

Public School Choice: An Economic Analysis

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Abstract

Public school choice programs, also known as open enrollment or intra-district choice programs, give households a free choice of public school and provide public schools with incentives to compete for students. Supporters of these programs argue that by injecting market forces into the public school system, they will improve the quality of the education that public schools provide. Critics counter that because households' perceptions of school quality depend on the composition of the student body as well as the efforts of school personnel (i.e., households have *peer preferences*), this market logic may not go through. In this paper we advance this debate by developing and analyzing an economic model of public school choice. A distinctive feature of our model is that households have peer preferences. Our analysis yields three main findings. First, we show that public school choice programs have ambiguous effects on welfare. Second, with respect to school quality, we show that public school choice programs generate a type of equity-efficiency trade-off. In particular, in the settings in which these programs generate the largest increases in average quality, they also exacerbate the gap between the quality of advantaged and disadvantaged schools. Third, we show that outcomes under these programs could be improved by auxiliary policies that help disadvantaged students to exercise choice and provide schools with stronger incentives to enroll disadvantaged students.

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1 Introduction

Public school choice programs, also known as open enrollment or intra-district choice programs, give households a free choice of public school and provide public schools with incentives to compete for students. Prominent examples include the nationwide programs introduced into the UK and New Zealand by market-oriented governments in the 1980s.¹ Other examples include programs specific to certain districts in Israel and the US.² Each program is different, but the defining characteristics are that parents have a free choice of school, schools must accept all applicants - at least up to some externally-determined capacity level - and school funding is proportionate to the number of students enrolled. Programs differ on how over-subscribed schools ration places (e.g., by lottery or by distance to school), whether students' transport costs are subsidized and whether schools receive additional funding for enrolling disadvantaged students.

Supporters of public school choice programs argue that by injecting market forces into the public school system, they will improve the quality of education that public schools provide. Some critics argue that these programs do not go far enough. In particular, they argue that these programs do nothing to enhance private school options and require public schools to compete within a straitjacket imposed by local and national schools authorities. For example, since they cannot control the number or composition of students that they enroll, schools cannot compete by specializing in teaching particular types of students. In some settings (e.g., the UK) they are required to teach a National Curriculum and hence cannot compete on any curricula dimension. This fact helps account for the lukewarm praise that Chubb and Moe (1992) give the British reform.³

A different concern with public school choice programs is that households' perceptions of school quality depend on the composition of the student body as well as the efforts of school personnel (i.e., households have *peer preferences*). Writing about the New Zealand public school choice program, Ladd (2002) argued that peer preferences would lead more-advantaged households to choose schools that enrolled more-advantaged students, thereby exacerbating educational inequality. Moreover, she claimed that peer preferences would blunt the incentive effects of this program, since both successful and unsuccessful schools would have limited incentives to improve quality. The concern for successful schools was that quality improvements would adversely affect the school's socioeconomic composition; the concern for unsuccessful schools was that despite quality improvements, they would be unable to attract parents.⁴

¹See Chubb and Moe (1992) for an account of the British reform and Ladd and Fiske (2000) for the New Zealand reform.

²See Lavy (2010) for Israel and Mikulecky (2013) for an overview of US programs.

³They argue that "The most glaring deficiency of the 1988 Education Reform Act is that it does almost nothing to liberate the supply of schools" (p.14). They concede that "Because of choice, success is now crucially dependent on pleasing parents, and the schools are doing what they can (which is not always much, given the constraints) to make themselves attractive." (p.16)

⁴She claims that "successful schools will be reluctant to expand if doing so requires lowering the average socioeconomic level of their students" (p.7), and that, since "no...educational strategy can make a school with a large proportion of disadvan-

In this paper we develop and analyze a model of public school choice. The model does not address the criticism that public school choice programs relax too few of the constraints facing public schools. We acknowledge that more radical programs would provide schools with very different incentives and might generate very different outcomes. But we wish to model public school choice programs as they operate in practice and hence we constrain schools to choose a single action - effort - that influences school quality as perceived by parents. The model does however incorporate peer preferences, since we assume that school composition is the second factor that influences school quality as perceived by parents. The question then becomes whether the competitive pressure exerted by parental choice improves school quality and household welfare, and how this process is influenced by peer preferences.

More specifically, our model features one community divided into two equal-sized neighborhoods, each containing one school. Households differ by their socio-economic status and one neighborhood contains more advantaged students than the other. The model is dynamic and the two schools are infinitely-lived; a new cohort of students is enrolled in every period. Initially, students attend their neighborhood school. After the public school choice program is introduced, households can enroll their children in either school, but face a cost of attending the non-neighborhood school. In accordance with how these programs operate, we assume schools must admit any student that wishes to enroll. To simplify the analysis, we assume no capacity constraints. It follows that households will choose the non-neighborhood school if the gain in expected school quality exceeds the cost. We assume that expected quality is the quality observed in the previous period, so that households are backward-looking. As noted above, we assume that quality depends on the efforts of school personnel and on the composition of the students enrolled. Schools obtain utility from the revenue that comes with enrolling more students. It follows that schools can increase revenue in the next period by exerting costly effort (and thereby increasing quality) in the current period. We study the Markov Perfect Equilibrium of this game. In particular, we derive the conditions that characterize the equilibrium and solve these conditions numerically.

Our analysis yields three main findings. First, we show that public school choice programs have ambiguous effects on welfare. That is because in equilibrium, households' school choices are driven by the preference for better peer groups. These choices are optimal from the household's perspective but wasteful from the perspective of society, since attending the non-neighborhood school is costly and one household's peer gain is another's peer loss. The wasteful effects of peer-driven school choices can overwhelm the benefits stemming from increased school quality. Second, with respect to school quality, we show that public school choice programs generate an equity-efficiency trade-off. That is because peer preferences influence schools' incentives tagged...students look effective" (p.7), these schools "have difficulty competing for students" (p.8).

to compete for marginal students. If marginal students are advantaged, schools have strong incentives to compete for them (thereby raising average quality) but in equilibrium these students will choose more-advantaged schools (thereby exacerbating inequality). Third, we show that outcomes under these programs could be improved by auxiliary policies that help disadvantaged students to exercise choice and provide schools with stronger incentives to enroll disadvantaged students. Versions of the first policy are often proposed (e.g., transport cost subsidies to help disadvantaged students to attend non-neighborhood schools), but we show that when implemented alone, they run afoul of the equity-efficiency trade-off and weaken incentives to improve quality. In contrast, outcomes can be improved when this policy is combined with a complementary policy that provides schools with stronger incentives (e.g., via subsidies) to enroll disadvantaged students.

The paper builds on three strands of empirical literature. The first includes studies of how households choose schools in public school choice systems (e.g., Hastings et al. (2006); Burgess et al. (2014)). The main message to emerge from these studies is consistent with our framework: parents weigh the costs and benefits of exercising choice.⁵ The second includes evaluations of the impacts of public school choice on various outcomes (e.g., Ladd and Fiske (2000); Lavy (2010)). The evidence (discussed in more detail below) is mixed. This is consistent with the efficiency-equity trade-off that we identify, since this implies that the effects of public school choice will be setting-specific. The third concerns how the effects of public school choice are shaped by peer preferences. Although this has not been analyzed formally, Ladd (2002) invokes peer preferences to explain the apparently disappointing effects of public school choice in New Zealand.

To our knowledge, Epple and Romano (2003) is the only previous theoretical analysis of public school choice. They analyze the equity implications of public school choice in a model in which parents choose schools, choose neighborhoods and vote on taxes, and in which schools are passive.⁶ To focus more clearly on the efficiency effects of public school choice, we model the behavior of public schools, but ignore private schools and abstract from the neighborhood and taxation decisions of households.⁷ Although the institutional environments are very different, our focus on the implications of peer preferences for efficiency as well as equity is related to some analyses of Tiebout choice and private school vouchers.⁸ Rothstein (2006) analyzes how

⁵In these studies, costs are typically measured by the distance from home to school, while quality is typically measured by school-average test scores. In our setup, costs can include psychic as well as transport costs, while school quality is simply the utility associated with attending a particular school.

⁶This complements Epple and Romano (1998), which analyzed how peer preferences shape the equity effects of private school vouchers in a model that features passive public schools and private schools that act as profit-maximizing clubs (Scotchmer (1985)).

⁷McMillan (2005) also models the behaviour of public schools but does not consider peer preferences. He models the competition between a single public school (or district) and a competitive private school sector and shows that public school effort can decrease when private school vouchers become more generous. Manski (1992) simulates a model similar to that studied by McMillan but which does feature peer preferences. These are not the main focus of his paper however and so he does not examine how they shape outcomes.

⁸Models of Tiebout choice share the property that some households must pay a (housing) cost to attend higher-quality schools, but must consider how districts make tax and spending decisions. Models of private school vouchers share the property that households trade higher quality against higher costs when deciding which school to attend, but must consider how private

peer preferences can dilute the incentives provided by Tiebout choice.⁹ Epple and Romano (2008) analyze a conditional private school voucher scheme that links vouchers to ability. They show that these can preserve the efficiency-enhancing effects of private school vouchers without the cream-skimming that occurs when vouchers are universal. Our model also bears some resemblance to the model of MacLeod and Urquiola (2013), although that is focused at the college level and student effort plays a key role.¹⁰

Our model is related to models in the industrial organization literature that deal with switching costs and network effects (Farrell and Klemperer (2007)). Whereas we consider the impacts of peer preferences, papers in that literature consider the implications of size preferences (i.e., how do two firms compete when consumers’ choices depend on each firm’s market share).¹¹ The idea is that consumers want to buy (e.g., telecommunications) products with larger market shares so that they can, for example, share the same network as friends and colleagues. An obvious and important difference is that there is no price-setting in our model. Hence while this literature finds that these size-dependent preferences can generate “fat cat” effects that soften competition between firms and increase price-cost mark-ups, it does not necessarily follow that peer preferences will blunt the incentive effects of public school choice.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 defines, characterizes, and solves for equilibrium under public school choice. Section 4, the heart of the paper, explores how public school choice impacts school quality and household welfare. Section 5 examines auxiliary policies that might improve outcomes under public school choice. Section 6 discusses our findings and Section 7 concludes.

2 The model

The model features a single community with a population of households of size 1. The community is divided into two neighborhoods, A and B , each containing one half of the population. There are two schools serving the community, one in each neighborhood. The school in neighborhood $J \in \{A, B\}$ is referred to as school J . The time horizon is infinite with periods indexed by $t = 1, \dots, \infty$.

Households differ in their socio-economic status. There are two types: rich (r) and poor (p). The aggregate fraction of rich households in the community is λ and the fraction in neighborhood A is μ . The parameter

schools admit students (e.g., whether or not they can turn voucher students away).

⁹The idea is that relative to a model without peer preferences, in which there is hierarchical sorting of wealthier households to communities with more-effective schools, peer preferences can induce coordination failures that prevent the Tiebout sorting process from rewarding school effectiveness.

¹⁰MacLeod and Urquiola (2013) model a college-level version of public school choice that features a specific source of peer preference - labor market signaling. In their model choice drives non-productive colleges from the market, but only provides strong incentives for student effort when colleges enroll students of mixed ability. Otherwise, wages will depend on the signal sent by college attended and there are lower returns to student effort.

¹¹It would be interesting to develop industrial organization models of the impacts of peer preferences, since these characterize many industries (e.g., gym memberships, dating agencies). To our knowledge, no such analyses exist.

μ is greater than $1/2$, implying that A is the richer neighborhood. The parameter λ measures *community heterogeneity* with λ equal to $1/2$ representing maximal heterogeneity. The parameter μ represents the degree of *neighborhood inequality* with higher values of μ representing greater inequality.¹²

In each period, each household has a child which it must send to one of the two schools.¹³ Households care about school quality (as they perceive it) but incur a cost if using the school not in their neighborhood. This cost captures the additional transaction costs arising from using the non-neighborhood school.¹⁴ The cost is denoted c and varies across the households. In our base specification, for both rich and poor households, costs are uniformly distributed on the interval $[0, \bar{c}]$, so that the fraction of households with cost less than or equal to $c \in [0, \bar{c}]$ is c/\bar{c} . Later in the paper, we will allow costs to differ across income groups.

Letting q_J denote the quality of school J , a household living in neighborhood A with cost c obtains a period payoff q_A from using school A and a payoff $q_B - c$ from using school B . Similarly, a household living in neighborhood B with cost c obtains a period payoff q_B from using school B and a payoff $q_A - c$ from using school A . The quality of school J in period t depends on the effort it exerts in period t and on the fraction of its children from rich households. Thus,

$$q_{Jt} = e_{Jt} + \alpha \lambda_{Jt}, \tag{1}$$

where e_{Jt} is school J 's effort in period t , λ_{Jt} is the fraction of its students from rich households, and α is a parameter measuring the importance of peer composition.

The district provides schools with a per-student payment that exceeds the costs that an additional student creates. We normalize the per student surplus to one, so that school J 's payoff from period t is given by

$$E_{Jt} - \gamma e_{Jt}^2/2, \tag{2}$$

where E_{Jt} denotes enrollment and γ is a parameter measuring the marginal cost of effort. Schools discount future payoffs at rate β .

2.1 Public school choice

We are interested in how public school choice impacts school quality and household welfare. To create a simple benchmark for comparison, we suppose that before the public school choice policy is introduced,

¹²Since each neighborhood contains half the population, $\lambda\mu$ must be less than $1/2$.

¹³One could imagine a world with turnover in which new households with children enter the community each period or a more static setting in which fixed households have a sequence of children to educate.

¹⁴These include any additional time taken to travel to school, additional expenses arising from higher transport costs, psychic costs resulting from loss of community, etc.

households must enroll their children in their neighborhood school. With the public school choice policy in place, households can choose either school in the community, irrespective of the neighborhood in which they live.¹⁵

Before the policy is introduced, each school's enrollment consists of the students in its neighborhood and thus is fixed at $1/2$. Since enrollment is fixed and effort is costly, schools exert zero effort. Each school's quality is therefore determined by the fraction of its students who are from rich households (see (1)). Thus, in each period school A 's quality is $\alpha 2\lambda\mu$ and school B 's is $\alpha 2\lambda(1 - \mu)$. It follows that a household living in neighborhood A obtains a payoff $\alpha 2\lambda\mu$ from enrolling their child in school A and a household living in neighborhood B obtains a payoff $\alpha 2\lambda(1 - \mu)$ from enrolling their child in school B .

Under public school choice, households have to select their school at the beginning of each period. They cannot know what school quality will be since it depends on schools' effort decisions during the period. We assume that households have *myopic expectations*, believing that school J 's quality in period t will equal its quality in period $t - 1$.¹⁶ It follows that a household living in neighborhood A with cost c will choose school A in period t if q_{At-1} is at least as big as $q_{Bt-1} - c$ and school B otherwise. Similarly, a household living in neighborhood B with cost c will choose school B in period t if q_{Bt-1} is at least as big as $q_{At-1} - c$ and school A otherwise.

Letting $\Delta q_{t-1} = q_{At-1} - q_{Bt-1}$ denote school A 's period $t - 1$ *quality differential*, it follows that under public school choice, enrollment in school A in period t is given by¹⁷

$$E(\Delta q_{t-1}) = \frac{1 + \frac{\Delta q_{t-1}}{\bar{c}}}{2}. \quad (3)$$

Enrollment in school B is given by $1 - E(\Delta q_{t-1})$. Rich enrollment in school A in period t is

$$E_r(\Delta q_{t-1}) = \begin{cases} \lambda \left(\mu + (1 - \mu) \frac{\Delta q_{t-1}}{\bar{c}} \right) & \Delta q_{t-1} \geq 0 \\ \lambda \mu \left(1 + \frac{\Delta q_{t-1}}{\bar{c}} \right) & \text{if } \Delta q_{t-1} < 0 \end{cases}. \quad (4)$$

Given this notation, the fraction of rich children in school A in period t , $\lambda_A(\Delta q_{t-1})$, equals $E_r(\Delta q_{t-1})/E(\Delta q_{t-1})$ and the fraction in school B , $\lambda_B(\Delta q_{t-1})$, is $(\lambda - E_r(\Delta q_{t-1})) / (1 - E(\Delta q_{t-1}))$. It follows that the quality

¹⁵We ignore capacity constraints, assuming that both schools can accommodate additional students.

¹⁶Concretely, one might imagine households visiting and researching schools in the period before they enroll each child. Effectively, what we are assuming is that households believe that whatever quality they encounter is predictive of what their child would experience if enrolled. Introspection convinced us that this was the natural assumption to make. Initial expectations in period 1 (i.e., q_{A0} and q_{B0}) are taken to be the school qualities prevailing before the public school choice program was introduced.

¹⁷This expression assumes that $\Delta q_{t-1} \in [-\bar{c}, \bar{c}]$.

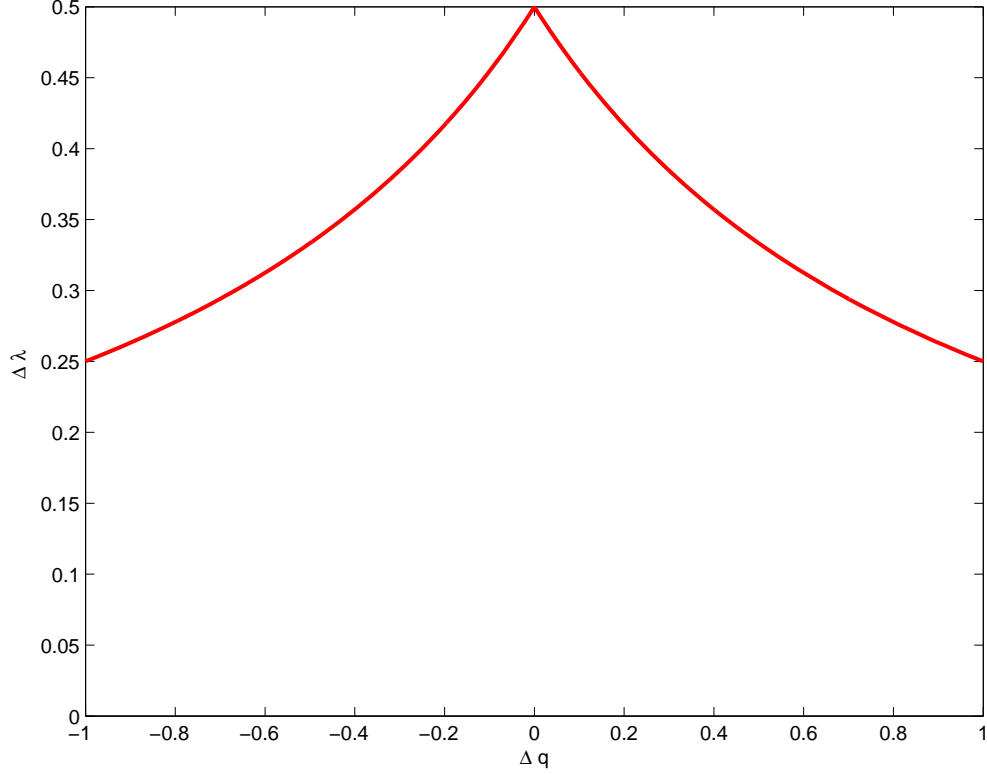


Figure 1: Peer differential

differential in period t is

$$\Delta q_t = e_{At} - e_{Bt} + \alpha (\lambda_A(\Delta q_{t-1}) - \lambda_B(\Delta q_{t-1})). \quad (5)$$

It will be convenient to let Δq denote the expected quality differential at the beginning of a period (i.e., the quality differential realized in the previous period) and $\Delta\lambda(\Delta q)$ denote the peer differential between the two schools under public school choice in that period; i.e., $\lambda_A(\Delta q) - \lambda_B(\Delta q)$. It is straightforward to show that

$$\Delta\lambda(\Delta q) = \begin{cases} \frac{\bar{c}2\lambda(2\mu-1)}{\bar{c}+\Delta q} & \text{if } \Delta q \geq 0 \\ \frac{\bar{c}2\lambda(2\mu-1)}{\bar{c}-\Delta q} & \text{if } \Delta q < 0 \end{cases}. \quad (6)$$

This function, which plays a key role in the analysis, is graphed in Figure 1 for our benchmark parameter values $(\bar{c}, \lambda, \mu) = (1, 0.5, 0.75)$.

The peer differential is increasing at an increasing rate for negative values of Δq and decreasing at a decreasing rate for positive values. To understand its behavior, note that when Δq equals 0, all students attend their neighborhood school and thus the peer differential just equals the difference between the fraction of rich in each neighborhood (i.e., $2\lambda(2\mu-1) = 0.5$). As Δq increases from 0, all students from neighborhood

A are enrolled in school A and additional enrolling students come from neighborhood B . Given that neighborhood B is the disadvantaged neighborhood, this reduces the peer differential between the two schools. As Δq approaches \bar{c} , more and more students from neighborhood B enroll in school A and the fraction rich approaches that in the whole community. Meanwhile, the fraction rich in school B remains constant at the fraction in neighborhood B . Thus, the difference approaches $\lambda(2\mu - 1) = 0.25$.

2.2 Discussion of the model

In developing a model with which to analyze public school choice, we faced several modeling decisions. To help readers understand the decisions made, we briefly discuss the main ones here.

- *School preferences.* We needed schools to value enrollment since, without this, public school choice could not be effective. For the same reason, it seemed clear that we should ignore the possibility that school personnel are intrinsically motivated to provide quality. To get enrollment to matter, we assumed that funding depends on enrollment and that the additional funds provided exceed the costs of higher enrollment. We have in mind that surplus funds can be used to provide things that school personnel value. Examples might include bonus payments, ancillary staff and professional development courses. An alternative approach would have been to tie the salaries of school personnel to enrollment, as they are in some public school choice programs (e.g., the UK).
- *School actions.* A key decision was to assume that schools only take one action: a choice of effort. This restricts how schools can compete for students. First, it means that they cannot compete by altering the size and composition of the school (other than via their effort choices which will influence subsequent enrollment decisions). For example, they cannot reject or encourage applications from specific types of students, (e.g., by various types of signaling) and they cannot claim to be full. A defining characteristic of public school choice programs is that these practices are not permitted, hence it seemed reasonable to restrict schools in this way. Second, it means that they cannot compete by differentiating horizontally, as they might if they made effort decisions over two dimensions of schooling (e.g., academic achievements and extra-curricula activities). This could be an interesting direction for future research, but would substantially complicate an already sophisticated model.
- *Household preferences.* It seemed natural to assume that households must incur costs when choosing their non-neighborhood school and that these costs would vary across households. This is consistent with Epple and Romano (2003) and the empirical literature on how households choose schools (Hastings et al. (2006), Burgess et al. (2014)). It also seemed sensible to abstract from households' choices of

neighborhood and of school financing because this would substantially complicate the model. This means that the only decision facing households is whether to attend the neighborhood or the non-neighborhood school. This depends whether the cost of attending the non-neighborhood school is smaller than the utility gain associated with attending that school.

- *School quality.* The utility that households derive from attending a particular school is what we refer to as school quality. Because we assume that school quality depends on school effort, school effort can be thought of as school actions and policies that increase the utility that households derive from attending a particular school. This begs the question of exactly what parents want from schools. It seems reasonable to suppose they care about several factors - academic achievements, personal safety, social networks, future labor market networks, etc. - but nothing was lost by formulating the model to abstract from this question.¹⁸ Because we want to allow households to have peer preferences, we assumed that households' utility also depends on the composition of a school. Again, we do not need to specify precisely why this is the case: the assumption would only be violated if households believe that all of the factors they care about are unaffected by composition.¹⁹ A more difficult modeling question concerned the exact specification of the relationship between school quality, school effort and school composition. Our assumption that effort and composition are separable in the production of school quality and that composition is "linear-in-means" ensures that average school quality is invariant to the distribution of students across schools. Instead, it just equals the enrollment-weighted average of the effort exerted by the two schools. Although more flexible specifications may be seen as superior, it is not obvious how effort and composition interact in the production of school quality. Nor is it obvious that quality is a non-linear function of average composition, or that there is a more natural measure of composition than the average.
- *Household heterogeneity.* Our assumption that there are two types of households yields a simple representation of community heterogeneity and neighborhood inequality. It seemed reasonable to assume that all households share the same view of school quality. This assumption is tractable and there is no clear-cut evidence on preference heterogeneity.²⁰
- *No capacity constraints.* We think it is realistic to suppose that schools cannot influence capacity but

¹⁸To the extent that parents care about several dimensions of schooling, not all of which could be observed by the econometrician, it follows that the best school as perceived by parents may not be the one that performs best in terms of test scores, value-added or any other metric that the econometrician could devise.

¹⁹It follows that there are many channels through which student composition could affect households utility even if there are no peer effects in the production of test scores (see Angrist (2013) for a recent review of the evidence).

²⁰Burgess et al. (2014) do not find strong preference differences by socio-economic status in the UK. Hastings et al. (2006) find stronger preference differences by socio-economic status in the US, in addition to preferences over more than one dimension of household type (income and race).

we admit it is unrealistic to assume there are no capacity constraints. A more realistic assumption would be that each school faces an externally-imposed capacity constraint. We chose not to make this assumption because it would complicate the analysis without yielding many additional insights. It seems obvious that public school choice programs cannot generate competitive pressure unless there is sufficient spare capacity in the public school system. In practice, there will be more spare capacity in some settings than others.²¹ By assuming unlimited spare capacity, we generate a best-case scenario for competitive pressure and can then focus on the complications introduced by peer preferences.

- *Dynamic framework.* To study the same forces in a static model, we would have to assume that schools commit up front to providing particular levels of effort and that parents make their enrollment decisions anticipating those of other parents. This is possible, but we find it more natural to assume that parents are backwards-looking in their evaluation of schools and that schools adjust effort on a period-by-period basis anticipating its impact on future enrollment. An intermediate set-up would be a partially-dynamic model in which schools make a one-time and permanent effort decision in the first period and backwards-looking households make enrollment decisions period-by-period in response. This would be more analytically tractable than our fully-dynamic model. However, we felt that the assumption of a one time effort decision was sufficiently unnatural that we would, in any case, have to check that any conclusions emerging from the partially-dynamic model were robust to relaxing it. This suggested that we might as well tackle the fully-dynamic model from the outset.

3 Equilibrium under public school choice

Public school choice creates a dynamic game between the two schools. The state variable in this game is the expected quality differential Δq which determines each school's enrollment. If the schools make effort decisions (e_A, e_B) this period, the expected quality differential in the next period $\Delta q'$ will equal $e_A - e_B + \alpha\Delta\lambda(\Delta q)$ which is the realized quality differential this period (see (5)). We will look for a Markov Perfect Equilibrium of this game. In such an equilibrium, schools' strategies depend only upon the state variable Δq . Let $V_J(\Delta q)$ denote school J 's value function and $e_J(\Delta q)$ denote school J 's strategy. Then, the two schools' strategies satisfy the requirements that

$$e_A(\Delta q) = \arg \max_{e_A} \left\{ E(\Delta q) - \gamma \frac{e_A^2}{2} + \beta V_A(e_A - e_B(\Delta q) + \alpha\Delta\lambda(\Delta q)) \right\}, \quad (7)$$

²¹For example, in the case of the UK reform, school capacity was specified to be the level that prevailed about ten years before, when the school-aged cohort was around one-third larger. But of course the fall in the school population was not equally distributed across the country and in some areas school capacity could be equal to twice current school enrollment (i.e., a lot of spare capacity) and in other areas it could be equal to current school enrollment (i.e., no spare capacity).

and that

$$e_B(\Delta q) = \arg \max_{e_B} \left\{ 1 - E(\Delta q) - \gamma \frac{e_B^2}{2} + \beta V_B(e_A(\Delta q) - e_B + \alpha \Delta \lambda(\Delta q)) \right\}. \quad (8)$$

Moreover, the two schools' value functions must satisfy the equations

$$V_A(\Delta q) = E(\Delta q) - \gamma \frac{e_A(\Delta q)^2}{2} + \beta V_A(e_A(\Delta q) - e_B(\Delta q) + \alpha \Delta \lambda(\Delta q)), \quad (9)$$

and

$$V_B(\Delta q) = 1 - E(\Delta q) - \gamma \frac{e_B(\Delta q)^2}{2} + \beta V_B(e_A(\Delta q) - e_B(\Delta q) + \alpha \Delta \lambda(\Delta q)). \quad (10)$$

An *equilibrium* consists of value functions $V_A(\Delta q)$ and $V_B(\Delta q)$ and strategies $e_A(\Delta q)$ and $e_B(\Delta q)$ satisfying these four equations.

3.1 Equilibrium characterization

Assuming that the value functions are differentiable, the first order conditions for the two schools' effort decisions imply that:

$$\gamma e_A(\Delta q) = \beta \frac{dV_A(\Delta q')}{d\Delta q'}, \quad (11)$$

and that

$$\gamma e_B(\Delta q) = -\beta \frac{dV_B(\Delta q')}{d\Delta q'}. \quad (12)$$

Equation (11) says that for school A , the marginal cost of effort (which is $\gamma e_A(\Delta q)$) is equal to the marginal benefit (which is $\beta dV_A(\Delta q')/d\Delta q'$). This marginal benefit represents the impact of a *higher* expected quality differential next period. Equation (12) says that for school B , the marginal cost of effort (which is $\gamma e_B(\Delta q)$) is equal to the marginal benefit (which is $-\beta dV_B(\Delta q')/d\Delta q'$). The marginal benefit is that stemming from a *lower* expected quality differential next period.

To interpret the first order conditions, more information on the derivative of the value functions is required. Applying the *Envelope Theorem* to equation (9) and assuming that the equilibrium strategies are differentiable at Δq , we have that for school A :

$$\frac{dV_A(\Delta q)}{d\Delta q} = \frac{dE(\Delta q)}{d\Delta q} + \beta \frac{dV_A(\Delta q')}{d\Delta q'} \alpha \frac{d\Delta \lambda(\Delta q)}{d\Delta q} - \beta \frac{dV_A(\Delta q')}{d\Delta q'} \frac{de_B(\Delta q)}{d\Delta q}. \quad (13)$$

Equation (13) reveals that a marginal increase in the expected quality differential has three effects on school A 's payoff. The first is to increase its enrollment, an effect which is always beneficial. The second is to

change the peer differential. This effect influences school A 's payoff via its impact on next period's expected quality differential. Figure 1 tells us that under the base cost specification, the change in the peer differential will be negative if Δq is positive and positive if Δq is negative. A negative change will reduce next period's expected quality differential and thereby reduce school A 's payoff, while a positive change will raise it. The third effect is to change school B 's effort, an effect that also impacts payoffs through its impact on next period's expected quality differential. The direction of change in school B 's effort depends on equilibrium behavior and is unclear *a priori*. A reduction will increase next period's expected quality differential and thereby increase school A 's payoff, while an increase will reduce it.

For school B , applying the *Envelope Theorem* to equation (10), we have that

$$-\frac{dV_B(\Delta q)}{d\Delta q} = \frac{dE(\Delta q)}{d\Delta q} - \beta \frac{dV_B(\Delta q')}{d\Delta q'} \alpha \frac{d\Delta\lambda(\Delta q)}{d\Delta q} - \beta \frac{dV_B(\Delta q')}{d\Delta q'} \frac{de_A(\Delta q)}{d\Delta q}. \quad (14)$$

This tells us that a marginal decrease in the expected quality differential has three effects on school B 's payoff. The first is to increase its enrollment. The second is to change the peer differential. Again, under the base cost specification, this change will be positive if Δq is positive and negative if Δq is negative. A positive change will increase next period's expected quality differential and thereby reduce school B 's payoff, while a negative change will raise it. The third effect is to change school A 's effort. An increase will raise next period's expected quality differential and thereby reduce school B 's payoff, while a decrease will increase it.

Combining these expressions for the value function derivatives with the first order conditions, we can write (11) and (12) as

$$\gamma e_A(\Delta q) = \beta \left[\frac{dE(\Delta q')}{d\Delta q'} + \gamma e_A(\Delta q') \left(\alpha \frac{d\Delta\lambda(\Delta q')}{d\Delta q'} - \frac{de_B(\Delta q')}{d\Delta q'} \right) \right], \quad (15)$$

and

$$\gamma e_B(\Delta q) = \beta \left[\frac{dE(\Delta q')}{d\Delta q'} + \gamma e_B(\Delta q') \left(\alpha \frac{d\Delta\lambda(\Delta q')}{d\Delta q'} + \frac{de_A(\Delta q')}{d\Delta q'} \right) \right]. \quad (16)$$

Moreover, we have that next period's quality differential is given by

$$\Delta q' = e_A(\Delta q) - e_B(\Delta q) + \alpha \Delta\lambda(\Delta q). \quad (17)$$

These are the three equations that characterize the equilibrium effort levels.

It is clear from these three conditions that equilibrium effort levels must in general depend on the state

Δq . If they were constant, then $de_A(\Delta q')/d\Delta q'$ and $de_B(\Delta q')/d\Delta q'$ would equal zero, and the effort levels would be equal (i.e., $e_A = e_B$). But then the common effort level would depend on $\alpha d\Delta\lambda(\alpha\Delta\lambda(\Delta q))/d\Delta q'$ which depends on Δq - a contradiction. The only exception to this is when peer preferences are irrelevant, as happens when there are no peer preferences ($\alpha = 0$) or there is no neighborhood inequality ($\mu = 1/2$). In this case, there exists an equilibrium under public school choice in which the two schools' effort levels are always $\beta/2\gamma\bar{c}$. In this equilibrium, the steady state quality differential will be zero.

It is also apparent from (15) and (16) that the two schools must in general choose *different* effort levels in equilibrium. If $e_A(\Delta q) = e_B(\Delta q)$, then $de_A(\Delta q')/d\Delta q'$ would equal $de_B(\Delta q')/d\Delta q'$. But the derivative of the other school's effort enters with a different sign in each first order condition which is inconsistent with the hypothesis of symmetric effort levels.

Finally, note that the only place that Δq enters these equations is through the peer differential term $\alpha\Delta\lambda(\Delta q)$ in (17). Intuitively, this period's expected quality differential Δq determines this period's peer differential which, along with the two schools' effort levels, determines next period's expected quality differential $\Delta q'$. Observe from Figure 1 that the peer differential is symmetric in the sense that for any Δq in the interval $[-\bar{c}, \bar{c}]$, $\alpha\Delta\lambda(\Delta q)$ equals $\alpha\Delta\lambda(-\Delta q)$. It follows that, assuming there is a unique equilibrium, the equilibrium effort levels for each school J will be such that for any Δq in the interval $[-\bar{c}, \bar{c}]$, $e_J(\Delta q)$ equals $e_J(-\Delta q)$.

Beyond these three points, (15) and (16) do not reveal much about the nature of schools' equilibrium effort levels. The conditions are complex because each school's first order condition includes the derivative of its rival's future strategy. This complexity makes it hard to develop analytical results.²² Thus, we will solve for equilibrium numerically.

3.2 Steady states

The evolution of the quality differential in equilibrium is described by the difference equation (17). A quality differential Δq_s is thus a *steady state* if

$$\Delta q_s = e_A(\Delta q_s) - e_B(\Delta q_s) + \alpha\Delta\lambda(\Delta q_s). \quad (18)$$

²² "The problem of unknown derivatives appearing in dynamic equilibrium conditions" (Klein et al. (2008)) is common in many dynamic economic environments such as models with multiple strategic agents or models in which the government cannot commit to future policies and hence "plays a game" against future selves. The core of the problem is that equilibrium conditions in these types of models typically boil down to a system of differential equations that do not have known closed form solutions and hence can only be solved numerically.

Assuming that the system converges to a unique steady state, we will use steady state outcomes to assess system performance. To simplify notation, we will let e_{As} and e_{Bs} denote the steady state effort levels $e_A(\Delta q_s)$ and $e_B(\Delta q_s)$.

3.3 Solving for equilibrium

The solution procedure is as follows. As noted earlier, the equilibrium effort levels for each school J will be such that for any Δq in the interval $[-\bar{c}, \bar{c}]$, $e_J(\Delta q)$ equals $e_J(-\Delta q)$. Thus, we just need to solve for the strategies on the interval $[0, \bar{c}]$. We first conjecture (i) that the strategies are continuous on $[0, \bar{c}]$ and differentiable everywhere on $(0, \bar{c})$, (ii) that the equilibrium converges to a steady state in which the quality differential lies in the interval $(0, \bar{c})$, and (iii) that equilibrium strategies are such that $\Delta q'$ belongs to the interval $(0, \bar{c})$ for all Δq in the interval $[0, \bar{c}]$. We then substitute equation (17) into the two first order conditions (15) and (16) and note that this yields a pair of differential equations in the unknown effort functions $e_A(\Delta q)$ and $e_B(\Delta q)$ defined on the compact interval $[0, \bar{c}]$. To solve this pair of equations, we approximate the equilibrium effort functions with higher order Chebyshev polynomials. To find the coefficients for these polynomials we use both Galerkin and collocation methods²³ and confirm that the solutions under both methods coincide. Given these solutions, we verify our assumptions (ii) and (iii) ex post (i.e., that the equilibrium converges to a steady state in which the quality differential lies in $(0, \bar{c})$ and that $\Delta q'$ belongs to $(0, \bar{c})$ for all Δq in the interval $[0, \bar{c}]$).

This procedure provides us with conjectured equilibrium strategies on the interval $[0, \bar{c}]$. We then use the symmetry property to deduce the strategies on the interval $[-\bar{c}, 0]$. Given the symmetry property of the strategies and the fact that for any Δq in the interval $[-\bar{c}, 0]$, $\alpha\Delta\lambda(\Delta q)$ equals $\alpha\Delta\lambda(-\Delta q)$, the $\Delta q'$ associated with any Δq in the interval $[-\bar{c}, 0]$ just equals that associated with $-\Delta q$. Accordingly, $\Delta q'$ belongs to $(0, \bar{c})$ and the equilibrium converges to a steady state in which the quality differential lies in $(0, \bar{c})$. Finally, given the strategies, we construct the value functions to verify that (7) and (8) are concave problems and, hence, solving (15) and (16) indeed amounts to solving (7) and (8) for all $\Delta q \in [-\bar{c}, \bar{c}]$.

3.4 Equilibrium for a benchmark case

To illustrate the workings of the model, we now solve for the equilibrium for a benchmark parameterization which assumes that the parameters $(\bar{c}, \lambda, \mu, \alpha, \gamma, \beta)$ equal $(1, 0.5, 0.75, 0.75, 10, 0.95)$. The choice of 0.95 for the discount rate β is standard. The choices of 0.5 for the degree of community heterogeneity λ and 0.75 for the degree of neighborhood inequality μ have no particular significance and we will discuss the implications

²³See Judd (1992).

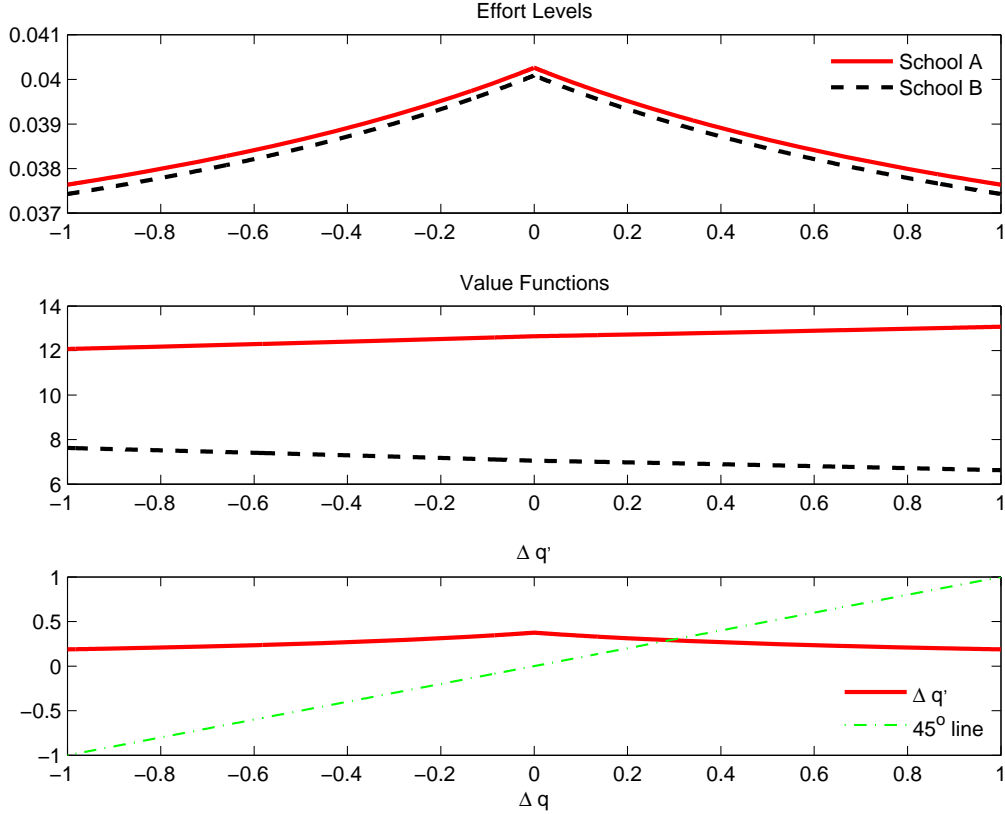


Figure 2: Solution for the benchmark case

of different choices below. The choice of 0.75 for the importance of peer composition α is just a starting point and we will discuss the implications of varying it extensively. The remaining parameters, which are the upper bound of the cost distribution \bar{c} and the school effort cost parameter γ will be held constant throughout the analysis.²⁴

Figure 2 describes the equilibrium solution. The top panel graphs the effort levels of the two schools as a function of the quality differential. Three points are noteworthy. First, while the equilibrium effort levels do not vary greatly, they are smaller the higher is the absolute value of the expected quality differential. Second, while the two schools' effort levels are similar, for any given quality differential, school *A* exerts marginally more effort than school *B*. Third, the two schools' effort levels are symmetric around $\Delta q = 0$.

The second panel of Figure 2 describes the value functions of the two schools. As expected, school *A*'s value function is increasing in the quality differential and school *B*'s is decreasing. While certainly not concave, the value functions are close to linear. When the convex cost of effort is factored in, this means that

²⁴The choice of \bar{c} is a normalization and, once the results are understood, it will be obvious how varying γ will impact the equilibrium.

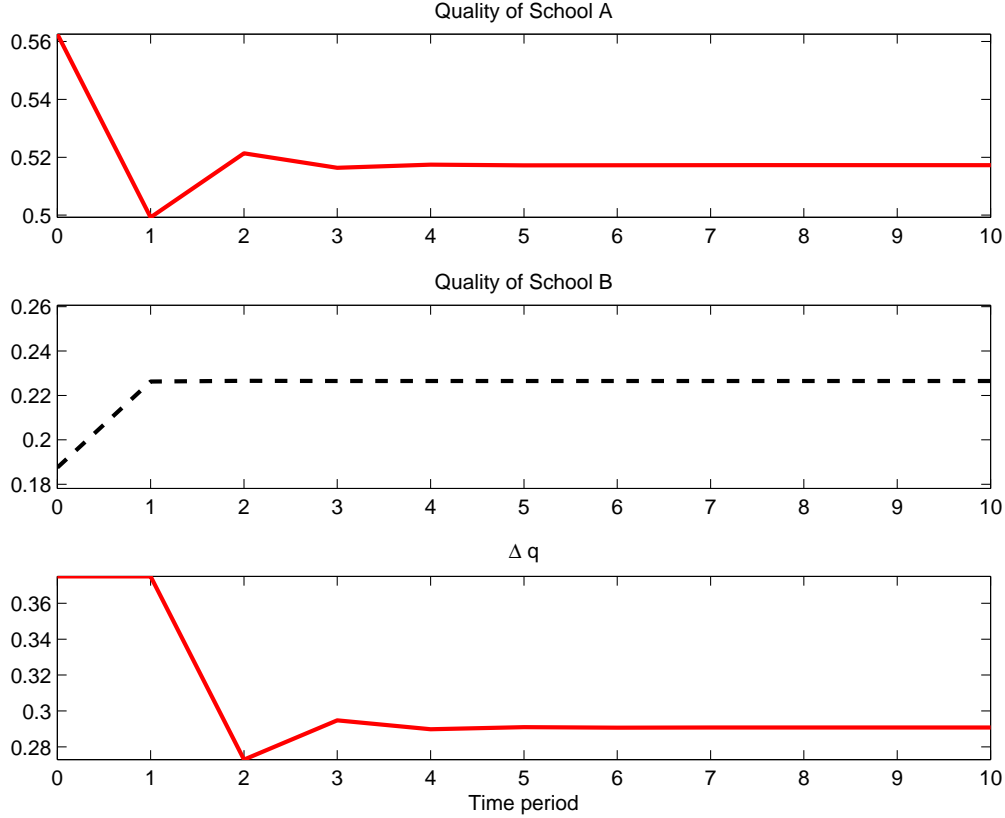


Figure 3: Dynamics for the benchmark case

the optimization problems described in (7) and (8) are concave problems. This implies that the solutions to the first order conditions (11) and (12) are optimal effort levels.

The bottom panel describes the dynamic evolution of the system, graphing how next period's quality differential depends on this period's (i.e., equation (17)). Notice that whatever this period's quality differential, next period's will always be positive. This is because the peer differential is always positive (see Figure 1) and, in addition, school A exerts slightly more effort. As the top panel reveals, the difference in the two schools' effort levels is approximately constant over the entire range of quality differentials. As a consequence, the shape of the function describing next period's quality differential reflects the shape of the peer differential illustrated in Figure 1. The symmetry properties of the two schools' effort levels and the peer differential imply that $\Delta q'$ is symmetric around $\Delta q = 0$. This implies that the dynamic evolution of the system starting from initial condition Δq is exactly the same as that starting from initial condition $-\Delta q$.

The steady state quality differential can be found where the function in the bottom panel intersects the 45° degree line. It is clear that there is a unique steady state and that the system converges monotonically to it for any given initial condition. The steady state quality differential and associated effort levels

$(\Delta q_s, e_{As}, e_{Bs})$ turn out to equal $(0.2907, 0.0392, 0.039)$.

Figure 3 illustrates the dynamic evolution of quality in the two schools starting from the initial condition before the public school choice programs introduced. Immediately after it is introduced, quality in school B increases. Thereafter, it varies sufficiently little that it appears flat. This reflects two considerations. First, since equal proportions of rich and poor households are attracted to school A , the fraction of rich children in school B is constant. Second, since the variation in Δq is small along the transition path (as illustrated in the bottom panel), equilibrium effort provided by school B is relatively constant. Even though school A exerts slightly more effort in steady state, quality in school A falls. That is because the fraction of rich students falls as students from the disadvantaged neighborhood enroll. In the first period, quality falls by more than the steady state amount because myopic households overestimate the quality differential between the two schools and too many switch in. In the second period, households overreact to the fall in the quality differential and too few switch in. This adjustment process continues until about the fifth period, when the steady state is reached. As illustrated in the bottom panel, the quality differential is reduced relative to that which prevailed before the public school choice program was introduced, but it still remains substantial.

To gain some intuition for the equilibrium effort levels described in the top panel of Figure 2, suppose that $\Delta\lambda(\Delta q)$ were piecewise linear rather than the curvy shape illustrated in Figure 1.²⁵ Then, conditions (15), (16), and (17) suggest the existence of an equilibrium with the following constant and symmetric equilibrium effort level

$$e^* = \frac{\beta}{\gamma 2\bar{c}} \left[\frac{1}{1 - \beta\alpha d\lambda/d\Delta q} \right], \quad (19)$$

where $d\lambda/d\Delta q$ is the constant negative slope of the peer differential on the interval $[0, \bar{c}]$. This candidate equilibrium effort level is increasing in the enrollment return to effort $(1/2\bar{c})$, decreasing in the cost of effort (γ) , and decreasing in the absolute value of the composition return to effort $\alpha d\lambda/d\Delta q$.

The nonlinearity of the peer differential accounts for the difference between this constant-symmetric effort solution and the equilibrium effort levels described in the top panel of Figure 2. In particular, equilibrium effort is decreasing in Δq on the interval $[0, \bar{c}]$ because next period's quality differential $\Delta q'$ is decreasing in Δq and because the composition return to effort is larger when $\Delta q'$ is smaller (i.e., the absolute value of $\alpha d\lambda(\Delta q')/d\Delta q$ is larger as illustrated in Figure 1).²⁶ Because equilibrium effort is decreasing in Δq , effort increases by school A (which increase next period's quality differential) elicit a weaker rival reaction than effort increases by school B (which reduce next period's quality differential). This explains why school A exerts slightly more effort than school B . Since this a second-order phenomenon relative to the other forces

²⁵Imagine in Figure 1, two line segments connecting $(-1, 0.25)$ to $(0, 0.5)$ and $(0, 0.5)$ to $(1, 0.25)$.

²⁶The first part follows provided effort levels are not "too asymmetric".

shaping effort incentives (enrollment effects, effort costs and the first derivative of the composition return), it is not surprising that the equilibrium effort described in the top panel of Figure 2 is approximately constant and symmetric.²⁷ This is true in every case that we consider in the paper and it underpins many of our findings.

Finally, note that this symmetry property explains why a substantial quality differential remains under the public school choice program. Because the disadvantaged school is located in the disadvantaged neighborhood and can never turn local students away, it can only neutralize its peer disadvantage by providing considerably more effort than the advantaged school. Such an asymmetry of effort decisions is not compatible with the near-symmetry of effort incentives facing the two schools. These incentives imply that the disadvantaged school remains lower quality in equilibrium.

3.5 Equilibrium with cost differences between income groups

As noted in the introduction, it is often argued that more advantaged households are better able to take advantage of public school choice. In our model, the way to capture this is to assume that rich households face a more favorable cost distribution. The simplest and most tractable way of introducing asymmetries between the groups' cost distributions is to assume that some fraction of each group of households (i_r and i_p respectively) are "immobile". Intuitively, these households are those who will never choose the non-neighborhood school (e.g., because they are unaware of their options, or have prohibitively high costs). By assuming that i_p exceeds i_r , we can examine how equilibrium changes when the rich are more mobile than the poor.

This extension complicates the equations of the model but in a way that is straightforward to analyze. Enrollment in school A is now given by

$$E(\Delta q) = \begin{cases} \frac{1}{2} + \frac{\Delta q}{\epsilon} [(1 - i_r)\lambda(1 - \mu) + (1 - i_p)(1/2 - \lambda(1 - \mu))] & \text{if } \Delta q \geq 0 \\ \frac{1}{2} + \frac{\Delta q}{\epsilon} [(1 - i_r)\lambda\mu + (1 - i_p)(1/2 - \lambda\mu)] & \text{if } \Delta q < 0 \end{cases} \quad (20)$$

Note that in contrast to (3) this function is not differentiable at $\Delta q = 0$. Rich enrollment in school A is now given by

$$E_r(\Delta q) = \begin{cases} \lambda \left(\mu + \frac{\Delta q}{\epsilon} (1 - i_r)(1 - \mu) \right) & \text{if } \Delta q \geq 0 \\ \lambda \mu \left(1 + \frac{\Delta q}{\epsilon} (1 - i_r) \right) & \text{if } \Delta q < 0 \end{cases} \quad (21)$$

²⁷Plugging in our benchmark parameters and assuming that $d\Delta\lambda/d\Delta q = -0.5$ (which is the slope of the line segment joining $(0, 0.5)$ to $(1, 0.25)$), we obtain $e^* = 0.04$.

and the peer differential is given by

$$\Delta\lambda(\Delta q) = \begin{cases} \frac{\lambda(\mu + \frac{\Delta q}{\bar{e}}(1-i_r)(1-\mu) - E(\Delta q))}{E(\Delta q)(1-E(\Delta q))} & \text{if } \Delta q \geq 0 \\ \frac{\lambda(\mu(1 + \frac{\Delta q}{\bar{e}}(1-i_r)) - E(\Delta q))}{E(\Delta q)(1-E(\Delta q))} & \text{if } \Delta q < 0 \end{cases} \quad (22)$$

Equilibrium is defined in the same way and equations (15), (16), and (17) still characterize the equilibrium effort levels. The difference is that the enrollment and peer differential functions are now given by (20) and (22).

To illustrate the implications of rich households being better able to exercise choice, we consider the case in which all the poor and half of the rich are immobile (i.e., $(i_p, i_r) = (1, 0.5)$).²⁸ The remaining parameters are as for the benchmark case. Figure 4 describes the equilibrium solution. The most striking difference between this and Figure 2 lies in the top panel which describes the equilibrium effort levels. These are *increasing* in the quality differential Δq when this is positive as opposed to decreasing in the basic model. Moreover, they are no longer symmetric around $\Delta q = 0$. Underlying this finding is the fact that the peer differential in this case is increasing when Δq is positive as opposed to decreasing in the basic model. However, when Δq is negative, the peer differential is increasing as in the basic model.²⁹ Intuitively, because only rich households in neighborhood B are mobile, a higher quality differential attracts more rich families and therefore increases the fraction of rich children in school A . Because the composition return is positive, schools have stronger incentives to exert effort. It is also the case that the effort levels of the two schools are even closer than in the baseline model and it is school B that puts in marginally more effort.³⁰

A further difference is evident in the bottom panel which describes the dynamics of the equilibrium. In particular, next period's quality differential is now increasing in the current period's when the latter is positive. Again, since the two school's exert almost identical levels of effort, next period's quality differential

²⁸The solution procedure is slightly more complicated because the equilibrium effort levels are no longer symmetric around $\Delta q = 0$. We deal with this by first constructing the equilibrium strategies on the interval $[0, \bar{c}]$ and then using these to construct the strategies on the interval $[-\bar{c}, 0]$. It should be noted that equilibrium will not exist for all possible choices of i_p and i_r . This is because there may exist Δq such that the $\Delta q'$ associated with any candidate equilibrium equals 0. But at that point the functions $E(\Delta q')$ and $\Delta\lambda(\Delta q')$ are not differentiable and thus the first order conditions (15) and (16) are not well-defined. This problem can be circumvented by assuming that i_p and i_r are such that for all Δq it is the case that $\Delta\lambda(\Delta q)$ is positive. Then, provided that the effort levels of the two schools are similar in the candidate equilibrium, (17) implies that $\Delta q'$ is always positive. Intuitively, this assumption means that whatever the expected quality difference between the two schools, after households have made their choices, school A still has better peers. It rules out, for example, a situation in which all the rich are mobile and all the poor immobile (i.e., $(i_p, i_r) = (1, 0)$). In this case, if $\Delta q = -\bar{c}$, all the rich would attend school B and only neighborhood A 's poor families would use school A . Clearly, then, $\Delta\lambda(-\bar{c})$ would be significantly negative. By continuity, with only a small difference between the two schools' effort levels, there would exist some $\Delta q \in (-\bar{c}, 0)$ at which $\Delta q'$ as described by (17) would equal 0 and the problem would arise.

²⁹The shape of the peer differential can be verified by plotting the function described in (22) with $(i_p, i_r) = (1, 0.5)$.

³⁰Again, the non-linearity in the peer differential accounts for the difference between these equilibrium effort levels and the constant symmetric effort level. Equilibrium effort levels are slightly increasing in Δq because the composition return to effort is positive and because the peer differential function is concave. This means that the composition return is largest when $\Delta q'$ is small (i.e., when Δq is large). School B exerts more effort because an increase in school B 's quality (which reduces the quality differential) elicits a weaker reaction from its rival.

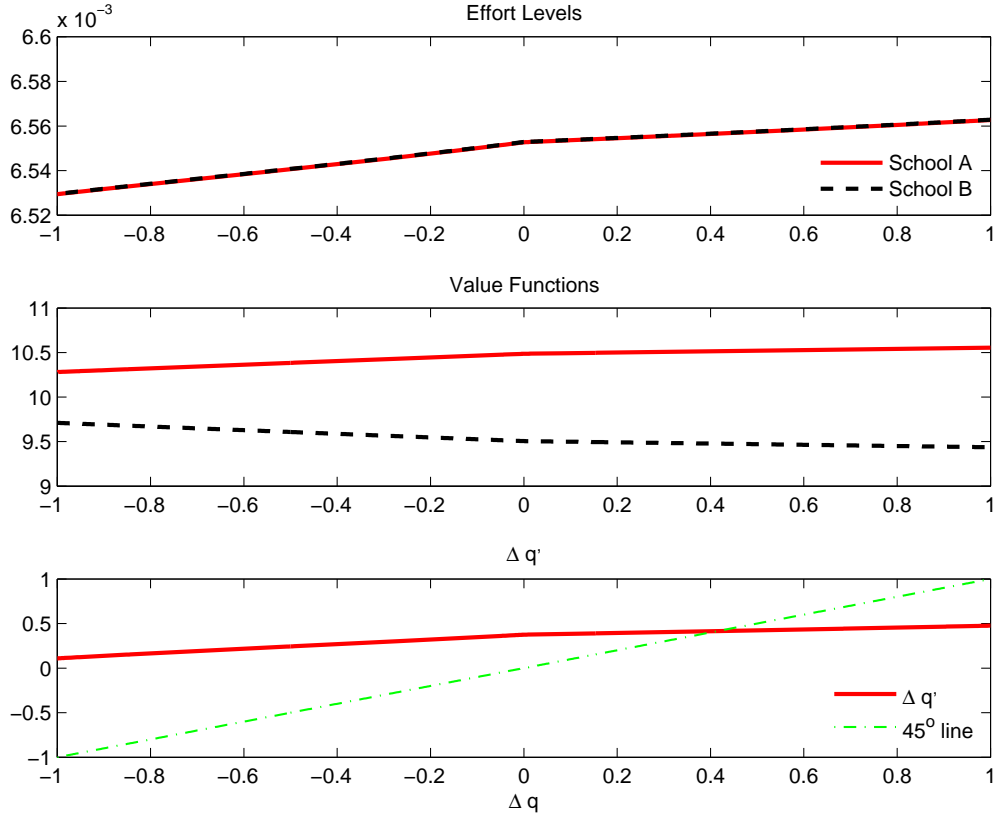


Figure 4: Solution for the asymmetric cost case

is just the peer differential (see (17)). As just discussed, the peer differential is increasing in the quality differential in this case. The change in shape results in the steady state quality differential being higher than in the basic model. The steady state quality differential and associated effort levels $(\Delta q_s, e_{A_s}, e_{B_s})$ equal $(0.415, 0.006565, 0.006567)$. Effort levels are lower despite the positive effect of the quality differential on the peer differential because so many households are immobile and hence the overall response of enrollment to the quality differential is lower. This underscores the point that the strength of any school effort response resulting from public school choice should be judged *relative* to the pool of mobile households.

4 The impact of public school choice

We are now ready to study the impact of introducing public school choice. We study its impacts on school quality and household welfare. The policy debate and the empirical literature have focused on the quality impacts of public school choice, both in the aggregate and across schools and communities. The focus on

household welfare is more in the spirit of traditional public economics.³¹ Measures of household welfare include the additional costs that households incur when they choose their non-neighborhood school, which we view as a legitimate part of the social calculus.

We begin by defining the precise quantities of interest. Recall that before the public school choice program is introduced, school A 's quality in each period is $\alpha 2\lambda\mu$ and school B 's is $\alpha 2\lambda(1 - \mu)$. Let $(\Delta q_s, e_{As}, e_{Bs})$ be the steady state associated with the equilibrium under public school choice and assume that Δq_s is positive. Using (1), the changes in the two schools' qualities are

$$dq_A = e_{As} + \alpha \left(\frac{E_r(\Delta q_s)}{E(\Delta q_s)} - 2\lambda\mu \right), \quad (23)$$

and

$$dq_B = e_{Bs} + \alpha \left(\frac{\lambda - E_r(\Delta q_s)}{1 - E(\Delta q_s)} - 2\lambda(1 - \mu) \right). \quad (24)$$

The enrollment-weighted average change in school quality, which we denote by dq , is³²

$$dq = E(\Delta q_s)e_{As} + (1 - E(\Delta q_s))e_{Bs}. \quad (25)$$

The expression in (25) is so simple because changes in school qualities resulting from student composition are zero sum and hence wash out of the analysis.

Turning to welfare, three variables are of particular interest: the steady state average change in per period welfare of households in the two neighborhoods, which we denote dW_A and dW_B , and the average change in per-period welfare, which we denote by dW . The per period welfare change from public school choice for households in neighborhood A is just

$$dW_A = dq_A. \quad (26)$$

This reflects the fact that, since Δq_s is positive, all households in neighborhood A continue to send their children to school A and hence the only impact on their welfare is how the quality of their school changes. Households in neighborhood B are more complicated because some switch to school A and some do not. The non-switchers obtain a per period welfare change of dq_B . Those who do switch obtain a per period welfare

³¹This said, we do not consider school payoffs in our welfare measure. This is because we see the policy problem to which public school choice is one possible answer as improving school performance for given levels of educational spending. Eliciting more effort from school personnel is not considered a social loss. In addition, our model of schools' payoffs is too reduced form to permit a satisfactory accounting of the surplus accruing to school personnel and stakeholders.

³²The enrollment weighted average change in school quality is defined to be

$$dq = E(\Delta q_s)q_{As} + (1 - E(\Delta q_s))q_{Bs} - \frac{1}{2}(\alpha 2\lambda\mu + \alpha 2\lambda(1 - \mu)),$$

where q_{Js} is the steady state quality of school J under public school choice.

change of $dq_A + \alpha 2\lambda(2\mu - 1) - c$. Averaging over switchers and non-switchers, we obtain

$$dW_B = (1 - 2\lambda(1 - \mu)) \left[i_p dq_B + (1 - i_p) \left(\int_0^{\Delta q_s} (dq_A + \alpha 2\lambda(2\mu - 1) - c) \frac{dc}{c} + \left(1 - \frac{\Delta q_s}{c}\right) dq_B \right) \right] \\ + 2\lambda(1 - \mu) \left[i_r dq_B + (1 - i_r) \left(\int_0^{\Delta q_s} (dq_A + \alpha 2\lambda(2\mu - 1) - c) \frac{dc}{c} + \left(1 - \frac{\Delta q_s}{c}\right) dq_B \right) \right] \quad (27)$$

Using (26) and (27), it is straightforward to show that the average welfare gain is

$$dW = dq - (E(\Delta q_s) - \frac{1}{2}) \frac{\Delta q_s}{2}. \quad (28)$$

This shows that the average welfare gain from public school choice depends on the difference between two terms. The first term is the change in school average quality, which we know is just the change in school average effort. The second term represents the additional costs incurred by households in neighborhood B who use school A . For public school choice to generate positive average welfare gains, increases in average quality must outweigh the additional costs incurred by switching households.

We are interested in the sign and magnitude of the six variables defined in (23)-(28) and in how they change with the importance of peer preferences and the mobility of the two groups. With respect to the former, we vary the parameter α from 0 to 1.5. With respect to the latter, we consider two scenarios: the benchmark case from the basic model in which all households are mobile ($(i_p, i_r) = (0, 0)$) and the asymmetric case in which only the rich are mobile ($(i_p, i_r) = (1, 0.5)$). Figure 5 presents our findings. Each of the six panels describes the level of one of the six variables as a function of α which is measured on the horizontal axis. The solid line in each panel describes what happens when all households are mobile, while the dashed line deals with the case in which only rich households are mobile. The parameters underlying the figure are those from the benchmark case.

4.1 School quality

Our findings concerning the impacts of public school choice on average school quality are summarized in:

Finding 1 i) *Public school choice increases average school quality. When all households are mobile, stronger peer preferences reduce the gain in average school quality. When only rich households are mobile, stronger peer preferences increase the gain in average school quality.*

The first part of this finding is unsurprising: average school quality depends solely on average school effort and schools exert zero effort before the public school choice program is introduced. The second part is more interesting and the intuition is as follows. When all households are mobile, there is a negative “composition

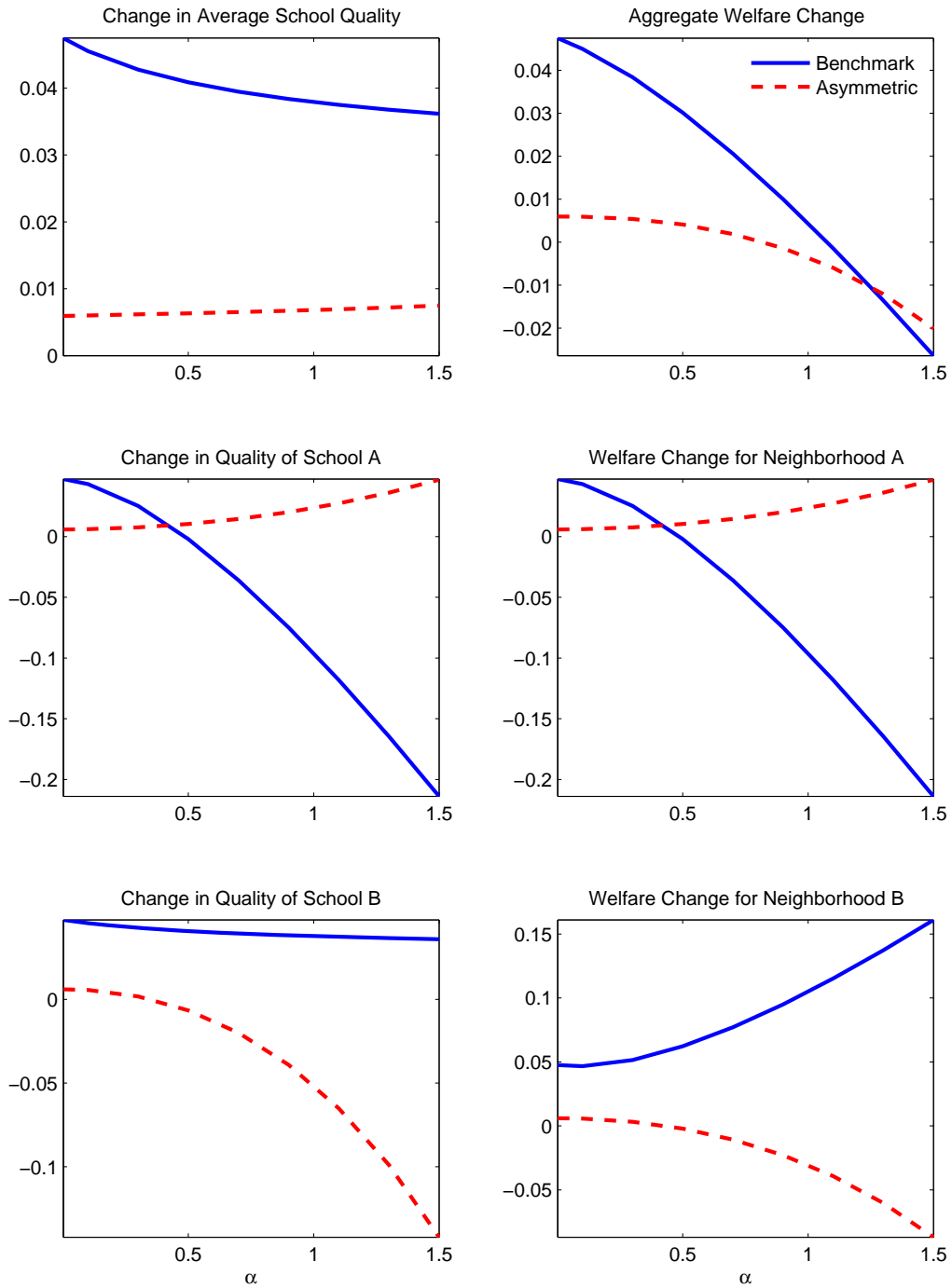


Figure 5: Impact of choice

return” to higher effort. In other words, all else equal, school A is reluctant to exert effort because this would decrease the peer differential between the two schools; school B is reluctant to exert effort because this would increase the peer differential between the two schools. When peer preferences are stronger, the composition return is larger, hence stronger peer preferences reduce effort incentives and lessen the increase in average school quality. When only rich households are mobile, the composition return is positive and stronger peer preferences increase effort incentives and generate larger increases in average school quality.³³

Turning to the impact of public school choice on the two schools qualities, our findings are summarized in:

Finding 1 ii) *In the advantaged school, when all households are mobile, public school choice increases quality when peer preferences are weak but reduces it once peer preferences exceed a critical level. Moreover, stronger peer preferences increase the reduction. When only rich households are mobile, public school choice increases quality and stronger peer preferences increase this quality gain.*

Finding 1 iii) *In the disadvantaged school, when all households are mobile, public school choice increases quality but stronger peer preferences reduce this quality gain. When only rich households are mobile, public school choice increases quality when peer preferences are weak but reduces it once peer preferences exceed a critical level. Moreover, stronger peer preferences increase the reduction.*

Irrespective of which households are mobile, weak peer preferences mean that few households in the disadvantaged neighborhoods exercise choice. It follows that in both schools, public school choice increases effort without changing school peer groups. As such, public school choice increases quality in both schools. This is what we would expect to see in a model in which parents did not have peer preferences.

When peer preferences are stronger, households in the disadvantaged neighborhood exercise choice. When all households are mobile, household switching decreases peer quality in school A and leaves it unchanged in school B . Hence for school B , stronger peer preferences reduce the quality gain (because they reduce the effort increase) but the quality gain remains positive. For school A , stronger peer preferences generate a large reduction in the quality gain (because they reduce effort and peer quality) and the quality gain is negative for peer preferences above some critical level. When only the rich are mobile, household switching increases peer quality in school A and decreases it in school B . For school A , this increases its peer advantage and hence its quality advantage. For school B , the costs of worse peer quality can exceed the benefits of higher effort, resulting in a decrease in quality.

³³Because effort levels depend on the quality differential, there is a second mechanism contributing to this finding. To see this, note that stronger peer preferences magnify the natural advantage of school A and induce some peer-driven choice. In the first case, this reduces the quality differential hence increases school effort (because effort is decreasing in the quality differential - see the top panel of Figure 2). In the second case, this increases the quality differential hence increases school effort (because effort is increasing the quality differential - see the top panel of Figure 4).

Findings 1i, 1ii and 1iii suggest that when households have strong peer preferences, the effects of public school choice on school quality exhibit an interesting type of equity-efficiency trade-off: when introduced into an environment in which rich and poor households are equally likely to take advantage of it, public school choice will have modest impacts on average school quality but will decrease the quality difference between the advantaged and disadvantaged school; when introduced into an environment in which rich households are more likely to take advantage of it, public school choice will have a greater impact on average quality, but will increase the quality difference between the advantaged and disadvantaged school.

4.2 Household welfare

Our findings concerning average household welfare are summarized in

Finding 2 i) *Public school choice increases average welfare when peer preferences are weak and decreases average welfare when peer preferences are strong. The decrease in average welfare associated with stronger peer preferences is greater when all households are mobile than when only rich households are mobile.*

To gain intuition for this finding, recall that the average welfare gain from public school choice is just the difference between the average quality gain and the costs incurred by the switching households (equation (28)). In the context of Finding 1i, we explained how peer preferences and mobility considerations impact the average quality gain. Thus, we just need to understand the switching costs.

When peer preferences are weak, school A 's natural peer advantage has little attraction. As a result, few households will exercise choice and hence these costs will be low. When peer preferences are stronger, school A 's peer advantage will drive more households to exercise choice. These switching households obviously benefit from their decision, but their benefit comes at the expense of households in the advantaged school and thus there is no aggregate gain. Welfare is reduced by the costs that these households incur and, for high enough peer preferences, these costs overwhelm the benefits of higher average quality. The decrease in average welfare associated with stronger peer preferences is greater when all households are mobile because in this case average quality is also dropping.

Turning to the impact on welfare in the two neighborhoods, our findings are summarized in:

Finding 2 ii) *In the advantaged neighborhood, when all households are mobile, public school choice increases welfare when peer preferences are weak, but reduces it once peer preferences exceed a critical level. When only rich households are mobile, public school choice increases welfare and stronger peer preferences increase this welfare gain.*

Finding 2 iii) *In the disadvantaged neighborhood, when all households are mobile, public school choice*

increases welfare and stronger peer preferences increase this welfare gain. When only rich households are mobile, public school choice increases welfare when peer preferences are weak but reduces it once peer preferences exceed a critical level.

Since no households in neighborhood A exercise choice, the change in welfare in neighborhood A equals the change in quality in school A . Thus, the intuition underlying the welfare change in the advantaged neighborhood is just that used to explain the quality change.

For neighborhood B , when all households are mobile, stronger peer preferences increase the welfare gain enjoyed by neighborhood B despite decreasing the quality of school B . This is because the benefits enjoyed by those households who switch to school A outweigh the costs (of lower school quality) experienced by those who do not. The benefits enjoyed by switching households also explains why, when only rich households are mobile, the neighborhood B welfare reduction associated with stronger peer preferences is milder than the school B quality reduction.

4.3 The role of neighborhood inequality

We have derived all these findings under the assumption that the fraction of rich in the community λ is equal to 0.5 and the fraction of rich in the advantaged neighborhood μ is equal to 0.75. It is worth discussing briefly how changing these values alters the impact of public school choice.

When all households are mobile, raising the fraction of rich in the community or the fraction of rich in the advantaged neighborhood has exactly the same implications for the impact of public school choice as increasing the strength of peer preferences. In other words, the impact of changing λ or μ is isomorphic to changing α . To see this, note that the equations characterizing the equilibrium (15), (16), and (17) depend on the peer differential (6) and its derivative multiplied by α , along with the derivative of the enrollment function (3). The enrollment function does not depend on λ and μ , while $\alpha\Delta\lambda(\Delta q)$ just depends on the product $\alpha\lambda(2\mu - 1)$. It follows that the equilibrium under public school choice and hence the steady state $(\Delta q_s, e_{As}, e_{Bs})$ just depends on $\alpha\lambda(2\mu - 1)$. From (25) and (28), this implies that the average change in school quality dq and the average change in household welfare dW just depends on $\alpha\lambda(2\mu - 1)$. Moreover, using this information along with (23), (24), (26), and (27), it is easy to verify that the other four variables only depend on the product $\alpha\lambda(2\mu - 1)$. This is the peer preference multiplied by the difference between the fraction of rich students in schools A and B in the period before the public school choice program was introduced. As such, it measures the advantage enjoyed by school A in the competition for students. It is interesting that in the baseline model in which all households are mobile, this is a sufficient statistic for the behavior of the model.

When only rich households are mobile, the equivalence between changing λ or μ and changing α no longer holds. To see this, note that the enrollment function depends on λ and μ and changing each of the three parameters has very different implications for $\alpha\Delta\lambda(\Delta q)$. Thus, to understand the implications of changing λ or μ , further analysis is required.³⁴ This analysis reveals that an increase in μ (i.e., a decrease in the fraction of the rich households that live in neighborhood B) leads to a reduction in all six variables. An increase in λ (i.e., an increase in the fraction of rich students in the community) has more complex effects, including increasing average quality and decreasing average welfare. None of these effects are not especially informative however, since in the scenario where only rich households are mobile, changes to these parameters will change the number of mobile households in neighborhood B . The resulting effort implications are somewhat mechanical, but will drive most of the effects of changing these parameters. The contrast between the model in which all households are mobile and the model in which only rich households are mobile is much better suited to the analysis of peer preferences.

5 Auxiliary policies

Proponents of public school choice often claim that the equity effects of these programs can be improved by policies that help disadvantaged students to exercise choice. In this Section, we consider public school choice combined with this policy. We also study public school choice combined with another policy that emerges naturally from our model: subsidizing schools for enrolling disadvantaged students. Since we show that first policy can weaken effort incentives, while the second can strengthen them, we then consider public school choice combined with a mix of these policies. Throughout, we take a balanced budget approach, requiring that the resources used to implement these policies come from within the school system.

5.1 Helping poor students to exercise choice

To permit a balanced budget analysis of policies that help poor students to exercise choice, we suppose that the school district has a fixed budget available for actions that encourage households in the disadvantaged neighborhood to take advantage of choice. In particular, we assume that these policies reduce the fractions of rich and poor students (i_r and i_p) that are immobile. The idea is that immobility is driven by high costs or ignorance of the program and is therefore responsive to policies such as subsidies and outreach. We further assume that by targeting its budget appropriately, the district can influence the extent of the reductions in i_r and i_p . For example, outreach targeted at the less-advantaged part of the neighborhood will cause a bigger

³⁴This analysis is available from the authors upon request.

reduction in i_p than i_r .

Formally, we assume that the combinations of i_p and i_r that the district can achieve with its budget satisfy an equation of the form

$$\left(\frac{1}{2} - \lambda(1 - \mu)\right)i_p + \lambda(1 - \mu)i_r = \frac{k}{2}. \quad (29)$$

The parameter k measures the fraction of households in the disadvantaged neighborhood that are immobile. This will depend on the size of the district’s subsidy and outreach budget and is thus exogenous for our purposes. Targeting more of the budget to helping poor students exercise choice will result in a decrease in i_p and a compensating increase in i_r to maintain (29). A key attraction of this formulation is that when we vary (i_r, i_p) in this way, we do not change the “direct” enrollment effect of a change in the quality differential. This can be seen from equation (20). The implicit assumption underlying this formulation is that a redistribution of outreach spending between rich and poor households will not change the overall fraction in neighborhood B that are immobile. Thus, the relationship between outreach dollars and immobility is linear.

We can now examine the implications of targeting resources to help poor students exercise choice. Given (29), such policies are equivalent to an increase in i_r and a compensating decrease in i_p . To illustrate the effect of such a policy, we assume that the district’s budget is such that 50% of the households in neighborhood B are immobile after its outreach efforts (i.e., $k = 0.5$). In addition, we assume that under the initial allocation of the budget, each group is equally mobile so that $i_r = i_p = 0.5$. We then consider a retargeting policy that increases i_r to 0.6 and reduces i_p to 0.4666. All other parameters are set at their benchmark levels.

Figure 6 illustrates the effects of this policy. Again, the six panels describe the level of the six variables identified in Section 4 as a function of α . The solid line in each panel describes what happens when $i_r = i_p = 0.5$ and the dashed line deals with the case in which (i_r, i_p) is equal to $(0.6, 0.4666)$. The policy reduces average school quality and quality in school A but increases quality in school B . This reflects the fact that both schools put in lower effort under the policy because a higher fraction of the marginal students are poor. Those households from neighborhood B switching into school A now contain a higher fraction of poor students which hurts school A ’s peer quality and helps school B ’s. Mitigating the impact on school A is the fact that there are fewer households switching in because the quality differential between the schools is smaller. The policy has very little effect on aggregate welfare but does lead to a small increase for high peer preferences (i.e., decreases the average welfare loss). This reflects the fact that less switching is occurring. Welfare for the disadvantaged neighborhood is increased. We summarize these findings as:

Finding 3 i) *Relative to a situation in which rich and poor students are equally mobile, targeting resources to help poor rather than rich students exercise choice decreases average school quality and quality in the*

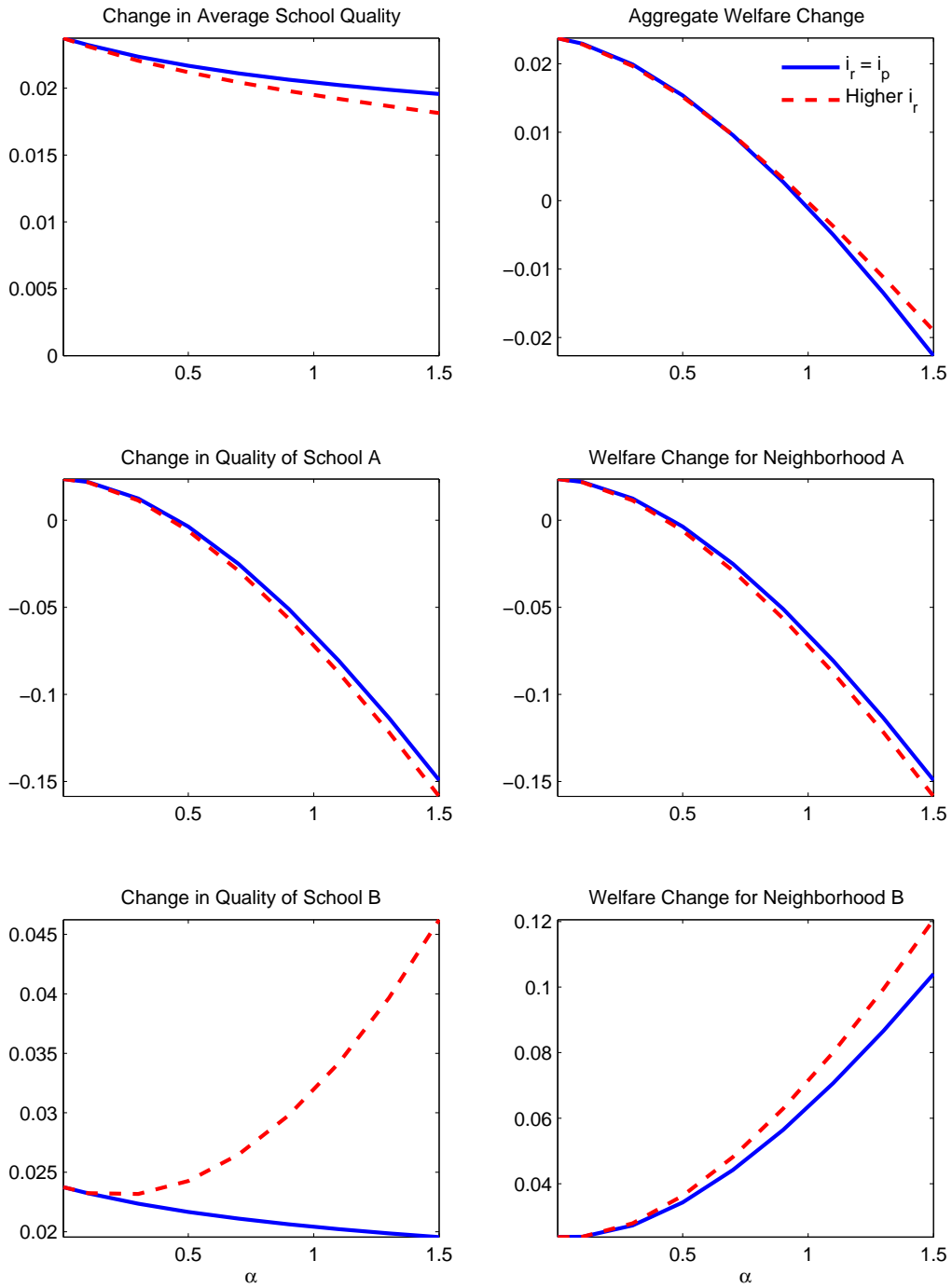


Figure 6: Helping the poor exercise choice

advantaged school, but increases quality in the disadvantaged school. When peer preferences are strong, the policy increases the welfare gain from public school choice in the disadvantaged neighborhood, decreases the welfare gain from public school choice in the advantaged neighborhood and decreases the average welfare loss from public school choice.

5.2 Incentivizing schools to enroll poor students

In light of the drawbacks of policies to help poor students to exercise choice, it is natural to instead work on the supply side by encouraging schools to enroll poor students. The basic model assumes that the school district allocates surplus to schools according to the formula of 1 unit per enrolled student. Suppose instead that it provides a surplus of $b_p > 1$ per enrolled poor student and $b_r < 1$ per enrolled rich student. Maintaining budget balance by holding constant the expenditure at 1 unit per student requires that

$$(1 - \lambda)b_p + \lambda b_r = 1. \quad (30)$$

This equation implies that b_r equals $(1 - (1 - \lambda)b_p) / \lambda$ and that the difference $b_r - b_p$ equals $(1 - b_p) / \lambda$. Ruling out negative surplus, the relevant range of b_p is therefore $[1, 1 / (1 - \lambda)]$. Setting $b_p = 1$ yields the benchmark case.

Under this policy, for any b_p in the range $[1, 1 / (1 - \lambda)]$, the two schools' strategies satisfy the requirements that

$$e_A(\Delta q) = \arg \max_{e_A} \left\{ b_p E(\Delta q) + (1 - b_p) \frac{E_r(\Delta q)}{\lambda} - \gamma \frac{e_A^2}{2} + \beta V_A(e_A - e_B(\Delta q) + \alpha \Delta \lambda(\Delta q)) \right\}, \quad (31)$$

and that

$$e_B(\Delta q) = \arg \max_{e_B} \left\{ b_p (1 - \lambda - E(\Delta q)) + (1 - b_p) \left(1 - \frac{E_r(\Delta q)}{\lambda} \right) - \gamma \frac{e_B^2}{2} + \beta V_B(e_A(\Delta q) - e_B + \alpha \Delta \lambda(\Delta q)) \right\}. \quad (32)$$

Following the same steps as before, we can write the first order conditions describing the equilibrium effort levels as:

$$\gamma e_A(\Delta q) = \beta \left[b_p \frac{dE}{d\Delta q'} + \frac{(1 - b_p)}{\lambda} \frac{dE_r}{d\Delta q'} + \gamma e_A(\Delta q') \left(\alpha \frac{d\Delta \lambda}{d\Delta q'} - \frac{de_B}{d\Delta q'} \right) \right], \quad (33)$$

and

$$\gamma e_B(\Delta q) = \beta \left[b_p \frac{dE}{d\Delta q'} + \frac{(1 - b_p)}{\lambda} \frac{dE_r}{d\Delta q'} + \gamma e_B(\Delta q') \left(\alpha \frac{d\Delta \lambda}{d\Delta q'} + \frac{de_A}{d\Delta q'} \right) \right]. \quad (34)$$

The only difference between (33) and (34) and (15) and (16), is that the value of a change in enrollment

is more complicated. This is the only place where the policy parameter b_p enters directly. Notice that when Δq is positive, then given the generalized cost structure introduced in Section 3.5, the enrollment value of a marginal increase in the quality difference is

$$b_p \frac{dE}{d\Delta q'} + \frac{(1-b_p)}{\lambda} \frac{dE_r}{d\Delta q'} = \frac{b_p}{\bar{c}} [(1-i_r)\lambda(1-\mu) + (1-i_p)(1/2 - \lambda(1-\mu))] + \frac{(1-b_p)}{\bar{c}}(1-i_r)(1-\mu). \quad (35)$$

Note that this is increasing in b_p if

$$\mu - 1/2 + i_r(1-\mu)(1-\lambda) > i_p(1/2 - \lambda(1-\mu)). \quad (36)$$

Since μ exceeds $1/2$, this condition will be satisfied unless the fraction of immobile poor significantly exceeds the fraction of immobile rich. Assuming inequality (36) is satisfied, the maximum incentive for schools is obtained by setting b_p equal to its maximum level $1/(1-\lambda)$.

Figure 7 depicts the benefits of setting b_p equal to its maximum level as opposed to its benchmark level when all households are mobile.³⁵ The solid line in each panel describes what happens when b_p is equal to its benchmark value (1) and the dashed line deals with the case in which b_p is equal to its maximum value (2). The important point to note is that in all dimensions, the policy improves outcomes under public school choice. The reason is simple: the policy raises schools' equilibrium effort levels as they have greater incentive to compete for the marginal students in the disadvantaged neighborhood. It is still the case, however, that for sufficiently strong peer preferences, the aggregate welfare change under public school choice is negative. Moreover, quality in school A and welfare in neighborhood A is harmed by public school choice. We record this finding as:

Finding 3 ii) *When all households are mobile, then relative to the benchmark policy, a public school choice policy that provides schools with the maximal incentive to enroll poor students increases quality in both schools and welfare in both neighborhoods. However, when peer preferences are sufficiently strong, public school choice still reduces average welfare, welfare in the advantaged neighborhood and quality in the advantaged school.*

5.3 Combining the two policies

What would happen if we combined the two policies just analyzed? Figure 8 illustrates the effects of a policy which simultaneously increases i_r to 0.6, reduces i_p to 0.5333, and raises b_p to 2. Because this policy improves schools' effort incentives, it is seen to generate more desirable outcomes than a policy that only

³⁵It goes without saying that if only rich households are mobile, paying schools for only enrolling poor students will completely backfire and provide no incentives for schools to exert effort.

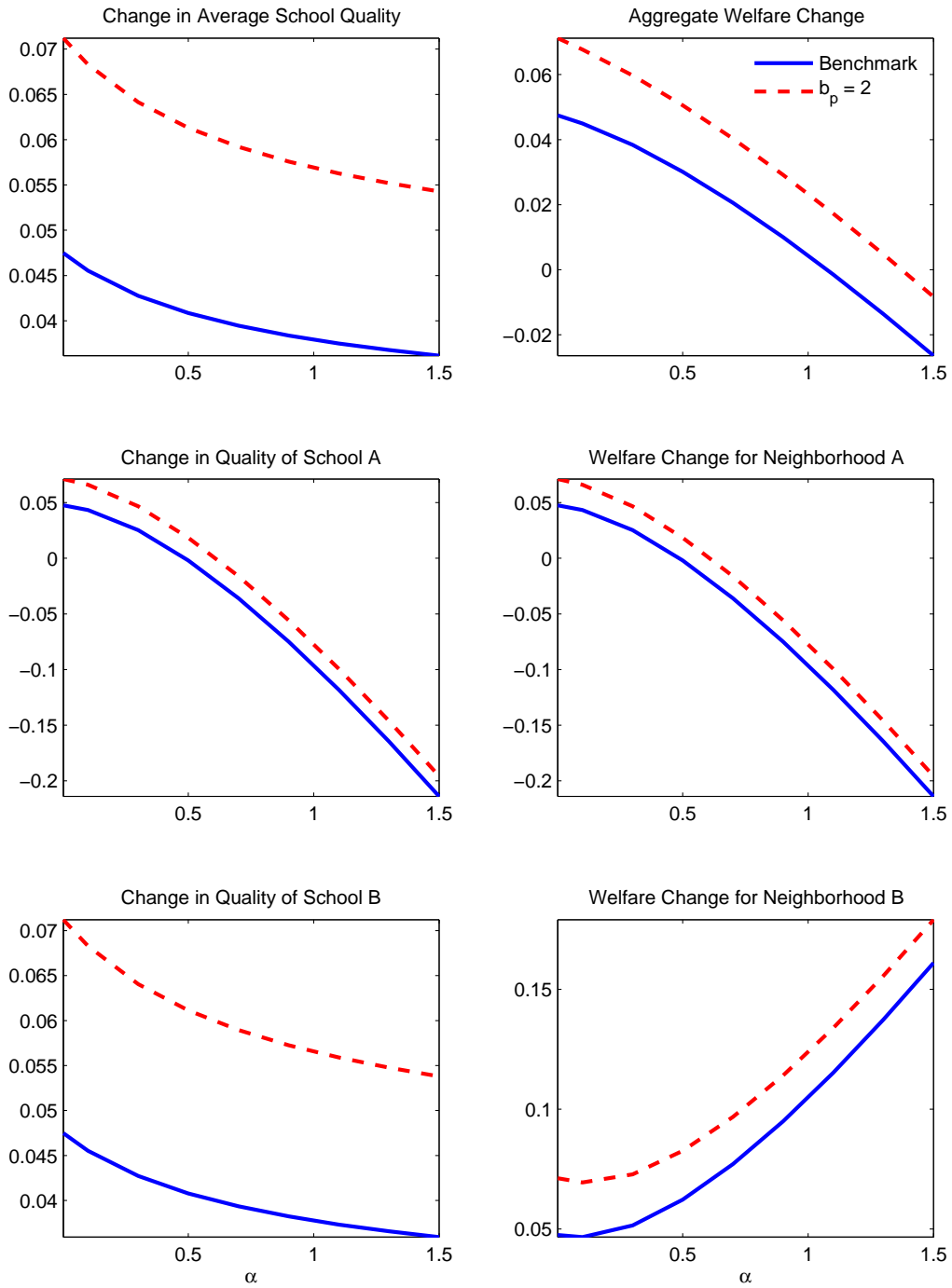


Figure 7: Incentivizing schools to enroll poor students

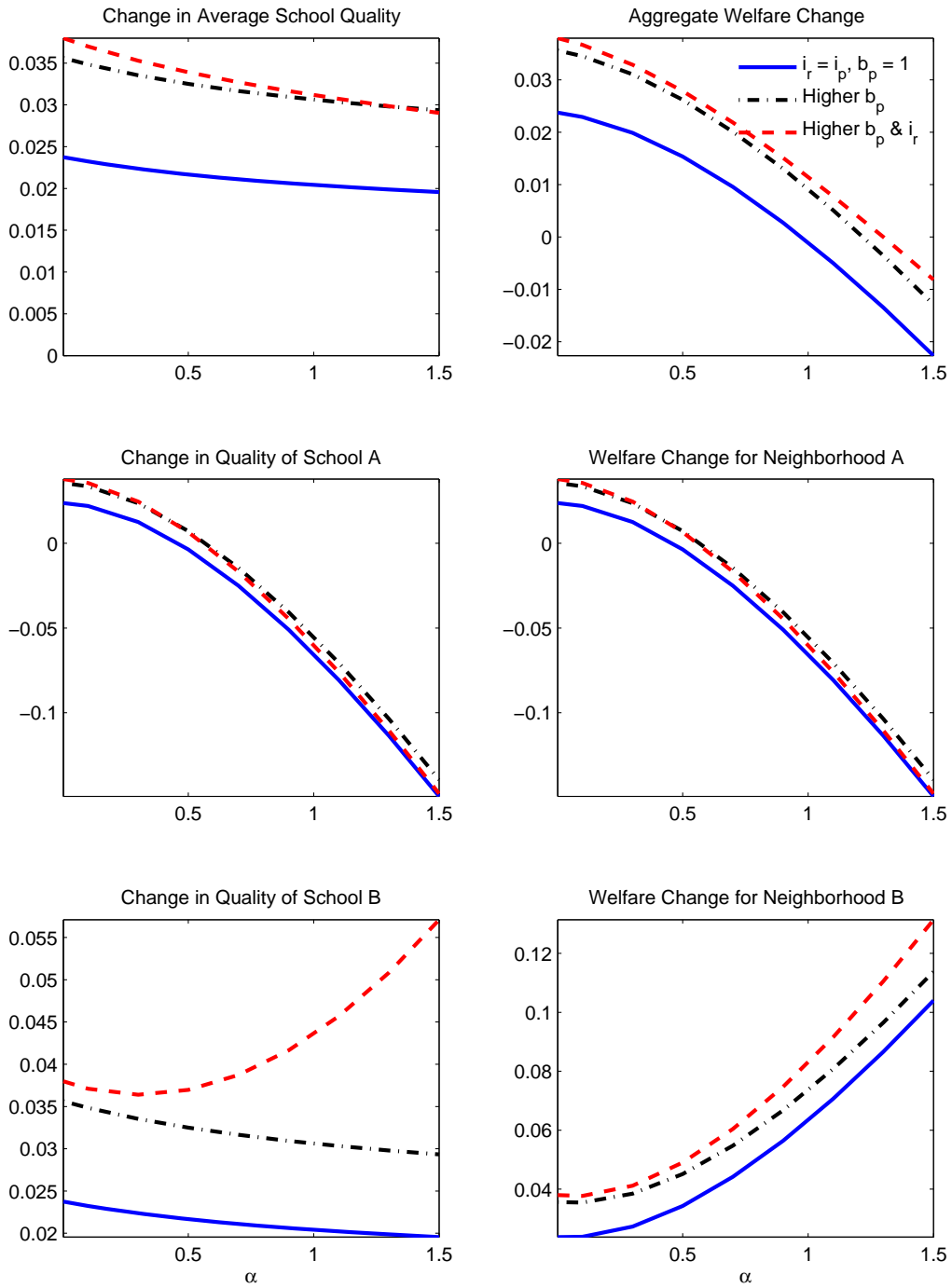


Figure 8: Helping the poor exercise choice and incentivizing schools to enroll poor students

helps poor students to exercise choice. Moreover, when compared with a policy that only incentivizes schools to enroll poor students, it improves school quality and household welfare in the disadvantaged neighborhood. Nonetheless, even under this two-pronged policy, when peer preferences are sufficiently strong, average welfare gains remain negative. Again, this is because the two schools always exert similar levels of effort, so that choice is driven by peer differentials and hence is socially wasteful.

6 Discussion

We believe that the three findings discussed above have interesting implications for the public school choice debate. This Section summarizes the findings and discusses these implications.

6.1 Welfare

Public school choice has ambiguous effects on welfare. When households have weak peer preferences or neighborhood inequality is small, public school choice will increase welfare. But when parents have strong preferences and neighborhoods are unequal (i.e., when schools compete on an uneven playing field), public school choice can reduce welfare. How is this possible given that public school choice increases average school quality? The answer is that since the two schools exert similar levels of effort, the school quality differences that cause households to exercise choice will be driven purely by peer differences. But peer-driven choice is wasteful, because it is costly to exercise choice and because peer quality is a zero-sum construct, such that one household's peer gain is another's peer loss.³⁶ For the parameters that we consider, public school choice always reduces welfare when peer preferences are sufficiently strong (i.e., the welfare cost of wasteful choice outweighs the welfare benefit of higher effort). Notably, this is true even when only rich students are mobile so that stronger peer preferences increase schools' incentive to exert effort.

The underlying problem is that while it is socially wasteful for households to exercise choice, the threat of choice is socially beneficial (since it elicits socially beneficial effort). When households have weak peer preferences, households threaten schools with choice, but do not exercise choice in equilibrium. That is because both schools exert similar levels of effort and hence provide similar levels of quality from the perspective of these peer-blind households. But when households have strong peer preferences, they threaten schools with choice and exercise choice in equilibrium.

One implication of this finding is that empirical evaluations of the impacts of public school choice on various indicators of school quality (e.g., test scores (Lavy (2010)) or the quality of teaching and learning

³⁶Put differently, we would like to subject schools to the threat of choice but would like to prevent households from actually exercising choice. The two goals are obviously incompatible.

(Ladd and Fiske (2003))) do not measure the full welfare effects of public school choice. That is because they exclude the costs incurred by households that exercise choice. These costs are as much a part of the welfare calculus as would be the costs of hiring superstar CEOs to run public schools. Hence even if public school choice increases school quality as proxied by these measures, an economist should not recommend the policy if it reduces average welfare.

6.2 School quality

With respect to school quality, public school choice programs generate a type of equity-efficiency trade-off. This trade-off stems from the dependence of effort incentives on the composition of the marginal students most likely to exercise choice. If the marginal students are richer than average (i.e., richer students are more mobile), schools will face strong incentives to recruit them and hence public school choice will have large impacts on average school quality. But it will also increase the gap between quality in the advantaged and the disadvantaged school, since the mobile rich students will leave the immobile poor students behind.

This trade-off implies that if public school choice programs are designed to improve school quality, then strong peer preferences may be problematic. These programs can increase average school quality by providing schools with strong incentives to compete for (advantaged) students. Alternatively, they can reduce the gap between the quality of advantaged and disadvantaged schools by providing disadvantaged students with access to “good” schools. But they may struggle to do both.

An implication of this trade-off is that the effects of public school choice programs will be setting-specific. This is consistent with the empirical literature, some of which finds positive effects of these programs on indicators of average school quality (e.g., the Lavy (2010) evaluation of an Israeli public school choice reform) and some of which does not (e.g., the Ladd and Fiske (2000) evaluation of New Zealand public school reforms).³⁷ One explanation for the conflicting findings that is consistent with our model is that the households that exercised choice in New Zealand were less advantaged than those that exercised choice in Israel, so that New Zealand schools had weaker incentives to recruit them. Lavy (2010) is concerned exclusively with average outcomes and hence it is difficult to evaluate this possibility, but it is consistent with the Ladd and Fiske account of the New Zealand reforms. First, as our model would predict, Ladd and Fiske (2001) use school enrollment and composition data to show that the program caused households to gravitate from low-

³⁷Ladd and Fiske (2003) find no statistically significant correlations between the principal reports of the impact of the policy on learning outcomes and principal reports of the level of local competition. Lavy (2010) evaluates an Israeli public school choice reform implemented in a single school district in Tel Aviv. He finds that relative to non-reforming control districts, a district that implemented a public school choice program enjoyed significant school productivity gains as measured by dropout rates, test scores and behavioral outcomes. Evidence from the UK is also mixed. Gibbons et al. (2008) exploit across-region variation in the availability of public school choice and find that it is associated with few productivity gains. Bradley and Taylor (2002) find stronger productivity gains using a difference-in-difference approach.

to high-quality schools (whether quality is measured by composition or achievement). Second, and also as our model would predict, Ladd and Fiske (2001) argue that when choosing schools, parents used “the mix of students in a school as a proxy for school quality” (p.49). Ladd’s interpretation is that: “(S)uccessful schools in urban areas had no desire to expand their enrollment. To the contrary, they did everything they could to maintain the mix of students that made them attractive to parents and students in the first place..(S)chools with large concentrations of disadvantaged students have difficulty competing for students” (Ladd (2003); p.70). Although the New Zealand reform deviated somewhat from the assumptions underlying our model, and although it is not clear whether the students that exercised choice decreased the peer quality differential (a condition for the composition return to be negative in our model), these mechanisms bear a striking resemblance to those at work in our model.³⁸

6.3 Auxiliary policies

Outcomes under public school choice programs could be improved by auxiliary policies that help disadvantaged students to exercise choice and provide schools with stronger incentives to enroll disadvantaged students. The first of these has been proposed as a means of improving the equity effects of public school choice by ensuring that choice can be exercised by disadvantaged households (e.g., by subsidizing the transport costs of poor students). But our analysis shows that this will blunt schools’ effort incentives and lessen the impact of public school choice on average school quality. More constructively, our findings suggest that when parents have strong peer preferences, outcomes under public school choice can be improved by a policy not usually considered in this context: incentivizing schools for enrolling poor students. This policy seems especially promising since “weighted-student financing” (WSF) already exists in many settings. Under these financing formulae, schools receive funds in proportion to the number of students taught with adjustments made for student composition. Some WSF formulas are fairly involved; some are more straightforward. In the UK for example, the school funding formula includes a “pupil premium” which attaches additional funds to each of a school’s students eligible for a free school meal. The policy that we have in mind would compensate schools for the additional costs associated with educating disadvantaged students (as at present) and provide an additional premium to incentivize them to exert the effort necessary to attract less-advantaged students.

Of course schools cannot attract less-advantaged students if it is costly for those students to move. That is why our analysis found that a two-pronged approach offers the best hope for achieving desirable outcomes under public school choice. First, policies that help poor students to exercise choice, with a view to ensuring

³⁸One important difference is that many schools in New Zealand appear to have been capacity constrained. Note that in our model, the successful schools would like to expand enrollment and the unsuccessful schools would have no difficult competing for marginal students. However, both schools would have limited incentives to exert effort because of the negative composition returns that they faced.

that the disadvantaged school and the disadvantaged households are not left behind. Second, policies that reward schools for enrolling poor students, thereby ensuring the largest possible efficiency gains from public school choice. Importantly though, even when accompanied by this two-pronged policy, public school choice can reduce welfare when peer preferences are sufficiently strong.

7 Conclusion

This paper has presented an economic model of household and school interaction in a system of public school choice and used it to assess the impact of public school choice programs on household welfare and school quality. While the model assumed that households have peer preferences, one could argue that the other assumptions were tilted towards finding that these programs had positive effects. In particular, before the program was introduced, these assumptions ensured that schools would exert zero effort. After the program was introduced, these assumptions ensured that schools would have incentives to increase enrollment and would never bump up against capacity constraints.

With this in mind, it is perhaps surprising that our analysis ends up highlighting the potential limitations of public school choice programs. We believe this speaks to the importance of peer preferences in shaping the effects of school choice policies. As we have stressed throughout, our analysis pertains to only one type of policy - public school choice. This is the most widely-used choice policy but many would argue that it does not go far enough. For example, one could argue the policy would be more effective if schools could choose which curricula to follow and if they could take other actions to shape the size and composition of the student body. While this type of analysis is beyond the scope of our paper, we think our paper demonstrates that any such analysis would have to take peer preferences into account.

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