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[figures available upon request]

## **On the Complexities of Complex Economic Dynamics**

J. Barkley Rosser Jr.

J. Barkley Rosser Jr. is Professor of Economics and Kirby L. Kramer Jr. Professor, James Madison University, Harrisonburg, Virginia. His email address is <rosserjb@jmu.edu>.

### Abstract:

Complex economic nonlinear dynamics endogenously do not converge to a point, a limit cycle, or an explosion. Their study developed out of earlier studies of cybernetic, catastrophic, and chaotic systems. Complexity analysis stresses interactions among dispersed agents without a global controller, tangled hierarchies, adaptive learning, evolution, and novelty, and out-of-equilibrium dynamics. Complexity methods include interacting particle systems, self-organized criticality, and evolutionary game theory, to simulate artificial stock markets and other phenomena. Theoretically, bounded rationality replaces rational expectations. Complexity theory influences empirical methods and restructures policy debates.

In recent years, considerable attention and publicity have emerged regarding studies of "complexity" in a variety of disciplines, including economics (Waldrop, 1992; Lewin, 1992). Some of this publicity has made dramatic claims that new approaches have arisen that are applicable across many disciplines and that promise fundamental new insights into these disciplines, with economics being one of the main disciplines drawing such research efforts and attention. These approaches evolved out of earlier work using nonlinear dynamics and have been used to explain such phenomena as path dependence in technological evolution and regional development and the appearance of discontinuities such as the crashes of speculative bubbles or the collapses of whole economic systems. Major centers of this research effort have included earlier the Free University in Brussels in Belgium, Stuttgart University in Germany, and later the Santa Fe Institute in New Mexico. This paper examines the nature and development of this research program in economics.

Referring to researchers at the Santa Fe Institute, Mitchell Waldrop (1992, pp. 12-13) declares, "they all share the vision of an underlying unity, a common theoretical framework for complexity that would illuminate nature and humankind alike ... They believe that their application of these ideas is allowing them to understand the spontaneous, self-organizing dynamics of the world in a way that no one ever has before -- with the potential for immense impact on the conduct of economics, business, and even politics ... They believe they are creating, in the words of Santa Fe Institute founder George Cowan, 'the sciences of the twenty-first century.'"

In contrast, John Horgan (1995, 1997) ridicules what he labels "chaoplexology," arguing

that complexity is the latest in a series of failed fads, the "four C's" of cybernetics, catastrophe, chaos, and complexity. He writes (1997, p. 226): "So far chaoplexologists have created some potent metaphors: the butterfly effect, fractals, artificial life, the edge of chaos, self-organized criticality. But they have not told us anything about the world that is both concrete and truly surprising, either in a negative or in a positive sense."

These sharply contrasting viewpoints reflect the controversy surrounding the multidisciplinary study of complexity, as well as its predecessors in a line of development within the broader science of nonlinear dynamics that includes the earlier C's. Is it (and them) a fundamental, paradigm-shifting breakthrough, or just a fallacious fad hyped by grant-grubbing academics and journalists seeking bestsellers? Waldrop and Horgan are both bestselling science journalists, which may explain the extremity of their views. Neither is an economist, although Waldrop writes more about economics than does Horgan. We shall attempt to sort out the substance from the hype in applications to economics of this claimed new "science of complexity."

### **What is Complexity? A Broad Tent View**

Unsurprisingly, there is no agreed-upon definition of such a complex term as "complexity." Indeed, MIT's Seth Lloyd has gathered over 45 such definitions, most of these listed in Horgan (1997, Chapter 8, footnote 11, p. 303), with many of these definitions emphasizing computational or informational measures. This plethora leads Horgan (1995) to complain that "we have gone from complexity to perplexity." Without doubt, this is a serious

problem.

A "broad tent" definition, following Richard H. Day (1994), is that a dynamical system is complex if it endogenously does not tend asymptotically to a fixed point, a limit cycle, or an explosion. Such systems can exhibit discontinuous behavior and can be described by sets of nonlinear differential or difference equations, possibly with stochastic elements. But not all such equation systems will generate complexity. The positive exponential function, basis for most growth models, is an example of a noncomplex, nonlinear system because it just explodes.

Despite its "broad tent" nature, this definition does not fit what some economists have called complexity. Indeed, in receiving comments on an earlier draft of this paper, no other issue elicited more commentary and more diversity of discussion. Pryor (1995) and Stodder (1995) emphasize a "structural" viewpoint on complexity, meaning simply that there are lots of complicated interrelationships and institutional structures within the economy. Leijonhufvud (1993), Stodder (1997), and Albin with Foley (1998) emphasize a computational definition, arguing that the situations exhibit complexity when there is an extreme difficulty of calculating solutions to optimization problems. For example, Albin (1982) emphasizes the problem of formal undecidability arising from agents trying to model other agents' modeling of them modeling those agents, ad infinitum, as a fundamental source of computational complexity.<sup>1</sup> Sargent (1993) emphasizes the computational aspect of complexity in his conversion from rational expectations to bounded rationality. Other views hold that complexity implies a completely new philosophical perspective on the relationship between humanity and nature, and

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<sup>1</sup>Formal undecidability derives from the work of the logician, Kurt Gödel, and is a deep problem in game theory (Binmore, 1987). See Conlisk (1996) and Koppl and Rosser (1997) for

that complexity cannot be deductively defined but can only inductively emerge from the modeling efforts of researchers. This short list does not exhaust the diversity of views on this subject.<sup>2</sup>

However, Day's (1994) broad-tent definition remains attractive, because it is sufficiently broad that it includes not only most of what is now generally labeled complexity, but also its nonlinear dynamics predecessors, cybernetics, catastrophe theory, and chaos theory. Horgan's (1995, 1997) criticism is partly right; all of these ideas have gone through intellectual boom-bust cycles to varying degrees, and all of them are in a sense related to each other. Although, for example, current advocates of complexity might sometimes wish to disassociate themselves from the terminology of "catastrophe theory," it might be that they should adopt the attitude of the Impressionist painters when their critics affixed that label upon them: accept it and go with it. There are common threads through all four of the "C" concepts that go beyond nonlinearity, threads that trace to common roots in late nineteenth century studies of mathematics and celestial mechanics by Henri Poincaré and others.

Although complexity is a multidisciplinary concept derived from mathematics and physics, the extra complications arising in economics because of the problem of interacting human calculations in decisionmaking add a layer of complexity that may not exist in other disciplines. This has led Donald Saari (1995, p. 222) to declare, "[W]hat we know indicates that even the simple models from introductory economics can exhibit dynamical behavior far more

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more general discussion.

<sup>2</sup>Other more general references on economic complexity include Anderson, Arrow, and Pines (1988); Barnett, Geweke, and Shell (1989); Day and Chen (1993); Krugman (1996); Puu (1997). and Arthur, Durlauf, and Lane (1997a).

complex than anything found in classical physics or biology." Indeed, this appears to be a fundamental source of complexity in virtually all dynamic economic models.

### **The Predecessors: Cybernetics, Catastrophe, Chaos**

Cybernetics was the brainchild of Norbert Wiener (1961). Although initially forbidden in the Soviet Union, it became especially popular with planners there due to its emphasis on feedback and control of systems, and eventually morphed into general systems theory (von Bertalanffy, 1975). Its most famous U.S. advocate in economics has been Jay Forrester (1961), who emphasized the possibility of "counterintuitive" and surprisingly discontinuous outcomes that can arise in systems of multiple nonlinear equations. Such ideas have carried through to current complexity models, including the self-organization and emergence of higher-ordered structures out of lower-level systems, such as the emergence of a pattern of distinct residential neighborhoods in a city out of the actions of localized individual agents. Much of the modeling that followed Forrester came to be criticized on various grounds, including arguments that it had sometimes used inappropriate aggregations and implausible or plain false assumptions, which in turn had led to incredible outcomes, predictions, and policy recommendations.<sup>3</sup> But the influence of cybernetics runs stronger and deeper in current modeling efforts than is often recognized.

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<sup>3</sup>Forrester (1969) made dubious (and Acounterintuitive $\cong$ ) recommendations about urban policy based on a very questionable such model. A much criticized effort derived from Forrester=s work was the Club of Rome=s model of limits to global economic growth (Meadows, et al, 1971).

The French mathematician, René Thom (1975), developed catastrophe theory out of the older theory of dynamical systems. A catastrophe is a particular kind of discontinuity in a dynamical system. The discontinuities depend on distinct multiple equilibria and involve jumping from one to another as some control parameter gradually changes. Figure 1 shows a canonical version due to Varian (1979) of the Kaldor (1940) trade cycle model. Savings is a linear function of income ( $y$ ). Investment is an S-shaped function of income and of the existing level of capital stock ( $k$ ). In equilibrium,  $S=I$ . Clearly, the middle investment function allows multiple equilibria, and one can lay out scenarios of jumping between these equilibria as the investment function rises and falls through the business cycle. Similar figures and dynamics appear in models of discontinuous dynamics in urban/rural balance equilibria (Casetti, 1980) and in foreign exchange markets (Krugman, 1984).

Catastrophe theory generated an even greater multidisciplinary fad than did cybernetics. Indeed, modeling discontinuities continues to be a major theme of more recent complexity models. However, some of the applications of catastrophe theory came in for sharp criticism, often because the theory was applied to explain all kinds of discontinuities, even when the rather restrictive assumptions necessary for applying the theory properly did not hold (Zahler and Sussman, 1977; Rosser, 1991). The major criticism of catastrophe theory in economics came from Zahler and Sussman (1977), who focused on Zeeman's (1974) model of stock market dynamics in which crashes happen when there are too many chartist relative to fundamentalist traders. In the heyday of rational expectations, Zahler and Sussman dismissed the idea of chartist traders as irrational and unscientific -- and catastrophe theory also. In its time, this critique was viewed as powerful enough that many economists have shied away from catastrophe theory ever

since. However, their dismissal of the very notion of chartists now looks absurd in a world where noise traders can survive (DeLong, Shleifer, Summers, and Waldmann, 1991).<sup>4</sup> Thus, Zahler and Sussman "threw the baby out with the bathwater" (Guastello, 1995).

Chaos theory has no single inventor, but a fad for it followed publication of the bestselling book by the journalist, James Gleick (1987), a fad further fueled in the popular mind by Jeff Goldblum's portrayal of a "chaotician" in the film *Jurassic Park*. From the late 1970s onward, there have been numerous applications of chaos theory in almost every area of economics.<sup>5</sup> Chaotic dynamics are generated by a deterministic process; however, both to the naked eye and to many more formal statistical examinations, they appear to be random. The empirical existence of truly chaotic dynamics has been the subject of intense and inconclusive debate; Dechert (1996) contains major relevant papers.

Two ideas commonly associated with chaotic dynamics are that of sensitive dependence on initial conditions and strange attractors. The former, more famously known as the "butterfly effect," notes that slight differences in a starting value or a parameter value can lead to very different trajectories over time, as with a butterfly causing a hurricane on the other side of the world by flapping its wings (even though the divergent trajectories will eventually almost reconverge, if just briefly). This implies a deep unpredictability that has been argued to undermine the possibility of rational expectations in economics (Grandmont, 1985). McCloskey (1991) suggests that this analysis applies to small historical events having enormous

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<sup>4</sup>Genotte and Leland (1990) suggest applying catastrophe theory to heterogeneous agent financial models, and Rosser (1997) provides such an application.

<sup>5</sup>Some overviews of such applications include Baumol and Benhabib (1989); Rosser (1991), and Puu (1997).



consequences, as in the famous homily of the want of a nail leading to the loss of the kingdom, with McCloskey noting relevant examples from the American Civil War.

The second idea is that many chaotic systems tend to follow paths of complicated and irregular geometric shapes known as "strange attractors."<sup>6</sup> Figure 2 presents an example of such a chaotic strange attractor, in particular the Rössler (1976) attractor<sup>7</sup> that has been found in several economics models exhibiting chaotic dynamics (Goodwin, 1990; Radzicki, 1990), with  $x$ ,  $y$ , and  $z$  possibly representing deviations from long-term trends of capital stock, GDP, and price level respectively. This pattern represents what might be an equilibrium trajectory in three variables in the sense that a deterministic system would follow it forever if it ever got onto the path. As can be seen in the case of the Rössler attractor there may be an overall pattern of apparent cycle or repetition. Thus it is not surprising that some have claimed to find chaos in dynamic trajectories that appear to possess some kind of cycle, as in milk prices that depend on the much-studied cattle cycle (Chavas and Holt, 1993). However, not all dynamical systems that

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<sup>6</sup>What defines the Astrangeness $\cong$  of a strange attractor is that it possesses a non-integer or Afractal $\cong$  dimensionality (Mandelbrot, 1983; Peitgen, Jürgens, and Saupe, 1994). Any attractor will have a Basin of attraction $\cong$  within which the system will asymptotically approach the attractor. It is possible for such basins to have boundaries that are of such irregular fractal shapes even if the attractor itself is not strange, thus implying further possibilities for complex dynamics. Examples in economic models are due to Lorenz (1992), Soliman (1996), Brock and Hommes (1997), and Puu (1997).

<sup>7</sup>The differential equation system that generates the Rössler (1976) attractor is given by the three equations shown below with  $a$ ,  $b$ , and  $c$  being adjustable constants. In Figure 2 they are fixed at  $a = 0.2$ ,  $b = 0.2$ , and  $c = 5.7$  (Peitgen, Jürgens, and Saupe, 1994, pp. 686-89). As can be readily seen these are not especially complicated equations:

$$\dot{x} = -(y+z)$$

$$\dot{y} = x + ay$$

$$\dot{z} = b + xz - cz.$$

follow strange attractors also exhibit the butterfly effect, they are strange but with a greater degree of local stability -- and thus are not truly chaotic. Lorenz (1992) demonstrated that another version of the Kaldor (1940) trade cycle model can follow such a nonchaotic strange attractor.

Despite questions and criticisms, chaos theory has not suffered nearly the avoidance by economists that catastrophe theory has since it was criticized by Zahler and Sussman (1977). Areas of recent research in chaotic economic dynamics include multidimensional chaotic models of financial markets (Brock and Hommes, 1997) and tâtonnement price adjustments (Goeree, Hommes, and Weddepohl, 1998), methods of controlling chaotic economic dynamics in general (Holyst, et al, 1996) as well as specifically in microeconomic markets (Kopel, 1997) and macroeconomic models (Kaas, 1998), the possibility of mimicking chaotic dynamics with simple, boundedly rational rules (Grandmont, 1998; Hommes and Sorger, 1998), and new econometric methods for estimating chaos (Dechert, 1996; Bask, 1998; Whang and Linton, 1999). Motivating much of this research is the hope that not only deeper understanding of the nature of dynamic processes will be achieved, but that improvements in forecasting in actual markets and economies will be achieved. The fact that many researchers of the more recent complexity models either were or still are researchers of chaos theory is what led Horgan (1997) to coin the terms "chaoplexity" and "chaoplexologist."

### **From Brussels and Stuttgart on the Way to Santa Fe**

There are many who see complexity as a newer and higher stage of analysis, distinct from

these earlier "C's" of cybernetics, catastrophe, and chaos. This stance begs for a "narrow tent" definition of complexity, but no tight definition exists. Rather, speaking for the "Santa Fe perspective," Arthur, Durlauf, and Lane (1997b, pp. 3-4) suggest that complexity consists of six characteristics: 1) dispersed interaction among heterogeneous agents acting locally on each other in some space (although some Santa Fe Institute models assume dispersed homogeneous agents); 2) no global controller that can exploit all opportunities or interactions in the economy even though there might be some weak global interactions; 3) cross-cutting hierarchical organization with many tangled interactions; 4) continual adaptation by learning and evolving agents; 5) perpetual novelty as new markets, technologies, behaviors, and institutions create new niches in the "ecology" of the system; and 6) out-of-equilibrium dynamics with either zero or many equilibria existing and the system unlikely to be near a global optimum. This is a world of bounded rationality, not rational expectations.

Although this view has been associated with Santa Fe Institute since the late 1980s, it was developed earlier in Brussels and Stuttgart by chemists and physicists concerned with questions of emergent structures and disequilibrium dynamics. The key figure in Brussels has been Nobel-Prize-winning physical chemist, Ilya Prigogine, and various associates (Prigogine and Stengers, 1984; Nicolis and Prigogine, 1989). In Stuttgart it has been theoretical physicist, Hermann Haken (1983, 1997), developer of "synergetics." In both of these locations, applications to economics have been made, with a heavy emphasis on urban and regional models in spatial contexts with many interacting agents experiencing both increasing and decreasing returns effects, with Peter Allen and coauthors important in Brussels (Allen and Sanglier, 1981) and Wolfgang Weidlich and coauthors important in Stuttgart (Weidlich and Haag, 1987; Weidlich

and Braun, 1992).

These pioneering efforts have influenced the work at the Santa Fe Institute, especially that involving increasing returns and path dependence, both in spatial and in technological lock-in contexts (Arthur, 1989, 1994). It is interesting that much of the early work in this area has involved spatial models, as they naturally allow for the "dispersed interaction of agents." Indeed, a canonical predecessor to the complexity approach is the model of emergent racial segregation in cities due to Schelling (1971), which in contrast to many of the current complexity models can be demonstrated without using computer simulations. Schelling considered a rectangle with many sub-rectangles, each occupied by a black or a white. He allowed for movement based on local interactions and discovered that even a very slight preference by members of either group for living with their own kind would eventually lead to a largely segregated city, even though agents were only acting in relation to their immediate neighbors. Young (1998) has updated the Schelling model to an evolutionary game theoretic context with elements of the interacting particle systems approach discussed below.

This model displays an emergent global structure from strictly local effects, one of the central ideas of modern complexity theory. One of the deeper arguments made by some complexity theorists is that such processes represent how life evolved out of nonliving molecules and how subsequently multicellular organisms evolved out of single-celled organisms (Kauffman, 1993). This idea of emergent structure is thus ultimately an application of biological analogies that have been popular among complexity theorists, in addition to the analogies that come from physics such as the interacting particle systems approach discussed below. The very process of economic development and economic institutional evolution is seen to arise from such

self-organizing emergent structure phenomena.

This approach of Schelling's has been mimicked more broadly by simulating systems of multiple heterogeneous agents that evolve strategies over time in response to the behavior of their neighbors. Computer scientist John Holland (1992) has developed a classifier system that judges strategic behavior and can generate adaptive mechanisms through genetic algorithms which evolve and select strategies over time. Dawid (1996) has applied these to various areas of economics such as interactions of firms in their R&D strategies. Closely related to this approach is that of "artificial life" using cellular automata that interact with each other and grow and evolve (Langton, 1989),<sup>8</sup> with these cellular automata possibly representing individuals or other interacting economic agents. Epstein and Axtell (1996) use this to model entire artificial societies with lots of individual agents forming into various classes interacting in various markets; Tesfatsion (1997) uses it to model the evolution of trading networks, and Albin with Foley (1998) in the evolution of market structures, problems of monetary policy, and the emergence of cooperation out of prisoners= dilemma games.

### **Recent Approaches and Research in Complexity Theory**

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<sup>8</sup>Considerable debate has erupted over the nature of these artificial life models. Their strongest advocates such as Langton argue that they are truly another form of life and that their evolutions are part of the broader evolutionary process of life in general. Such arguments are linked to the debates over artificial intelligence in computer science, with the strong view being that computers have real intelligence and are evolving and could eventually replace humans as the dominant species on earth.

An increasingly popular approach to modeling discontinuities in complex systems involves adapting models of phase transitions of interacting particle systems, analogous to the magnetization of a bar of metal or the melting of ice or freezing of water.<sup>9</sup> Hans Föllmer (1974) initiated this approach in economics by specifying local interactions dependent on a probability structure conditional on agent characteristics. Idiosyncratic shocks can generate aggregate consequences, an idea developed further by Durlauf (1993).

Blume (1993), Brock (1993), and Brock and Durlauf (1995) introduced models using discrete choice theory within this framework, generally with only two sets of choices available to agents, usually interpreted as optimistic and pessimistic, although Brock (1993) presents a solution for more than two choices. For the two-choice case, Brock (1993) develops the "mean-field" approach, with  $n$  individuals choosing from discrete choice set  $\{1, -1\}$ , with  $m$  being the average of agents' choices,  $J$  a strength of interaction between the agents,  $\beta$  the intensity of choice (equivalent to temperature in the original interacting particle systems models),  $h$  the utility gain from switching to a positive attitude which shows the probabilistic state of the system, and an exogenous stochastic process. Figure 3 depicts the solutions for  $m$  in such a model with  $h = 0$  and  $\beta = 1$ . A discontinuity and multiplication of solutions occur at  $\beta J = 1$ .<sup>10</sup>

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<sup>9</sup>When negative interactions occur then these systems can be known as Aspin glass models (Durlauf, 1997), a term used widely in the statistical mechanics literature of physics from which these models were originally derived and occasionally appearing in the economics literature as well.

<sup>10</sup>More specifically, in this model utility maximization implies  $m = \tanh(\beta J m + \beta h)$ , where  $\tanh$  is the hypertangent. At  $\beta J = 1$  a phase transition occurs with  $m = 0$  if  $h = 0$  below this value and above it there being three solutions with two stable ones,  $m = -m_+$  and an unstable one at  $m = 0$ , the case depicted in Figure 3. If  $h \neq 0$  then for  $\beta J > 1$  there will be a threshold  $H$  dependent on  $\beta J$  such that if  $|\beta h| < H$  there will be three solutions, one with the same sign as  $h$  and two

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opposite, and if  $\beta h^* > H$  there will be a single solution with the same sign as  $h$  (Brock and Durlauf, 1995; Durlauf, 1997).

In this sort of an economy with interacting agents, gradual changes in the degree of interaction (or coordination) or gradual changes in the willingness of agents to change their attitudes (intensity of choice) can lead to discontinuous changes, in which suddenly agents will be moving together in some very different direction, as in the takeoff or crash of a speculative bubble or the emergence or disappearance of "animal spirits" or coordination in a Keynesian macro model. One can imagine applications to the cases of fads and information contagion and cascades, or revolutions arising from a brave individual speaking out, although such models have not yet been applied in these cases.

Applications in economics of this approach have been multiplying. Examples include: Brock (1993) on volume and price dynamics in asset markets and asymmetric information macro models; Brock and Hommes (1997) on complex dynamics that can also imply chaotic asset bubble dynamics; Durlauf (1994) on persistent differences in national economic growth rates; Durlauf (1996) on endogenous neighborhood stratification; Ioannides (1997) and Kirman (1997) on the evolution of spatial trading networks; Blume (1997) on fashion dynamics and also on coordination failure in Keynesian models; Rosser and Rosser (1997a) on macroeconomic collapse in transitional economies; Kulkarni, Stough, and Haynes (1997) on automobile and pollution dynamics; and Rosser (1998a) on regional endogenous growth.

A variation on the interacting particle systems mean-field approach uses simulated annealing (Kollman, Miller and Page, 1997), in which a process of "heating up" a system raises the probability of locally anti-optimal movements, with some of these movements moving the system away from local optima to search for the global optimum, in a form of dynamic learning from errors.



A competitor to the interacting particle systems approach is that of self-organized criticality, also known as the "sandpile model." This colorful term indicates that the dynamics are like those in a pile of sand being built up by randomly dropped grains of sand. As the sandpile grows, it gets further away from its long-run equilibrium of being flat, thus following the Brussels School tradition of out-of-equilibrium dynamics. From time to time, a dropped grain triggers an avalanche that restructures the sandpile to a new state of self-organized criticality. The distribution of these avalanches follows a power law; that is, even when the pattern of the sand dropping is random and normally distributed, the distribution of avalanches will exhibit greater variance than the distribution of sand dropping. The most prominent application of this approach is a model of macrodynamics due to Bak et al. (1993). They posit a lattice structure, depicted in Figure 4, of buyers and sellers with a set of primary producers and a set of final buyers. Final demands are randomly arriving and set off chains of response through the economy that can occasionally trigger "avalanches" larger than the initial changes in final demand.

This borderline instability of self-organized criticality has led some observers to relate it to models of the "edge of chaos," identified by popularizers as the central idea of complexity theory (Waldrop, 1992; Lewin, 1992). This idea came from modelers of artificial life such as Langton (1989) and those seeking to explain the origins of life such as Stuart Kauffman (1993). This group defines chaos as disorder (different from the view presented above and usually taken of chaos) and sees self-organization in evolving systems of interacting agents as occurring at the edge between chaos and order, hence the "edge of chaos." This idea has had scattered applications in economics with Darley and Kauffman (1997) applying it to the evolution of rational decisionmaking and Jin and Haynes (1997) applying to the transition process in the

Chinese economy.

This Aedge of chaos $\cong$  idea involves a specific form of the more general idea of structures emerging from lower level interactions, with the argument that this higher order self-organizing emergence is most likely to occur in certain zones of the system, the borderland between rigid order and total disorder. It has engendered much controversy among the researchers associated with the Santa Fe Institute, with Horgan (1995, 1997) recording some of their debates and with some of its original advocates backing away from it somewhat more recently. Despite this idea=s publicity in the more popular literature, it has had much less application in economics than has the interacting particle systems mean-field approach described above.

A substantial body of work in economics has looked at the prisoner's dilemma, and at how cooperation in that game may be maintained or disturbed if the game is repeated. The complexity approach looks at what happens when the play is repeated, but instead of having pairs of agents face each other with a single strategy repeatedly, a group of agents with different strategies play against their nearest neighbors in a spatial framework, and the question of interest is how the play of the group evolves. Lindgren (1997) examines the evolution of finitely repeated prisoner's dilemma games in which agents make mistakes from time to time, a question pursued in depth also by Albin with Foley (1998), following initial work on evolutionary games in stochastic settings by Foster and Young (1990). Lindgren uses both the interacting particle systems mean-field approach, which has agents interacting with a broad average of the decisions made by others, and cellular automata models with local spatial interactions that resemble the self-organized criticality models. Agents have finite memory and evolve strategies over time, usually starting out from a fairly simple set such as cooperate, defect, and tit-for-tat.

The typical results are that the system rarely settles down, but rather presents a pattern of constantly changing distributions of the strategies. Figure 5 shows the evolution of the distribution of strategies for a particular mean-field interacting particle systems formulation that runs for 26,000 generations (Lindgren, 1997, p. 349). Each line in Figure 5 indicates a particular strategy and its value on the vertical axis indicates the proportion of the (many) players who are using that strategy. Clearly there is no settling down to any equilibrium pattern. It is worth noting that new strategies are constantly emerging as the games iteratively proceed, which partly reflects the role of noise constantly bombarding the system. Lindgren finds that the two competing approaches of mean-field interacting particle systems and self-organized criticality both exhibit this pattern of not settling down, even as they differ from each other in their details.

A nearly archetypal example of the Santa Fe Institute complexity approach is found in an artificial stock market model that has been developed at the Santa Fe Institute since 1989 (Arthur et al., 1997). This model has numerous agents, numerous market predictors, and a stochastic dividend process. Agents begin with a random set of expectations and then search among predictors, adopting and discarding them according to their accuracy. From time to time, there is an updating of each agent's predictor set, with the 20 percent of predictors that have performed most poorly replaced. At each iteration, the market clears.

This study broadly produces two competing outcomes. In one, agents do not search much for predictors and there is convergence on a homogeneous rational expectations outcome that tracks the stochastic dividend process fairly closely. In the other, a "rich psychological regime" of complex behavior emerges, with all kinds of technical trading strategies appearing and remaining and periods of bubbles and crashes occurring. In such a regime, "the market becomes

driven by expectations that adapt endogenously to the ecology these expectations cocreate" (Arthur et al., 1997, p. 38). This second world is one where "noise traders" survive.

An analytical analogue to this is the model of asset market behavior of Brock and Hommes (1997). Again, their model evolves to either a stable or an unstable outcome. In the unstable version of their model, agents oscillate between two strategies, a convergent one that is informationally costly when they are far from the equilibrium and a divergent one that is informationally cheap when they are near the equilibrium. Given sufficiently high intensity of choice (equivalent to willingness to search in Arthur et al., 1997), a variety of complex dynamics can happen, including chaos, strange attractors, fractal basin boundaries, and more, as the agents jerk back and forth between strategies.

### **Implications: Theory**

Probably the most obvious implication of the study of complexity in its various forms is that a general assumption of rational expectations is very unlikely to hold. This is seen most clearly from models that assume rational expectations but then generate chaotic dynamics (Benhabib and Day, 1982; Grandmont, 1985). Because of the existence of the butterfly effect in chaotic dynamics, it is impossible for agents in such a setting to obtain adequate information to form rational expectations in the first place. Rosser (1990, 1998b) argues that such models are "Strong New Keynesian" in that they force a direct abandonment of a key assumption of the New Classical approach, in contrast to the "Weak New Keynesian" models of Mankiw and Romer (1991) that essentially buy in to the New Classical model by positing market imperfections

within a framework of rational expectations as the source of the few Keynesian results they actually succeed in obtaining. It can be argued that by assuming rational expectations, such models essentially accept the New Classical paradigm, even though their purpose in doing so is to discredit the rational expectations assumption (Davidson, 1996).

Even in the absence of outright chaotic dynamics, the likely existence of multiple equilibria and the high likelihood that systems can either converge to any equilibrium or might not converge to any at all, especially as seen in the dispersed multiple agent models of the more recent complexity approaches, imply that neither rational expectations nor continuous Walrasian market clearing -- two cornerstones of the New Classical model -- are realistic assumptions. Colander (1998) argues that this implication of general complexity suggests the need for a new broad tent view he calls "Post Walrasian economics."

Indeed, this argument has been essentially accepted by one of the early leading advocates of rational expectations, Thomas Sargent (1993). But even as Sargent has seen the need to accept that people mostly use adaptive expectations within a bounded rationality context, because of the computational difficulties arising from inherent complexity, he argues that often people will use adaptive mechanisms that will converge on rational expectations, if only asymptotically, an argument also made by Heiner (1989). In debates at the Santa Fe Institute, Sargent has argued for the likelihood of the solution to the artificial stock market (Arthur et al., 1997) that tends to converge on rational expectations, even as those authors argue that the nonconvergent solution fits certain facts of asset market dynamics better, such as the leptokurtotic "fat tails" of the distribution of asset returns that are larger than one would expect from a normal distribution.

An alternative response involves the recent work on "consistent expectations equilibria"

(Grandmont, 1998; Hommes and Sorger, 1998). This perspective takes the position that in some cases people may be able to mimic an underlying true chaotic dynamic by the use of some simple adaptive mechanism. Indeed, people may even be able to converge on such a mechanism, a process that Sorger (1998) labels "learning to believe in chaos." Hommes and Sorger (1998) show this possibly happening with a positive probability for a model with a piecewise linear underlying process known as the asymmetric tent map. Hommes and Rosser (1999) extend these results to more general, smooth underlying processes, although how broadly these results apply remains an open question.

This last discussion suggests that a likely future direction of such a complex APost Walrasian $\cong$  research program may increasingly focus upon investigating the specific nature of the outcomes of more specifically denoted kinds of boundedly rational behavior. Of course, defenders of the rational expectations approach can be expected to complain that this will simply lead economists down a slippery slope of adhocism into a morass of alternative cases and situations among which nobody can reasonably distinguish, the original complaint against adaptive expectations. However, hopefully if more clear links are identified between certain kinds of behavior and certain kinds of outcomes, a better understanding of the actual functioning of the economy will emerge.

### **Implications: Methodology and Empirics**

One of the more important outcomes of the effort to study and model complex economic dynamics has been a shift of method as well as of focus. Especially with the more recent

emphasis on modeling the behavior of dispersed individuals, the use of computer simulation, which was already going on in the study of chaotic dynamics, has increased substantially. Of course, the use of computer simulation in economics is hardly new, as the earlier experience with cybernetics shows, which was hardly an unalloyed success. Is there reason to believe that the new efforts can avoid the mistakes of the earlier ones?

A major difference between the older cybernetics and the newer complexity models is that in cybernetics, aggregations and relationships were assumed that sometimes had little foundation, whereas in the newer models, generally only local relationships among individual actors are assumed, and aggregate behaviors or structures emerge out of self-organization rather than simply being imposed or assumed. What may emerge in the aggregate may well not be a simple addition of what happens at the individual level. Thus, the new complexity admits of the fallacy of composition, in contrast with the approach of representative agent models in which the individual equals the aggregate. However, this advantage alone is no guarantee that the newer complexity models will be any more accurate or useful than what is generated or predicted by other approaches.

There remains the problem of empirical testing of what has been predicted or modeled. Here the criticism of Horgan (1997, p. 220) that "chaoplexologists" have discovered "nothing concrete or truly surprising" beyond their mighty metaphors remains largely unanswered. Studies which use a complexity approach often end up justifying themselves by how they correspond with already-observed facts, rather than by the new insights they provide. For example, Arthur et al. (1997) argue that their artificial stock market model is useful because it mimics some well-known facts about asset market dynamics, rather than because the model comes up with

anything "truly surprising." Indeed, despite some work on catastrophe theory econometrics (Guastello, 1995) and the recent work on testing for chaotic dynamics (Dechert, 1996; Bask, 1998; Whang and Linton, 1999), there has been little work on empirical techniques for testing dispersed agent complexity models.

The most promising approach, it appears, is to test the predictive capability of particular complexity models against various alternatives; for example, Baak (forthcoming) uses such methods to estimate the fraction of agents in the U.S. cattle market that are boundedly rational and shows that such a model has better predictive capability than one assuming purely rational expectations. But deep problems of identification face any effort to determine whether a particular model of endogenous complexity is "truly accurate" (Manski, 1995). More generally, some argue that complexity implies a need to seriously rethink the nature of empirical testing in economics. Such a project threatens to lead into deeply philosophical issues such as induction versus deduction, objectivism versus subjectivism, and other difficult conundra.

### **Implications: Policy and Institutions**

Old debates about government intervention repeat themselves in the context of complexity, albeit with some new twists. It may be that we can dismiss rational expectations and markets that continuously clear and attain the global optimum, but in the face of empirical uncertainty and multiple equilibria we do not know what is the appropriate model or adjustment mechanism. This certainly makes life highly uncertain for policymakers, whether this uncertainty is ontologically Keynesian (Davidson, 1996) or merely epistemological due to



computational difficulties (Hayek, 1967; Leijonhufvud, 1993; Rosser, 1998b).

The deep uncertainty over how outcomes will evolve, and even what strategies will be used by economic agents at different times, has led to a number of arguments from complexity theorists that governments have a welfare-improving role to play to reduce uncertainty (Shubik, 1996), to coordinate to select among equilibria (Guesnerie, 1993), to stabilize potentially chaotic dynamics (Grandmont, 1985), to engender some limits to fluctuations so that even boundedly rational calculations and decisions can be made (Leijonhufvud, 1997), and more generally to impose desirable institutional structures on the economy (Colander and van Ees, 1996; Albin with Foley, 1998). The problem of how to determine the selection among multiple equilibria is an especially vexing one that has attracted much attention. An example of this difficulty appears in the literature on evolutionary game theory, where multiple equilibria are ubiquitous. Efforts have been made to derive mechanisms that would select among potential equilibria, but these efforts have failed to achieve a general answer, despite some interesting specialized models (Mailath, 1998).

Moreover, critics of the argument that complex dynamics imply a role for government intervention point out that efforts to carry out stabilization policy may itself generate unpleasant complex dynamics when they might not otherwise occur (Dwyer, 1992; DeCoster and Mitchell, 1992). Economists of the Austrian school argue that indeed the economy is complex,<sup>11</sup> but that this is a good thing, in that the economy effectively self-organizes (Hayek, 1948; Lavoie, 1989).

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<sup>11</sup>Hayek (1967) was an early and independent developer of complexity theory in something resembling its current form, albeit without computers. He had significant communication with both Ilya Prigogine and Hermann Haken, respectively the founders of the Brussels and Stuttgart schools.

But even the Austrians accept that there must be some institutional structure, even if it is a relatively minimal "night watchman" state, although Hayek (1948, 1967) argued that the most effective institutional structures are those that have themselves evolved in a "natural" self-organizing manner.

Debates over the application of complexity to these disputes over the role of government might usefully begin with the recognition that much of the time there is a great deal of stability and ordinariness about what happens in many markets, much stability and apparent noncomplexity, perhaps more than we have any reason to expect. Thus, the classic debate can be reframed as a clash between the view that this apparent stability arises naturally from the self-organizing nature of the market economy, as the Austrians argue, and the view that it depends on the institutional structures of the economy that bound its tendency to complex dynamic fluctuations, as the Post Keynesians argue. In either case, the real world apparently has mechanisms which keep economies within certain bounds most of the time, if not necessarily convergent on equilibria, much less optimal equilibria. This view is consistent with the "corridor of stability" notion, that the economy can fluctuate reasonably well within some bounds, possibly in a quite complex manner, but will behave in a severely dysfunctional manner if it exceeds those bounds (Leijonhufvud, 1981).

In some cases, there may be a problem of stability versus resilience, an idea found in ecology (Holling, 1973). To put it differently, there may be cases when global and local stability may be in conflict. Too great an effort to create local stability may lead to the undermining of the very institutions that support global stability, with the result at some point being a total systemic collapse. An explanation along these lines -- how attempts to assure local stability set the

conditions for global collapse, may help to explain the breakdowns we have seen in many of the formerly socialist economies (Rosser and Rosser, 1997b).

### **Complexity and Controversy: Conclusions**

Let us return to the debate with which this paper began. With regard to the views of Waldrop (1992), we have sidestepped the question of whether "underlying unity" between many disciplines exists or can be found, by largely avoiding discussion of topics substantially outside of economics. However, the reports of deep controversies and debates among "chaoplexologists" suggests that even if such a unity exists, there is not agreement on what its nature is. It is worth remembering Saari's (1995) argument that economic models are prone to an even greater complexity of dynamics than are models in other disciplines. Even so, the emphasis on dispersed agent models in economics provides an alternative way of thinking about the economy that has significantly contributed to undermining important elements of conventional thinking such as the likelihood of rational expectations, continual market-clearing, and reaching global optima. It is likely that subsequent development of this approach will be influential within economics, even if it is not the ultimate panacea suggested by some advocates.

With regard to Horgan's (1995, 1997) skepticism concerning the ultimate value of complexity, he must be granted that it is hard to identify a concrete and surprising discovery (rather than "mere metaphor") that has arisen due to the emergence of complexity analysis. Rather, complexity theory has shifted the perspective of many economists towards thinking that what was viewed as anomalous or unusual may actually be the usual and expected, especially in

the realm of asset markets where the unusual seems increasingly to be commonplace! Indeed, there is a strain of common perspective that has been accumulating as the four C's of cybernetics, catastrophe, chaos and complexity emerged, which may now be reaching a critical mass in terms of influencing the thinking of economists more broadly.

Such supposed anomalies were understood and recognized by many earlier economists, ranging from Malthus and Marx to Walras and Marshall, and were often discussed when they would speak of the "real world" as opposed to the world of theory. As a telling example of such discussion, I conclude by quoting one of the leading founders of neoclassical orthodoxy, Alfred Marshall (1920, p. 346), regarding price dynamics:

But in real life such oscillations are seldom as rhythmical as those of a stone hanging freely from a string; the comparison would be more exact if the string were supposed to hang in the troubled waters of a mill-race, whose stream was at one time allowed to flow freely, and at another partially cut off. Nor are these complexities sufficient to illustrate all the disturbances with which the economist and the merchant alike are forced to concern themselves.

If the person holding the string swings his hand with movements partly rhythmical and partly arbitrary, the illustration will not outrun the difficulties of some very real and practical problems of value.

With the recent work on complexity, we are now learning how to understand these very real and practical problems both theoretically and empirically.

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