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An Application of Filtering Techniques For the Tracking Control of Mobile Robots with Slipping

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Abstract: This paper presents an application of nonlinear filtering techniques for the tracking control design of tracked mobile robot under slip condition. The slip is represented only by the longitudinal wheels slip that is described by just an unknown parameter. The extended Kalman filter (EKF), the unscented Kalman filter (UKF) and the particle filter (PF) are used to estimate the states of the system, when measurements are assumed all available. Two adaptive tracking control design for tracked mobile robots are proposed. The first controller is based on the kinematic model and provides angular velocities as the control input. The second controller, based on the dynamic model, consists in a feedback control law that provides torque as control input. Numerical results show the performance of the proposed adaptive control laws using the EKF, UKF and the PF filtering techniques.

Keywords: Nonholonomic systems, tracked mobile robots, nonlinear filters, adaptive control.

NOMENCLATURE

b = distance between the wheels, m
B = input transformation matrix
k_i = controller gains, dimensionless
i = longitudinal slip ratio, dimensionless
J = moment of inertia, $kg.m^2$
m = mass of the robot, kg
M = inertia matrix
n = number of system states
p = relative to slip ratio, dimensionless
r = radius of the robot wheels, m
α = size of the sigma point distribution
β = weighting parameter
γ = auxiliar weighting parameter
κ = secondary scaling parameter
ρ = update law gain, dimensionless
π = posterior probability density, dimensionless

INTRODUCTION

Autonomous mobile robots has received renewed attention in the last years because of its increasing use in tasks as forestry, mining, agriculture, military applications, space exploration, etc (Nourbakhsh and Siegwart, 2004). All of these applications require an efficient solution to the autonomous navigation problem, which has motivated various works in the area due to its theoretical challenges. Furthermore, these applications usually require the robot to travel across unstructured environments, where the precise localization of the robot is an important key for feedback control purposes.

Feedback control for mobile robots need knowledge of the robot's state vector. In general, the estimation of the robot's state vector from measurement system can be obtained using filtering techniques. It is well known that mobile robots are typical examples of nonlinear systems. In general, two types of filtering approaches for nonlinear systems can be found (Thrun et al., 2005). The first class, known as Gaussian filters, includes the extended Kalman filter (EKF), Gauss-Hermite filter (GHF) and unscented Kalman filter (UKF). The other class consists of the nonparametric filters, in which the main algorithm is the particle filter (PF). Several works has been developed in the literature to deal with the estimation problem applied to mobile robot motion. For instance, Jetto et al. (1999) developed the adaptive EKF for the localization of mobile robots, Kwon et al. (2005) proposed a robust localization method for mobile robot based on the combination of Kalman filter and perturbation estimator, and Rigatos (2010) compared the EKF and PF techniques for sensor fusion in motion control of mobile robots. In the same line as Rigatos (2010), this paper studies the performance of the EKF, UKF and PF algorithms applied to the proposed tracking control methods.

Morin and Samson (2006) present a review of the most recents tracking control methods for mobile robots. Other studies on tracking control designs using Lyapunov analysis can be found in Lee et al. (2009), Wu et al. (2009) and Ju et al. (2009). All these control design techniques are based on the assumption that the wheels roll without slipping. However, the slip has a critical influence on the performance of mobile robots that cannot be neglected. Thus, to attain higher performance, in addition to estimation of the state vector, the slip parameters is incorporate into the model of the robot. Many papers have addressed the slip phenomenon in the navigation of mobile robots (Matyukhin, 2007; Wang and Low, 2008). However, in such works, the slip parameters are considered as disturbance or noise (Scaglia et al., 2009) or are estimated using some filtering technique (Zhou et al., 2007). In Iossaqui et al. (2010b,a), an adaptive law is proposed to estimate the longitudinal slip parameter for two different tracked mobile robot. The first adaptive control design, taken from Iossaqui et al. (2010b), is based on the kinematic model and provides angular velocities as the control input. The second adaptive control design, taken from Iossaqui et al. (2010a), based on the dynamic model, consists in a feedback control law that provides torque as control input. Even as in several other works in the literature these two control designs consider the perfect measurement of the states.

The main contribution of this paper is to incorporate the nonlinear filtering techniques to the adaptive control designs proposed by Iossaqui et al. (2010b,a). To address the nonlinear filtering problem, two group of filtering approaches are studied and compared: Gaussian approximation and nonparametric simulation. The former group, consisting of the EKF and UKF algorithms, uses either a single Gaussian distribution to match the first and second-order moments of the required density to different accuracy levels (Cui et al., 2005). In the EKF algorithm, the state distribution is propagated analytically through the first-order linearization of the nonlinear system. In the UKF algorithm, the state distribution is represented using a minimal set of carefully chosen sample points and propagated through the true nonlinear system. The latter group, represented by the PF algorithm, does not make any assumption on the measurement noise distribution. Instead, the nonparametric filters approximate posterior probability distribution by finite number of values, each corresponding to a region in the state space.

The paper is organized as follows. First, the adaptive controllers for a tracked mobile robot under longitudinal slip condition are reviewed. Next, the nonlinear filtering techniques used to estimate the states of the tracked mobile robot are presented. Then, the results obtained by numerical simulations of the controlled systems using the filtering techniques are showed and compared. Concluding remarks follow afterwards.

THE PROPOSED ADAPTIVE TRACKING CONTROLS

In this section, two adaptive control techniques for tracked mobile robots proposed by Iossaqui et al. (2010b,a) are presented. First, the model of the robot is presented and the equations that characterize the tracking problem are established. Then, the first adaptive control law that provides velocities as input is described. Finally, the second adaptive control law that provides torque as input is reviewed.

As presented in (Iossaqui et al., 2010b), the kinematic equation of the tracked robot under slip condition is given by

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} r\cos\psi/2p & r\cos\psi/2p \\ r\sin\psi/2p & r\sin\psi/2p \\ -r/bp & r/bp \end{pmatrix} \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} \Leftrightarrow \dot{q} = S(q)\xi \tag{1}$$

where $q = (X, Y, \psi)^T$ denotes the states of the robot, which is given by the robot position (X, Y) and its orientation ψ in an appropriate inertial frame. The angular velocities of the left and the right wheels are respectively ω_L and ω_R. The radius of the robot wheels is r and the distance between the wheels is b. The parameter p is defined as

$$p = \frac{1}{(1-i)}$$

with i, the longitudinal slip ratio of the two wheels, given by

$$i = \frac{(r\omega_L - v_L)}{r\omega_L} = \frac{(r\omega_R - v_R)}{r\omega_R}, \quad 0 \le i < 1$$

where v_L and v_R are the linear velocities of the left and the right wheels with relation to the terrain.

As presented in (Iossaqui et al., 2010b), the dynamic equation of the tracked robot is given by

$$\overline{M}\dot{\xi} = \overline{B}(q)\tau \tag{2}$$

where $q = (X, Y, \psi)^T$ has been defined before, the input torque in left and right wheels is given by $\tau = (\tau_L, \tau_R)^T$, $\overline{M} = S^T(q)MS(q)$ and $\overline{B}(q) = S^T(q)B(q)$, with the matrices M and $B(q)$ given by

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{pmatrix}, \quad B(q) = \frac{1}{r}\begin{pmatrix} \cos\psi & \cos\psi \\ \sin\psi & \sin\psi \\ b/2 & b/2 \end{pmatrix}$$

where m is the total mass of the robot and J is the moment of inertia about the vertical axis through geometric center of the robot.

In order to deal with the tracking control problem, we need to define the reference trajectory, $q_r = (X_r, Y_r, \psi_r)^T$, which is generated using the kinematic model

$$\begin{pmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\psi}_r \end{pmatrix} = \begin{pmatrix} \cos\psi_r & 0 \\ \sin\psi_r & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_r \\ \omega_r \end{pmatrix} \Leftrightarrow \dot{q}_r = S_r(q_r)\eta_r \tag{3}$$

where v_r and ω_r are constant reference inputs. It is assumed that the signals η_r and $\dot{\eta}_r$ are bounded.

In addition, to analyze the tracking problem, the error is defined as

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_r - X \\ Y_r - Y \\ \psi_r - \psi \end{pmatrix} \tag{4}$$

Adaptive velocity-based control

Figure 1 shows the scheme of the adaptive control used in Iossaqui et al. (2010b). The numbering inside the blocks in Fig. 1 indicate the corresponding equation number. Note that the state vector is composed of the three states X, Y, and ψ.

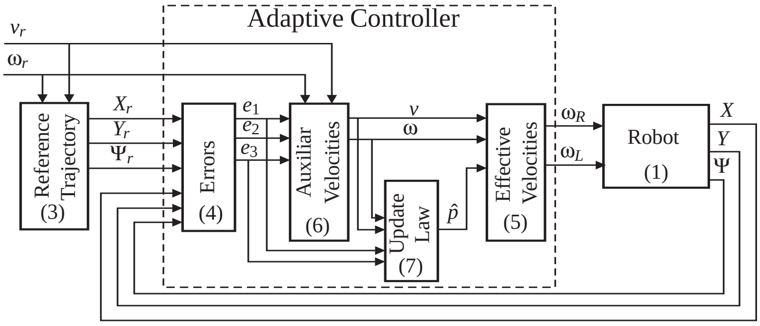

Figure 1: Adaptive velocity-based control.

The velocity control input $\xi = (\omega_L, \omega_R)^T$ is given by

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \frac{\hat{p}}{2r} \begin{pmatrix} 2 & -b \\ 2 & b \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{5}$$

with auxiliary velocity

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_r \cos e_3 + k_1 e_1 \\ \omega_r + v_r k_2 e_2 + k_3 \sin e_3 \end{pmatrix} \tag{6}$$

and update law

$$\dot{\hat{p}} = \rho \left(v e_1 + \frac{\omega \sin e_3}{k_2} \right) \tag{7}$$

where $\hat{p}$ is the estimation of the parameter p, $k_i > 0$ and $\rho > 0$ are controller gains.

Adaptive torque-based control

Figure 2 shows the scheme of the adaptive control used in Iossaqui et al. (2010a). The numbering inside the blocks in Fig. 2 indicate the corresponding equation number. Observe that the state vector is composed of the five states X, Y, ψ, ω_L, and ω_R.

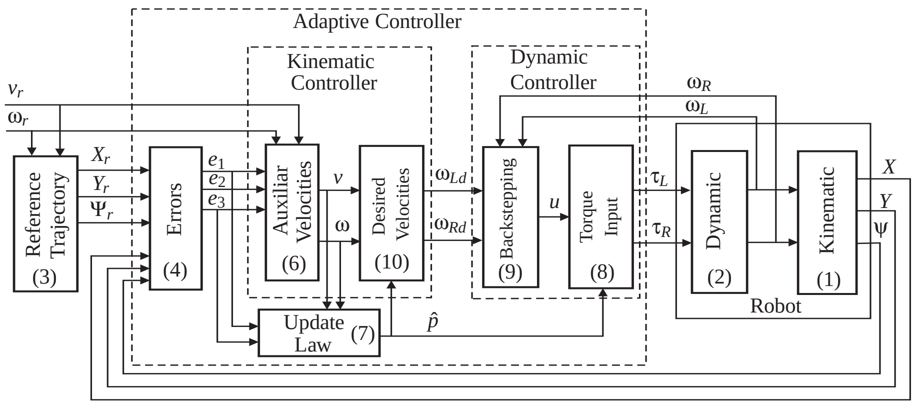

Figure 2: Adaptive torque-based control.

The torque control input $\tau = (\tau_L, \tau_R)^T$ is given by

$$\tau = \overline{B}(q)^{-1} \overline{M} u \tag{8}$$

with

$$u = \dot{\xi}_d + \begin{pmatrix} k_4 & 0 \\ 0 & k_5 \end{pmatrix} (\xi_d - \xi) \tag{9}$$

and the desired velocities ξ_d given by

$$\xi_d = \begin{pmatrix} \omega_{Ld} \\ \omega_{Rd} \end{pmatrix} = \frac{\hat{p}}{2r} \begin{pmatrix} 2 & -b \\ 2 & b \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{10}$$

where $k_i > 0$ are given constant and $(v, \omega)^T$ is given by (6). The update law that provides the estimate $\hat{p}$ is given by (7).

NONLINEAR FILTERING TECHNIQUES

This section presents the EKF and UKF Gaussians algorithms and the nonparametric PF algorithm (Haykin, 2001). The basic structure for the EKF, UKF and for the PF involves estimation of the state of a discrete-time nonlinear dynamic system of the form

$$x_{k+1} = f(x_k, u_k, w_k)$$
$$y_k = h(x_k, v_k)$$

where x_k is the state vector of the system, u_k is a control input and y_k is the measured signal. The process and measurement noises are respectively given by w_k and v_k. It is assumed that w_k and v_k are independent zero-mean Gaussian random variables with respectively covariance matrices Q and R. Note that the PF algorithm does not require any assumption on the measurement noise distribution.

The EKF algorithm

The EKF algorithm (Haykin, 2001) is based on a first order Taylor series expansion of the nonlinear functions f and h at the estimate $\hat{x}_{k|k}$ and the propagation $\hat{x}_{k+1|k}$. The EKF algorithm is given below:

Initialize with

$$\hat{x}_0 = \mathbb{E}[x_0],$$

$$P_0 = \mathbb{E}[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T].$$

For $k \in \{1, \ldots, \infty\}$, the time-update equations are

$$\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, \bar{w}_k),$$

$$P_{x_k}^- = A_{k-1} P_{x_{k-1}} A_{k-1}^T + B_k Q_k B_k^T,$$

and the measurement-update equations are

$$K_k = P_{x_k}^- C_k^T \left(C_k P_{x_k}^- C_k^T + D_k R_k D_k^T \right)^{-1},$$

$$\hat{x}_k = \hat{x}_k^- + K_k \left[y_k - h(\hat{x}_k^-, v_k) \right],$$

$$P_{x_k} = (I - K_k C_k) P_{x_k}^-,$$

with

$$A_k \triangleq \left. \frac{\partial f(x, u_k, \bar{w}_k)}{\partial x} \right|_{\hat{x}_k}, \quad B_k \triangleq \left. \frac{\partial f(\hat{x}^-, u_k, w_k)}{\partial w_k} \right|_{\bar{w}_k}, \quad C_k \triangleq \left. \frac{\partial h(x, \bar{v}_k)}{\partial x} \right|_{\hat{x}_k}, \quad D_k \triangleq \left. \frac{\partial h(\hat{x}^-, v_k)}{\partial v_k} \right|_{\bar{v}_k},$$

with $\bar{w}_k = \mathbb{E}[w_k]$ and $\bar{v}_k = \mathbb{E}[v_k]$, where $\mathbb{E}[\cdot]$ is the expectation. The means $\bar{w}_k$ and $\bar{v}_k$ are usually zero.

The EKF can achieve satisfactory results for many applications, but may suffer from large estimate errors when systems have strong nonlinearities. As stated in Thrun et al. (2005), the EKF is a widely used technique in nonlinear state estimation and, in spite of its theoretical weakness, i.e., the lack of a formal proof of convergence, a number of applications exists, giving satisfactory results, in a large broad of technological areas.

The UKF algorithm

The UKF algorithm (Haykin, 2001) does not approximate the nonlinear process and measurement models. Instead, it uses the true nonlinear models and approximates the distribution of the state random variable. The UKF, which does not need to compute the Jacobian, uses the so-called unscented transform (UT) to obtain the sigma points. These sigma points are propagated through the nonlinear function. The UKF algorithm is given below:

Initialize with

$$\hat{x}_0 = \mathbb{E}[x_0]$$

$$P_0 = \mathbb{E}[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$$

For $k \in \{1, \ldots, \infty\}$

Calculate the sigma points

$$\mathcal{X}_{k-1} = \left[\hat{x}_{k-1} \quad \hat{x}_{k-1} + \gamma\sqrt{P_{k-1}} \quad \hat{x}_{k-1} - \gamma\sqrt{P_{k-1}}\right]$$

The time-update equations are

$$\mathcal{X}^*_{k|k-1} = f(\mathcal{X}_{k-1}, u_{k-1})$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{X}^*_{i,k|k-1}$$

$$P_k^- = \sum_{i=0}^{2n} W_i^{(c)} \left[\mathcal{X}^*_{i,k|k-1} - \hat{x}_k^-\right] \left[\mathcal{X}^*_{i,k|k-1} - \hat{x}_k^-\right]^T + Q_k$$

$$\mathcal{X}_{k|k-1} = \left[\mathcal{X}^*_{k|k-1} \quad \mathcal{X}^*_{0,k|k-1} + \rho\sqrt{Q_k} \quad \mathcal{X}^*_{0,k|k-1} - \rho\sqrt{Q_k}\right]$$

$$\mathcal{Y}_{k|k-1} = h(\mathcal{X}_{k|k-1})$$

$$\hat{y}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \mathcal{Y}_{i,k|k-1}$$

The measurement-update equations are

$$P_{\tilde{y}_k\tilde{y}_k} = \sum_{i=0}^{2n} W_i^{(c)} \left(\mathcal{Y}_{i,k|k-1} - \hat{y}_k\right) \left(\mathcal{Y}_{i,k|k-1} - \hat{y}_k\right)^T + R_k$$

$$P_{x_k y_k} = \sum_{i=0}^{2n} W_i^{(c)} \left(\mathcal{X}_{i,k|k-1} - \hat{x}_k^-\right) \left(\mathcal{Y}_{i,k|k-1} - \hat{y}_k^-\right)^T$$

$$K_k = P_{x_k y_k} P_{\tilde{y}\tilde{y}}^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k \left(y_k - \hat{y}_k^-\right)$$

$$P_k = P_k^- - K_k P_{y_k y_k} K_k^T$$

with the weights

$$W_0^{(m)} = \lambda/(n+\lambda), W_0^{(c)} = \lambda/(n+\lambda) + (1 - \alpha^2 + \beta)$$

$$W_i^{(m)} = W_i^{(c)} = 1/2(n+\lambda), \quad i = 1, 2, \ldots, n$$

where $\lambda = \alpha^2(n+\kappa) - n$, $\gamma = \sqrt{n+\lambda}$ and with $\kappa \geq 0$. The dimension of the state vector is n, the size of the sigma point distribution is regularized by non-negative weighting terms α and β, which can be used to compensate for the information of the higher order moments of the distribution.

The PF algorithm

The PF algorithm (Haykin, 2001) is based on Monte Carlo simulation with sequential importance sampling. The key idea is to directly represent the required probability density function as a set of particles. These particles are propagated and updated from one discrete time to the next to represent the latest posterior density. The PF algorithm is given below:

1. Initializion: $k=0$
   - For $i=1,...,N$, draw the states $x_0^{(i)}$ from prior $p(x_0)$
2. For $k=1,2,...$
   (a) Importance sampling step
   - For $i=1,...,N$, sample $x_k^{(i)} \sim \pi(x_k|x_{0:k-1}^{(i)}, Y_0^k)$, where π represents the posterior probability density
   - For $i=1,...,N$, evaluate the importance weights up to a normalizing constant:

$$w_k^{(i)} = w_{k-1}^{(i)} \frac{p(y_k|x_k^{(i)})p(x_k^{(i)}|x_{k-1}^{(i)})}{\pi(x_k^{(i)}|x_{0:k-1}^{(i)}, Y_0^k)}$$

   - For $i=1,...,N$, normalize the importance weights:

$$\tilde{w}_k^{(i)} = \frac{w_k^{(i)}}{\sum_{j=1}^N w_k^{(i)}}$$

   (b) Selection step (resampling)
   - Multiply/suppress samples $x_k^{(i)}$ with high/low importance weights $\tilde{w}^{(i)}$, respectively, to obtain N random samples $x_k^{(i)}$ approximately distributed according to $p(x_k^{(i)}|Y_0^k)$
   - For $i=1,...,N$, set $w_k^{(i)} = \tilde{x}_k^{(i)} = N^{-1}$

   (c) Output: The output of the algorithm is a set of samples that can be used to approximate the posterior distribution as follows:

$$\hat{p}(x_k|Y_0^k) = \frac{1}{N}\sum_{i=1}^{N} \delta(x_k - x_k^{(i)})$$

   The optimal MMSE estimator is given as

$$\hat{x}_k = \mathbb{E}(x_k, Y_0^k) \approx \frac{1}{N}\sum_{i=1}^{N} x_k^{(i)}$$

In general, the PF algorithm presents better accuracy of the Gaussians filters, but this occurs at the cost of greater computational effort (Thrun et al., 2005).

NUMERICAL RESULTS

Two simulation scenarios are presented in this section using the filtering techniques EKF, UKF and PF together with the adaptive velocity-based control and the adaptive torque-based control. Figure 3 shows the schematic representation of the close-loops with controller, filter and noises characterization for the two scenarios studied. Observe that the variables without subscript, with subscripts "r", "m" and "e" describe respectively real, reference, measured and estimated states.

Note that in the first scenario the angular velocities are not used in the feedback control, that is, the velocities do not need to be measured. The velocities are provided directly by the control law. In the second scenario, the angular velocities need to be measured and estimated for control proposes. The velocity motion model that uses robot's velocity to compute posterior over poses (position and orientation) is considered in the filtering implementation. The alternative solution is odometry motion model, commonly obtained by integrating wheel encoder information.

In order to applied the nonlinear filtering techniques, we discretize the continuous time equations (1) and (2) of the mobile robot with slipping using first-order difference. Then the nonlinear system dynamics at discrete time, for the first and second scenario, can be described as

$$x_k = f(x_k, u_k) + w_k$$

where w_k is a zero-mean Gaussian noise vector with covariance Q_k. The states vector for the first and second scenarios are respectively $x_k = (X, Y, \Psi)^T$ and $x_k = (X, Y, \Psi, \omega_L, \omega_R)^T$. The input vector for the first and second scenarios are respectively $u_k = (\omega_L, \omega_R)^T$ and $u_k = (\tau_L, \tau_R)^T$. The state noise w_k is considered zero for the two scenarios.

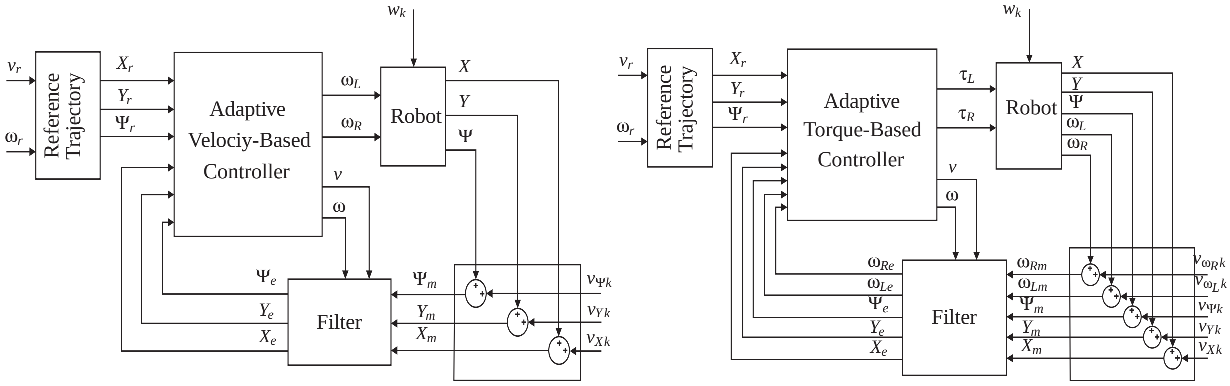

Figure 3: The representation of the close-loops studied.

For the first scenario, a simplify model used to represent the measurement model is given by

$$\begin{pmatrix} X_m \\ Y_m \\ \Psi_m \end{pmatrix} = \begin{pmatrix} X \\ Y \\ \Psi \end{pmatrix} + \begin{pmatrix} v_{Xk} \\ v_{Yk} \\ v_{\Psi k} \end{pmatrix} \Leftrightarrow y_k = f(x_k) + v_k$$

and for the second scenario by

$$\begin{pmatrix} X_m \\ Y_m \\ \Psi_m \\ \omega_{Lm} \\ \omega_{Rm} \end{pmatrix} = \begin{pmatrix} X \\ Y \\ \Psi \\ \omega_L \\ \omega_R \end{pmatrix} + \begin{pmatrix} v_{Xk} \\ v_{Yk} \\ v_{\Psi k} \\ v_{\omega_L k} \\ v_{\omega_R k} \end{pmatrix} \Leftrightarrow y_k = f(x_k) + v_k$$

where v_k is an additive zero-mean Gaussian noise vector with covariance R_k given respectively by

$$R_k = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{pmatrix} \quad \text{and} \quad R_k = \begin{pmatrix} 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0.01 \end{pmatrix}$$

for the first and second scenarios. To perform comparison between the filtering methods, it is assumed an additive zero-mean Gaussian noise in all simulations.

The numerical simulations were performed using MATLAB. The physical parameters for the model, taken from Iossaqui et al. (2010a), are given by $b = 0.65$ m, $r = 0.35$ m, $m = 0.80$ kg and $I = 0.0608$ kg.m^2. The total time of the simulation is chosen as $t = 60$ s. The control parameters of the controller are chosen as $k_1 = k_3 = 6$, $k_2 = 8$ and $k_4 = k_5 = 4$. The parameter of the adaptive rule is chosen as $\rho = 3$. The initial conditions of the equations that generates the reference trajectory are taken as $q_r(0) = (0,0,0)^T$. The initial conditions of the adaptation law is $\hat{p}(0) = 1$. The initial conditions of the robot for the first and second scenario are respectively $q(0) = (0,-1.5,\pi/4)^T$ and $q(0) = (0,-1,\pi/6,0,0)^T$. In order to demonstrate the tracking performance, the slip parameter changes from $i = 0$ to $i = 0.25$ during the time period 22.5 s $\leq t \leq 45$ s.

The reference inputs v_r, w_r are chosen as following

$$\begin{aligned} 0\text{s} \leq t < 15\text{s}: &\quad v_r = 0.5\text{m/s} \quad \text{and} \quad w_r = 0\text{rad/s} \\ 15\text{s} \leq t < 33\text{s}: &\quad v_r = 0.5\text{m/s} \quad \text{and} \quad w_r = -0.4\text{rad/s} \\ 33\text{s} \leq t < 51\text{s}: &\quad v_r = 0.5\text{m/s} \quad \text{and} \quad w_r = 0.4\text{rad/s} \\ 51\text{s} \leq t: &\quad v_r = 0.7\text{m/s} \quad \text{and} \quad w_r = 0\text{rad/s} \end{aligned}$$

The three constant parameters used in the UKF are chosen as $\alpha = 0.01$, $\beta = 2$ and $\kappa = 0$. The initial state covariance used in the first and second scenarios are respectively $P(0) = 10I_{3\times 3}$ and $P(0) = 10I_{5\times 5}$, being I the identity matrix. The number of particles, necessary in PF method, is chosen as $N = 100$.

Scenario 1: Adaptive velocity-based control

In the first scenario, the nonlinear kinematic used in the estimation is given by

$$\begin{pmatrix} x \\ y \\ \phi \end{pmatrix}_{k+1} = \begin{pmatrix} x \\ y \\ \phi \end{pmatrix}_{k} + h \begin{pmatrix} v\cos\phi_k \\ v\sin\phi_k \\ \omega \end{pmatrix}$$

where x_k, y_k and ϕ_k corresponds to robot pose, v and ω are respectively the linear and angular velocities. The sampling step is taken as $h = 0.005$.

Figures 4(a), 4(b) and 4(c) show the posture error $e = (e_1, e_2, e_3)^T$ obtained using the EKF, UKF and PF methods. The reference trajectory and robot trajectory in the inertial frame for each method is depicted in Fig. 4(d). All three filters methods show consistent and similar results.

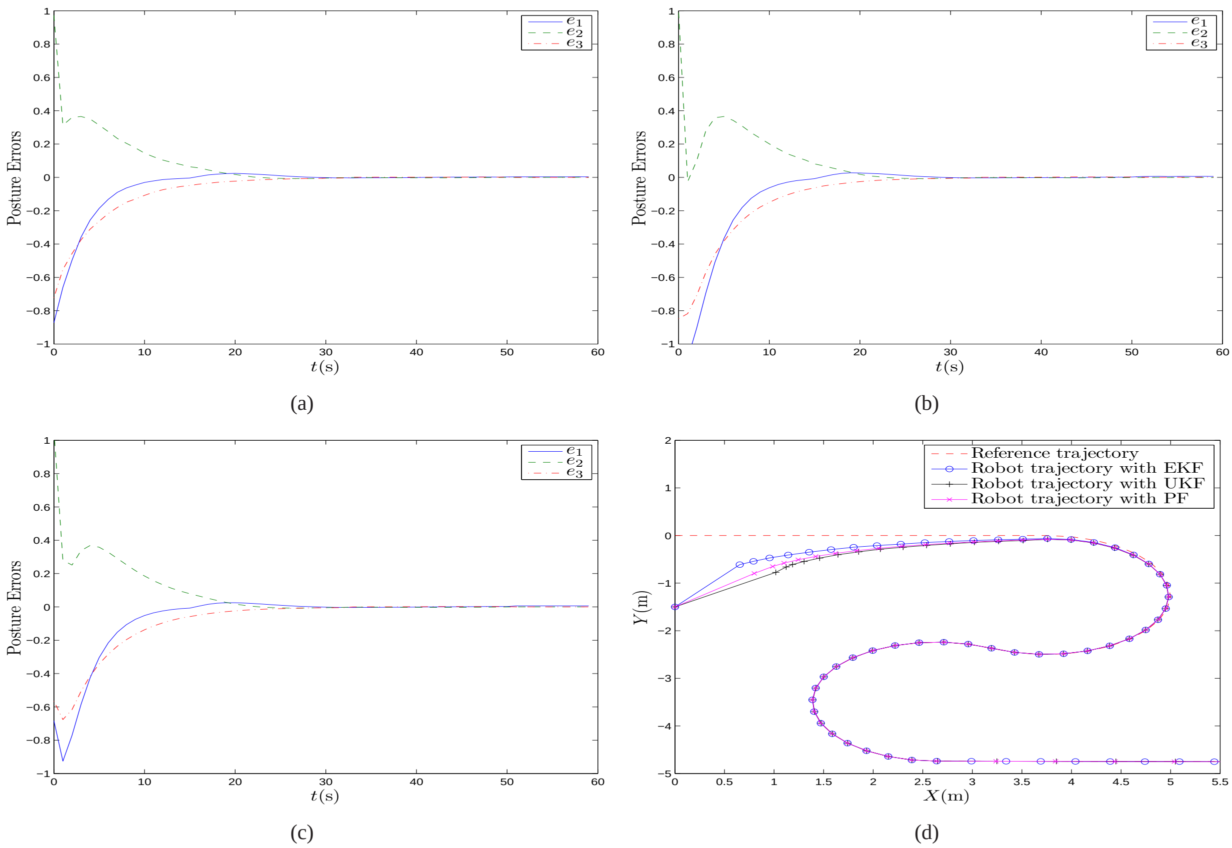

Figure 4: Posture errors (using the filters: (a) EKF; (b) UKF and (c) PF) and (d) Trajectory of the robot for 1st scenario.

Scenario 2: Adaptive torque-based control

The nonlinear kinematic used in the filter is given by

$$\begin{pmatrix} x \\ y \\ \phi \\ \omega_L \\ \omega_R \end{pmatrix}_{k+1} = \begin{pmatrix} x \\ y \\ \phi \\ \omega_L \\ \omega_R \end{pmatrix}_{k} + h \begin{pmatrix} v\cos\phi_k \\ v\sin\phi_k \\ \omega \\ 0 \\ 0 \end{pmatrix}$$

where x_k, y_k and ϕ_k corresponds to robot pose, w_L and w_R are respectively left and right angulares velocities of the wheels, v and ω are respectively the linear and angular velocities, h is the sampling time. The sampling step is taken as $h = 0.005$.

Figures 5(a), 5(b) and 5(c) show the posture error $e = (e_1, e_2, e_3)^T$ obtained using respectively the EKF, UKF and PF methods. Figure 5(d) show the reference trajectory and comparison between robot trajectory using the EKF, UKF and PF methods. As in the first scenario, all three filters methods show consistent and similar results.

Figures 6(a), 6(b) and 6(c) show the velocity error $e = (e_4, e_5)^T$ obtained using respectively the EKF, UKF and PF filters.

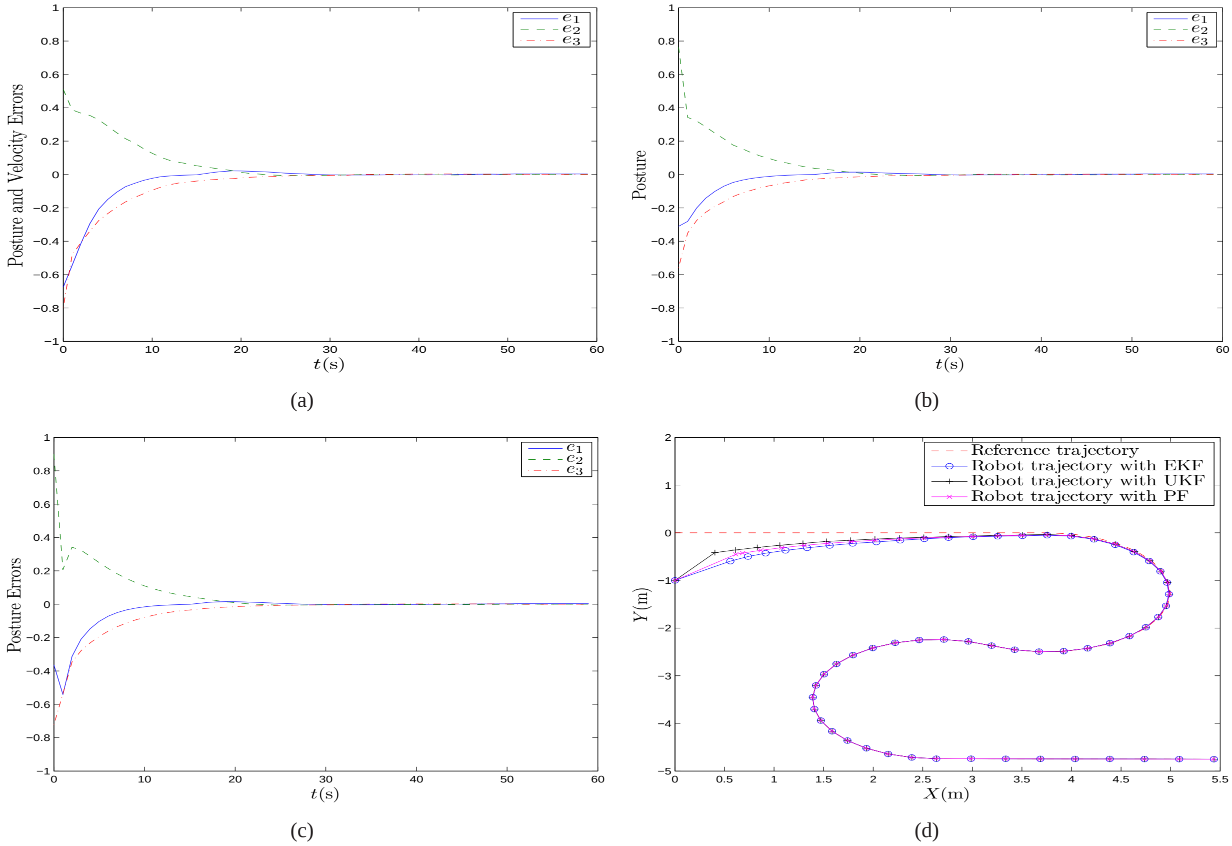

Figure 5: Posture errors (using the filters: (a) EKF; (b) UKF and (c) PF) and (d) Trajectory of the robot for 2nd scenario.

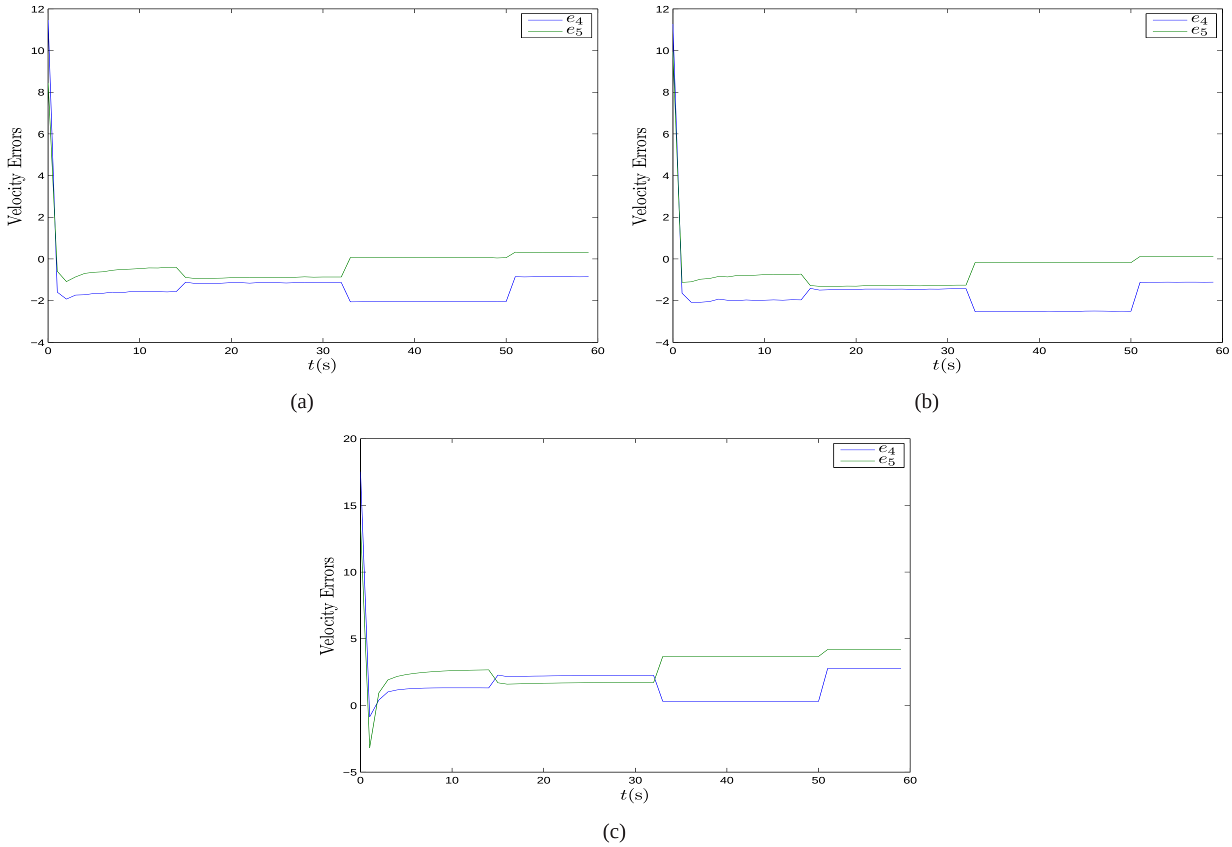

Figure 6: Velocity errors using the filters: (a) EKF, (b) UKF and (c) PF.

CONCLUSIONS

Two different adaptive tracking control for tracked mobile robot under slip condition using the extended Kalman filter (EKF), the unscented Kalman filter (UKF) and the particle filter (PF) to estimate all states of the robot are presented. The EKF, UKF and PF techniques are analyzed in two scenarios. In the first scenario, the controller is based on the kinematic model and provides angular velocities as the control input. Furthermore, the states estimates are position and orientation of the robot. In the second scenario, the controller, based on the dynamic model, consists in a feedback control law that provides torque as control input. In this case, in addition of the position and orientation the angular velocities of the wheels are estimated. Numerical results show the performance of the adaptive control laws using the EKF, UKF and PF filtering techniques. In future works, the sensors models should be included in the close-loop and the fusion datas should be studied. Others nonlinear filtering approaches will be compared.

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