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Some multiplier difference sequence spaces defined by a sequence of modulus functions [1]

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Abstract

In this paper we introduce and investigate the multiplier difference sequence spaces $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, $c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ and $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ defined by a sequence $\boldsymbol{F}= (f_k)$ of modulus functions and $p = (p_k)$ be any bounded sequence of positive real numbers. We study their different properties like completeness, solidity, monotonicity, symmetricity etc. We also obtain some relations between these spaces as well as prove some inclusion results.

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1 Introduction

Throughout the paper w, ℓ_∞, c and c_0 denote the spaces of *all*, *bounded*, *convergent* and *null* sequences $x = (x_k)$ with complex terms respectively. The zero sequence is denoted by $\theta = (0, 0, \dots)$.

The notion of difference sequence space was introduced by Kizmaz [4], who studied the difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$. The notion was further generalized by Et and Colak [1] by introducing the spaces $\ell_\infty(\Delta^n)$, $c(\Delta^n)$ and $c_0(\Delta^n)$. Another type of generalization of the difference sequence spaces is due to Tripathy and Esi [10], who studied the spaces $\ell_\infty(\Delta_m)$, $c(\Delta_m)$ and $c_0(\Delta_m)$. Tripathy, Esi and Tripathy [11] generalized the above notions and unified these as follows:

Let m, n be non-negative integers, then for Z a given sequence space we have

$$Z(\Delta_m^n) = \{x = (x_k) \in w : (\Delta_m^n x_k) \in Z\},$$

where $\Delta_m^n x = (\Delta_m^n x_k) = (\Delta_m^{n-1} x_k - \Delta_m^{n-1} x_{k+m})$ and $\Delta_m^0 x_k = x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta_m^n x_k = \sum_{v=0}^{n} (-1)^v \binom{n}{v} x_{k+mv}.$$

Taking $m = 1$, we get the spaces $\ell_\infty(\Delta^n)$, $c(\Delta^n)$ and $c_0(\Delta^n)$ studied by Et and Colak [1]. Taking $n = 1$, we get the spaces $\ell_\infty(\Delta_m)$, $c(\Delta_m)$ and $c_0(\Delta_m)$ studied by Tripathy and Esi [10]. Taking $m = n = 1$, we get the spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ introduced and studied by Kizmaz [4].

Let $\Lambda = (\lambda_k)$ be a sequence of non-zero scalars. Then for E a sequence space, the multiplier sequence space $E(\Lambda)$ associated with the multiplier sequence Λ is defined as

$$E(\Lambda) = \{(x_k) \in w : (\lambda_k x_k) \in E\}$$

The scope for the studies on sequence spaces was extended by using the notion of associated multiplier sequences. Goes and Goes [2] defined the differentiated sequence space dE and integrated sequence space $\int E$ for a given sequence space E, using the multiplier sequences (k^{-1}) and (k) respectively. A multiplier sequence can be used to accelerate the convergence of the sequences in some spaces. In some sense, it can be viewed as a catalyst, which is used to accelerate the process of chemical reaction. Sometimes the associated multiplier sequence delays the rate of convergence of a sequence.

A function $f : [0, \infty) \longrightarrow [0, \infty)$ is called a *modulus* if

(a) $f(x) = 0$ if and only if $x = 0$,

(b) $f(x + y) \leq f(x) + f(y)$, for $x \geq 0$, $y \geq 0$,

(c) f is increasing,

(d) f is continuous from the right at 0.

Hence f is continuous everywhere in $[0, \infty)$.

The following inequality will be used throughout the article. Let $p = (p_k)$ be a positive sequence of real numbers with $0 < p \leq \sup p_k = G$, $D = \max(1, 2^{G-1})$. Then for all $a_k, b_k \in C$ for all $k \in N$, we have

$$|a_k + b_k|^{p_k} \leq D\{|a_k|^{p_k} + |b_k|^{p_k}\}$$

and for $\lambda \in C$,

$$|\lambda|^{p_k} \leq \max(1, |\lambda|^G)$$

The studies on paranormed sequence spaces were initiated by Nakano [8] and Simons [9] at the initial stage. Later on it was further studied by Maddox [7], Lascardies [5], Lascardies and Maddox [6], Ghosh and Srivastava [3] and many others.

2 Definitions and Preliminaries

A sequence space E is said to be *solid* (*or normal*) if $(x_k) \in E$ implies $(\alpha_k x_k) \in E$ for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$ for all $k \in N$.

A sequence space E is said to be *monotone* if it contains the canonical preimages of all its step spaces.

A sequence space E is said to be *symmetric* if $(x_{\pi(k)}) \in E$, where π is a permutation on N.

A sequence space E is said to be *convergence free* if $(y_k) \in E$ whenever $(x_k) \in E$ and $y_k = 0$ whenever $x_k = 0$.

A sequence space E is said to be *sequence algebra* if $(x_k.y_k) \in E$ whenever $(x_k) \in E$ and $(y_k) \in E$.

Let $p = (p_k)$ be any bounded sequence of positive real numbers and $\Lambda = (\lambda_k)$ be a sequence of non-zero scalars. Let m, n be non-negative integers, then for a sequence $\boldsymbol{F} = (f_k)$ of modulus functions we define the following sequence spaces:

$$c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) = \{x = (x_k) : \lim_{k\to\infty} (f_k(|\Delta^n_{(m)} \lambda_k x_k|))^{p_k} = 0\},$$

$$c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) = \{x = (x_k) : \lim_{k\to\infty} (f_k(|\Delta^n_{(m)} \lambda_k x_k - L|))^{p_k} = 0,$$

for some $L \in C\}$,

$$\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) = \{x = (x_k) : \sup_{k \geq 1}(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{p_k} < \infty\},$$

where $(\Delta^n_{(m)}\lambda_k x_k) = (\Delta^{n-1}_{(m)}\lambda_k x_k - \Delta^{n-1}_{(m)}\lambda_{k-m}x_{k-m})$ and $\Delta^0_{(m)}\lambda_k x_k = \lambda_k x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta^n_{(m)}\lambda_k x_k = \sum_{v=0}^{n}(-1)^v \binom{n}{v} \lambda_{k-mv}x_{k-mv}.$$

In the above expansion it is important to note that we take $x_{k-mv} = 0$ and $\lambda_{k-mv} = 0$, for non-positive values of $k - mv$. Also it is obvious that $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) \subset c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) \subset \ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$.

The inclusions are strict follows from the following examples.

Example 1 *Let $m = 2, n = 2, f_k(x) = x^6$, for all k odd and $f_k(x) = x^2$ for all k even, for all $x \in [0, \infty)$ and $p_k = 1$ for all $k \geq 1$. Consider the sequences $\Lambda = (k^8)$ and $x = (\frac{1}{k^6})$. Then x belongs to $c(\mathbf{F}, \Delta^2_{(2)}, \Lambda, p)$, but x does not belong to $c_0(\mathbf{F}, \Delta^2_{(2)}, \Lambda, p)$.*

Example 2 *Let $m = 2, n = 2, f_k(x) = x^2$, for all $k \geq 1$ and $x \in [0, \infty)$ and $p_k = 2$ for all k odd and $p_k = 3$ for all k even. Consider the sequences $\Lambda = \{1, 1, 1, ...\}$ and $x = \{1, 3, 2, 4, 5, 7, 6, 8, 9, 11, 10, 12, ...\}$. Then x belongs to $\ell_\infty(\mathbf{F}, \Delta^2_{(2)}, \Lambda, p)$, but x does not belong to $c(\mathbf{F}, \Delta^2_{(2)}, \Lambda, p)$.*

Lemma 1 *If a sequence space E is solid, then E is monotone.*

3 Main Results

In this section we prove the main results of this article. The proof of the following result is easy, so omitted.

Proposition 1 *The classes of sequences* $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, $c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *and* $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *are linear spaces.*

Theorem 1 *For* $Z = \ell_\infty, c$ *and* c_0*, the spaces* $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *are paranormed spaces, paranormed by*

$$g(x) = \sup_{k\geq 1}((f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{p_k})^{\frac{1}{H}},$$

where $H = \max(1, \sup_{k\geq 1} p_k)$.

Proof. Clearly $g(x) = g(-x)$; $x = \theta$ implies $g(\theta) = 0$.

Let (x_k) and (y_k) be any two sequences belong to anyone of the above spaces. Then we have,

$$\begin{aligned} g(x+y) &= \sup_{k\geq 1}(f_k(|\Delta^n_{(m)}\lambda_k x_k + \Delta^n_{(m)}\lambda_k y_k|))^{\frac{p_k}{H}} \\ &\leq \sup_{k\geq 1}(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{\frac{p_k}{H}} + \sup_{k\geq 1}(f_k(|\Delta^n_{(m)}\lambda_k y_k|))^{\frac{p_k}{H}} \end{aligned}$$

This implies that

$g(x+y) \leq g(x) + g(y)$.

The continuity of the scalar multiplication follows from the following inequality:

$$\begin{aligned} g(\alpha x) &= \sup_{k\geq 1}((f_k(|\Delta^n_{(m)}\alpha\lambda_k x_k|))^{p_k})^{\frac{1}{H}} \\ &= \sup_{k\geq 1}((f_k(|\alpha||\Delta^n_{(m)}\lambda_k x_k|))^{p_k})^{\frac{1}{H}} \end{aligned}$$

$\leq (1 + [|\alpha|])g(x)$, where $[|\alpha|]$ denotes the largest integer contained in $|\alpha|$.

Hence the spaces $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, for $Z = \ell_\infty, c, c_0$ are paranormed spaces paranormed by g.

Theorem 2 *For* $Z = \ell_\infty, c$ *and* c_0*, the spaces* $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *are complete paranormed spaces, paranormed by* g *as defined above.*

Proof. We prove the result for the case $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ and for the other spaces it will follow on applying similar arguments.

Let (x^i) be any Cauchy sequence in $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$. Let $\varepsilon > 0$ be given and let $\varepsilon_1 = \sup_k (f_k(\varepsilon))^{\frac{p_k}{H}}$. Then there exists a positive integer n_0 such that

$$g(x^i - x^j) < \varepsilon_1, \text{ for all } i,\ j \geq n_0.$$

Using the definition of paranorm, we get

$$\sup_{k\geq 1}((f_k(|\Delta^n_{(m)}\lambda_k x^i_k - \Delta^n_{(m)}\lambda_k x^j_k|))^{p_k})^{\frac{1}{H}} < \varepsilon_1, \text{ for all } i,\ j \geq n_0.$$

It follows that

$$|\Delta^n_{(m)}\lambda_k x^i_k - \Delta^n_{(m)}\lambda_k x^j_k| < \varepsilon, \text{ for each } k \geq 1 \text{ and for all } i,\ j \geq n_0.$$

Hence $(\Delta^n_{(m)}\lambda_k x^i_k)$ is a Cauchy sequence in C for all $k \in N$. This implies that $(\Delta^n_{(m)}\lambda_k x^i_k)$ is convergent in C for all $k \in N$. Let $\lim_{i\to\infty} \Delta^n_{(m)}\lambda_k x^i_k = y_k$ for each $k \in N$.

Let $k = 1$, we have

$$\text{(1)} \qquad \lim_{i\to\infty} \Delta^n_{(m)}\lambda_1 x^i_1 = \lim_{i\to\infty} \sum_{v=0}^{n} (-1)^v \binom{n}{v} \lambda_{1-mv} x^i_{1-mv} = y_1$$

Similarly we have,

$$\text{(2)} \qquad \lim_{i\to\infty} \Delta^n_{(m)}\lambda_k x^i_k = \lim_{i\to\infty} \lambda_k x^i_k = y_k, \text{ for } k = 1, ..., nm$$

Thus from (1) and (2) we have $\lim_{i\to\infty} x^i_{1+nm}$ exists. Let $\lim_{i\to\infty} x^i_{1+nm} = x_{1+nm}$. Proceeding in this way inductively, we have $\lim_{i\to\infty} x^i_k = x_k$ exists for each

$k \in N$.

Now we have for all $i, j \geq n_0$,

$$\sup_{k\geq 1}((f_k(|\Delta^n_{(m)}\lambda_k x^i_k - \Delta^n_{(m)}\lambda_k x^j_k|))^{p_k})^{\frac{1}{H}} < \varepsilon$$

Then we have

$$\lim_{j\to\infty}[\sup_{k\geq 1}((f_k(|\Delta^n_{(m)}\lambda_k x^i_k - \Delta^n_{(m)}\lambda_k x^j_k|))^{p_k})^{\frac{1}{H}}] < \varepsilon, \text{ for all } i \geq n_0$$

This implies that

$$\sup_{k\geq 1}((f_k(|\Delta^n_{(m)}\lambda_k x^i_k - \Delta^n_{(m)}\lambda_k x_k|))^{p_k})^{\frac{1}{H}} < \varepsilon$$

It follows that

$$(x^i - x) \in \ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p).$$

Since $(x^i) \in \ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ and $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ is a linear space, so we have $x = x^i - (x^i - x) \in \ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$.

This completes the proof of the Theorem.

Theorem 3 *If* $0 < p_k \leq q_k < \infty$ *for each* k*, then* $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) \subseteq Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, q)$*, for* $Z = c_0$ *and* c.

Proof. We prove the result for the case $Z = c_0$ and for the other case it will follow on applying similar arguments.

Let $(x_k) \in c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$. Then we have

$$\lim_{k\to\infty}(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{p_k} = 0.$$

This implies that

$$f_k(|\Delta^n_{(m)}\lambda_k x_k|) < \varepsilon (0 < \varepsilon \leq 1) \text{ for sufficiently large } k.$$

Hence we get

$$\lim_{k\to\infty}(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{q_k} \le \lim_{k\to\infty}(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{p_k} = 0$$

This implies that $(x_k) \in c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, q)$

This completes the proof.

The following result is a consequence of Theorem 3.

Corollary 1 **(a)** *If* $0 < \inf p_k \le p_k \le 1$, *for each* k, *then* $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p) \subseteq Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda)$, *for* $Z = c_0$ *and* c.

(b) *If* $1 \le p_k \le \sup p_k < \infty$, *for each* k, *then* $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda) \subseteq Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, *for* $Z = c_0$ *and* c.

Theorem 4 $Z(\boldsymbol{F}, \Delta^{n-1}_{(m)}, \Lambda, p) \subset Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *(in general* $Z(\boldsymbol{F}, \Delta^i_{(m)}, \Lambda, p) \subset Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, *for i=1,2,...,n-1), for* $Z = \ell_\infty, c$ *and* c_0.

Proof. We prove the result for $Z = c_0$ and for the other cases it will follow on applying similar arguments.

Let $x = (x_k) \in c_0(\boldsymbol{F}, \Delta^{n-1}_{(m)}, \Lambda, p)$. Then we have

$$\lim_{k\to\infty}(f_k(|\Delta^{n-1}_{(m)}\lambda_k x_k|))^{p_k} = 0 \tag{3}$$

Now we have

$$f_k(|\Delta^n_{(m)}\lambda_k x_k|) \le f_k(|\Delta^{n-1}_{(m)}\lambda_k x_k|) + f_k(|\Delta^{n-1}_{(m)}\lambda_{k-m} x_{k-m}|)$$

Hence we have

$$(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{p_k} \le D\{(f_k(|\Delta^{n-1}_{(m)}\lambda_k x_k|))^{p_k} + (f_k(|\Delta^{n-1}_{(m)}\lambda_{k-m} x_{k-m}|))^{p_k}\}$$

Then using (3), we get

$$\lim_{k\to\infty}(f_k(|\Delta^n_{(m)}\lambda_k x_k|))^{p_k}=0$$

Thus $c_0(\boldsymbol{F},\Delta^{n-1}_{(m)},\Lambda,p)\subset c_0(\boldsymbol{F},\Delta^n_{(m)},\Lambda,p)$.
The inclusion is strict follows from the following example.

Example 3 *Let* $m=3, n=2, f_k(x)=x^{10}$, *for all* $k\geq 1$ *and* $x\in[0,\infty)$ *and* $p_k=5$ *for all* k *odd and* $p_k=3$ *for all* k *even. Consider the sequences* $\Lambda=(\frac{1}{k^3})$ *and* $x=(k^4)$. *Then* $\Delta^2_{(3)}\lambda_k x_k=0$, *for all* $k\in N$. *Hence* $x\in c_0(\mathbf{F},\Delta^2_{(3)},\Lambda,p)$. *Again we have* $\Delta^1_{(3)}\lambda_k x_k=-3$, *for all* $k\in N$. *Hence* x *does not belong to* $c_0(\mathbf{F},\Delta^1_{(3)},\Lambda,p)$. *Thus the inclusion is strict.*

Theorem 5 *The spaces* $c_0(\boldsymbol{F},\Delta^n_{(m)},\Lambda,p)$, $c(\boldsymbol{F},\Delta^n_{(m)},\Lambda,p)$ *and* $\ell_\infty(\boldsymbol{F},\Delta^n_{(m)},\Lambda,p)$ *are not monotone and as such are not solid in general.*

Proof. The proof follows from the following example.

Example 4 *Let* $n=2, m=3, p_k=1$ *for all* k *odd and* $p_k=2$ *for all* k *even and* $f_k(x)=x^4$, *for all* $x\in[0,\infty)$ *and* $k\in N$. *Then* $\Delta^2_{(3)}\lambda_k x_k=\lambda_k x_k-2\lambda_{k-3}x_{k-3}+\lambda_{k-6}x_{k-6}$, *for all* $k\in N$. *Consider the* J^{th} *step space of a sequence space* E *defined as, for* $(x_k),(y_k)\in E^J$ *implies that* $y_k=x_k$ *for* k *odd and* $y_k=0$ *for* k *even. Consider the sequences* $\Lambda=(k^3)$ *and* $x=(\frac{1}{k^2})$. *Then* $x\in Z(\boldsymbol{F},\Delta^2_{(3)},\Lambda,p)$ *for* $Z=\ell_\infty, c$ *and* c_0, *but its* J^{th} *canonical pre-image does not belong to* $Z(\boldsymbol{F},\Delta^2_{(3)},\Lambda,p)$ *for* $Z=\ell_\infty, c$ *and* c_0. *Hence the spaces* $Z(\boldsymbol{F},\Delta^2_{(3)},\Lambda,p)$ *for* $Z=\ell_\infty, c$ *and* c_0 *are not monotone and as such are not solid in general.*

Theorem 6 *The spaces* $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, $c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *and* $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *are not symmetric in general.*

Proof. The proof follows from the following example.

Example 5 *Let* $n = 2, m = 2, p_k = 2$ *for all* k *odd and* $p_k = 3$ *for all* k *even and* $f_k(x) = x^2$, *for all* $x \in [0, \infty)$ *and for all* $k \geq 1$. *Then* $\Delta^2_{(2)}\lambda_k x_k = \lambda_k x_k - 2\lambda_{k-2}x_{k-2} + \lambda_{k-4}x_{k-4}$, *for all* $k \in N$. *Consider the sequences* $\Lambda = (1, 1, 1, \dots)$ *and* $x = (x_k)$ *defined as* $x_k = k$ *for* k *odd and* $x_k = 0$ *for* k *even. Then* $\Delta^2_{(2)}\lambda_k x_k = 0$, *for all* $k \in N$. *Hence* $(x_k) \in Z(\boldsymbol{F}, \Delta^2_{(2)}, \Lambda, p)$ *for* $Z = \ell_\infty, c$ *and* c_0. *Consider the rearranged sequence,* (y_k) *of* (x_k) *defined as*

$$(y_k) = (x_1, x_3, x_2, x_4, x_5, x_7, x_6, x_8, x_9, x_{11}, x_{10}, x_{12}, ...)$$

Then (y_k) *does not belong to* $Z(\boldsymbol{F}, \Delta^2_{(2)}, \Lambda, p)$ *for* $Z = \ell_\infty, c$ *and* c_0.
Hence the spaces $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *for* $Z = \ell_\infty, c$ *and* c_0 *are not symmetric in general.*

Theorem 7 *The spaces* $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, $c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *and* $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ *are not convergence free in general.*

Proof. The proof follows from the following example.

Example 6 *Let* $m = 3, n = 1, p_k = 6$ *for all* k *and* $f_k(x) = x^3$, *for* k *even and* $f_k(x) = x$, *for* k *odd, for all* $x \in [0, \infty)$. *Then* $\Delta^1_{(3)}\lambda_k x_k = \lambda_k x_k - \lambda_{k-3}x_{k-3}$, *for all* $k \in N$. *Let* $\Lambda = (\frac{7}{k})$ *and consider the sequences* (x_k) *and* (y_k) *defined as* $x_k = \frac{4}{7}k$ *for all* $k \in N$ *and* $y_k = \frac{1}{7}k^3$ *for all* $k \in N$. *Then*

(x_k) belongs to $Z(\boldsymbol{F}, \Delta^1_{(3)}, \Lambda, p)$, but (y_k) does not belong to $Z(\boldsymbol{F}, \Delta^1_{(3)}, \Lambda, p)$ for $Z = \ell_\infty, c$ and c_0. Hence the spaces $Z(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ for $Z = \ell_\infty, c$ and c_0 are not convergence free in general.

Theorem 8 *The spaces $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, $c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ and $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ are not sequence algebra in general.*

Proof. The proof follows from the following examples.

Example 7 *Let $n = 2, m = 1, p_k = 1$ for all k and $f_k(x) = x^{22}$, for each $k \in N$ and $x \in [0, \infty)$. Then $\Delta^2_{(1)}\lambda_k x_k = \lambda_k x_k - 2\lambda_{k-1}x_{k-1} + \lambda_{k-2}x_{k-2}$, for all $k \in N$. Consider $\Lambda = (\frac{1}{k^4})$ and let $x = (k^5)$ and $y = (k^6)$. Then x, y belong to $Z(\boldsymbol{F}, \Delta^2_{(1)}, \Lambda, p)$ for $Z = \ell_\infty, c$, but $x.y$ does not belong to $Z(\boldsymbol{F}, \Delta^2_{(1)}, \Lambda, p)$ for $Z = \ell_\infty, c$. Hence the spaces $c(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$, $\ell_\infty(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ are not sequence algebra in general.*

Example 8 *Let $n = 2, m = 1, p_k = 3$ for all k and $f_k(x) = x^5$, for each $k \in N$ and $x \in [0, \infty)$. Then $\Delta^2_{(1)}\lambda_k x_k = \lambda_k x_k - 2\lambda_{k-1}x_{k-1} + \lambda_{k-2}x_{k-2}$, for all $k \in N$. Consider $\Lambda = (\frac{1}{k^7})$ and let $x = (k^8)$ and $y = (k^8)$. Then x, y belong to $c_0(\boldsymbol{F}, \Delta^2_{(1)}, \Lambda, p)$, but $x.y$ does not belong to $c_0(\boldsymbol{F}, \Delta^2_{(1)}, \Lambda, p)$ for $Z = \ell_\infty, c$. Hence the space $c_0(\boldsymbol{F}, \Delta^n_{(m)}, \Lambda, p)$ is not sequence algebra in general.*

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