Computational Strategies in Optimizing a Real-Time Grad-Shafranov PDE Solver Using High-Level Graphical Programming and COTS Technology

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Abstract

This paper describes an alternative approach based on LabVIEW that solves the critical plasma shape and position control problems in tokamaks. Input signals from magnetic probes and flux loops are the constraints for a non-linear Grad-Shafranov PDE solver to calculate the magnetic equilibrium. An architecture based on off-the-shelf multi-core hardware and graphical software is described with an emphasis on seamless deployment from development system to real-time target. A number of mathematical challenges were addressed and several generally applicable numerical and mathematical strategies were developed to achieve the timing goals. Several benchmarks illustrate what can be achieved with such an approach.

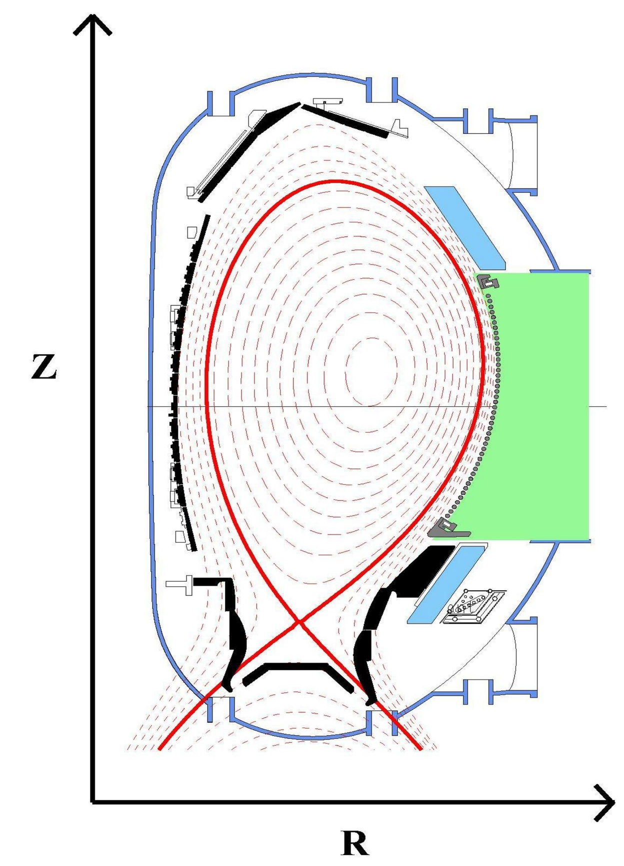
- ❑ commercial-off-the-shelf (COTS) multi-core computers
- ❑ Grad-Shafranov PDE
- ❑ real-time algorithm

Grad-Shafranov PDE

$$R\frac{\partial}{\partial R^2}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial Z^2}=-\mu_0 Rj(R,Z)$$

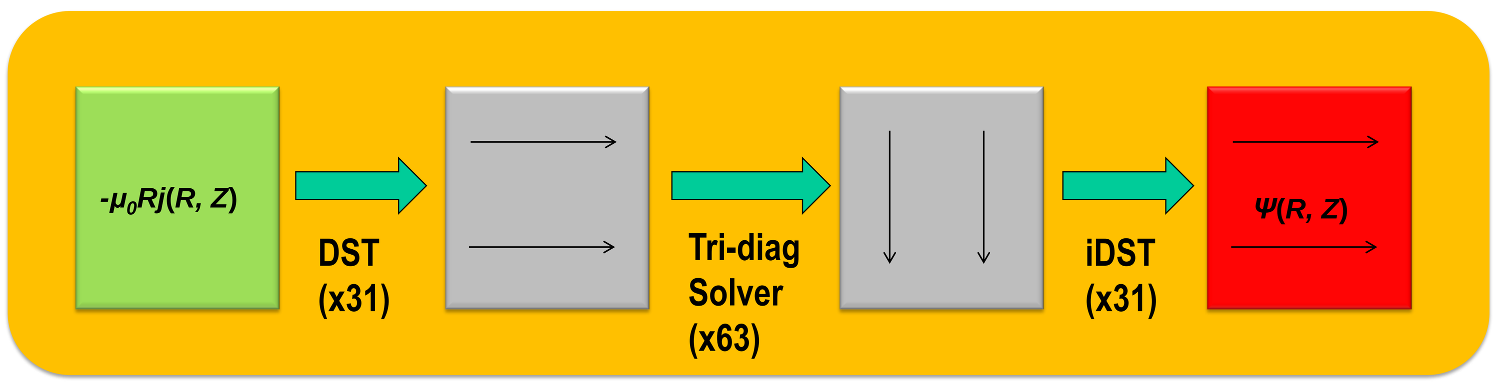
- Ψ is the poloidal flux function;
- j is the current density;
- R is the radial component;
- Z is the axial component.

- 33x65 grid



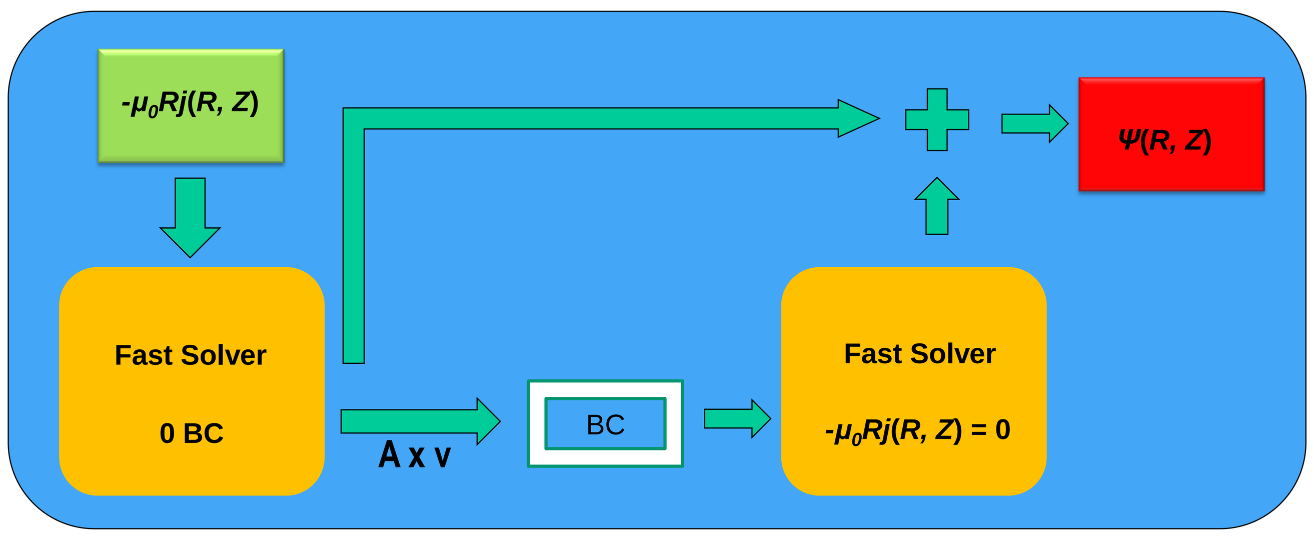
G-S Solver in Bounded Domain (Fast Solver)

- ❑ Dirichlet Boundary Condition: given $\Psi(R, Z)$ on boundary.
- ❑ Spectral Method:
  1. Multi-channel DST
  2. Tri-diagonal Solver
  3. Multi-channel Inverse DST
- ➢ An alternative of Cyclic Reduction Algorithm [1, 2, 3].
- ➢ Easy to program and parallelize in LabVIEW.



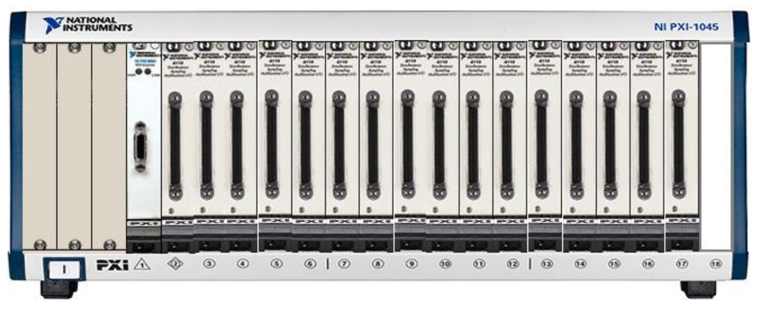
G-S Solver in Unbounded Domain

- ❑ Two fast solver steps [1]:
  1. Fast solver with zero Dirichlet BC;
  2. Compute BC based on the solution from step 1;
  3. Another fast solver with Dirichlet BC from step 2.
- ❑ Take advantage of linearity:



Implementation

- ❑ Algorithm
  - In the first step, compute reduced iDST instead of full iDST.
  - In the second step, use optimized DST leveraging sparsity.
- ❑ Hardware and Software



18 slot PXI-1045 chassis

16 National Instruments PXI 6143 S Series cards with 16 bit ADC's
128 channels sampled at 10 kHz

Dell T5500 and LabVIEW RT 2009

2 quad-core Intel Xeon X5677 (3.46 GHz)
PCIe VMIC 5565 Reflective Memory
NI PCIe-8362 MXI extension to PXI chassis

Benchmarks

Benchmarks for the real-time Grad-Shafranov solver and simultaneous function paramerisation and Grad-Shafranov solvers using 8 cores:

Platform	GS (ms)	FP+GS Time (ms)
Xeon X5365 @ 3.0 GHz	1.13	2.78
Xeon X5677 @ 3.46 GHz	0.63	1.03

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