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Fuzzy economic production in inventory model without shortage

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Abstract In this paper, an inventory model without shortage has been considered in fuzzy environment, using the new hexagonal fuzzy numbers. Our goal is to determine the fuzzy production quantity and fuzzy minimum total cost for the proposed inventory model. The storage cost, production cost and total demand quantity are taken as in terms of hexagonal fuzzy numbers. New arithmetic operations are defined and applied in sensitivity analysis. A relevant numerical example is also included, to justify the notion.

Keywords: Hexagonal Fuzzy Numbers, Fuzzy Production Quantity, Fuzzy Minimum Total cost.

2010 MSC: 03E72 and 90B05

1 Introduction

In 1915, the first inventory model was developed by Harris [3]. Later in 1965, first time the concept of fuzzy sets was introduced by Zadeh [11]. Fuzzy set theory is an extension of classical set theory where elements have degrees of membership. The theory sets attracted the attention of many researches. In 1983, Urgeletti Tinarelli [9] proposed the inventory control models and problems. In 1987, Park [5] proposed the model on fuzzy set theoretical interpretation of economic order quantity inventory problem. In 2002, Hsieh [4] proposed an approach for the optimization of fuzzy production inventory models. In 2003, Yao and Chiang [10] introduced an inventory without back order with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. In 2012, Dutta and Pavan Kumar [2] presented a fuzzy inventory model without shortage using trapezoidal fuzzy number with sensitivity analysis. In 2000, Der-Chen Lin and Jing-Shing Yao [1] proposed the fuzzy economic production for production inventory models. In 2014, Stephen and Rajesh [6] have developed an optimal total cost and optimal order quantity for fuzzy inventory model without shortage using hexagonal fuzzy numbers. In 2015, Stephen and Rajesh [7] have developed the fuzzy inventory model with allowable shortage using hexagonal fuzzy numbers. In 2015, Stephen and Rajesh [8] introduced inventory model with allowable shortage using LR-type hexagonal fuzzy numbers.

In this paper, a new fuzzy number called 'Hexagonal fuzzy number' is utilized in developing the notion of Inventory model. The main aim of the authors is to estimate the fuzzy optimal solution for the fuzzy economic production inventory model. The production Inventory model both crisp and fuzzy sense have been derived. A numerical example is added to illustrate the process of obtaining the fuzzy optimal production quantity and the fuzzy minimal total inventory cost.

In this article, in Section 2, some basic definitions and arithmetic operations on hexagonal fuzzy numbers are presented. In Section 3, we describe in brief the notions and assumptions used in the developed Fuzzy inventory model. In Section 4, Proposed Inventory Model in Crisp Sense, fuzzy sense and algorithm are presented. In Section 5, a numerical example is given to justify the proposed notions. In Section 6, the concluding remarks are also added.

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2 Definitions and preliminaries

Definition 2.1 (Fuzzy set). *A Fuzzy set $\tilde{A}$ in a universe of discourse x is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.*

Definition 2.2 (Convex fuzzy set). *A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} \subseteq X$ is called convex fuzzy set if all A_α are convex sets i.e. for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0,1] \lambda x_1 + (1-\lambda)x_2 \in A_\alpha \forall \lambda \in [0,1]$. Otherwise the fuzzy set is called non convex fuzzy set.*

Definition 2.3 (Hexagonal Fuzzy Number). *A fuzzy number on $\tilde{A}_h$ is a Hexagonal fuzzy number denoted by $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $(a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ are real numbers Satisfying $a_2 - a_1 \le a_3 - a_2$ and $a_5 - a_4 \ge a_6 - a_5$ and its membership function $\mu_{\tilde{A}_h}(x)$ is given as;*

$$\mu_{\tilde{A}_h}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0, & x > a_6 \end{cases}$$

Remark 2.1.

1. *The Hexagonal fuzzy numbers $\tilde{A}_h$ becomes trapezoidal fuzzy numbers if $a_2 - a_1 = a_3 - a_2$ and $a_5 - a_4 = a_6 - a_5$.*
2. *The Hexagonal fuzzy numbers $\tilde{A}_h$ becomes non-convex fuzzy numbers if $a_2 - a_1 > a_3 - a_2$ and $a_5 - a_4 < a_6 - a_5$.*

Definition 2.4 (Equality of two Hexagonal fuzzy numbers). *Two Hexagonal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are said to be equal i.e. $\tilde{A} = \tilde{B}$ if and only if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5, a_6 = b_6$.*

Definition 2.5 ((New Arithmetic Operations)). *The new arithmetic operations between hexagonal fuzzy numbers are proposed given below. Let us consider $\tilde{A}_1 = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{A}_2 = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers. Then,*

(i) The addition of $\tilde{A}_1$ and $\tilde{A}_2$ is
$\tilde{A}_1(+)\tilde{A}_2 = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$.

(ii) The subtraction of $\tilde{A}_1$ and $\tilde{A}_2$ is
$\tilde{A}_1(-)\tilde{A}_2 = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$.

(iii) The multiplication of $\tilde{A}_1$ and $\tilde{A}_2$ is
$\tilde{A}_1(\times)\tilde{A}_2 = \left(\frac{a_1}{6}\sigma_b, \frac{a_2}{6}\sigma_b, \frac{a_3}{6}\sigma_b, \frac{a_4}{6}\sigma_b, \frac{a_5}{6}\sigma_b, \frac{a_6}{6}\sigma_b\right)$, *where* $\sigma_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6)$.

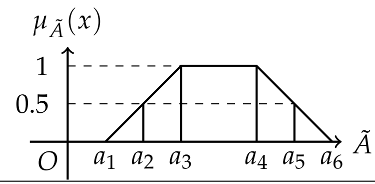
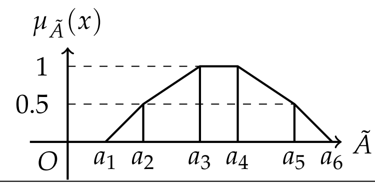
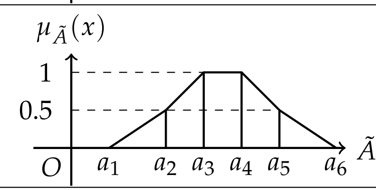
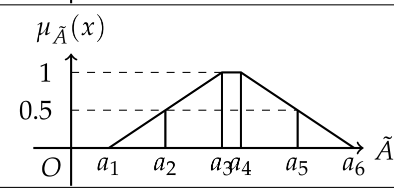
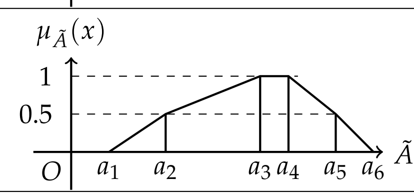
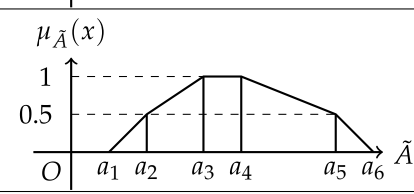
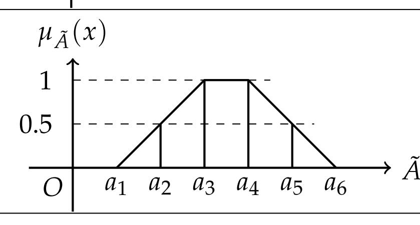
(iv) The division of $\tilde{A}_1$ and $\tilde{A}_2$ is
$\tilde{A}_1(\div)\tilde{A}_2 = \left(\frac{6a_1}{\sigma_b}, \frac{6a_2}{\sigma_b}, \frac{6a_3}{\sigma_b}, \frac{6a_4}{\sigma_b}, \frac{6a_5}{\sigma_b}, \frac{6a_6}{\sigma_b}\right)$, *if* $\sigma_b \neq 0$, *where* $\sigma_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6)$.

(v) If $k \neq 0$ is a scalar $k\tilde{A}$ is defined as
$$k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6), & \text{if } k > 0 \\ (ka_6, ka_5, ka_4, ka_3, ka_2, ka_1), & \text{if } k < 0. \end{cases}$$

(vi) $\sqrt{\tilde{A}} = \sqrt{a_1, a_2, a_3, a_4, a_5, a_6} = (\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}, \sqrt{a_5}, \sqrt{a_6})$, *where $a_1, a_2, a_3, a_4, a_5, a_6$ are non zero positive real numbers.*

Definition 2.6. *We define a ranking function $R : f(R) \to R$ which maps each fuzzy numbers to the real line: $f(R)$ represents the set of all hexagonal fuzzy numbers. If R be any linear ranking function, then* $R(\tilde{A}) = \left(\frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}\right)$.

Table 1: Different types of hexagonal fuzzy numbers.

Types of Hexagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$	Conditions	Pictorial Representations
Hexagonal fuzzy numbers $\tilde{A}_h$ becomes trapezoidal fuzzy numbers	$a_2 - a_1 = a_3 - a_2$ $a_5 - a_4 = a_6 - a_5$	
Convex Hexagonal fuzzy numbers $\tilde{A}_h$	$a_2 - a_1 \leq a_3 - a_2$ $a_5 - a_4 \geq a_6 - a_5$	
Non convex Hexagonal fuzzy numbers $\tilde{A}_h$	$a_2 - a_1 > a_3 - a_2$ $a_5 - a_4 < a_6 - a_5$	
Symmetric Hexagonal fuzzy numbers $\tilde{A}_h$	$a_3 - a_1 = a_6 - a_4$	
Non symmetric Hexagonal fuzzy numbers type-1 $(l_s > r_s)$	$a_3 - a_1 > a_6 - a_4$	
Non symmetric Hexagonal fuzzy numbers type-2 $(l_s < r_s)$	$a_3 - a_1 < a_6 - a_4$	
Perfect symmetric Hexagonal fuzzy numbers	$a_2 - a_1 = a_3 - a_2$ $a_3 - a_2 = a_4 - a_3$ $a_4 - a_3 = a_5 - a_4$ $a_5 - a_4 = a_6 - a_5$	

Definition 2.7 (Equivalent hexagonal fuzzy number)**.** *A fuzzy number $\tilde{A}$ is said to be equivalent to a fuzzy number $\tilde{B}$ if their values of the Ranking functions are the same. i.e. $\tilde{A} \approx \tilde{B}$ if $R(\tilde{A}) = R(\tilde{B})$*

3 Notations and Assumptions

3.1 Notations

We define the following symbols:

a : storage cost per unit

b : production cost per unit

r : demand quantity

d : production quantity

R : Total demand quantity

T : Length of the plan

q^* : Optimal order quantity

TC^* : Minimum total cost

$\tilde{a}$: Fuzzy storage cost per unit quantity per unit time

$\tilde{b}$: Fuzzy production cost per unit

$\tilde{R}$: Fuzzy Total demand over the planning time period $[0, T]$

$\tilde{Q}$: Fuzzy Order quantity per cycle

$\tilde{Q}^*$: Fuzzy Optimal order quantity

$\tilde{T}_c$: Fuzzy total cost for the period $[0, T]$

$\tilde{F}(Q)^*$: Minimum Fuzzy total cost for $[0, T]$

3.2 Assumptions

In this paper the following assumptions are considered as

(i) Total demand is fuzzy nature

(ii) Time plan is constant

(iii) Production cost, storage cost are fuzzy in nature

(iv) Shortages are not allowed.

4 Proposed Inventory Model in Crisp Sense

Assume r and d, are respectively, the daily demand quantity and the daily production quantity. The production quantity in each section $t_s(OB, CB')$ is q (Fig. 4.1).During the period OB, CB', if production and sales are kept at point B, B', etc.; then production stops, and in period BC, BC', only sales go on. The cycle is maintained for T days, where T represents planning period. S denotes the largest inventory,R the total demand quantity, a the daily storage cost per unit quantity and b the production cost per section. Thus, Production time per section $t_S = \frac{q}{d}$; inventory consumption during period t_s is $rt_s = \frac{rq}{d}$. Inventory during end of production period (B, B', etc.) is $S = q - rt_s = q(1 - r/d)(> 0)$. Assume that the time for each section is $t_q(= OC = CC')$. During the period BC, $t_q - t_S$, inventory consumption is S; average inventory in $t_q = \frac{Tq}{R}$. In planning period T, the number of production time is $\frac{R}{q}$.

In this model, the production inventory model without shortages, in crisp environment. The economic lot size is obtained by the following model equation:

$$q = \sqrt{\frac{2bRC}{Ta}} \text{where } C = (1 - r/d). \tag{4.1}$$

The total cost for the period $[0, T]$ is

$$TC = \frac{CQTa}{2} + \frac{bR}{Q} \text{where } C = (1 - r/d). \tag{4.2}$$

The optimum q^* and TC^* can be obtained by equating the first partial derivatives of TC to zero, and solving the resulting equations:

$$\text{Optimal order quantity } q^* = \sqrt{\frac{2bR}{aCT}} \tag{4.3}$$

$$\text{Minimum total cost } TC^* = \sqrt{2baCRT} \tag{4.4}$$

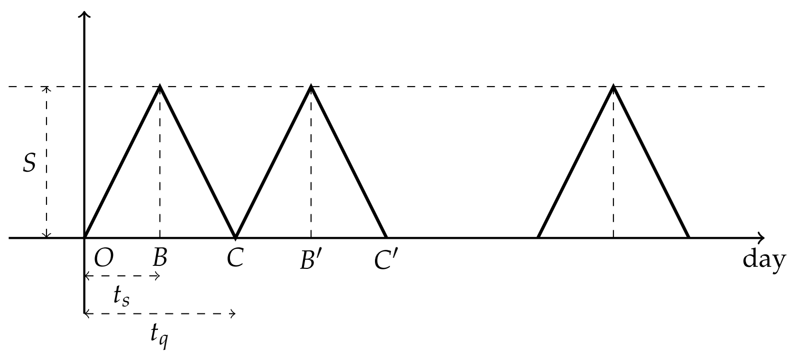


Figure 4.1: Production inventory model.

4.1 Proposed Inventory Model in Fuzzy Sense

We consider the model in fuzzy environment. Since the daily storage cost per unit quantity, production cost per cycle and total demand quantity are fuzzy nature, we represent them by hexagonal fuzzy numbers.
Let
$\tilde{a}$ = daily storage cost per unit quantity
$\tilde{b}$ = production cost per cycle
$\tilde{R}$ = total demand quantity

The daily demand and production quantity, time of plan are consider as constants. Now we fuzzify total cost given in (4.2), the fuzzy total cost is given by:

$$\tilde{T}_c = \frac{C\tilde{Q}T\tilde{a}}{2} + \frac{\tilde{b}\tilde{R}}{Q} \text{ where } C = (1 - r/d). \tag{4.5}$$

Our aim is to apply the hexagonal fuzzy number fuzzy total cost and obtain the fuzzy optimal order quantity by using the simple calculus technique.
Suppose
$\tilde{a} = (a_1, a_2, a_3, a_4, a_5, a_6)$
$\tilde{b} = (b_1, b_2, b_3, b_4, b_5, b_6)$
$\tilde{R} = (R_1, R_2, R_3, R_4, R_5, R_6)$ are hexagonal fuzzy numbers.

By using new arithmetic operations and simplifying we get, From (4.5), We have:

$$\tilde{T}_c = \Big(\frac{CT\tilde{Q}}{2}a_1 + \frac{R_1}{6\tilde{Q}}\tau_a, \frac{CT\tilde{Q}}{2}a_2 + \frac{R_2}{6\tilde{Q}}\tau_a, \frac{CT\tilde{Q}}{2}a_3 + \frac{R_3}{6\tilde{Q}}\tau_a, \frac{CT\tilde{Q}}{2}a_4 + \frac{R_4}{6\tilde{Q}}\tau_a, \frac{CT\tilde{Q}}{2}a_5 + \frac{R_5}{6\tilde{Q}}\tau_a, \frac{CT\tilde{Q}}{2}a_6 + \frac{R_6}{6\tilde{Q}}\tau_a\Big)$$

$$\tilde{T}_c = F(\tilde{Q}). \tag{4.6}$$

The fuzzy optimal order quantity $\tilde{Q}^*$ which is minimize the total inventory cost $\tilde{T}_c = F(\tilde{Q})$ is obtained as the solution of the first order fuzzy differential equation $\frac{d}{d\tilde{Q}}(\tilde{T}_c) = \tilde{0}$ and it is found as

$$\tilde{Q}^* = \sqrt{\frac{2b_1\tau_b}{CT\tau_c}, \frac{2b_2\tau_b}{CT\tau_c}, \frac{2b_3\tau_b}{CT\tau_c}, \frac{2b_4\tau_b}{CT\tau_c}, \frac{2b_5\tau_b}{CT\tau_c}, \frac{2b_6\tau_b}{CT\tau_c}},$$

where
$\tau_a = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6)$
$\tau_b = (R_1 + R_2 + R_3 + R_4 + R_5 + R_6)$
$\tau_c = (a_1 + a_2 + a_3 + a_4 + a_5 + a_6)$
$\tau_d = (Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6)$.

Also $\tilde{Q} = \tilde{Q}^*$ we have $\frac{d^2F(\tilde{Q})}{d\tilde{Q}^2} > 0$; this show that $F(\tilde{Q})$ is minimum at $\tilde{Q} = \tilde{Q}^*$ and from (4.6)

$$F(\tilde{Q}^*) = \Big(\frac{CTa_1\tau_d}{12} + \frac{R_1\tau_a}{\tau_d}, \frac{CTa_2\tau_d}{12} + \frac{R_2\tau_a}{\tau_d}, \frac{CTa_3\tau_d}{12} + \frac{R_3\tau_a}{\tau_d}, \frac{CTa_4\tau_d}{12} + \frac{R_4\tau_a}{\tau_d}, \frac{CTa_5\tau_d}{12} + \frac{R_5\tau_a}{\tau_d}, \frac{CTa_6\tau_d}{12} + \frac{R_6\tau_a}{\tau_d}\Big).$$

4.2 Algorithm for finding fuzzy total cost and fuzzy optimal order quantity

Step 1: Calculate the model fuzzy total cost for the fuzzy values of $\tilde{a}, \tilde{b}, \tilde{R}$ and T.
Step 2: Now determine fuzzy total cost using new arithmetic operations fuzzy holding cost, fuzzy ordering cost, fuzzy shortage cost and fuzzy demand taken in terms of hexagonal fuzzy numbers.
Step 3: Find the fuzzy optimal order quantity which can be obtain by putting the first derivative of $F(\tilde{Q})$ equal to zero and second derivative is positive at $\tilde{Q} = \tilde{Q}^*$

5 Numerical example

Crisp Model

Let a = Rs. 5 per unit, b = Rs. 10 per unit, d = Rs. 10 per unit, R = Rs. 100 per unit, r = Rs. 2 per unit, $C = (1 - \frac{r}{d}) = 0.8$ per unit, T = Rs. 80 per unit. Then $Q^* = 2.5$ units. TC^* = Rs. 800.

Fuzzy model

Let $d = 10$, $r = 2$, $T = 80$, $C = 0.8$, $\tilde{R} = (20, 40, 80, 120, 160, 180)$, $\tilde{b} = (2, 4, 8, 12, 16, 18)$ $\tilde{a} = (1, 2, 4, 6, 8, 9)$, then
$\tilde{Q}^* = (1.118, 1.581, 2.236, 2.738, 3.162, 3.354)$
$R(\tilde{Q}^*) = 2.44$ units
$f(\tilde{Q}^*) = (160.24, 320.48, 640.98, 961.79, 1281.59, 1442.22)$
$R(f(\tilde{Q}^*))$ = Rs. 801.2.

It is observed that (Table 2),

1. The fuzzy optimal order quantity is closer to crisp optimal order quantity.
2. The fuzzy total cost is closer to crisp total cost.
3. For different values of $\tilde{a}$ and $\tilde{b}$, changing only middle two spreads the fuzzy optimal order quantity remains fixed. The same is true for fuzzy total cost.

6 Conclusion

In this paper, we have studied the fuzzy economic production in inventory model with the aid of hexagonal fuzzy numbers. The various fuzzy optimal quantities, the Demand, Production cost, and storage cost using hexagonal fuzzy numbers have been estimated. A new arithmetic operations of hexagonal fuzzy numbers are proposed to get the expected result. Also it is observed that the fuzzy estimates are closer to the crisp estimates of the real systems.

Table 2: Sensitivity analysis.

S.No	Demand ($\tilde{R}$)	$\tilde{a} = (1,2,4,6,8,9)$ $\tilde{b} = (2,4,8,12,16,18)$		$\tilde{a} = (1,2,3,7,8,9)$ $\tilde{b} = (2,4,7,13,16,18)$	
		$\tilde{Q}^*$	$F(\tilde{Q}^*)$	$\tilde{Q}^*$	$F(\tilde{Q}^*)$
1	(10,30,70, 110,150,170)	(1.060,1.500, 2.121,2.598, 3.000,3.182)	(116.35, 277.29, 599.17, 921.06, 1242.93, 1403.88)	(1.060,1.500, 2.121,2.598, 3.000,3.182)	(116.35, 277.29, 599.17, 921.06, 1242.93, 1403.88)
2	(15,35,75, 115,155,175)	(1.089,1.541, 2.179,2.669, 3.082,3.269)	(138.82, 299.36, 620.43, 941.51, 672.93, 1423.12)	(1.089,1.541, 2.179,2.669, 3.082,3.269)	(138.82, 299.36, 620.43, 941.51, 672.93, 1423.12)
3	(20,40,80, 120,160,180)	(1.118,1.581, 2.236,2.738, 3.162,3.354)	(160.24,320.48, 640.98, 961.79, 1281.59, 1442.22)	(1.118,1.581, 2.236,2.738, 3.162,3.354)	(160.24,320.48, 640.98, 961.79, 1281.59, 1442.22)
4	(25,45,85, 125,165,185)	(1.145,1.620, 2.291,2.806, 3.240,3.436)	(180.21, 340.23, 660.24, 980.27 1300.47, 1460.29)	(1.145,1.620, 2.291,2.806, 3.240,3.436)	(180.21, 340.23, 660.24, 980.27 1300.47, 1460.29)
5	(30,50,90, 130,170,190)	(1.172,1.658, 2.345, 2.872, 3.316,3.517)	(200.32, 360.33, 680.34, 1000.35, 1320.36, 1480.36)	(1.172,1.658, 2.345, 2.872, 3.316,3.517)	(200.32, 360.33, 680.34, 1000.35, 1320.36, 1480.36)

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