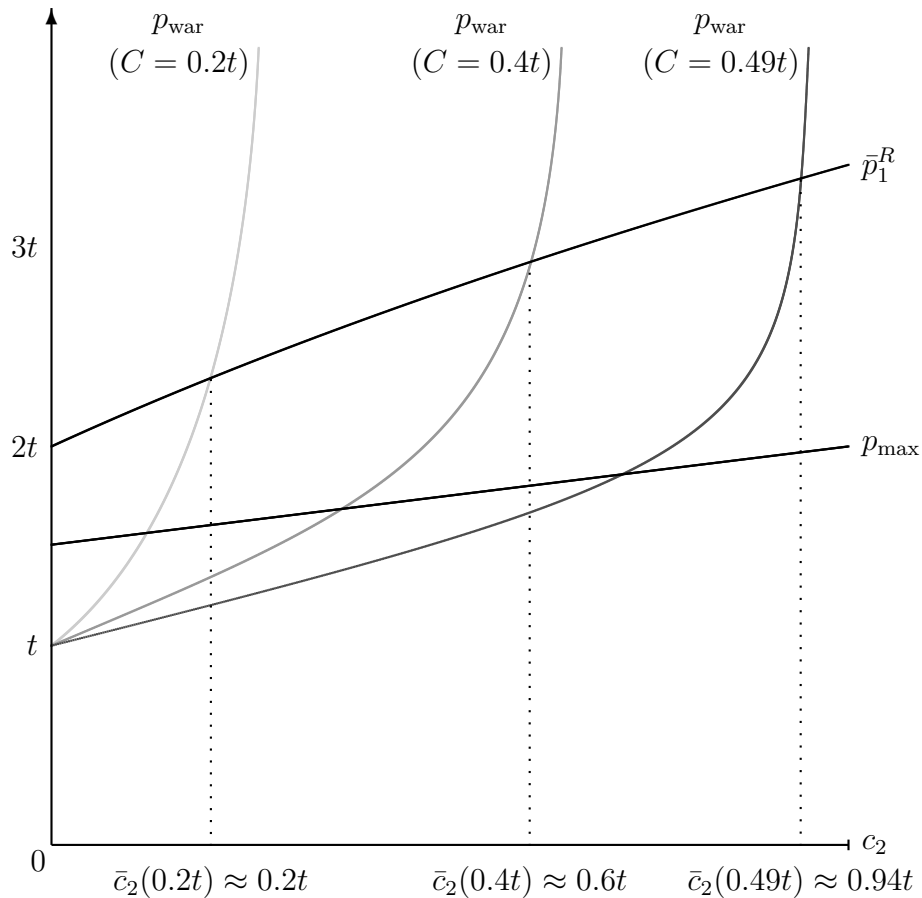


Figure 2: Constraints on  $p_1^R$

Here,  $p_{\max}$  is the price that maximises firm 1's equilibrium path profits subject to the assumption that firm 2 will play its best response.<sup>13</sup> While, the second price,  $p_{\text{war}}$ , is the lowest price that delivers firm 2 the necessary profits to win a war of attrition against an efficient entrant.

The relationship between  $p_{\max}$  and  $c_2$  is illustrated in Figure 2. Firm 1's profit maximising price is linear and strictly increasing in  $c_2$ . Moreover,  $p_{\max}$  is independent of the common fixed cost.

Figure 2 also shows the relationship between  $p_{\text{war}}$  and  $c_2$ , for three different levels of the fixed cost. These curves are convex and strictly increasing in  $c_2$ , while  $p_{\text{war}}$  is strictly decreasing in the fixed cost  $C$ .

<sup>13</sup>Essentially,  $p_{\max}$  is the price firm 1 would choose as a Stackelberg leader. Meza and Tomback (2009) explore the notion that in an entry game leadership may be endogenous, although their setting is essentially static.

Notice that  $p_{\max} > p_{\text{war}}$  in the neighbourhood of  $c_2 = 0$ , and as such  $p_1^R = p_{\max}$  where firm 2's marginal cost is relatively low. As  $c_2$  increases, a point is reached at which  $p_{\text{war}}$  crosses  $p_{\max}$ , and thereafter  $p_1^R = p_{\text{war}}$ . This crossing must occur as  $p_{\text{war}}$  tends to infinity as  $c_2$  approaches the level that would cause  $C$  to violate the lower bound established in (1). Intuitively, as  $C$  approaches its lower bound, an entrant almost breaks even during a war of attrition. This means that the entrant is willing to wait a very long time to achieve victory. To counter this advantage, firm 2 must receive arbitrarily high profits when it wins. But this requires firm 1 to set an arbitrarily high price in the duopoly phase. The value of  $c_2$  at which the crossing occurs is, itself, an increasing function of  $C$ , as  $p_{\text{war}}$  is a decreasing function of  $C$ .

Of course, the relational contracting equilibrium also requires that firm 1's price satisfies the finite upper bound in (4). The upper bound is illustrated in figure 2 as the finite and strictly increasing function  $\bar{p}_1^R$ . As shown in figure 2, for every  $C$  there exists an upper bound  $\bar{c}_2(C) \in (0, t)$ , on the relative inefficiency of firm 2 that can be accommodated in a relational contracting equilibrium. If firm 2's marginal cost exceeds this threshold, a relational contracting equilibrium does not exist for any discount factor. The upper bound is an increasing function of  $C$  indicating that the robustness of a relational contracting equilibrium improves as the fixed cost of operating in the market increases.

**Proposition 3.** *In the duopoly phase of a relational contracting equilibrium,*

- (a) *For all  $c_2$  and  $C$ , firm 1's duopoly phase price  $p_1^R$  is strictly greater than firm 2's duopoly phase price  $R_2(p_1^R)$ , and therefore, firm 1's market share is strictly less than that of firm 2.*
- (b) *If  $p_1^R = p_{\max}$  (resp.  $p_1^R = p_{\text{war}}$ ), firm 1's market share increases (resp. decreases) in  $c_2$ .*
- (c) *If  $p_1^R = p_{\max}$ , firm 1's profit increases in  $c_2$ , while firm 2's profit decreases.*
- (d) *If  $p_1^R = p_{\text{war}}$ , firm 2's profit increases in  $c_2$ , while firm 1's profit is a concave function of  $c_2$ , initially increasing before decreasing to  $\pi_1 = t/2 - C$  at  $c_2 = \bar{c}_2(C)$ .*

*Proof.* For (a) it is sufficient to show that  $p_1^r > R(p_1^R)$ . The two cases are,

$$R_2(p_{\max}) = \frac{5t + 3c_2}{4} < p_{\max},$$

where the inequality follows from the assumption that  $c_2 < t$ , and,

$$R_2(p_{\text{war}}) = \sqrt{\frac{C}{2} \cdot \frac{(2t - c_2)(t - c_2)}{C - c_2 \frac{3t - c_2}{4t}}} + c_2 + \frac{\varepsilon}{2} < p_{\text{war}}.$$

Note also that,

$$\frac{\partial p_1^R}{\partial c_2} < \frac{\partial R_2(p_1^R)}{\partial c_2},$$

where  $p_1^R = p_{\text{max}}$ , while the reverse holds where  $p_1^R = p_{\text{war}}$ , proving the market share result in (b). For the profit result in (c) note that where  $p_1^R = p_{\text{max}}$ ,

$$\pi_1 = \frac{1}{t} \left( \frac{3t + c_2}{4} \right)^2 - C \quad \text{and} \quad \pi_2 = \frac{1}{2t} \left( \frac{5t - c_2}{4} \right)^2 - C.$$

In this region,  $\pi_1$  (resp.  $\pi_2$ ) is unambiguously increasing (resp. decreasing) in  $c_2$ . For (d), where  $p_1^R = p_{\text{war}}$ , firm 2's profit is,

$$\pi_2 = \frac{C}{4t} \cdot \frac{(2t - c_2)(t - c_2)}{C - c_2 \frac{3t - c_2}{4t}},$$

which is unambiguously increasing in  $c_2$ . While,

$$\frac{\partial}{\partial c_2} \pi_1(p_{\text{war}}, R_2(p_{\text{war}})) = \frac{p_{\text{war}}}{4t} + \frac{\partial p_{\text{war}}}{\partial c_2} \cdot \frac{3t - 2p_{\text{war}} + c_2}{4t}.$$

The first term on the RHS is unambiguously positive. The second term on the RHS is zero where  $p_{\text{max}} = p_{\text{war}}$ , and negative thereafter, tending to  $-\infty$  as  $c_2$  approaches the level that would cause  $C$  to violate its lower bound.  $\square$

Proposition 3 develops the surprising result that, within the duopoly phase of a relational contracting equilibrium, the inefficient firm has a greater market share than its efficient rival. Firm 1's market share is highest where  $p_{\text{max}} = p_{\text{war}}$ . At low levels of  $c_2$  firm 1 loses market share because firm 2 is relatively aggressive. In the region where  $p_1^R = p_{\text{war}}$ , firm 1 must sacrifice significant market share to firm 2 as  $c_2$  increases, in order to keep firm 2 competitive in a war of attrition.

The relationship between  $c_2$  and firm profits is illustrated in Figure 3 for the case in which  $C = 0.4t$ . As stated in Proposition 3, firm 1's profits are initially increasing while firm 2's are declining. Firm 1's profits pass firm 2's at  $c_2 = \frac{5-3\sqrt{2}}{1+\sqrt{2}}t \approx 0.31t$ . This crossing point does not depend on  $C$  as  $p_{\text{max}}$  is independent of  $C$ . Firm 2's profit does not exceed that of firm 1 again until after the point at which  $p_{\text{max}} = p_{\text{war}}$ . In this example the second crossing occurs where  $c_2 \approx 0.44t$ .<sup>14</sup>

<sup>14</sup>Note that if  $C$  is small,  $p_{\text{war}}$  crosses  $p_{\text{max}}$  before the profit functions cross. In this case  $\pi_1 < \pi_2$  for all  $c_2$ .

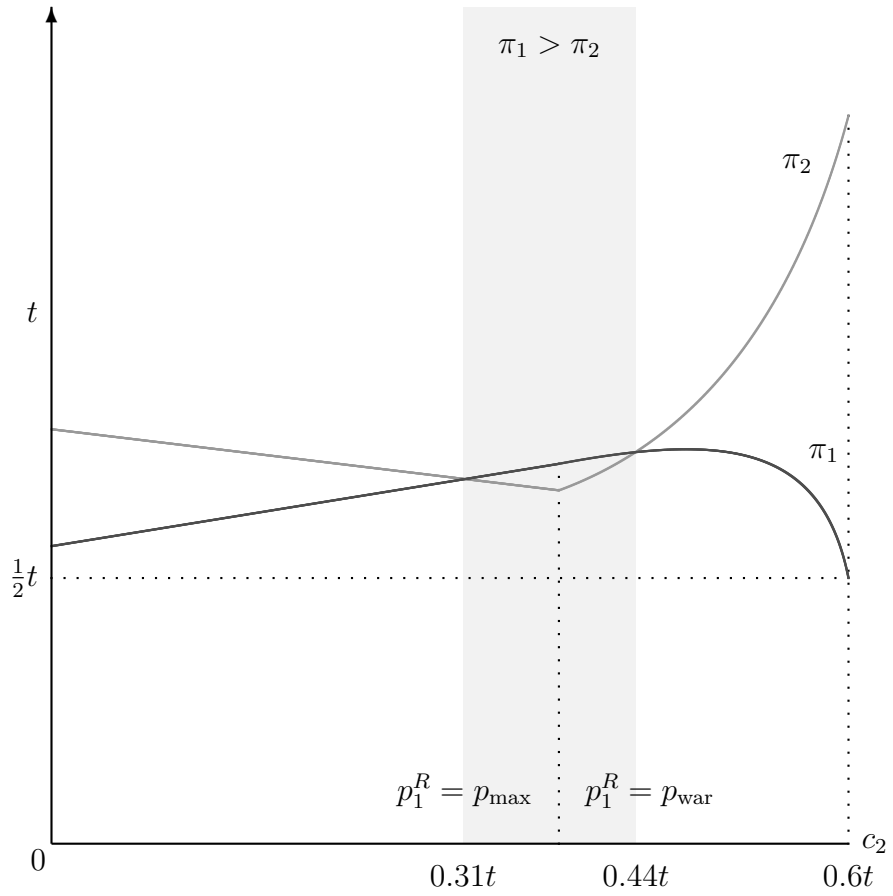


Figure 3: Firm profits ( $C = 0.4t$ )

C	$p_{\max} = p_{\text{war}}$	$\pi_1 > \pi_2$	$\max \pi_1$	$\bar{c}_2(C)$
0.20t	$c_2 \approx 0.12t$	—	$c_2 \approx 0.14t$	$c_2 \approx 0.20t$
0.40t	$c_2 \approx 0.38t$	$c_2 \in (0.31t, 0.44t)$	$c_2 \approx 0.47t$	$c_2 \approx 0.60t$
0.49t	$c_2 \approx 0.75t$	$c_2 \in (0.31t, 0.92t)$	$c_2 \approx 0.87t$	$c_2 \approx 0.94t$

Table 1: Critical values of  $c_2$

Figure 3 shows that, for the case in which  $C = 0.4t$ , firm 1's profit is maximised where  $c_2 \approx 0.47t$ . By contrast, the best outcome for firm 2 occurs where  $c_2$  approaches  $\bar{c}_2(0.4t) \approx 0.6t$ . In order to show how these relationships vary with  $C$ , Table 1 details the value of  $c_2$  at which  $p_{\max} = p_{\text{war}}$ , the range over which  $\pi_1 > \pi_2$  and the value of  $c_2$  that maximises  $\pi_2$ , for the values of  $C$  illustrated in Figure 2.

### 4.3 Consumer welfare

The welfare of any given consumer depends both on the distance they have to travel to purchase the good and on the price of the good. A consumer who travels a distance  $x$ , and pays a price  $p$ , receives a surplus of,

$$u = v - p - xt.$$

Figure 4 illustrates consumer welfare under a relational contracting equilibrium, and contrasts it with the surplus consumers receive under the various baselines of the model.

The horizontal axis in Figure 4 represents a consumer's location on the unit line. The four lines show the surplus consumers receive under various scenarios. From top to bottom these are: The competitive equilibrium with two efficient firms; the competitive equilibrium with one efficient and one inefficient firm; the relational contracting equilibrium with  $p_1^R = p_{\max}$ ,<sup>15</sup> and the price a monopoly would charge if it had a presence at each end of the market.

Each line initially slopes down because a consumer's surplus decreases at a rate of  $t$  per unit of distance traveled. In any given equilibrium, the consumer with the lowest surplus is the consumer who is indifferent between the offerings of the two firms. The location of this consumer also defines the market shares of the competing firms.

Figure 4 shows that, from the perspective of consumers, the best case is when two efficient firms compete. This equilibrium both has the lowest prices and minimises travel costs. The next best equilibrium is competition between one efficient and one inefficient

<sup>15</sup>Where  $p_1^R = p_{\text{war}}$  this line would be even lower.

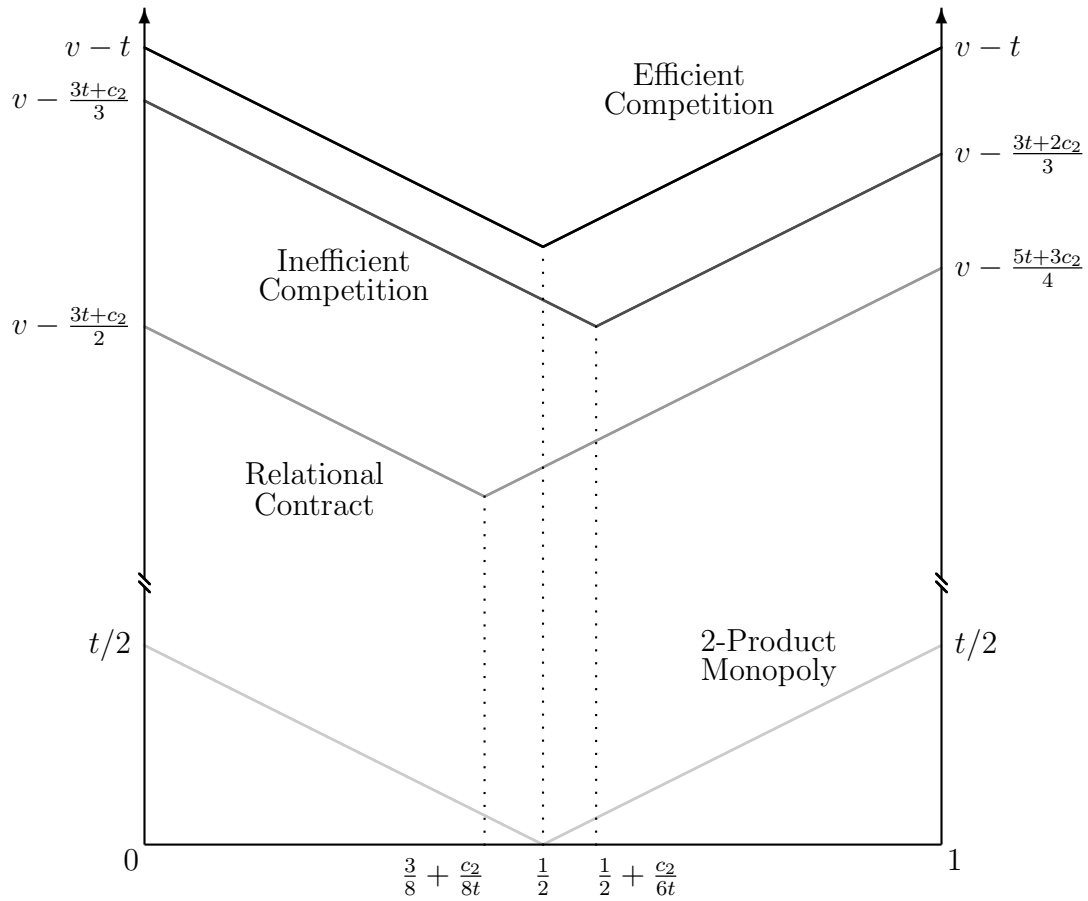


Figure 4: Consumer welfare

firm. Because the efficient firm prices more aggressively, some consumers to the right of centre choose to purchase from the more distant, but cheaper, firm. As shown in Proposition 3(a), the relational contracting equilibrium reverses this relationship. The inefficient firm now has the lowest price and captures the largest market share. The only case that is worse for consumers than the relational contracting equilibrium is where a monopoly controls both locations and sets a price of  $v - t/2$ .

## 4.4 Empirical implications

As described in the introduction, we view our model here as complementary to existing theories of PPDs that are based on frictions to competitive processes in markets. That said, there are some implications of the model that may be amenable to direct empirical testing should the right opportunity present itself.

First, the relational contracting equilibrium does not exist if there are barriers to entry. To see this, suppose that there were no entrants available with a higher efficiency level than the inefficient incumbent. In this case, the efficient incumbent would have no incentive to offer softer prices in the market as this would reduce their ongoing profits without the benefit of forestalling entry by more efficient firms. Thus, if we imagined a shock that suddenly reduced or eliminated entry barriers (e.g., deregulation of an industry or trade liberalisation), then our model predicts that a distribution of firm performance could emerge thereafter. Examining such an event could generate a test of the model's prediction.<sup>16</sup>

Second, the mechanism we identify here suggests that dispersion in firm performance is associated with stability in the identity of firms in the market. Specifically, dispersion in firm performance is associated with less churn amongst the firms – including the less efficient. While some analyses<sup>17</sup> have found that some of the worst performing firms exit the market (something consistent with our model), it would be instructive to look at the relationship between dispersion amongst the long-lived firms in the industry and how this relates to exogenous events that may change those firm's identities (e.g.,

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<sup>16</sup>Relatedly, suppose that, initially, all firms have marginal cost  $c_2 > 0$ . One day, firm 1 develops and patents a production technology that yields  $c_1 = 0$ . During the life of the patent the market behaves according to the stationary equilibrium in Proposition 1. Once the patent expires, firm 1 implements a relational contracting equilibrium with firm 2 to protect its advantage. What changes? All prices go up and the order of prices is reversed ( $p_1 < p_2$  in the stationary equilibrium is replaced by  $p_1 > p_2$ ). This results in firm 1 yielding market share to firm 2. Unless firm 2 is very inefficient, both firms' profits increase. (If  $c_2$  is close to  $\bar{c}_2(C)$  then firm 1's profits may fall.)

<sup>17</sup>Syverson (2004).



acquisition, mergers, technological shocks, etc).

## 4.5 Endogenous efficiency

Thus far, we have shown that when two firms in a natural duopoly start from differing levels of efficiency, there can exist an equilibrium whereby that distribution of efficiency is preserved even in the face of entry. Here, we analyse the outcomes when firms can choose their efficiency level, say, for some upfront cost.

First, suppose that, for a cost of  $F$ , a firm can reduce its marginal cost from  $c_2$  to 0. Then if firm 1 has a cost of 0 while firm 2 has a cost of  $c_2$ , if firm 2 expends  $F$  (regardless of how small  $F$  is), then its profits would be strictly lower compared with what it earns in the relational contracting equilibrium. This is intuitive as the same outcome could occur if an entrant acquired firm 2 and, in a relational contracting equilibrium, an entrant has no incentive to do that (even when transaction costs,  $k$ , are 0).

Second, suppose that both incumbents in an industry are inefficient and have marginal costs of  $c_2$ . Suppose also that prior to entry, each can simultaneously choose whether to expend  $F$  and reduce their marginal costs to 0. If neither do so, then at least one of them would be replaced by the first efficient entrant. Following that, if a relational contracting equilibrium emerges (which is now possible given that there are asymmetries in efficiency), then firm differences will persist thereafter with the efficient firm earning  $\pi_1$  and the inefficient firm earning  $\pi_2$ . Thus, at the outset, each incumbent's expected payoff is around  $\pi_2/2$  as it is random where entry might occur and the acquisition price would reflect the entrant's option to enter. By contrast, if both incumbents reduce their marginal costs, then each has an expected profit that would be earned under competition between efficient firms less  $F$ . Finally, if one firm invests while the other does not, then the investing firm earns  $\pi_1 - F$  and the other firm earns  $\pi_2$ . From this, it can be seen that, so long as  $\pi_1 - F > \pi_2/2$ , then there exists two pure strategy equilibria involving one or the other firm choosing to become efficient with the other remaining inefficient. Thus, from symmetrical starting conditions, a distribution of efficiencies emerges.

Finally, it is worth remarking that if entrants had differing levels of efficiency, then we can imagine models whereby an efficient incumbent allows entry to occur until the point where the entrant whose efficiency level generates the most profits for the efficient

incumbent in an relational contracting equilibrium appears.<sup>18</sup> This might assist in pinning down the precise expected distribution of firm efficiencies.

## 5 Extensions

We now consider some extensions to our baseline model. We consider the implications of additional firms being part of the natural oligopoly before demonstrating that there are equilibria that expand the range of parameters under which PPDs arise. Finally, we comment on whether the equilibrium is robust to other forms of within market competition such as quantity competition.

### 5.1 Three firms

A natural extension of our model is the case in which firm 1 must protect multiple rivals of varying efficiencies. Consider the case of three firms, firms 1, 2 and 3, evenly spaced around the unit circle. Suppose that the marginal costs of the firms satisfy  $0 = c_1 < c_2 \leq c_3 < t/3$ . As before, all potential entrants are assumed to be efficient with marginal costs of zero.

The reduced lower bound on  $c_3$  is required by the higher intensity of competition on the unit circle. It is also necessary to place tighter bounds on the common fixed cost such that  $C \in \left( c_3 \frac{c_2 - 2c_3 + 5t/3}{3t}, \frac{t}{9} \right)$ . The interpretation of these bounds is the same as for those in (1) above.

Along the short arc of the circle between firms  $i$  and  $j$ , the indifferent consumer is located at a distance of,

$$x_{ij}^* = \frac{1}{6} + \frac{p_j - p_i}{2t},$$

from firm  $i$ . It follows that firm  $i$ 's profit can be written,

$$\pi_i(p_i, p_j, p_k) = (p_i - c_i) \left( \frac{1}{3} + \frac{p_j + p_k - 2p_i}{2t} \right).$$

While firm  $i$ 's best response function is,

$$R_i(p_j, p_k) = \frac{p_j + p_k + 2c_i + \frac{2t}{3}}{4}.$$

As a baseline, it is useful to note that for a market in which three efficient firms compete, equilibrium prices and profits are  $t/3$  and  $t/9$ , respectively.

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<sup>18</sup>These marginal costs are detailed in table 1 for a range of values of  $C$ .

**Definition 3.** A relational contracting equilibrium is an SPE of the three firm game, satisfying definition 2, with condition (b) replaced by,

(b') If firms 1, 2 and 3 are the only firms active in the market, firm 1 sets the price,

$$p_1^T = \max \left\{ \frac{5}{12}t + \frac{c_2 + c_3}{4}, \sqrt{C \frac{(t - 2c_3)(t - 3c_3) - 3c_2c_3}{C - c_3 \left( \frac{c_2 - 2c_3 + \frac{5t}{3}}{3t} \right)}} + \frac{7c_3 - 2c_2}{5} - \frac{2t}{3} + \varepsilon \right\}, \quad (7)$$

for some arbitrarily small  $\varepsilon > 0$ , and firms 2 and 3 play their best responses to  $p_1^T$ .

As previously stated, the first term on the RHS of 7 is the price that maximises firm 1's profits subject to the assumption that firms 2 and 3 will play their respective best responses. While the second term on the RHS is the minimum price that guarantees firm 3 victory in a war of attrition. It is not necessary to develop a similar term for firm 2 as, given that firm 2 is (weakly) more efficient than firm 3, the critical price for firm 2 will be (weakly) lower.

**Proposition 4.** For sufficiently high  $\delta \in (0, 1)$ , a relational contracting equilibrium exists if and only if,

$$p_1^T < \frac{1}{4} \left( \sqrt{\frac{t^2}{9} + \frac{10}{3}(c_2 + c_3)t + (c_2 + c_3)^2 + c_2 + c_3 + \frac{5t}{3}} \right). \quad (8)$$

*Proof.* The proof follows immediately from the proof of Proposition 2. Note that step 1 requires,

$$\theta_3 = \frac{\frac{1}{t} \left( \frac{5p_1^T - 7c_3 + 2c_2 + \frac{10t}{3}}{15} \right)^2 - C}{C} > \frac{\frac{t}{9} - C}{C - c_3 \left( \frac{c_2 - 2c_3 + \frac{5t}{3}}{3t} \right)} = \theta_4,$$

while step 3 requires,

$$p_1^T \left( \frac{c_2 + c_3 - 2p_1^T + \frac{5t}{3}}{3t} \right) - C > \frac{t}{9} - C.$$

Solving for  $p_1^T$  yields the required constraints.  $\square$

The price firm 1 sets in the three-firm case displays essentially the same relationship with the marginal cost of the most inefficient firm, and the common fixed cost, as was

the case in the two-firm relational contracting equilibrium discussed above. Of interest here is the role that firm 2's marginal cost plays in the equilibrium.

From the perspective of firm 1, the best case is where the two remaining incumbents share the same inefficiency. A high  $c_2$  increases firm 1's price and profit when the first term on the RHS of (7) binds. It also increases the price ceiling in (8). At the same time, the second constraint on the RHS of (7) is decreasing in  $c_2$ . This means that as  $c_2$  rises relative to  $c_3$ , the first term on the RHS of (7) binds for a wider range of parameter values.

The intuition behind this outcome is straightforward. Firm 2 plays no role in protecting firm 3 from entry or acquisition. As firm 2 becomes more efficient, it both reduces firm 1's profits through more intense direct competition and reduces the effectiveness of firm 1's efforts to protect firm 3 by competing away a portion of firm 3's profits. This demonstrates that it is possible to enrich the model by adding more firms into the market. This, however, raises issues as to the appropriate product space which is a direction we leave open to future research.

## 5.2 Firm 1's efficiency

While we have considered the role that firm 2's marginal cost plays in determining the stability of a relational contracting equilibrium, throughout the paper it was always assumed that firm 1 operates at the technological frontier. The consequences of relaxing this assumption depend on where firm 1's marginal cost sits relative to the marginal costs of the potential entrants.

Similar to [Byford and Gans \(2014\)](#), allowing for firm 1 to be more efficient than the potential entrants only strengthens a relational contracting equilibrium. Intuitively, as an entrant's efficiency falls relative to that of the efficient incumbent, so too does its type in a war of attrition. It follows that  $p_{\max}$  will bind over a larger range of  $c_2$  (weakly) increasing firm 1's duopoly-phase profits. This demonstrates that our potential driver of PPDs is a complement to existing theories that provide reasons why an incumbent may have other efficiency advantages over potential entrants. If those theories hold, the prediction that there will be a dispersion in performance is strengthened. Moreover, the relational contracting equilibrium could be sustained without the additional implication that the efficient firm's market share is less than the inefficient firm's share.

On the other hand, if firm 1 is less efficient than the potential entrants ( $0 < c_1 < c_2$ ), there are two possible outcomes. If the costs of entry and acquisition ( $K$  and  $k$ ) are

sufficiently high, or equivalently if  $c_1$  is sufficiently close to zero, firm 1 will remain in the market. However, by the reverse of the logic outlined above, a relational contracting equilibrium will be weaker and possibly cease to exist. If entry or acquisition is viable, then an efficient entrant maximises its profits by displacing firm 1, and then protecting firm 2 as the new efficient incumbent, in the manner described in Section 4.

### 5.3 A relational contract with predation

The relational contracting equilibrium shows how an efficient incumbent can unilaterally improve an inefficient rival's type, thereby eliminating the threat of de novo entry. As noted above, for some parameter values there does not exist a price  $p_1^R$  that simultaneously satisfies (3) and (4) (or equivalently  $c_2 \geq \bar{c}_2(C)$ ). Where this is the case, any price high enough to give firm 2 the edge in a war of attrition reduces firm 1's profits below the level it would receive if firm 2 were replaced by an efficient entrant.

The advantage the entrant enjoys when firm 2 has a high marginal cost can be reduced if the inefficient incumbent employs *predatory pricing* during a war of attrition. By pricing at the entrant's marginal cost, firm 2 guarantees that the entrant will incur a loss of  $C$  in each period of a war of attrition. Of course, this requires firm 2 to make sales at a price below its own marginal cost, increasing the loss firm 2 incurs during the war.

**Definition 4.** A predatory equilibrium is an SPE of the game, satisfying definition 2, with conditions (b) and (c) replaced by,

(b') In a duopoly phase in which firms 1 and 2 are active in the market, firm 1 sets the price,

$$p_1^P = \max \left\{ \frac{3t + c_2}{2}, \sqrt{4t^2 - 6tc_2 + \frac{3t^2c_2}{C}} - t + c_2 + \varepsilon \right\}, \quad (9)$$

for some arbitrarily small  $\varepsilon > 0$ , and firm 2 plays its best response to  $p_1^P$ .

(c') During a war of attrition firm 2 sets the price  $p_2 = 0$ , firm 1 plays its best response to this price, and firm 3 does not offer units for sale. If firm 2 charges a price in excess of  $p_2 = 0$ , thereafter each firm plays its best response to the prices chosen by its rivals.

The interpretation of condition (b') is the same as that of condition (b) in definition 2. The change to firm 1's duopoly-phase price is due to the higher losses both firms

incur during a war of attrition. The second value in (9) is derived in the proof of Proposition 5 below.

Condition (c') captures the key difference between the relational contracting equilibrium and the predatory equilibrium. In a predatory equilibrium, firm 2 sets the price  $p_2 = 0 < c_2$  during a war of attrition. Firm 1 still plays its best response, and the entrant does not offer its goods for sale. (Refusing to trade maximises the cost to firm 2 of its predatory pricing.)

Note that during a war of attrition, firm 2 has an incentive to raise its price above zero. At a higher price, firm 2's losses are lower. Condition (c') states that if firm 2 fails to implement below cost pricing, behaviour during the war of attrition reverts to that described by condition (c) of definition 2. This change in behaviour alters the types of the collocated firms because subsequent losses in the war of attrition are lower. Intuitively, in this equilibrium, the failure of firm 2 to follow through on the threat of predatory pricing undermines firm 2's credibility, and as such all firms return to playing their best responses.

**Proposition 5.** *For sufficiently high  $\delta \in (0, 1)$ , if  $c_2 \geq \bar{c}_2(C)$  then a predatory equilibrium exists.*

*Proof.* Steps 2 and 3 from the proof to Proposition 2 ensure that firm 1 will not deviate during the duopoly phase, and that entry by acquisition is not viable so long as (4) holds.

To see that firm 2's type is higher than an entrant's, note that the entrant makes a loss of  $C$  during a war of attrition as it makes no sales, while firm 2's loss is  $C + \frac{3}{4}c_2$ . The inequality,

$$\theta_2 = \frac{\frac{1}{2t} \left( \frac{p_1^P + t - c_2}{2} \right)^2 - C}{C + \frac{3}{4}c_2} > \frac{\frac{t}{2} - C}{C} = \theta_3$$

implies,

$$p_1^P > \sqrt{4t^2 - 6tc_2 + \frac{3t^2c_2}{C}} - t + c_2,$$

which is satisfied by (9). Moreover, for sufficiently high  $\delta$ , firm 2 will not deviate during a war of attrition as a deviation results in the inefficient incumbent losing the war, while maintaining  $p_2 = 0$  results in victory and a positive payoff.

To see that  $p_1^P$  satisfies (4), first note that  $p_1^P$  is continuous and decreasing in  $C$ . It follows that the worst case is where  $C$  approaches the lower bound defined in (1).

Substituting for  $C$  into (9),

$$\begin{aligned}
\sqrt{4t^2 - 6tc_2 + \frac{3t^2c_2}{c_2\frac{3t-c_2}{4t}}} - t + c_2 &= \sqrt{8t^2 - 2tc_2 \left(3 - \frac{2t}{3t - c_2}\right)} - t + c_2 \\
&< \sqrt{8t^2} - t + c_2 \\
&< 2t + c_2 \\
&< \frac{1}{2} \left( \sqrt{t^2 + 6tc_2 + c_2^2} + 3t + c_2 \right),
\end{aligned}$$

which is the upper bound on  $p_1^P$  from (4).  $\square$

Proposition 5 proves that the damage done to an entrant's type as a result of predatory pricing by firm 2 during a war of attrition, is sufficient to guarantee firm 2 victory in the war. It is unclear how regulators would regard firm 2's behaviour. While it is true that firm 2's unilateral conduct following entry involves it pricing below its own marginal cost, it does not price below the entrant's marginal cost. Indeed, if the entrant matches firm 2's price, its profit on the marginal unit is zero. The losses of the collocated firms could still be attributed to the natural duopoly characteristic of the market.

## 5.4 Other competitive environments

Our model is set on the Hotelling line as this model of competition illustrates how a strong firm can protect a weak rival from entry or acquisition. It remains an open question as to what other competitive environments can support equilibria in the vein of definitions 2 and 4. While we do not provide a definitive answer in this paper, we can identify two characteristics that are likely to be necessary.

First, our model depends on the natural oligopoly structure of the market. The mechanism requires that when an efficient entrant enters de novo, attempting to displace the inefficient incumbent, the result is a war of attrition. If an entrant could compete against the inefficient incumbent without incurring a loss, as would be the case in our model if  $C$  were below the lower bound in (1), there would be no incentive for the entrant to exit the market, regardless of firm 1's duopoly-phase behaviour.

Second, it is important that market-stage actions are strategic complements. Intuitively, when firm 1 raises its price to direct a stream of profits to firm 2, firm 2 responds by likewise increasing its price. This reduces the cost to firm 1 of engaging in accommodating behaviour. In contrast, where the actions of firms are strategic substitutes

(as they are in Cournot quantity competition), firm 2 responds to firm 1's largesse with increased aggression. This increases the cost to firm 1 of accommodating firm 2 and may preclude a relational contracting equilibrium for any parameter values.<sup>19</sup>

## 6 Conclusion and future directions

This paper has presented a new approach to examining how persistent performance differences amongst firms might arise in the face of strong competitive pressures (in particular, entry). It has done so by considering which actors have both the ability and the incentive to give an inefficient firm permission to exist. We identified the inefficient firm's efficient incumbent rivals as a natural candidate. We then proceeded to demonstrate, using a special but commonly examined environment, that an efficient incumbent could unilaterally present conditions that permitted its inefficient counterpart to either act strongly in the face of entry or hold a position that was too expensive for an efficient entrant to acquire.

While our approach here was to provide a 'proof of concept' rather than a general treatment, the foundation of the model is on familiar elements. Future research could easily examine alternative modes of oligopolistic competition with richer specifications and elements, wars of attrition that involved information asymmetries and uncertainty and finally alternative mechanisms to transfer rents from efficient to inefficient incumbents such as openness and disclosure as well as the licensing of key technologies. Thus, we believe that our model presented in this paper offers many directions that could lead to formal models tailored to specific strategic circumstances in particular industries. At the same time, this offers the possibility for more nuanced predictions to further empirical examination of PPDs.

## References

- Barney, J. (1991) Firm resources and sustained competitive advantage, *Journal of Management*, 17, 99.
- Bartelsman, E.J. and M. Doms (2000), Understanding productivity: Lessons from longitudinal microdata, *Journal of Economics Literature*, 38 (3), 569–594.

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<sup>19</sup>A similar result is derived by [Byford and Gans \(2014\)](#).



- Bishop, D., C. Canning and J. Maynard Smith (1978), The war of attrition with random rewards, *Journal of Theoretical Biology* 74, 377–388.
- Byford, M.C. and J.S. Gans (2014), Exit deterrence, *Journal of Economics and Management Strategy* 23, 651–668.
- Gans, J.S. and J. Quiggin (2003), A technological and organisational explanation of the size distribution of firms, *Small Business Economics*, 21(3), 243–256.
- Gibbons, R. and R. Henderson (2013), What do managers do? in R. Gibbons and J. Roberts (eds.), *The Handbook of Organizational Economics*, Princeton University Press: Princeton, Chapter 17.
- Kreps, D. and R. Wilson (1982), Reputation and imperfect information, *Journal of Economic Theory* 27, 253–279.
- Levin, J. (2004), Wars of attrition, *mimeo.*, Stanford.
- MacDonald, G. and M.D. Ryall (2004), How do value creation and competition determine whether a firm appropriates value? *Management Science* 50, 1319–1333.
- Makadok, R. (1998), Can firstmover and earlymover advantages be sustained in an industry with low barriers to entry/imitation? *Strategic Management Journal*, 19(7), 683–696.
- Meza, S. and M. Tombak (2009), Endogenous location leadership, *International Journal of Industrial Organization* 27, 687–707.
- Myatt, D.P. (1999), Instant exit from the asymmetric war of attrition, *mimeo.*, Oxford.
- Powell, M. (2013), Productivity and credibility in an industry equilibrium, *mimeo.*, Northwestern.
- Riley, J. (1980), Strong evolutionary equilibrium and the war of attrition, *Journal of Theoretical Biology* 82, 383–400.
- Sutton, J. (1991), *Sunk Costs and Market Structure*, MIT Press: Cambridge.
- Syverson, C. (2004), Market structure and productivity: A concrete example, *Journal of Political Economy*, 112(6), 1181–1222.

Syverson, C. (2011), What determines productivity? *Journal of Economic Literature*, 49(2), 326–365.

Wernerfelt, B. (1984), A resource-based view of the firm, *Strategic Management Journal* 5, 171–180.