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# EXPERIMENTAL NONLINEAR VIBRATION ANALYSIS OF PIEZOELECTRICALLY ACTUATED MICROCANTILEVERS

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## ABSTRACT
With daily growth of using microcantilevers in microelectromechanical systems, comprehensive analysis on their dynamical behavior is necessary since they are mostly utilized as the main sensing device. In this paper, the out-of-plane vibrations of the piezoelectrically actuated microcantilever are experimentally investigated. The microcantilever is covered with a piezoelectric layer on its top surface through which it can be excited by applying the voltage to the piezoelectric actuator. The nonlinear frequency response of the microcantilever is studied and shift in natural frequency due to nonlinearity is examined. By observing the subharmonics of the fundamental frequencies at 2X and 3X, it is experimentally shown that there exist cubic and quadratic nonlinearities in the microcantilever. A mathematical model based on these experimental tests is then proposed and verified. The out-of-plane measurements provide the ability to observe both transversal and torsional modes. In addition, the modes in which the microcantilever acts like a plate are observed.

## INTRODUCTION
Microcantilevers find many applications in nanomechanical sensors/actuators and especially piezoelectrically-actuated microcantilevers have recently received considerable attention since they are capable of better actuation. The sensing/actuating operation is based on static and dynamic deflections of the microcantilevers. However, measurement of the dynamic vibrations of the microcantilever is the base sensing strategy. Therefore, a nonlinear comprehensive experimental study on the frequency response of these microcantilevers seems to be essential since in such small scale even very small excitations can provide large amplitude and consequently nonlinear vibrations [1, 2]. Applications of this research can be expanded to several instruments, e.g., scanning force microscopy [3, 4], chemical/biological mass and surface stress sensing [5-7], precision optical sensing [8] and other applications [9].
Piezoelectric composite microcantilevers were first utilized for sensing purposes [10] and then utilized as actuators [11]. The structure of this type of microcantilever consists of a main metallic layer which is usually silicon based layer such as Si or $SiO_2$ and the piezoelectric layer usually covers a part of one side of the microcantilever (monolayer piezoelectric microcantilever) such as ZnO. Fig. 1 shows a microscopic image of the piezoelectrically-actuated microcantilever and its components.

The nonlinear-flexural vibrations of piezoelectrically-actuated microcantilever for fundamental resonance were studied [2, 12, 13], and it was shown, as expected for cantilevers, that there are cubic nonlinearities due to inertia and stiffness in the model and for large excitations jump phenomenon is expected. It is shown in this paper that these cubic nonlinearities produce subharmonic resonance. It is also expected to observe subharmonic in the response due to quadratic nonlinearity. This type of nonlinearity is predicted to be added to the system due to nonlinearity of the piezoelectric material which is induced in the response through the electromechanical coupling into the piezoelectric actuator [14, 15]. Therefore, by experimentally investigating the response of the microcantilever, a general theoretical model can be obtained and verified. The results are valuable not only for modeling the systems but also for control purposes [16].

In this paper, vibration and frequency response of the microcantilever are investigated. First, three natural frequencies of the microcantilever are obtained and linear and nonlinear phenomena occurred during frequency response sweep are studied to find the nonlinearities in the system to present a general model for the microcantilever. In addition, mode shapes of the microcantilever are experimentally obtained and presented.

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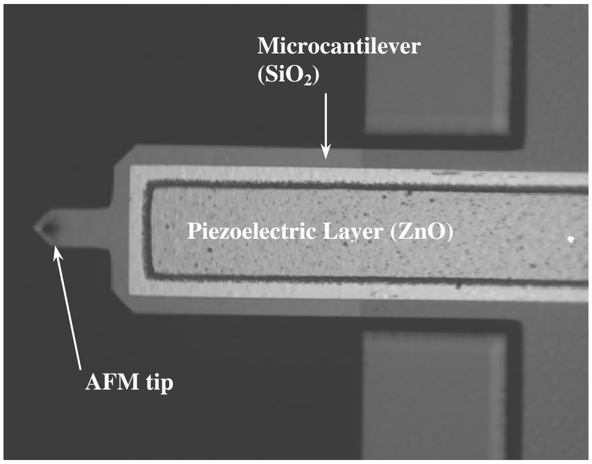


**Figure 1.** Microcantilever with piezoelectric layer for Atomic Force Microscopy (AFM) applications.

## EXPERIMENTAL SETUP AND METHOD

The experimental investigation is performed utilizing the state-of-the-art MSA-400 microsystem analyzer which is equipped with laser interferometry measurement system to provide both in-plane and out-of-plane displacement and velocity measurements at nanoscale. Eight microcantilevers are utilized for the test and they all have the same shape as shown in Fig. 1. The piezoelectric layer is made of one 3.5μm Zinc Oxide (ZnO) layer and two 0.25μm Titanium-Gold (Ti/Au) layers [2]. The mechanical properties of all specimens slightly vary; however, due to precision manufacturing the geometrical dimensions of the microcantilevers are quite similar. This causes some changes in natural frequencies of the microcantilevers but the overall response properties show similar behavior. These properties are presented in Table 1.

**Table 1.** Geometrical and mechanical properties of the microcantilever.

| *Property* | *Value* |
|---|---|
| Density of piezoelectric layer | 6390 $kg/m^3$ |
| Density of Si | 2330 $kg/m^3$ |
| Length of the piezoelectric layer | 375±5 μm |
| Length of the Si microcantilever | 500±5 μm |
| Thickness of the piezoelectric layer | 4±0.5 μm |
| Thickness of the Si microcantilever | 4±0.5 μm |
| Width of the microcantilever tip part | 55±2 μm |
| Width of the piezoelectric layer | 130±5 μm |
| Width of the Si microcantilever | 250±5 μm |
| Young's Modulus of piezoelectric layer | 130±5 GPa |
| Young's Modulus of Si | 180±5 GPa |

Each microcantilever is installed in a holder as shown in Fig. 2. The holder provides the sockets for connecting the piezoelectric layer to a voltage source for actuation of the microcantilever. The voltage source for experiments is provided by the MSA-400 microsystem analyzer which can produce different signal types in the range of 0 to 10 volts and up to 20 MHz. The piezoelectric layer of the microcantilever is also limited to 10 Volts input. In the experiments presented here, the excitation voltage is in the range of 2 to 9 volts.

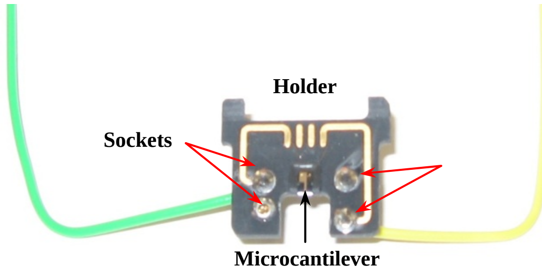


**Figure 2.** Piezoelectric actuated microcantilever on the holder.

The holder is then assembled on a goniometer which can provide the ability to rotate the microcantilever to make it completely horizontal under the laser and microscope. Fig. 3 demonstrates the microcantilever holder installed in the goniometer.

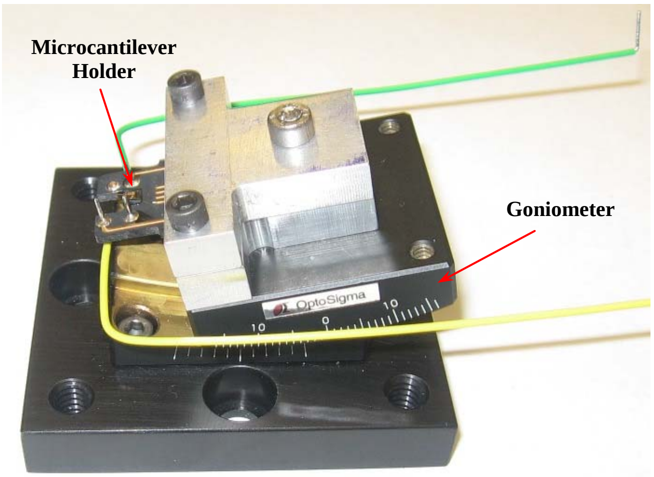


**Figure 3.** Microcantilever holder on the goniometer.

The complete setup including microcantilever, the holder and the goniometer is then placed on a microstage under the laser sensor of the MSA-400 as shown in Fig. 4. The microstage system provides the 3D motions to place the microcantilever tip in proper range and position under the laser sensor for vibration measurement.

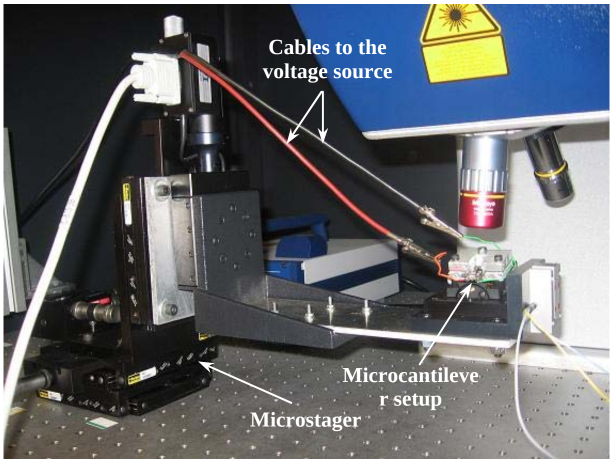


**Figure 4.** Test platform.

The experimental results are produced by MSA-400 compatible software. The excitation voltage for the piezoelectric layer is provided by the MSA-400 in different formats and amplitudes. For these experiments, chirp signal with 2-9 Volts amplitudes are applied to the piezoelectric layer. Once the microcantilever is placed under the laser, the piezoelectric layer is connected to the excitation voltage through connection cables. Scanning points on the microcantilever for measuring the vibration can be defined for the MSA-400 software; then the laser automatically reads the vibrations of each point. Therefore, one can define large numbers of points on the microcantilever to almost cover the entire surface of the microcantilever, as shown in Fig. 5. This way, three dimensional (out-of-plane) motion of the microcantilever can be measured. Therefore, not only flexural but also torsional vibrations can be recognized simultaneously. The measured signal is then plotted in frequency domain where the natural frequencies and mode shapes can be investigated. This also provides shapes and frequencies of subharmonics and interaction of modes in the frequency response as it will be discussed next.

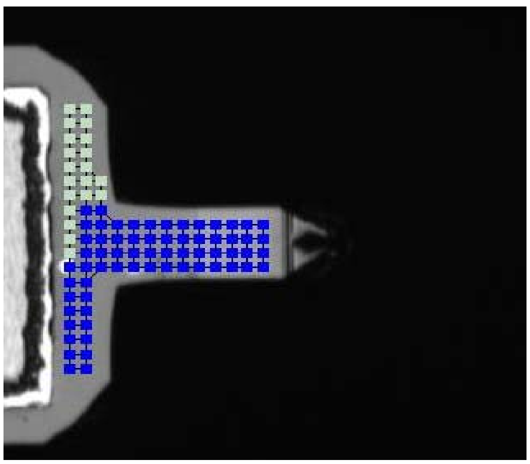

**Figure 5.** Test procedure.

## EXPERIMENTAL RESULTS AND THEORETICAL MATCHING

In this section, the experimental results for frequency response of the piezoelectrically actuated microcantilever are presented, followed by predicting the nonlinear model of the systems using experimental observation and theoretical background. By applying a chirp periodic signal with an amplitude of 9 Volts to the piezoelectric and sweeping the frequencies of 0 to 400 kHz, the frequency response can be obtained as shown in Fig. 6.

Here, each peak in the obtained frequency response of Fig. 6 is investigated. First, natural frequency of the microcantilever occurs at 52.03 kHz while the second one is at 203.0 kHz. Figs. 7 and 8 depict the mode shapes of the first and second natural frequencies, respectively. The velocity of all nodes are experimentally measured then the software produced the three dimensional representation of the mode shape. The laser vibrometer measures velocity so all the amplitudes are based on velocity of the vibration.

The mode shape, as expected is similar to linear mode, however it will be shown that there is a difference between linear and nonlinear natural frequencies. The linear mode shapes, $\phi(x)$, and natural frequencies, $\omega_n$, can be theoretically obtained using the following relations [17].

$$\phi_n(s) = \cosh(\beta_n s) - \cos(\beta_n s) + [\sin(\beta_n s) - \sinh(\beta_n s)]\frac{\cosh(\beta_n l) + \cos(\beta_n l)}{\sin(\beta_n l) + \sinh(\beta_n l)}, \quad (1)$$

$$\omega_n = \beta_n \sqrt{\frac{EI}{ml^4}}, \quad (2)$$

where $s$ is displacement variable, $l$ is length of the microcantilever, $n$ is the number of mode, $m$ is linear mass density, $E$ is Young's modulus of elasticity and $I$ is the mass moment of inertia of the microcantilever and $\beta_n$ is obtained by

$$1 + \cos(\beta_n l)\cosh(\beta_n l) = 0. \quad (3)$$

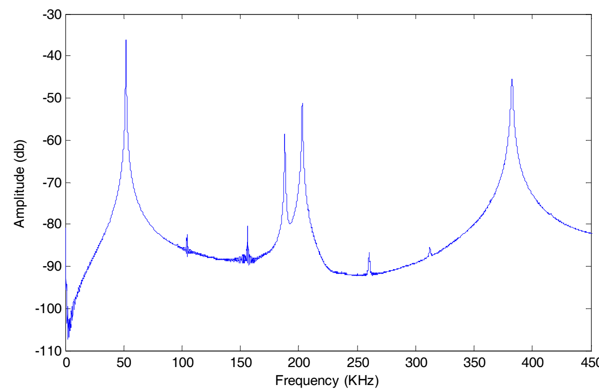


**Figure 6.** Frequency response of the piezoelectrically actuated microcantilever by a 9 Volts excitation.

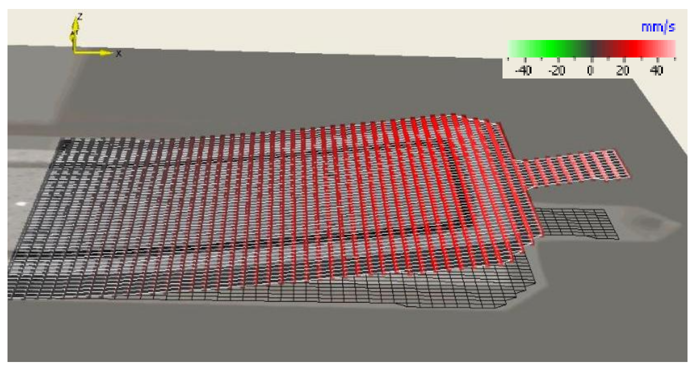


**Figure 7.** 3D response of the first mode at 52.03 kHz.

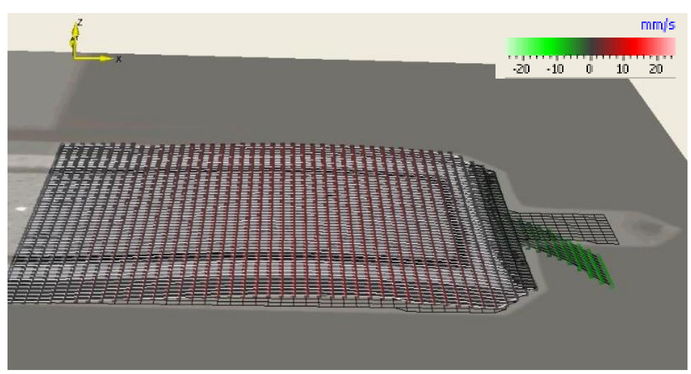


**Figure 8.** 3D response of the second mode at 203.0 kHz.

However, it should be noted that since the microcantilever has two layers with variable width along its length, its properties are not constant and should be taken into account. More detailed calculations are given in [2, 18].

The nonlinear phenomena can be observed by a closer look at the nonlinear frequency response as depicted in Fig. 9. It shows the response when the excitation frequency, Ω, is 9 Volts and is close to fundamental natural frequency, $\omega_1$=52028 Hz. It demonstrates that the response is nonlinear and there is a shift of frequency to the left. This is due to quadratic and/or cubic nonlinearities. In addition, Fig. 6 shows that there are two subharmonic resonances of the first natural frequency at 104 kHz and 156 kHz. Referring the details to [2, 15, 19], the 2X subharmonic is due to quadratic and 3X subharmonic is the result of cubic nonlinearities in the system.

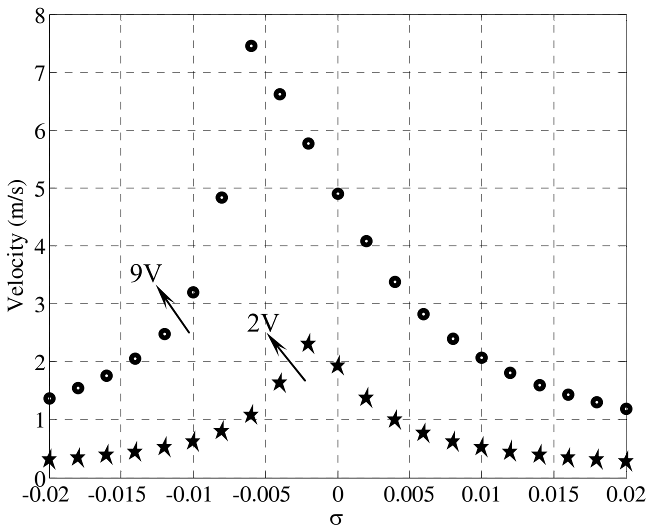


**Figure 9**. Frequency response of piezoelectrically actuated microcantilever.

The 2X and 3X subharmonics are demonstrated in Figs. 10 and 11, respectively. Knowing the two reasons mentioned in the preceding paragraph and utilizing the models presented in [15], the dynamic model of the microcantilever can be obtained as

$$m(s)\ddot{v} + \left(EI(s)v''\right)'' + \left(\alpha(s)v''^2\right)'' + \left[v'\left(EI(s)v'v''\right)'\right]' + \left[v'\int_l^s m(s)\int_0^s\left(\ddot{v}'v' + \dot{v}'^2\right)ds\,ds\right]' - \left[\frac{1}{2}v'\left[K_p(s)v'P(t)\right]'\right]' + \left[\frac{1}{4}K_p(s)v'^2P(t)\right]'' + HOT(v) = \left[\frac{1}{2}K_p(s)P(t)\right]'' \quad (4)$$

where $v(x,t)$ is the transversal vibration, $P(t)$ is the applied voltage to the piezoelectric layer, $K_p$ and α depend on the properties of the piezoelectric layer and silicon layers, respectively, and *HOT* stands for higher order nonlinear terms. Looking at the response in Fig. 6, there is a peak at 188.0 kHz, close to second mode, where the microcantilever performs torsional vibration combined with small flexural vibration as depicted in Fig. 12. This phenomenon can be due to two reasons; the interaction between flexural and torsional modes or because of lateral force produced by the piezoelectric layer.

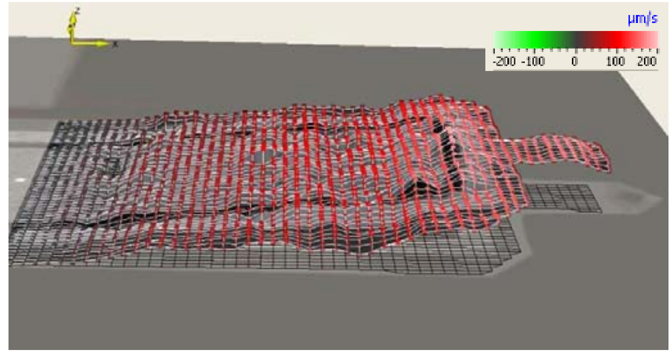


**Figure 10**. 3D response of the 2×fundamental frequency at 104.2 kHz.

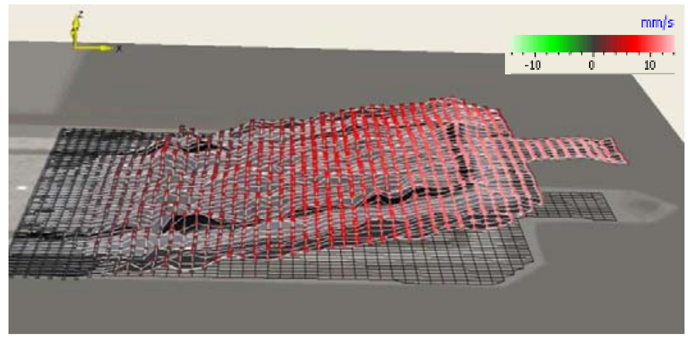


**Figure 11**. 3D response of the 3×fundamental frequency at 156.2 kHz.

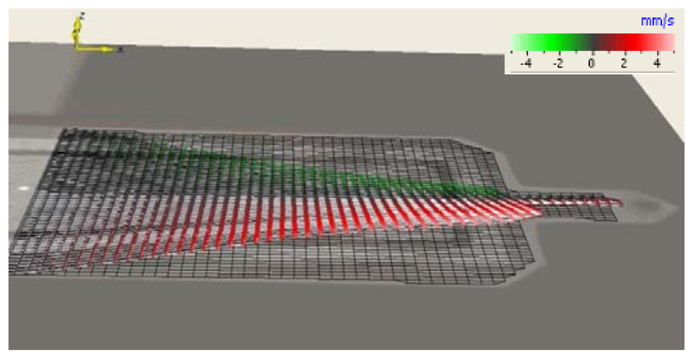


**Figure 12**. 3D response of the torsional frequency at 188 kHz.

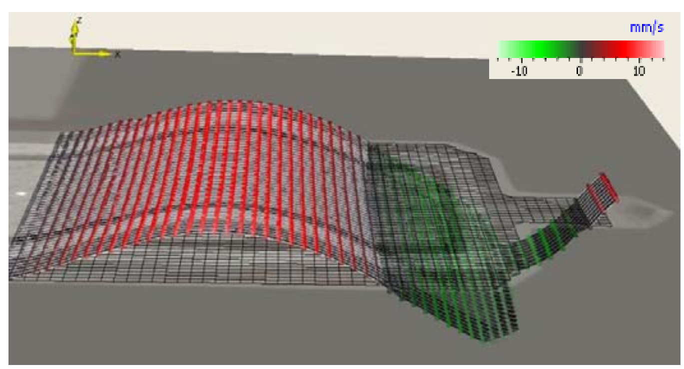


**Figure 13**. 3D response of the third mode at 382.5 kHz.

Fig. 13 demonstrates the third mode shape of the microcantilever which occurs at 382.5 kHz. Above the second natural frequency due to large width to length ratio, the microcantilever starts to show plate-like response. Fig. 14 depicts the microcantilever response at 260.2 kHz. It is, therefore, not recommended to use the microcantilever at this high frequency if one expects behaviors similar to beam. Between first and second natural frequencies, there are two peaks due to subharmonics. However, these peaks are not clear, and in addition, near the second natural frequency there is a torsional mode which should be avoided if only bending vibration is utilized for sensing applications. Higher than second mode, the response has lower amplitude and can produce plate- like response.

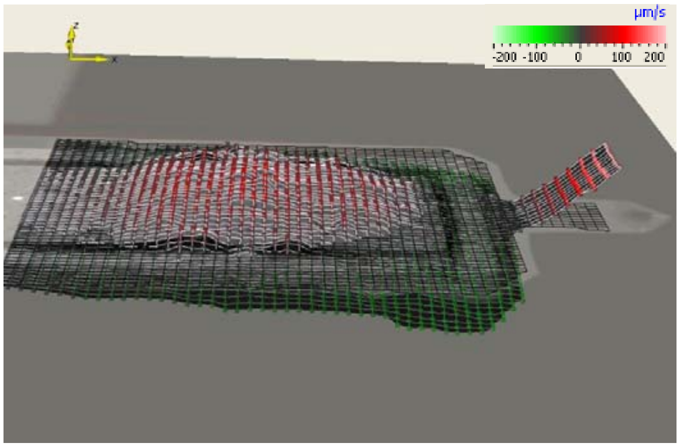

**Figure 14**. 3D response of the plate mode at 260 kHz.

## CONCLUSIONS

A comprehensive experimental investigation on nonlinear response of the piezoelectrically actuated microcantilever has been performed. The results demonstrated a large amplitude flexural vibration due to excitation of the piezoelectric layer which leads to nonlinear vibration of the microcantilever. The nonlinearity appeared as frequency shift at natural frequencies and subharmonic resonances at 2X and 3X of natural frequencies which is due to presence of quadratic and cubic nonlinearities in the system. The out-of-plane measurements demonstrated a torsional vibration near second flexural natural frequency and a plate-like mode vibration above this frequency. These interesting phenomena show the interaction between flexural and torsional modes and plate modes which needs more investigations.

## ACKNOWLEDGMENT

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## REFERENCES

[1] Xie, W.C. and Lee H.P. and Lim S.P., 2003, "Nonlinear Dynamic Analysis of MEMS Switches by Nonlinear Modal Analysis", *Nonlinear Dynamics,* **31**, 243–256.

[2] Mahmoodi, S.N. and Jalili, N., 2007, "Non-linear vibrations and frequency response analysis of piezoelectrically driven microcantilevers", International *Journal of Non-Linear Mechanics*, **42**, 577 – 587.

[3] Giessibl, F.J., 2003, "Advances in Atomic Force Microscopy**",** *Reviews of Modern Physics*, **75**, 949-983.

[4] Garcia, R. and Perez, R., 2002, "Dynamic Atomic Force Microscopy Methods"**,** *Surface Science Reports***,** **47**(6-8), 197-301.

[5] Mahmoodi, S.N., Afshari, M., and Jalili, N., 2008, "Nonlinear vibrations of piezoelectric microcantilevers for biologically-induced surface stress sensing", *Communications in Nonlinear Science and Numerical Simulation*, **13**, 1964-1977.

[6] Gunter, R.L. , Zhine, R., Delinger, W.G., Manygoats, K., Kooser, A. and Porter, T.L., 2004, "Investigation of DNA Sensing Using Piezoresistive Microcantilever Probes"**,** *IEEE Sensors Journal*, **4**, (4), 430-433.

[7] Van Steenkiste, F., Baert, K., Debruyker, D., Spiering, V., Van Der Schoot, B., Arquint, P., Born, R. and Schumann, L., 1997, "A Microsensor Array for Biochemical Sensing**",** *Sensors and Actuators B (Chemical)*, **B44**(1-3), Oct. 1997, 409-412.

[8] Ramos, D., Tamayo, J., Mertens, J. and Calleja, M., 2006, "Photothermal excitation of microcantilevers in liquids", *Journal of Applied Physics*, **99**, 124904.

[9] Adams, J.D., Rogers, B. and Whitten, R., 2005, "Microcantilever Technology for Law Enforcement and Anti-Terrorism Applications: Chemical, Biological and Explosive Material Detection", *Proc. of SPIE*, *Sensors, and Command, Control, Communications, and Intelligence (C3I) Technologies for Homeland Security and Homeland Defense IV,* **5778** (Part I), 139-150.

[10] Itoh, T., and Suga, T., 1994, "Scanning Force Microscpes Using A Piezoelectric Microcantilever", *Journal of Vacuum Science and Technology*, **12**(3), 1581-1585.

[11] Itoh, T., Lee, C. and Suga, T., 1996, "Deflection Detection and Feedback Actuation Using A Self-Excited Piezoelectric Pb(Zr,Ti)$O_3$ Microcantilever for Dynamic Scanning Force Microscopy", *Applied Physic Letter*, **69** (14), 2036-2038.

[12] Mahmoodi, S.N., and Jalili, N., 2008, "Coupled Flexural-Torsional Nonlinear Vibrations of Piezoelecrically-actuated Microcantilevers with Application to Friction Force Microscopy", *ASME Journal of Vibration and Acoustics*, **130**, 1-10.

[13] Zhang, W. and Guang Meng, G., 2005, "Nonlinear Dynamical System of Micro-Cantilever under Combined Parametric and Forcing Excitations in MEMS**",** *Sensors and Actuators A (Physical)*, **119**(2), 291-299.

[14] Guyomar, D., Aurelle, N. and Eyraud, L., 1997, "Piezoelectric ceramics nonlinear behavior. Application to Langevin transducer", *Journal De Physique III*, **7**(6), 1197-1208.

[15] Mahmoodi, S.N. Jalili, N., and Daqaq, M.F., 2008, "Modeling, Nonlinear Dynamics, and Identification of a Piezoelectrically-Actuated Microcantilever Sensor", *ASME/IEEE Transactions on Mechatronics*, **13**, 58-65.

[16] Zhang, W., Meng, G. and Li, H., 2006, "Adaptive Vibration Control of Micro-cantilever Beam with Piezoelectric Actuator in MEMS", *International Journal of Advanced Manufacturing Technology*, **28**, 321–327.

[17] Meirovitch, L., *Analytical Methods in Vibrations*, Prentice Hall, 1997.

[18] Dadfarnia, M., Jalili, N., Xian, B., and Dawson, D. M., 2004, "A Lyapunov-Based Piezoelectric Controller for

Flexible Cartesian Robot Manipulators", *ASME Journal of Dynamic Systems, Measurement, and Control*, **126**, 347-358.

[19] Nayfeh, A.H. and Nayfeh, S.A., 1995, "Nonlinear normal modes of a continuous system with quadratic nonlinearities", *Transactions of the ASME, Journal of Vibration and Acoustics*, **117**(2), 199-205.