

Aircraft Controller Design to Enhance Longitudinal Maneuvers

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Abstract: In many cases, we can linearize the dynamical equations of a nonlinear system depending on a parameter, for each value of that parameter around any equilibrium point. As a result, the obtained matrices of the linear model also depend on that parameter and the equilibrium point. Thus, corresponding to any value of the parameter, there exists a linear controller that can achieve satisfactory efficiency metrics at that point and its neighborhood. However, outside this neighborhood there isn't any guarantee to fulfill the minimal performance requirements. In the gain-scheduling method, we linearize the system for several points within the spanning domain of the parameter and for any of these linear models we will design controllers to switch into them depending on some criteria. In this paper, the problem of determining multiplicity of equilibrium points in gain-scheduling method for an aircraft to have longitudinal movement is considered. Also we seek to find a criterion to tell us when each particular controller should be used which is carried out through stability margin observations and the *v-Gap* metric.

Key words: Longitudinal manoeuvring, Steady State Rectilinear Flight, Gain-Scheduling, Stability Margin, *v-gap* metric

INTRODUCTION

Suppose an aircraft that has a steady state turning fly and wants to have longitudinal movement. Due to some criteria such as instability, model uncertainty and disturbances, the movement of this aircraft is virtually impossible with just the direct adjustment of inputs (e.g. aileron, rudder and elevator angle) and without the help of a controller. Analogous to classical control theory, it is necessary to have a controller that, in a closed-loop system, processes the error between the reference inputs and the plants (aircraft's) output to produce an acceptable control signal. In classic control theory the design of a controller is based on some templates or models for the dynamical behavior of the system and also uncertainties and/or disturbances. In the continuous time domain analysis, the dynamical behavior of the plant can be modeled by differential equations. In the case of an aircraft, these equations are nonlinear and time varying in many instances during the flight experience. Linearizing these equations around the equilibrium point leads to Linear Time Invariant (LTR) equations, which can be utilized to design controllers for the system. But finally these controllers must be placed within the nonlinear plant in a closed loop system. Hence, if the system works far away from the equilibrium point, there is no level of guarantee for the performance of the system's output. One of the best solutions for this problem is that instead of only linearizing the system around one equilibrium point and extracting one LTR model and designing one controller, the nonlinear equations be linearized around several different equilibrium points. Hence, we can obtain several LTR models and controllers for our nonlinear system. It is quite obvious that this procedure must be carried out off-line. Having done that, at any instance of time, the best controller designed for the nearest equilibrium point to the current operating point of the system is chosen and switched into the closed loop system.

In fact, this method is one of the conventional methods for designing controllers in nonlinear systems that is named gain-scheduling. In this paper, we use this method for the control of an aircraft movement.

In (Fujimori, A., 2005), authors use gain-scheduling for flight control of an unmanned space vehicle named HOPE-X. They introduce some basic procedures that can be used in the control of many manned and unmanned vehicles. In (Fujimori, A., 2005) authors mixed some context (e.g. Linear Parameter Varying (LPV), Linear Matrix Inequality (LMI), Gain Margin and *v-Gap* Metric (Vinnicombe, G., 2001)) to design a controller.

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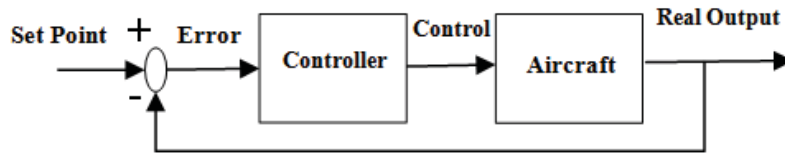


Fig. 1: The closed-loop system for controlling an Aircraft

In their paper, they first determine the number of equilibrium points and then calculate the place of these equilibrium points by the numerical minimizing method. By this procedure, they introduce a method for the implementation of the gain-scheduling method. Two main questions in gain-scheduling are as follows:

1. How to choose the equilibrium points.
2. How to switch or program linear controllers into the nonlinear system to construct a closed-loop system.

Although in (Fujimori, A., 2005), the numbers of equilibrium points were initially determined, in this paper, we calculate the place of equilibrium points without any constraint on the multiplicity of equilibrium points.

The rest of the paper is organized as follows: In section 2 the nonlinear equations of the aircraft are introduced. Also, in this section nonlinear equations are linearized and are prepared to design a Linear Quadratic Regulator (LQR) controller. In section 3 the design procedure is implemented on the system. In this part concepts of Stability Margin and v-Gap are utilized. In section 4 the simulation results and discussions are shown. Finally, section 5 concludes the paper.

2. Aircraft Dynamical Equations:

Aircraft is a rigid body and is assumed symmetric along the longitudinal vertical plane. Because the fuel consumption is low in the interval time of one longitudinal movement, we relinquish the variation of its weight. According to Newton's second law the resultant of all forces applied to a rigid body is equal to the derivation of its moment relative to time. In other words, this law can be presented as below (Roskam, J., 1979):

$$\frac{d}{dt} \vec{V}_p = m(\dot{\vec{V}}_p + \vec{\omega} \times \vec{V}_p) = m \vec{g} + \vec{F}_A + \vec{F}_T \quad (1)$$

In (1), \vec{g} is the acceleration due to gravity, ω represents the angular velocity of the aircraft, V_p is the linear velocity of the aircraft, m is the mass of the aircraft and r is the distance between each part of aircraft to its center of mass. Similarly, relation of angular movement amplitude can be written as below:

$$\int_V \vec{r} \times (\dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) \rho_A dv = M_A + M_T \quad (2)$$

In which ρ_A is the density and v is the volume of aircraft. Now, we must rewrite vector equations (1) and (2) according to their basis components. For this purpose, we must define each vector separately. Table (1) shows the definition of each vector parameter in terms of its basis components in the Cartesian coordinates. Components of g can be obtained from equation (3):

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi & \cos \theta \sin \phi \\ 0 & -\sin \phi & \cos \theta \sin \phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (3)$$

Table 1: Definition of each vector parameters according to its basis component

Vector Definition	Vector
$\vec{F}_A = iF_{Ax} + jF_{Ay} + kF_{Az}$	Aerodynamic Force
$\vec{F}_T = iF_{Tx} + jF_{Ty} + kF_{Tz}$	Thrust Force
$\vec{g} = ig_x + jg_y + kg_z$	acceleration due to gravity
$\vec{M}_A = iL_A + jM_A + kN_A$	Aerodynamic Moment
$\vec{M}_T = iL_T + jM_T + kN_T$	Thrust Moment
$\vec{\omega} = iP + jQ + kR$	Angular Velocity
$\vec{V}_p = iU + jV + kW$	Linear Vector
$\vec{r} = ix + jy + kz$	Distance

In (3), θ is the pitch angle, ϕ the roll angle and ψ the yaw angle (ψ doesn't appear in (3)). These angles are Euler angles and represent the rotation of the body coordinate system according to the ground ones (Roskam, J., 1979). By substituting (3) and the vector components of the movement equation into (1) and (2), we have:

$$\begin{aligned}
 m(\dot{U} - VR + WQ) &= mg \sin \theta + F_{Ax} + F_{Tx} \\
 m(\dot{V} + UR - WP) &= mg \cos \theta \sin \phi + F_{Ay} + F_{Ty} \\
 m(\dot{W} - UQ + VP) &= mg \cos \theta \cos \phi + F_{Az} + F_{Tz}
 \end{aligned} \tag{4}$$

Due to the voluminal integral in (2), the extension of this equation is complicated. In equation (5) we only mention the final result:

$$\begin{aligned}
 I_{xx} \dot{P} - I_{xz} \dot{R} - I_{xz} PQ + (I_{zz} - I_{yy})RQ &= L_A + L_T \\
 I_{yy} \dot{Q} + (I_{zz} - I_{xx})PR + I_{xz}(P^2 - R^2) &= M_A + M_T \\
 I_{zz} \dot{R} - I_{xz} \dot{P} + (I_{yy} - I_{xx})PQ + I_{xz}QR &= N_A + N_T
 \end{aligned} \tag{5}$$

It is important to note that to attain these equations we use the assumption of aircraft symmetry with respect to the xz plane. Also, we use the definition of inertial moments ($I_{xx}, I_{yy}, I_{zz}, I_{xz}$) in our calculations. To find the direction of flight we have:

$$\begin{aligned}
 P &= \dot{\phi} - \dot{\psi} \sin \theta \\
 Q &= \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi \\
 R &= \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi
 \end{aligned}
 \tag{6}$$

If all of the forces and moments in the aircraft system are known, we can determine the position of the aircraft. Equations (4)-(6) are general. We can derive two other important equations: 1- Continual flight state equations, 2- Perturbed flight state equations.

2.1. Continual Flight:

Based on definition, continual flight is the flight experience in which all of the flight parameters remain constant in the body coordinate system. This definition can be mathematically expressed as below:

$$\vec{\dot{V}}_p = \vec{0} \quad , \quad \vec{\dot{\omega}} = \vec{0}
 \tag{7}$$

Based on this definition continual flight cannot last for infinite time. One example of continual flight is the steady state rectilinear flight. In a continual flight, usually all of the flight variables are determined with capital letters and index 1. The mathematical expression of a steady state rectilinear flight is as below:

$$\dot{\omega} = \dot{U} = \dot{V} = \dot{W} = 0
 \tag{8}$$

$$P = Q = R = 0
 \tag{9}$$

By substituting (8) and (9) in (4) and (5), the continual rectilinear flight equations are derived. The steady state maneuver flight, which is considered in this paper, is of the same kind. Moreover, it is assumed that the steady state linear velocity is in the direction of the promontory point of the aircraft, which is in the direction of the x axis of aircraft body. Also, it's assumed that the rolling angle in steady state is not zero; which means

$$\text{that the velocity in the direction of the } z \text{ axis nonzero } V_1 = W_1 = \Phi_1 = \Theta_1 = 0$$

2.2. Perturbed Flight:

For better description of perturbed flight, suppose an aircraft in continual flight. Now, if for any reason the aircraft leaves this state, we can suppose that each of the flight variables is formed by two components: the initial condition that exists in continual flight, plus a perturbed value. The values of continual flight are always shown by capital letters and index 1; while the perturbed values are shown by small characters. In general, each flight that is not continual can be considered as perturbed. Based on the information from the continual flight, each of the state variables can be written as below:

$$\begin{aligned}
 U &= U_1 + u & V &= V_1 + v & W &= W_1 + w \\
 P &= P_1 + p & Q &= Q_1 + q & R &= R_1 + r \\
 \Psi &= \Psi_1 + \psi & \Theta &= \Theta_1 + \theta & \Phi &= \Phi_1 + \phi
 \end{aligned}
 \tag{10}$$

$$\begin{aligned}
 F_{Ax} &= F_{Ax1} + f_{Ax} & F_{Tx} &= F_{Tx1} + f_{Tx} \\
 F_{Ay} &= F_{Ay1} + f_{Ay} & F_{Ty} &= F_{Ty1} + f_{Ty} \\
 F_{Az} &= F_{Az1} + f_{Az} & F_{Tz} &= F_{Tz1} + f_{Tz}
 \end{aligned}
 \tag{11}$$

$$\begin{aligned}
 L_A &= L_{A1} + l_A & L_T &= L_{T1} + l_T \\
 M_A &= M_{A1} + m_A & M_T &= M_{T1} + m_T \\
 N_A &= N_{A1} + n_A & N_T &= N_{T1} + n_T
 \end{aligned}
 \tag{12}$$

By substituting these equations into (4) and (5), and using the aforementioned assumptions during the flight, we can extract the nonlinear equations of the flight movement. So we have:

$$\begin{aligned}
 m \left[\dot{u} - vr + wq \right] &= -mg \sin \theta + f_{Ax} + f_{Tx} \\
 m \left[\dot{v} + ur + U_1 r - wp \right] &= mg \sin \phi \cos \theta + f_{Ay} + f_{Ty} \\
 m \left[\dot{w} - uq - U_1 q + vp \right] &= mg \cos \phi \cos \theta + f_{Az} + f_{Tz} - mg \\
 I_{xx} \dot{p} - I_{xz} \dot{r} - I_{xz} pq + (I_{zz} - I_{yy}) rq &= l_A + l_T \\
 I_{yy} \dot{q} - (I_{xx} - I_{zz}) pr + I_{xz} (p^2 - r^2) &= m_A + m_T \\
 I_{zz} \dot{r} - I_{xz} \dot{p} + (I_{yy} - I_{xx}) pq + I_{xz} qr &= n_A + n_T \\
 p = \dot{\phi}, \quad q = \dot{\theta}, \quad r = \dot{\psi}
 \end{aligned}
 \tag{13}$$

To obtain the linear equations, we usually use some simplifying assumptions. For our case we use three assumptions as follows:

1. Each of the flight variables are introduced using the linear portion of Taylor expansion.
2. Perturbation of Yaw and Pitch angles are considered less than 15 degrees. So:

$$\begin{aligned}
 \cos \theta &= 1 & \sin \theta &= \theta \\
 \cos \phi &= 1 & \sin \phi &= \phi
 \end{aligned}
 \tag{14}$$

3. Deviations about nominal values are supposed to be very small. Hence, their product and second powers are neglected.

Under these conditions, the linearized perturbed equations are:

$$\begin{aligned}
 m \dot{u} &= -mg\theta + f_{Ax} + f_{Tx} \\
 m \dot{v} &= mg\phi + f_{Ay} + f_{Ty} \\
 m \dot{w} &= f_{Az} + f_{Tz} \\
 I_{xx} \dot{p} - I_{xz} \dot{r} &= l_A + l_T
 \end{aligned}$$

$$\begin{aligned}
 I_{yy} \dot{q} &= m_A + m_T \\
 I_{zz} \dot{r} - I_{xz} \dot{p} &= n_A + n_T \\
 p &= \dot{\phi}, \quad q = \dot{\theta}, \quad r = \dot{\psi}
 \end{aligned}
 \tag{15}$$

The forces and moments in the RHS of (15) are equal to equations (16) and (17), where q is the dynamical pressure, S is the surface wing, \bar{c} is the mean aerodynamic chord, b is mouth of wing, α is the attack angle and is equal to $\alpha = wU_1^{-1}$, β is side-slip angle and is equal to $\beta = vU_1^{-1}$, δ_A is aileron angle, δ_R is the rudder angle and δ_E is elevator angle. The matrices in the RHS of (16) and (17) are the derivation matrices of stability and control or the aerodynamic coefficients matrix of the aircraft. These coefficients are related to the geometrical shapes, velocity and the altitude of the aircraft.

$$\begin{bmatrix} \frac{f_{A_x} + f_{T_x}}{\bar{q}S} \\ \frac{f_{A_z} + f_{T_z}}{\bar{q}S} \\ \frac{m_A + m_T}{\bar{q}S\bar{c}} \end{bmatrix} = \begin{bmatrix} (C_{T_{x_u}} + 2C_{T_{x1}}) - (C_{D_\alpha} + 2C_{D1}) & (-C_{D_\alpha} + C_{L1}) & -C_{D_\alpha} & -C_{D_q} & -C_{D_{\delta E}} \\ (C_{T_{z_u}} + 2C_{T_{z1}}) - (C_{L_\alpha} + 2C_{L1}) & (-C_{L_\alpha} - C_{D1}) & -C_{L_\alpha} & -C_{L_q} & -C_{L_{\delta E}} \\ (C_{m_{r_u}} + 2C_{m_{r1}}) - (C_{m_\alpha} + 2C_{m1}) & C_{m_\alpha} + C_{m_{r\alpha}} & C_{m_\alpha} & C_{m_q} & C_{m_{\delta E}} \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \dot{\alpha}\bar{c} \\ \frac{q\bar{c}}{2U_1} \\ \frac{q\bar{c}}{2U_1} \\ \delta_E \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} \frac{f_{A_y} + f_{T_y}}{\bar{q}S} \\ \frac{l_A + l_T}{\bar{q}Sb} \\ \frac{n_A + n_T}{\bar{q}Sb} \end{bmatrix} = \begin{bmatrix} C_{y_\beta} & C_{y_\beta} & C_{y_p} & C_{y_r} & C_{y_{\delta A}} & C_{y_{\delta R}} \\ C_{l_\beta} & C_{l_\beta} & C_{l_p} & C_{l_r} & C_{l_{\delta A}} & C_{l_{\delta R}} \\ C_{n_\beta} + C_{n_{T\beta}} & C_{n_\beta} & C_{n_p} & C_{n_r} & C_{n_{\delta A}} & C_{n_{\delta R}} \end{bmatrix} \begin{bmatrix} \beta \\ \dot{\beta}b \\ \frac{\beta b}{2U_1} \\ \frac{pb}{2U_1} \\ \frac{rb}{2U_1} \\ \delta_A \\ \delta_R \end{bmatrix} \tag{17}$$

Now by substituting the values of the forces and moments in (13) and selecting the state variable as follows, the nonlinear equations are obtained.

$$\vec{x} = \left[u \quad \alpha \quad \theta \quad \dot{\theta} \quad \beta \quad \dot{\psi} \quad \phi \quad \dot{\phi} \right]^T \tag{18}$$

Using (15)-(18) nonlinear state equations of the aircraft are as follows:

$$\begin{aligned} \dot{x}_1 &= U_1 x_5 x_6 - U_1 x_2 x_4 - g \sin x_3 + \left(X_u + X_{T_u} \right) x_1 + X_\alpha x_2 + X_{\delta_E} \delta_E \\ \dot{x}_2 &= \frac{U_1 x_4 + x_1 x_4}{U_1 - Z_\alpha} - \frac{U_1 x_5 x_8}{U_1 - Z_\alpha} + \frac{g \cos x_7 \cos x_3}{U_1 - Z_\alpha} + \frac{Z_u}{U_1 - Z_\alpha} x_1 + \frac{Z_\alpha}{U_1 - Z_\alpha} x_2 + \frac{Z_q}{U_1 - Z_\alpha} x_4 + \frac{Z_{\delta_E}}{U_1 - Z_\alpha} \delta_E \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{I_{zz} - I_{xx}}{I_{yy}} x_6 x_8 + \frac{I_{xz}}{I_{yy}} (x_6^2 - x_8^2) + \left(M_u + M_{T_u} + \frac{Z_u M_\alpha}{U_1 (U_1 - Z_\alpha)} \right) x_1 + \left(M_\alpha + M_{T_\alpha} + \frac{Z_\alpha M_\alpha}{U_1 (U_1 - Z_\alpha)} \right) x_2 + \\ &\quad \frac{M_\alpha}{U_1 (U_1 - Z_\alpha)} (U_1 x_4 + x_1 x_4) - \frac{M_\alpha}{(U_1 - Z_\alpha)} (x_8 x_5) + \frac{M_\alpha}{U_1 (U_1 - Z_\alpha)} g \cos x_7 \cos x_3 + \left(M_q + \frac{Z_q M_\alpha}{U_1 (U_1 - Z_\alpha)} \right) x_4 \\ &\quad \left(M_{\delta_E} + \frac{Z_{\delta_E} M_\alpha}{U_1 (U_1 - Z_\alpha)} \right) \delta_E \tag{19} \\ \dot{x}_5 &= -\frac{x_6 (x_1 + U_1)}{U_1} + x_2 x_8 + \frac{g \sin x_7 \cos x_3}{U_1} + \frac{Y_p x_8}{U_1} + \frac{Y_r x_6}{U_1} + \frac{Y_{\delta_A} \delta_A}{U_1} + \frac{Y_{\delta_R} \delta_R}{U_1} \\ \dot{x}_6 &= \frac{1}{A_1 B_1 - 1} \left(\frac{I_{yy} - I_{xx}}{I_{zz}} - A_1 B_1 \right) x_8 x_4 + \frac{1}{A_1 B_1 - 1} \left(\frac{I_{zz} - I_{yy}}{I_{xx}} B_1 + B_1 \right) x_6 x_4 + \frac{1}{1 - A_1 B_1} (B_1 L_\beta + N_\beta + N_{T_\beta}) x_5 \\ &\quad + \frac{1}{1 - A_1 B_1} (B_1 L_p + N_p) x_8 + \frac{1}{1 - A_1 B_1} (B_1 L_r + N_r) x_6 + \frac{1}{1 - A_1 B_1} (B_1 L_{\delta_A} + N_{\delta_A}) \delta_A + \frac{1}{1 - A_1 B_1} (B_1 L_{\delta_R} + N_{\delta_R}) \delta_R \\ \dot{x}_8 &= \frac{1}{A_1 B_1 - 1} \left(\frac{I_{yy} - I_{xx}}{I_{zz}} A_1 + A_1 \right) x_8 x_4 + \frac{1}{A_1 B_1 - 1} \left(\frac{I_{zz} - I_{yy}}{I_{xx}} + A_1 B_1 \right) x_6 x_4 + \frac{1}{1 - A_1 B_1} (A_1 N_\beta + A_1 N_{T_\beta} + L_\beta) x_5 \\ &\quad + \frac{1}{1 - A_1 B_1} (A_1 N_p + L_p) x_8 + \frac{1}{1 - A_1 B_1} (A_1 N_r + L_r) x_6 + \frac{1}{1 - A_1 B_1} (A_1 N_{\delta_A} + L_{\delta_A}) \delta_A + \frac{1}{1 - A_1 B_1} (A_1 N_{\delta_R} + L_{\delta_R}) \delta_R \end{aligned}$$

In which $A_1 = \frac{I_{xz}}{I_{xx}}$, $B_1 = \frac{I_{xz}}{I_{zz}}$ (refer to the appendix, table 2,3)

To obtain linear equations, one can assume two distinct movements for aircrafts: Longitudinal Movement and Lateral-Directional Movement. Longitudinal Movement is the movement in the direction of the x axis in the body coordinate system and rotates around y axis. The states and inputs are as follows:

$$\vec{x} = [u \quad \alpha \quad \theta]^T, \quad U_{input} = \delta_E \tag{20}$$

Note that all of these variables are located in the xz plane of the body coordinate system.

Lateral-Directional Movement is the rotation about the x or z axes that are known Roll and Yaw respectively. As it is known from its name, this movement is used for the rotation of aircraft and is consisted of the following states and input:

$$\vec{x} = [\beta \quad \dot{\psi} \quad \phi \quad \dot{\phi}]^T, \quad U_{input} = [\delta_A \quad \delta_R]^T \tag{21}$$

The reason that we divided the aircraft movement into Longitudinal Movement and Lateral-Directional Movement is that if the nonlinear model of the aircraft is linearized around an operation point, we will observe that the states in equations (20) and (21) are excessively uncorrelated. So, when we design a controller for the aircraft, we suppose that aircraft movement belongs to one of these movements and we design the controller for that movement type. Now by substituting equations (16) and (17) into (15) and selecting the state and input variables of (20), we can obtain the Longitudinal equations and in the same way, by selecting the state and input variables of (21), we achieve the Lateral-Directional equations.

$$\begin{aligned} \dot{u} &= -g\theta + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_E} \delta_E \\ U_1 \dot{\alpha} - U_1 \dot{\theta} &= Z_u u + Z_\alpha \alpha + Z_\alpha \dot{\alpha} + Z_q \dot{\theta} + Z_{\delta_E} \delta_E \\ \ddot{\theta} &= M_u u + M_{T_u} u + M_\alpha \alpha + M_\alpha \dot{\alpha} + M_\alpha \alpha + M_q \dot{\theta} + M_{\delta_E} \delta_E \end{aligned} \tag{22}$$

Using equations (15)-(20) and (22) longitudinal-linear state equations are as follows:

$$\begin{aligned} \dot{x}_1 &= (X_u + X_{T_u})x_1 + X_\alpha x_2 - g x_3 + X_{\delta_E} u \\ \dot{x}_2 &= \left(\frac{Z_u}{U_1 - Z_\alpha} \right) x_1 + \left(\frac{U_1 + Z_q}{U_1 - Z_\alpha} \right) x_4 + \left(\frac{Z_\alpha}{U_1 - Z_\alpha} \right) x_2 + \left(\frac{Z_{\delta_E}}{U_1 - Z_\alpha} \right) \delta_E \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \left(M_u + M_{T_u} + \frac{M_\alpha Z_u}{U_1 - Z_\alpha} \right) x_1 + \left(M_\alpha + M_{T_\alpha} + \frac{M_\alpha Z_\alpha}{U_1 - Z_\alpha} \right) x_2 \end{aligned}$$

$$+ \left(M_q + \frac{M \cdot (U_1 + Z_q)}{\alpha} \right) x_4 + \left(M_{\delta_E} + \frac{M \cdot Z_{\delta_E}}{U_1 - Z_{\alpha}} \right) u \quad (23)$$

3. Control Design:

The dynamical model of our plant is nonlinear; hence it is obvious that when the domain of aircraft operation area is large, using just one linear controller around an operation point does not lead to good results. Hence, using nonlinear controllers is necessary. One of the conventional approaches in the aircraft control field is the gain-scheduling method. In the gain-scheduling method, linearizing the system is done around some operation point. Moreover it designs linear controllers that will be switched into the system based on the system's current operation point. Indeed gain-scheduling method leads to a nonlinear controller. To design a gain-scheduling controller we must consider two main points: First is the design of linear controllers which are the components of the gain-scheduling controller; and second is determining a law for appropriately switching linear controllers into the closed loop system.

3.1. Selecting Linear Components of a Gain-scheduling Controller:

Necessary linear controllers are designed based on optimal control methods and named *designated controllers*. These are close relatives of a prominent family of controllers that are known as Linear Quadratic Regulator (LQR) controllers. The difference between them is that the reference input in the former is not necessarily zero. The system is defined by the below state equations:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \quad (24)$$

The performance index that must be minimized is as follows:

$$\begin{aligned} J &= \frac{1}{2} [x(t_f) - r(t_f)]^T W [x(t_f) - r(t_f)] \\ &+ \frac{1}{2} \int_{t_0}^{t_f} \{ [x(t) - r(t)]^T Q(t) [x(t) - r(t)] + u^T(t) R(t) u(t) \} dt \\ &= \frac{1}{2} \|x(t_f) - r(t_f)\|_W^2 \\ &+ \frac{1}{2} \int_{t_0}^{t_f} \{ \|x(t) - r(t)\|_{Q(t)}^2 + \|u(t)\|_{R(t)}^2 \} dt \end{aligned} \quad (25)$$

In (25), $r(t)$ is the reference or desired input of the state vector. The final time (t_f) is constant but $x(t_f)$ is free. Also, all of the states and control effort values are unlimited and unbounded. W and Q are symmetric positive semi definite matrices and R is a symmetric positive definite matrix. A physical explanation for performance index is that the system states go to the desired states without any large control inputs. The control law for this problem is as follows:

$$u(t) = F(t)x(t) + v(t) \quad (26)$$

Where:

$$\begin{aligned} F(t) &= -R^{-1}(t)B^T(t)K(t) \\ v(t) &= -R^{-1}(t)B^T(t)s(t) \end{aligned} \quad (27)$$

And $K(t)$ and $s(t)$ are the solutions of below equations:

$$\begin{aligned} \dot{K}(t) &= -K(t)A(t) - A^T(t)K(t) - Q(t) \\ &\quad + K(t)B(t)R^{-1}(t)B^T(t)K(t) \\ \dot{s}(t) &= -[A^T(t) - K(t)B(t)R^{-1}(t)B^T(t)]s(t) \\ &\quad + Q(t)r(t) \end{aligned} \tag{28}$$

$F(t)$ is the gain feedback matrix and $v(t)$ is the order signal. Note that $v(t)$ is related to system parameters and the reference signal. Moreover, measurement of all state variables is necessary for implementation of the control input law.

- (1) System is completely controllable
- (2) $W=0$
- (3) A, B, R, Q are constant matrices and

$$t_f \rightarrow \infty, \quad k(t) \rightarrow k(\text{constant matrices})$$

If the above assumptions apply, the optimal control law for the unlimited time process will be constant (Kalman, R.E., 1960).

Matrix k can be resulted from the solution of the rickety equation:

$$-KA - A^T K - Q + KBR^{-1}B^T K = 0 \tag{29}$$

Setting $s(t)$ to zero in (28), while $r(t)$ is a constant non-zero value, we have:

$$s = [A^T - KBR^{-1}B^T]^{-1}Qr \tag{30}$$

The optimal control law equals:

$$u(t) = Fx(t) + v = -R^{-1}B^T K x(t) - R^{-1}B^T s \tag{31}$$

3.2. Programming the Linear Controllers:

In this section, before determining the law for programming linear controllers, the contexts of stability margins, v-Gap metrics and the relation between them are introduced. Afterwards, using these contexts, the controllers are programmed.

3.2.1. Stability Margin:

The pair [P, C] is stable if

$$\begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix} \in H_\infty \tag{32}$$

This is a matrix-form transfer function for a closed loop system between the measured noise and the reference signal with respect to system output and input as shown in Figure 1.

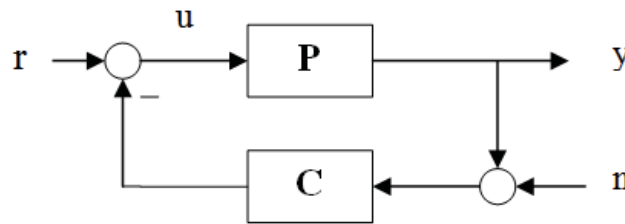


Fig. 2: Standard Feedback Block Diagram

If the pair $[P, C]$ was stable, the stability margin is defined as follows:

$$b_{P,C} = \left(\left\| \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix} \right\| \right)^{-1} \tag{33}$$

(33) is always a number between 0 and 1. This means that the closer this number is to zero, the higher is the stability margin. If the pair $[P, C]$ was not stable, then the stability margin is zero.

3.2.2. v-Gap Metrics:

For the constant metrics $X, Y \in \mathcal{C}^{p \times q}$, we define:

$$\kappa(X, Y) = \bar{\sigma} \left[\left(I + YY^* \right)^{-\frac{1}{2}} (X - Y) \left(I + YY^* \right)^{-\frac{1}{2}} \right] \tag{34}$$

Now, if P_1, P_2 were two matrix-form transfer functions for two different models, then the v-Gap metric is defined as below:

$$\delta_v(P_1, P_2) = \sup_{\omega} \kappa(P_1(j\omega), P_2(j\omega)) \tag{35}$$

In SISO equations will be:

$$\delta_v(P_1, P_2) = \sup_{\omega} \frac{|P_1(j\omega) - P_2(j\omega)|}{\sqrt{1 + |P_1(j\omega)|^2} \sqrt{1 + |P_2(j\omega)|^2}} \tag{36}$$

The performance index (or stability margin) and measurement index (or uncertainty), are related to each other by the following inequality:

$$b_{P_2,C} \geq b_{P_1,C} - \delta_v(P_1, P_2) \tag{37}$$

In (37), C is the controller for the model of P_1 . Application of this inequality when the stability is important can be expressed as follows:

1- Suppose that the nominal model $P_1 \in \mathcal{R}^{p \times q}$ and controller c and the number β are given. Then the pair of $[P_2, C]$ is stable for all P_2 that satisfy $\delta_v(P_1, P_2) \leq \beta$ if and only if:

$$b_{P_1,C} > \beta \tag{38}$$

2- Suppose the nominal model $P_1 \in \mathcal{R}^{p \times q}$ and perturbed model $P_2 \in \mathcal{R}^{p \times q}$ and the number $\beta < b_{opt}(P_1)$ are given. Then the pair of $[P_2, C]$ is stable for all controllers that $b_{P_1,C} > \beta$ if and only if:

$$\delta_v(P_1, P_2) \leq \beta \quad \text{where} \quad b_{opt}(P) = \sup_C b_{P,C} \tag{39}$$

So, each model that is less than a distance β from the nominal model can be stabilized by a controller which stabilizes the nominal plant with stability margin β . Moreover, each model that is more than a distance β from the nominal model can be made unstable by the controller which stabilizes the nominal plant with stability margin β . Now, using these definitions we can determine a law for programming linear controllers in the gain-scheduling method. Suppose the Nonlinear Parameter Varying (NPV) system as below:

$$\dot{x}(t) = f(x(t), u(t), \theta(t)) \quad (40)$$

We can linearize this system and obtain a Linear Parameter Varying (LPV) model:

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) \quad (41)$$

This linear model varies as the parameter θ varies. In this manner, for each value of θ in time t , there is a linear controller that performs well in that point and its neighborhood. But this controller does not guarantee to exhibit good performance outside of this neighborhood. Hence, we must use the best linear controller in the nearest neighborhood of each value of θ . One method is choosing various points for linearizing the nonlinear system at, and designing controllers for each. Controller design must be done off-line for each point.

3.3. Selecting the Operation Points:

The algorithm for determining the number and places of the operation points are as follows:

1. The domain of θ is divided to $n-1$ subintervals. The n point are $\{\theta_1, \dots, \theta_n\}$
2. Set these parameters: $s = \theta_1, d = 0, \theta = \theta_1$
3. $i = 0$
4. $\theta = \theta + 1$
5. $key = 0, \theta = \theta + d$
6. The linearized model is calculated around the points S and θ
7. Design a controller (k) for the system
8. Calculate the stability margin of the linear model in the point θ . ($b_{P_\theta, K}$)
9. Measure the difference between the two linear models in two points θ and s using *v-gap* and $\delta_v(P_\theta, P_s)$.
10. If $b_{P_\theta, K} > \delta_v(P_\theta, P_s)$, then $Q = \theta$
11. If $b_{P_\theta, K} > \delta_v(P_\theta, P_s)$, then
 - I. Consider Q as an operation point and save it in the matrices of operation points.
 - II. $j = 0$
 - III. $z = Q + j$
 - IV. If $key = 0$ then
 - a) Calculate linear model at point Q .
 - b) Design a controller (K_Q) to linearize the model in Q
 - c) Calculate the linearized model at point z .
 - d) Calculate the difference between two models:

(a) & (c), $\delta_v(P_Q, P_z)$

e) Calculate the stability margin of linear model at point Q .

f) If $\delta_v(P_\theta, P_s) > b_{P_\theta, K}$, then $b_{P_\theta, K}$, $b_{P_\theta, K}$ and $key = 1$

V. $j = j + 1$

VI. If $Q + j \leq 250$ go to (III)

12. $i = i + 1$

13. If $Q + j \leq 250$ go to (4)

14. The Saved operation point matrices are determined as the whole operation points of system.

15. End

In this method, both the number and the places of the operation points are determined simultaneously. Moreover, the stability margin of all values of θ in its domain is satisfied. This means that the difference between the model for each sample and the model of one operation point is less than the stability margin of that specific operation point.

3.4. Determining the Controller Coefficients in Each Point of the Domain of θ :

The only remaining problem is to determine the coefficients of each controller as θ varies. In this section we determined the controller coefficients using the specifications of the proposed algorithm and the relationship that exists between the stability margin and v-gap metrics. These coefficients are equal to those of a controller that exists at the nearest operation point.

Suppose that the obtained operation point matrices are w with dimension m . Now, we should determine the coefficient of the controller for an optional point θ . For this purpose we must find the nearest operation point in W so that stability margin in this point is greater than the difference between the optional point model and the operation point model. Consequently, based on the index between the stability margin and v-Gap metrics and its application we can use the controller of operation point instead of our controller.

The algorithm of this approach is as follows:

1. $E = 0$

2. $i = 0$

3. If $E = 0$ then:

I. determine i^{th} element of $W (P_{W(i)})$

II. Calculate the linear model in optional point θ

III. Design a controller for the linear model of i^{th} element of $W (K_{W(i)})$.

IV. Calculate difference between (3-I) and (3-III) using v-Gap ($\delta_v(P_{W(i)}, P_\theta)$)

V. Determine the stability margin of the i^{th} element of $W(b_{P_{W(i)}, K_{W(i)}})$ using model of (3-I) and controller of (3-III).

VI. If the controller of the i^{th} element of W is appropriate To be used in point θ save it and set $E = 1$

4. $i = i + 1$

5. If $i \leq m$ go to 3

6. Saved coefficients are used for the coefficients of the controller of optional point θ .
7. End

Finally, all of the steps in gain-scheduling are accomplished and we can determine the controller coefficients in any time just by measuring the parameter θ .

4. Simulation Results:

In this section we adopt our proposed algorithm into an aircraft. In our simulations, our purpose is to control the longitudinal movement of the aircraft. Longitudinal maneuver in our simulation is based on a referenced signal that forces the aircraft to move in a rectilinear flight fashion with a constant rotation in its pitch angle. Output performances of our system are based on flight parameter conditions, geometrical parameters, coefficients of steady state and stability. The values of these parameters are approximate but are reasonable since they are obtained by interpolating the data of a sample Boeing 747 (Roskam, J., 1979).

At first we verify whether the linear controller performs satisfactorily for the nonlinear system. In this paper, we suppose that the velocity of the sample aircraft (Boeing 747) is limited to the interval [150 250] m/s. This assumption is reasonable (Roskam, J., 1979). Consequently, when the velocity of the aircraft is increased from 150 to 250 (m/s), the closed loop system consisting of the nonlinear plant and the designed LQR controller are developed and the pitch angle is set to 30 degrees as an order to movement. The results of the designed LQR controller for the linear model are shown in Figures 3 and 4.

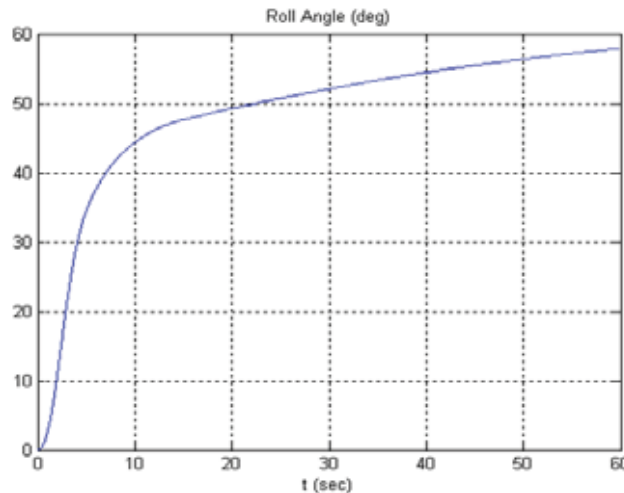


Fig. 3: Pitch Angle that resulted from LQR controller in the velocity of 20(m/s) and desired pitch angle of 30 degree.

As shown in Figures 3 and 4 none of the two LQR controllers have good results, because in both tests the Pitch Angle diverges and cannot converge to the desired angle. To converge, we can use the gain-scheduling controller. As we mentioned previously, we must determine the appropriate operation points. This is accomplished by using our proposed algorithm. The operation points that are extracted from our algorithm are as follows:

- 1th operation point at velocity 209 (m/s)
- 2th operation point at velocity 218 (m/s)
- 3th operation point at velocity 231 (m/s)
- 4th operation point at velocity 245 (m/s)
- 5th operation point at velocity 256 (m/s)
- 6th operation point at velocity 267 (m/s)
- 7th operation point at velocity 275 (m/s)
- 8th operation point at velocity 286 (m/s)
- 9th operation point at velocity 295 (m/s)

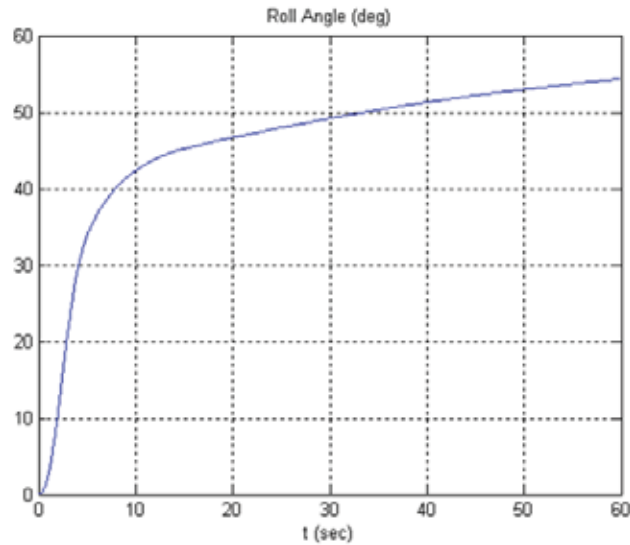


Fig. 4: Pitch Angle that resulted from LQR controller in the velocity of 250(m/s) and desired pitch angle of 30 degree.

Now, we can form the closed loop system and see the behavior of the longitudinal maneuver of the aircraft. The simulation results are shown in figures (5)-(10).

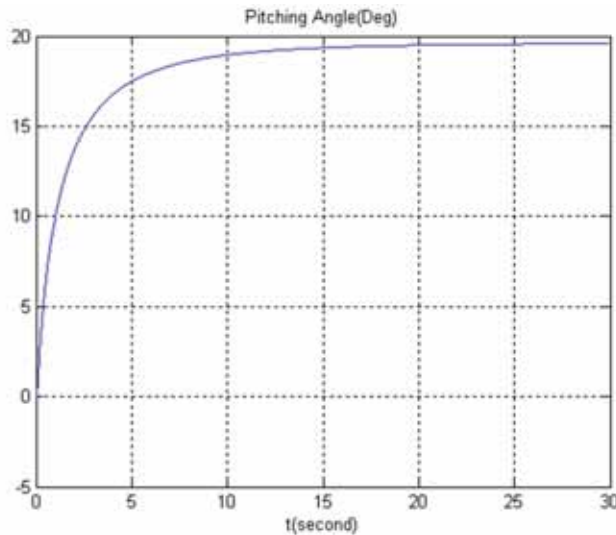


Fig. 5: Pitch Angle

As can be seen in the figures, the Pitch angles are tracking the desired value with acceptable error (about 2%). Also other states have a good behavior. Moreover the angle of elevation is a reasonable value and can be implemented in a real plant.

5. Conclusion:

We designed a set of linear controllers to be appropriately utilized when controlling the longitudinal movement of an aircraft which has a nonlinear nature. Because of the strict nonlinearity of the plant and its dependence on velocity, the linear controller did not exhibit satisfactory results. To overcome this problem, using nine operation points in the domain of the aircraft's velocity and exploiting our proposed algorithm, a controller was designed based on the gain-scheduling method. Further, by appropriately deploying the proposed controllers into the nonlinear model, promising results were obtained.

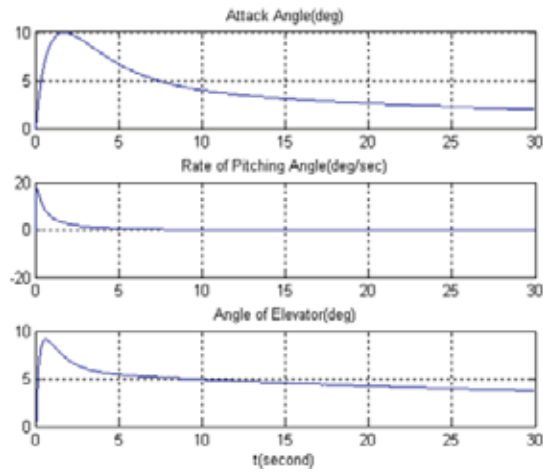


Fig. 6: Attack angle, Rate of Pitching Angle and Angle of Elevator

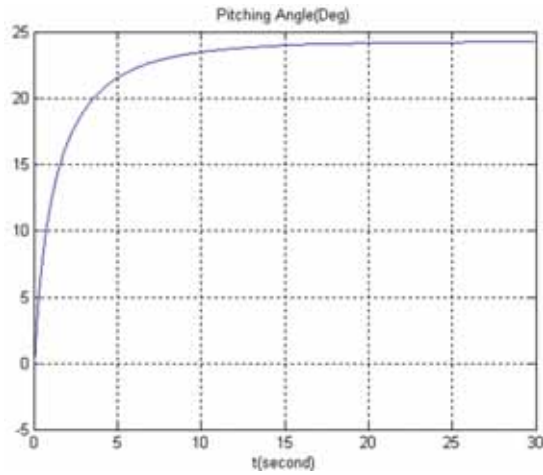


Fig. 7: Pitch Angle

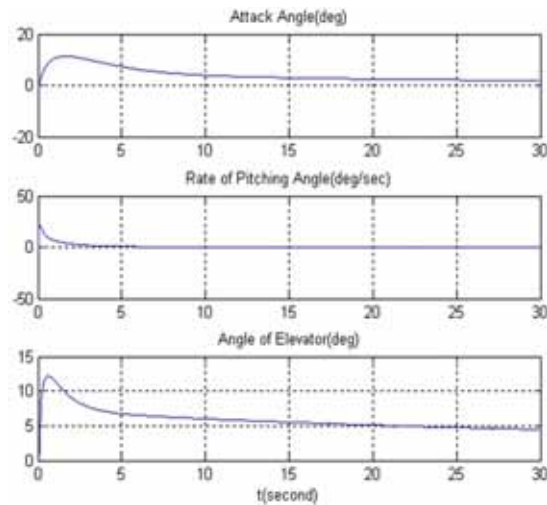


Fig. 8: Attack angle, Rate of Pitching Angle and Angle of Elevator

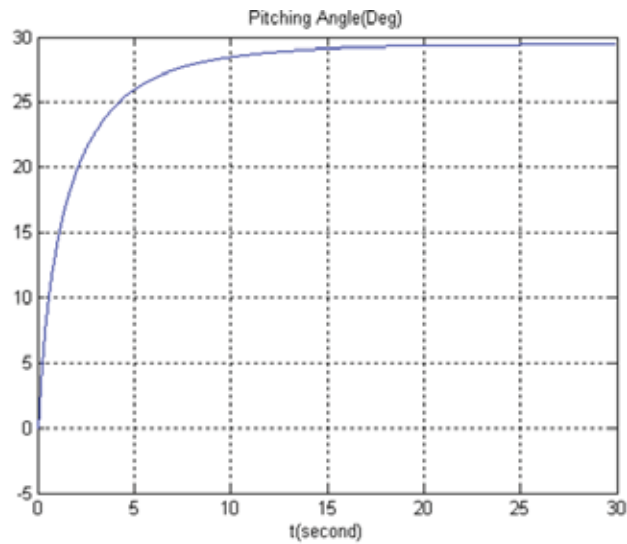


Fig. 9: Pitch angle

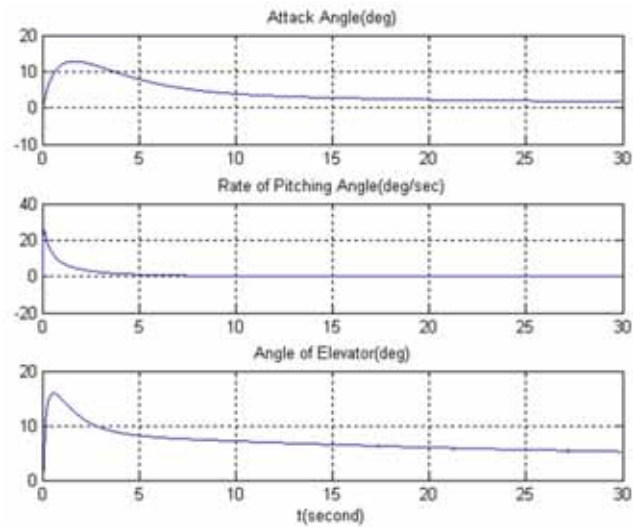


Fig. 10: Attack Angle, Rate of pitching Angle and Angle of elevator

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Appendix

Table 2: dimensional longitudinal stable derivations

$X_u = \frac{-\bar{q}_1 s(C_{D_u} + 2C_{D_1})}{mU_1} (\text{sec}^{-1})$	$Z_{\delta_g} = \frac{-\bar{q}_1 s C_{L_{\delta_g}}}{m} (ft \text{ sec}^{-2})$
$X_{T_u} = \frac{-\bar{q}_1 s(C_{T_{T_u}} + 2C_{T_{T_1}})}{mU_1} (\text{sec}^{-1})$	$M_u = \frac{\bar{q}_1 s \bar{c}(C_{m_u} + 2C_{m_1})}{I_{yy} U_1} (ft^{-1} \text{ sec}^{-1})$
$X_\alpha = \frac{-\bar{q}_1 s(C_{D_\alpha} + 2C_{D_1})}{m} (ft \text{ sec}^{-2})$	$M_{T_u} = \frac{\bar{q}_1 s \bar{c}(C_{m_{T_u}} + 2C_{m_{T_1}})}{I_{yy} U_1} (ft^{-1} \text{ sec}^{-1})$
$X_{\delta_g} = \frac{-\bar{q}_1 s C_{\delta_g}}{m} (ft \text{ sec}^{-2})$	$M_\alpha = \frac{\bar{q}_1 s \bar{c} C_{m_\alpha}}{I_{yy}} (\text{sec}^{-2})$
$Z_u = \frac{-\bar{q}_1 s(C_{L_u} + 2C_{L_1})}{mU_1} (\text{sec}^{-1})$	$M_{T_\alpha} = \frac{\bar{q}_1 s \bar{c} C_{m_{T_\alpha}}}{I_{yy}} (\text{sec}^{-2})$
$Z_\alpha = \frac{-\bar{q}_1 s(C_{L_\alpha} + 2C_{D_1})}{m} (ft \text{ sec}^{-2})$	$M_{T_u} = \frac{\bar{q}_1 s \bar{c}^2 C_{m_\alpha}}{2I_{yy} U_1} (\text{sec}^{-1})$
$Z_\alpha = \frac{-\bar{q}_1 s C_{L_\alpha} \bar{c}}{2mU_1} (ft \text{ sec}^{-1})$	$M_q = \frac{\bar{q}_1 s \bar{c}^2 C_{m_q}}{2I_{yy} U_1} (\text{sec}^{-1})$
$Z_q = \frac{-\bar{q}_1 s C_{L_q} \bar{c}}{2mU_1} (ft \text{ sec}^{-1})$	$M_{T_\alpha} = \frac{\bar{q}_1 s \bar{c} C_{m_{\delta_g}}}{I_{yy}} (\text{sec}^{-2})$

Table 3: dimensional lateral directional derivations

$Y_{\beta} = \frac{\bar{q}_1 s C_{y_{\beta}}}{m} (ft \sec^{-2})$	$L_{\delta_A} = \frac{\bar{q}_1 s b C_{l_{\delta_A}}}{I_{xx}} (\sec^{-2})$
$Y_p = \frac{\bar{q}_1 s b C_{y_p}}{2mU_1} (ft \sec^{-1})$	$L_{\delta_R} = \frac{\bar{q}_1 s b C_{l_{\delta_R}}}{I_{xx}} (\sec^{-2})$
$Y_r = \frac{\bar{q}_1 s b C_{y_r}}{2mU_1} (ft \sec^{-1})$	$N_{\beta} = \frac{\bar{q}_1 s b C_{n_{\beta}}}{I_{zz}} (\sec^{-2})$
$Y_{\delta_A} = \frac{\bar{q}_1 s C_{y_{\delta_A}}}{m} (ft \sec^{-2})$	$N_{T_{\beta}} = \frac{\bar{q}_1 s b C_{n_{T_{\beta}}}}{I_{zz}} (\sec^{-2})$
$Y_{\delta_R} = \frac{\bar{q}_1 s C_{y_{\delta_R}}}{m} (ft \sec^{-2})$	$N_p = \frac{\bar{q}_1 s b^2 C_{n_p}}{2I_{zz}U_1} (\sec^{-1})$
$L_{\beta} = \frac{\bar{q}_1 s b C_{l_{\beta}}}{I_{xx}} (\sec^{-2})$	$N_r = \frac{\bar{q}_1 s b^2 C_{n_r}}{2I_{zz}U_1} (\sec^{-1})$
$L_p = \frac{\bar{q}_1 s b^2 C_{L_p}}{2I_{xx}U_1} (\sec^{-1})$	$N_{\delta_A} = \frac{\bar{q}_1 s b C_{n_{\delta_A}}}{I_{zz}} (\sec^{-2})$
$L_r = \frac{\bar{q}_1 s b^2 C_{L_r}}{2I_{xx}U_1} (\sec^{-1})$	$N_{\delta_R} = \frac{\bar{q}_1 s b C_{n_{\delta_R}}}{I_{zz}} (\sec^{-2})$

The specifications and derivations of stable and approximate control of a passenger jumbo jet:

Table 4: Flight conditions

Altitude(km)	6.07
(kg / m^3) Air specific mass	0.6535

Table 5: inertia geometrical specifications

wing surface(m^2)	510.97
wing span(m)	59.74
wing mean cord(m)	8.32
Weight kg ()	288723.81
$I_{xx}(kgm^2)$	24.68×10^6
$I_{yy}(kgm^2)$	44.88×10^6
$I_{zz}(kgm^2)$	67.38×10^6
$I_{xz}(kgm^2)$	1.32×10^6

Table 6: steady state coefficient

$C_{L_1} = 6 \times 10^{-5} v^2 - 0.0262 v + 3.2521$
$C_{D_1} = 2 \times 10^{-5} v^2 - 0.0005 v + 0.0507$
$C_{T_1} = 2 \times 10^{-5} v^2 - 0.0005 v + 0.0507$
$C_{m_1} = 0$
$C_{mT_1} = 0$

Table 7: longitudinal derivations

$C_{L_\alpha} = 7 \times 10^{-5} v^2 - 0.0176 v + 7.58$	$C_{m_u} = -6 \times 10^{-6} v^2 + 0.0013 v + 0.0097$
$C_{L_q} = 7 \times 10^{-5} v^2 - 0.011 v + 6.09$	$C_{m_\alpha} = -7 \times 10^{-5} v^2 + 0.0214 v - 2.5914$
$C_{D_\alpha} = 6 \times 10^{-5} v^2 - 0.023 v + 2.40$	$C_{m_\alpha} = -0.0004 v^2 + 0.1019 v - 8.3817$
$C_{D_u} = 2 \times 10^{-5} v^2 - 0.005 v + 0.25$	$C_{m_q} = -0.0005 v^2 + 0.1295 v - 28.74$
$C_{T_{\alpha}} = 0$	$C_{mT_u} = 0$
$C_{L_{\delta E}} = -0.0003 v + 0.3806$	$C_{mT_\alpha} = 0$
$C_{D_{\delta E}} = 0$	$C_{L_u} = -2 \times 10^{-5} v^2 + 0.0085 v - 0.692$
$C_{m_{\delta E}} = 5 \times 10^{-6} v^2 - 0.0006 v - 1.384$	$C_{L_\alpha} = 0.0001 v^2 - 0.047 v + 8.2$

Table 8: lateral directional derivations

$C_{n_{\delta A}} = 3 \times 10^{-7} v^2 - 0.0001 v + 0.0154$	$C_{l_{\beta}} = 2 \times 10^{-5} v^2 - 0.0037 v - 0.1067$
$C_{n_{\delta R}} = -6 \times 10^{-8} v^2 + 0.0001 v - 0.12$	$C_{l_{\gamma}} = -4 \times 10^{-6} v^2 + 0.0023 v - 0.64$
$C_{y_{\beta}} = 6 \times 10^{-5} v^2 - 0.015 v - 0.323$	$C_{l_{\gamma}} = 8 \times 10^{-6} v^2 - 0.0027 v + 0.34$
$C_{y_{\rho}} = 0$	$C_{l_{\delta A}} = 2 \times 10^{-6} v^2 - 0.0007 v + 0.094$
$C_{y_{\gamma}} = 0$	$C_{l_{\delta R}} = 7 \times 10^{-8} v^2 + 2 \times 10^{-5} v - 0.0015$
$C_{y_{\delta A}} = 0$	$C_{n_{\beta}} = 5 \times 10^{-6} v^2 - 0.0016 v + 0.2657$
$C_{y_{\delta R}} = -3 \times 10^{-6} v^2 + 0.0004 v + 0.1684$	$C_{n_{\rho}} = -3 \times 10^{-6} v^2 + 0.0023 v - 0.3634$
	$C_{n_{\gamma}} = -7 \times 10^{-6} v^2 + 0.0025 v - 0.4974$