THE CAPM AND VALUE AT RISK AT DIFFERENT TIME SCALES

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Abstract

Wavelet analysis, a refinement of Fourier analysis that was developed in the late 1980's, is a powerful tool for decomposing time series data into orthogonal components with different frequencies. Each frequency is localized in the time domain, which makes it possible to quantify correlations between time series at different time horizons.

In this article, we focus on the estimation of the capital asset pricing model (CAPM) at different time scales for Chile's stock market. Our sample is comprised of twenty four stocks that were actively traded on the Santiago Stock Exchange over 1997–2002. We find evidence in support of the CAPM at a medium–term horizon. We extend the literature in this area to analyze the impact of time scaling on the computation of value at risk. We conclude that risk is concentrated at the higher frequencies of the data.

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I Introduction

The basic capital asset pricing model (CAPM), a corner-stone of modern finance, states that the risk premium of an individual asset equals its beta times the risk premium on the market portfolio. Beta measures the degree of co-movement between the asset's return and the return on the market portfolio. In other words, beta quantifies the systematic risk of an asset—the amount of risk that cannot be diversified away. The CAPM was the result of independent work by Sharpe (1964), Lintner (1965), and Mossin (1966).

In recent years, however, the CAPM has been questioned by several empirical studies. In particular, Fama and French (1992)'s work announced the death of beta. Using a sample for 1963-1990, they found that beta does a poor job of explaining the cross-section variation of average returns, as opposed to the book-to-market ratio and market capitalization (firm size). Kothari and Shanken (1998), however, conclude that Fama and French's results hinge on using monthly rather than yearly returns. Kothari and Shanken argue that the use of annual returns to estimate betas helps to circumvent measurement problems caused by non-synchronous trading, seasonality in returns, and trading frictions. Based on annual returns over 1927-1990, the authors conclude that the betas are statistically significant, and that the incremental contribution of size to explaining cross-section differences in returns, beyond beta, is small.

Simultaneously, several authors have worked on theoretical extensions of the CAPM: the after-tax CAPM that accounts for the fact that investors have to pay higher taxes on high-dividend yield stocks, and, therefore, must be compensated with higher pre-tax returns; the intertemporal CAPM that deals with a multi-period setting; the consumption CAPM that states that security returns will be closely correlated with aggregate economic output, as investors are concerned with protecting their consumption over economic slowdowns; the international asset pricing model (IAPM) that establishes the conditions under which fully integrated capital markets are in equilibrium (see Megginson, 1997, for a through discussion and citations).

Another topic that has been covered in the empirical literature of the CAPM, and that it is connected with our research, is the testing of asset pricing models that allow for a time-varying beta, a time-varying risk premium, or both. This research area has received the name of tests of the conditional CAPM because the next period expected return and/or variance is routinely updated conditional on the most recent past information. Typically, the machinery used in such testing is generalized autoregressive conditional heteroscedastic (GARCH) and GARCH-in-mean (GARCH-M) processes (e.g., Engle, Lilien and Robins, 1987; Bollerslev, Engle, and Wooldridge, 1988). An alternative, but promising, approach is wavelet methods.

Wavelet analysis is a refinement of Fourier analysis that was developed in the late 1980's, and which offers a powerful methodology for processing signals, images, and other types of data. In particular, the discrete wavelet transform allows for the decomposition of time series data into orthogonal components with different frequencies. This makes it possible to quantify correlations between markets at different time horizons. As discussed in the next sections, wavelets will allow us to estimate the CAPM for different time horizons. Recent applications of wavelet methods in economics and finance are Ramsey and Lampart (1998), Norsworthy, Li and Gorener (2000), Lee (2001a, 2001b), Li and Stevenson (2001), and Gençay, Whitcher, and Selcuk (2003).

Ramsey and Lampart (1998) study the permanent income hypothesis, and conclude that time–scale decomposition is very important to analyzing economic relationships. In particular, they find that an appropriate way to model the consumption–income relationship during the post–war period is at the time scale dominated by a trend. At lower scales (i.e., higher frequencies), the degree of fit and the slope of the consumption–income relationship declines monotonically, except for the lowest scale.

Norsworthy, Li and Gorener (2000) and Gençay, Whitcher, and Selçuk (2003) apply wavelet analysis to the estimation of the CAPM. The main conclusion of Norsworthy et al. is that the major part of the market's influence on an individual asset return is at higher frequencies. In other words, the beta coefficient will generally decrease when regressing an individual asset return on the smoother components of the market portfolio. Moreover, the R^2 of such regressions will generally decline as the frequency of the market portfolio decreases.

Unlike Norsworthy et al., who model an individual asset return on different time scales of the market portfolio, Gençay et al. focus on a portfolio and calculate the wavelet variance of the market return and the wavelet covariance between the market return and the portfolio return at each scale to obtain the corresponding portfolio beta. Their finding is that the relationship between the return on a portfolio and its beta becomes stronger as the scale increases. That is to say, the predictions of the CAPM model are more relevant at the medium-term than at short-time horizons.

Lee (2001a) studies the interaction between the U.S. and the South Korean stock markets. Using the KOSPI and the DJIA, and the KOSDAQ and the NASDAQ, he finds evidence of price and volatility spillover effects from the U.S. to South Korea, but not vice versa. Lee concludes that his findings confirm the importance of innovations in developed stock markets to the determination of stock returns and volatility in emerging economies. In turn Lee (2001b) illustrates the use of wavelet analysis for seasonality filtering of time-series data.

Li and Stevenson (2001) use wavelets to study the relation between futures and spot prices. They find that the lead-lag relationship between the spot and the futures index prices, well-documented in the literature, is more persistent when more detailed information is used for price reconstruction. Therefore, if the non-contemporaneous relationship between the spot and the futures indices is due to market imperfections, investors should concentrate exclusively on those imperfections that are likely to take place in the very short run.

Further discussion of the use of wavelets in economics and finance can be found in Ramsey (2002).

This article is organized as follows. Section II presents a brief theoretical background on the CAPM and wavelets. Section III shows our estimation results for the CAPM and value at risk at different time scales, using a sample of firms that traded actively on the Santiago Stock Exchange over 1997-2002. Finally, Section IV presents our main conclusions.

II Theoretical Background

2.1 A Market Equilibrium Model: The CAPM

The capital asset pricing model states that the equilibrium rate of return on all risky assets is a function of their covariance with the market portfolio. The derivation of the CAPM equation is based upon the assumptions of risk-averse investors, frictionless markets, absence of information costs and information asymmetries, unlimited borrowing and lending at the risk-free rate, and perfect divisibility and marketability of financial assets (see, for instance, Copeland, Weston, and Shastri, 2004; Megginson, 1997).

The CAPM establishes that the expected return on any risky satisfy the equation:

$$E(R_i) = R_f + \beta_i E(R_m - R_f), \tag{1}$$

where R_i is the return on asset i, R_f is the risk-free rate, R_m is the return on the market portfolio, and $\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$ is the asset's beta.

The $(E(R_m)–R_f)$ term is referred to as the market risk premium, given that it represents the return over the risk-free rate required by investors to hold the market portfolio.

Equation (1) can be re-written as

$$E(R_i) - R_f = \beta_i E(R_m - R_f). \tag{2}$$

This says that the risk premium on an individual asset equals its beta time the market risk premium.

The empirical version of equation (2) is given by

$$R_i - R_f = \alpha_i + \beta_i (R_m - R_f) + \varepsilon_i, \tag{3}$$

where ε_i is a random error term. From this expression, we can obtain the following relationship for the variance of the return on asset i:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2. \tag{4}$$

Equation (4) says the total risk of an asset (variance) can be partitioned into systematic risk (a measure of how the asset covariates with the economy), $\beta_i^2\sigma_m^2$, and unsystematic risk (independent of the economy), σ_ε^2.

2.2 Wavelet Analysis in a Nutshell

Wavelets or short waves are similar to sine and cosine functions in that they also oscillates about zero. However, as its name indicates, oscillations of a wavelet fade away around zero, and the function is localized in time or space.[2] In wavelet analysis, a signal (i.e., a sequence of numerical measurements) is represented as a linear combination of wavelet functions.

Unlike Fourier series, wavelets are suitable building-block functions for signals whose features change over time, and for non-smooth signals. A wavelet allows for decomposing a signal into multi–resolution components: fine and coarse resolution components.

There are father wavelets ϕ and mother wavelets ψ such that

$$\int \phi(t)dt = 1 \qquad\qquad \int \psi(t)dt = 0\,. \tag{1}$$

Father wavelets are good at representing the smooth and low-frequency parts of a signal, whereas mother wavelets are good at representing the detailed and high-frequency parts of a signal. The most commonly used wavelets are the orthogonal ones (i.e., haar, daublets, symmelets, and coiflets). In particular, the orthogonal wavelet series approximation to a continuous signal f(t) is given by

$$f(t) \approx \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \ldots + \sum_k d_{1,k}\psi_{1,k}(t)\,, \tag{2}$$

where J is the number of multi-resolution components or scales, and k ranges from 1 to the number of coefficients in the corresponding component. The coefficients $s_{J,k}$, $d_{J,k}$,..., $d_{1,k}$ are the wavelet transform coefficients, whereas the functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are the approximating wavelet functions. These functions are generated from ϕ and ψ as follows

$$\phi_{j,k}(t) = 2^{-j/2}\phi\left(\frac{t-2^j k}{2^j}\right) \qquad\qquad \psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right). \tag{3}$$

The wavelet coefficients can be approximated by the following integrals

$$s_{J,k} \approx \int \phi_{J,k}(t)f(t)dt \qquad\qquad d_{j,k} \approx \int \psi_{j,k}(t)f(t)dt\,,\ j=1, 2, ..., J. \tag{4}$$

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[2] Mathematically, a function $\varpi(.)$ defined over the entire real axis is called a wavelet if $\varpi(t)\to 0$ as $t\to\pm\infty$.

These coefficients are a measure of the contribution of the corresponding wavelet function to the total signal. On the other hand, the approximating wavelet functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are scaled and translated versions of ϕ and ψ. As equation (3) indicates, the scale or dilation factor is 2^j, whereas the translation or location parameter is $2^j k$. As j gets larger, so does the scale factor 2^j, and the functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ get shorter and more spread out. In other words, 2^j is a measure of the width of the functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$. Likewise, as j increases, the translation step gets correspondingly larger in order to match the scale parameter 2^j.

Most applications of wavelet analysis make use of a discrete wavelet transform (DWT). The DWT calculates the coefficients of the approximation in (2) for a discrete signal of final extent, $f_1, f_2,.., f_n$. That is, it maps the vector $\mathbf{f}=(f_1, f_2,\ldots,f_n)'$ to a vector ω of n wavelet coefficients that contains $s_{J,k}$ and $d_{j,k}$, j=1,2,…, J. The $s_{J,k}$ are called the smooth coefficients and the $d_{j,k}$ are called the detail coefficients. Intuitively, the smooth coefficients represent the underlying smooth behavior of the data at the coarse scale 2^J, whereas the detail coefficients provide the coarse scale deviations from it.

When the length of the data n is divisible by 2^J, there are n/2 coefficients $d_{1,k}$ at the finest scale $2^1=2$. At the next finest scale, there are $n/2^2$ coefficients $d_{2,k}$. Similarly, at the coarsest scale, there are $n/2^J$ $d_{J,k}$ coefficients and $n/2^J$ $s_{J,k}$ coefficients. Altogether, there are $n\left(\sum_{i=1}^{J}\frac{1}{2^i}+\frac{1}{2^J}\right)=n$ coefficients. The number of coefficients at a given scale is related to the width of the wavelet function. For instance, at the finest scale, it takes n/2 terms for the functions $\psi_{1,k}(t)$ to cover the interval $1\le t\le n$.

The wavelet coefficients are ordered from coarse scales to fine scales in the vector ω. If n is divisible by 2^J, ω will be given by

$$\boldsymbol{\omega}=\begin{pmatrix}\mathbf{s}_J\\ \mathbf{d}_J\\ \mathbf{d}_{J-1}\\ \vdots\\ \mathbf{d}_1\end{pmatrix}, \qquad (5)$$

where

$$\begin{aligned}\mathbf{s}_J &= (s_{J,1}, s_{J,2},\ldots, s_{J,n/2^J})'\\ \mathbf{d}_J &= (d_{J,1}, d_{J,2},\ldots, d_{J,n/2^J})'\\ \mathbf{d}_{J-1} &= (d_{J-1,1}, d_{J-1,2},\ldots, d_{J-1,n/2^{J-1}})'\\ \vdots & \qquad\qquad \vdots\\ \mathbf{d}_1 &= (d_{1,1}, d_{1,2},\ldots, d_{1,n/2})'\end{aligned}$$

Each of the sets of coefficients $\mathbf{s}_J$, $\mathbf{d}_J,\ldots,\mathbf{d}_1$ is called a crystal.

Expression (2) can be rewritten as

$$f(t) \approx S_J(t)+D_J(t)+D_{J-1}(t)+...+D_1(t), \quad (6)$$

where

$$S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t) \quad (7a)$$

$$D_J(t) = \sum_k d_{j,k}\psi_{J,k}(t) \quad (7b)$$

are denominated the smooth signal and the detail signals, respectively.

The terms in expression (6) represent a decomposition of the signal into orthogonal signal components $S_J(t)$, $D_J(t)$, $D_{J-1}(t)$, ...,$D_1(t)$ at different scales. These terms are components of the signal at different resolutions. That is why the approximation in (6) is called a multi-resolution decomposition (MRD).

2.3 Computation of Wavelet Variance and Covariance

Wavelet variance analysis consists in partitioning the variance of a time series into pieces that are associated to different time scales. It tells us what scales are important contributors to the overall variability of a series (see Percival and Walden, 2000). In particular, let x_1, x_2,..., x_n be a time series of interest, which is assumed to be a realization of a stationary process with variance σ_X^2. If $\upsilon_X^2(\tau_j)$ denotes the wavelet variance for scale $\tau_j \equiv 2^{j-1}$, then the following relationship holds:

$$\sigma_X^2 = \sum_{j=1}^{\infty} \upsilon_x^2(\tau_j) \quad (8)$$

This relationship is analogous to that between the variance of a stationary process and its spectral density function (SDF):

$$\sigma_X^2 = \int_{-1/2}^{1/2} S_X(f)df \quad (9)$$

where $S_X(f)$ is the SDF at the frequency $f \in [-1/2, 1/2]$.

The SDF for a stationary process decomposes the variance across different frequencies, whereas the wavelet variance decomposes it across different scales. Given that the scale τ_j can be related to range of frequencies in the interval $[1/2^j, 1/2^{j-1}]$, the wavelet variance usually leads to a more succinct decomposition. Moreover, unlike the SDF, the square root of the wavelet variance is expressed in the same units as the original data.

Another advantage of the wavelet variance is that it replaces the sample variance with a sequence of variances over given scales. That is, it offers a scale–by–scale decomposition of variability, which makes it possible to analyze a process that exhibits fluctuations over a range of different scales.

Let $n'_j = \lfloor n/2^j \rfloor$ be the number of discrete wavelet transform (DWT) coefficients at level j, where n is the sample size, and $L'_j \equiv \left\lceil (L-2)(1-\frac{1}{2^j}) \right\rceil$ be the number of DWT boundary coefficients[3] at level j (provided that $n'_j > L'_j$), where L is the width of the wavelet filter[4]. An unbiased estimator of the wavelet variance is defined as

$$\hat{\upsilon}_X^2(\tau_j) \equiv \frac{1}{(n'_j - L'_j)2^j} \sum_{t=L'_j-1}^{n'_j-1} d_{j,t}^2 \qquad (10)$$

Given that the DWT decorrelates the data, the non-boundary wavelet coefficients in a given level ($\mathbf{d}_j$) are zero–mean Gaussian white noise process.

Similarly, the unbiased wavelet covariance between the time series X and Y, at scale j, can be defined as

$$\hat{\upsilon}_{XY}^2(\tau_j) \equiv \frac{1}{(n'_j - L'_j)2^j} \sum_{t=L'_j}^{n'_j-1} d_{j,t}^{(X)} d_{j,t}^{(Y)} \qquad (11)$$

provided that $n'_j > L'_j$.

In the CAPM model, as proposed by Gençay, Whitcher and Selçuk (2003), the wavelet beta estimator for asset i, at scale j, is defined as

$$\hat{\beta}_i(\tau_j) = \frac{\hat{\upsilon}_{R_i R_m}^2(\tau_j)}{\hat{\upsilon}_{R_m}^2(\tau_j)} \qquad (12)$$

where $\hat{\upsilon}_{R_i R_m}^2(\tau_j)$ is the wavelet covariance of asset i and the market portfolio at scale j, and $\hat{\upsilon}_{R_m}^2(\tau_j)$ is the wavelet variance of the market portfolio at scale j.

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[3] $\lfloor x \rfloor$ and $\lceil x \rceil$ represent the greatest integer ≤x and the smallest integer x≥, respectively. Boundary coefficients are those that are formed by combining together some values from the beginning of the sequence of scaling coefficients with some values from the end.

[4] In practical applications, we deal with sequences of values (i.e., time series) rather than functions defined over the entire real axis. Therefore, instead of using actual wavelets, we work with short sequences of values named wavelet filters. The number of values in the sequence is called the width of the wavelet filter, and it is denoted by L.

III Data and Estimation Results

In this section, we focus on estimating the CAPM at different time scales for a group of stocks regularly traded on the Santiago Stock Exchange. We selected those stocks that were traded at least 85 percent of all business days over the sample period (January 1997-Sepetmber 2002). Table 1 presents descriptive statistics of excess returns on the 24 stocks in the sample and on the proxy for the market portfolio, the Price Index of Selected Stocks (IPSA). The latter gathers the 40 most actively-traded stocks on the Santiago Stock Exchange over the past year. The risk-free rate of return is proxied by the 30-day nominal interest paid on bank deposits. This choice is based on data availability of interest rates at a daily frequency, and on the fact that longer-maturity interest rates are inflation indexed. Given that we work with nominal returns, we use a nominal proxy for the risk-free rate.

[Table 1]

The figures in Table 1 show that, as is the case with most financial assets, excess returns on the stocks and the market portfolio exhibit little skewness but high excess kurtosis.

The next step consists of running some exploratory regressions to study the relationship between the excess return on each individual stock and the time scales of the market portfolio. In particular, we estimate a linear regression of each stock excess return $(R_i – R_f)$ on each recomposed crystal j of the market portfolio $(R_m – R_f)^j$:

$$R_i - R_f = \alpha_i^j + \beta_i^j (R_m - R_f)^j + \varepsilon_i^j \equiv \alpha_i^j + \beta_i D_m^j + \varepsilon_i^j \qquad j=1, 2,.., 6. \qquad (13)$$

Given that we work with daily data, wavelet scales are such that scale 1 is associated with 2-4 day dynamics, scale 2 with 4-8 day dynamics, scale 3 with 8-16 day dynamics, scale 4 with 16-32 day dynamics, scale 5 with 32-64 day dynamics, and scale 6 with 64-128 day dynamics. Scale 7 corresponds with 128-256 day dynamics, that is, approximately one year. The linear model in (13) is estimated up to scale 6 of the market portfolio because scale 7 includes not only D7 but also S7. Therefore, when recomposing the market excess returns at scale 7, we cannot separate D7 from S7.

For illustrative purposes, Figure 1 shows the recomposed crystals D1 and D6 of the excess return on the market portfolio at scales 1 and 6, respectively. As we see, D1 depicts the high frequency movements of the market portfolio, whereas D6 depicts its long-term behavior.

[Figure 1]

Table 2 reports the regression results. When looking at individual excess returns, the mean contribution of D_m^j tends to decline as the scale increases, and so does its explanatory power measured by R^2. This implies that the major part of the market portfolio's influence on individual stocks is at higher frequencies (i.e., lower scales). Put another way, the systematic-risk component of the excess returns on individuals stocks is captured to greater extent by the detailed components of the market portfolio.

[Table 2]

An alternative way of analyzing the same issue is by regressing each individual excess return on the recomposed residues of the market portfolio. The residue of the market portfolio at scale j is defined as $S_m^j = R_m - \sum_{i=1}^{j} D_m^i$, j=1, …, 6. By construction, as the scale increases, the variation of the market portfolio left out in the recomposed residue decreases. And, in consequence, the fraction of systematic risk of each stock explained by the recomposed residue of the market portfolio falls monotonically with frequency, on average (Table 3). Similar conclusions are drawn by Nortsworthy, Li and Gorener (2000) for a sample of 99 stocks of the S&P 500 in 1998.

[Table 3]

Our above findings can be visualized in Figure 2, for the particular case of the CAP stock. Panel (a) shows the CAP excess return on scales 1 to 6 of the market portfolio. As the frequency of the excess return on the market portfolio declines, the relationship between the two variables departures more from linearity. A similar pattern is observed in Panel (b), where the CAP excess return is plotted against scales 1 to 6 of the market portfolio residues.

[Figure 2]

Strictly speaking, β_i^j in equation (13) is not a true beta because the excess returns on the individual stock and the market portfolio are measured at different time scales. Therefore, a more accurate measure of beta, at scale j, is given by expression (12). Table 4 shows our estimation results. We also report an R^2 for each scale, which is computed as

$$R_i^2(\tau_j) = \hat{\beta}_i(\tau_j)^2 \frac{\hat{\upsilon}_{R_m}^2(\tau_j)}{\hat{\upsilon}_{R_i}^2(\tau_j)} \qquad (14)$$

[Table 4]

Unlike the results reported in Tables 2 and 3, the linear relationship between the excess return and the market portfolio becomes in general stronger at higher scales of the two variables. This is illustrated in Figure 3, where each recomposed crystal of the excess return on the CAP stock is plotted on the corresponding recomposed crystal of the market portfolio. The linear association between the two variables is particularly strong at scales 2 and 3. This evidence implies that the fraction of systematic risk contained in an individual stock at lower frequencies has a higher association with lower frequencies of the market portfolio.

[Figure 3]

The betas reported in Table 4 allow us to compute an average market premium at each time scale. Specifically, we run a regression on the average risk premium of each stock $\overline{R_i - R_f}$ on its wavelet beta estimate at scale j, $\hat{\beta}_i(\tau_j)$. The regression is also estimated for the betas obtained from the raw data. Table 5 reports the ordinary least-squared estimates, whereas Figure 4 shows the average risk premium for each of the 24 stocks in the sample on its corresponding beta at scales 1 to 6.

[Table 5, Figure 4]

From the sample, the average market premium was –9.06 percent per year. The slope estimate at scale 2, which is statistically significant at the 10 percent level, is the one closest to this figure: –11.5 percent per year. At scales 4 to 6, the relationship between the average risk premium and the wavelet beta of each stock is statistically insignificant. Meanwhile, the first row of Table 5 shows that the average market premium yielded by the raw data (–13.7 percent per year) underestimates the sample average market premium. These results, along with those reported in Table 4, indicate that the CAPM model tends to be more statistically significant at scales 2 and 3 of data. In other words, its predictions are more meaningful for investment horizons of 4-16 days.

IV An application to Value at Risk

From the CAPM,

$$R_i - R_f = \alpha_i + \beta_i (R_m - R_f) + \varepsilon_i . \qquad k=1, 2,...,k. \qquad (15)$$

Then, the variance of the excess return i and the covariance between the excess returns i and j are given, respectively, by

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2 , \text{ i=1, 2,..,k,}$$

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 ,$$

where $E(\varepsilon_i^2) = \sigma_{\varepsilon_i}^2$ and $E(\varepsilon_i \varepsilon_j)=0$, $\forall i \neq j$.

Consequently, the variance-covariance matrix of excess returns is given by

$$\mathbf{\Omega} = \boldsymbol{\beta\beta}' \sigma_m^2 + \mathbf{E} \qquad (16)$$

where $\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix}$ and $\mathbf{E} = \begin{pmatrix} \sigma_{\varepsilon_1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_{\varepsilon_k}^2 \end{pmatrix}$.

The (1–α) %-value at risk (VaR) of a portfolio of k assets is then

$$\text{VaR}(\alpha) = V_0 l(\alpha) \sqrt{\boldsymbol{\omega}' (\boldsymbol{\beta\beta}' \sigma_m^2 + \mathbf{E}) \boldsymbol{\omega}} \qquad (17)$$

where ω is a k x 1 vector of portfolio weights, V_0 is the initial value of the portfolio, and $l(\alpha)=\Phi^{-1}(1-\alpha)$, where $\Phi(.)$ is the cumulative distribution function of the standard normal.

For an equally-weighted portfolio, $\omega_i=1/k$, $\forall$ i, the VaR boils down to

$$\mathrm{VaR}(\alpha) = V_0 l(\alpha)\sqrt{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i / k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2} \ . \qquad (18)$$

As k becomes large, $\mathrm{VaR}(\alpha) \approx V_0 l(\alpha)\sqrt{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i / k\right)^2}$. That is, for a well-diversified portfolio, all that matters to computing VaR is systematic risk.

We use equation (18) to compute the value at risk at different time scales. In particular, the VaR at scale j can be obtained by evaluating (18) at the j-scale components of the variance of the market portfolio return, the betas of the k stocks, and of the variances of the error terms that capture non-systematic risk.

$$\mathrm{VaR}_{\tau_j}(\alpha) = V_0 l(\alpha)\sqrt{\sigma_m^2(\tau_j)\left(\sum_{i=1}^{k}\beta_i(\tau_j) / k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2(\tau_j)} \ . \qquad (19)$$

In order to obtain $\sigma_{\varepsilon_i}^2(\tau_j)$, we use the relation $\sigma_i^2(\tau_j) = \beta_i^2(\tau_j)\sigma_m^2(\tau_j) + \sigma_\varepsilon^2(\tau_j)$. That is,

$$\sigma_\varepsilon^2(\tau_j) = \sigma_i^2(\tau_j) - \beta_i^2(\tau_j)\sigma_m^2(\tau_j) \qquad (20)$$

The variance of stock i at scale j, $\sigma_i^2(\tau_j)$, the beta of stock i return at scale j, $\beta_i(\tau_j)$, and the variance of the market portfolio at scale j, $\sigma_m^2(\tau_j)$, can be computed using equations (10) and (12).

Now it should be the case that

$$\mathrm{VaR}(\alpha) \approx V_0 l(\alpha)\sqrt{\sum_{j=1}^{J-1}\left(\sigma_m^2(\tau_j)\left(\sum_{i=1}^{k}\beta_i(\tau_j) / k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2(\tau_j)\right)} \qquad (21)$$

The right-hand side of (21) is an approximation to the VaR of the raw data (equation (18)) because we do not have a beta estimate for the highest scale J. However, the rounded error should be negligible, in general, as most energy is concentrated at the lower scales.[5]

[5] The energy in a given crystal is calculated as the sum of squares of all of its elements over the sum of squares of all observations in the original time series. One appealing characteristic of the discrete wavelet

From expressions (18) and (21), we have

$$1 \approx \frac{\sum_{j=1}^{J-1} \sigma_m^2(\tau_j)\left(\sum_{i=1}^{k} \beta_i(\tau_j)/k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2(\tau_j)}{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i/k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2}.$$

We can interpret the ratio

$$\frac{\sigma_m^2(\tau_j)\left(\sum_{i=1}^{k} \beta_i(\tau_j)/k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2(\tau_j)}{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i/k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2} \qquad (22)$$

as the contribution of scale j to total value at risk.

Table 6 shows our computations. First, as expected, value at risk generally decreases as the time scale increases. Second, the contribution to total risk is higher at the lower scales. That is to say, potential portfolio losses at a 1-day horizon are larger when looking at the detailed components of the data. Finally, the last two rows of the table show the value at risk computed by equations (18) and (21), respectively. As we see, the discrepancy between the two figures shows up only at the fifth decimal.

[Table 6]

IV Conclusions

The basic capital asset pricing model (CAPM) states that the risk premium of an individual asset equals its beta times the risk premium on the market portfolio. Beta measures the degree of co-movement between the asset's return and the return on the market portfolio. In recent years, however, the CAPM has been questioned by several empirical studies.

One strand of the literature has built asset pricing models that allow for a time-varying beta, a time-varying risk premium on the market portfolio, or both. This research area has received the name of tests of the conditional CAPM because the next period expected return and/or variance is updated conditional on the most recent past information. Typically, such testing resorts to GARCH and GARCH-in-mean processes. An alternative approach is wavelet analysis, which is a refinement of Fourier analysis. Wavelets are a

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transform (DWT) is that it is an energy preserving transform. This means that the energy in all the DWT coefficients equals the energy in the original time series. For instance, for the excess return on the IPSA, the first three crystals d1, d2, and d3 concentrate altogether 79 percent of the total energy, whereas crystals d1 to d6 concentrate approximately 98 percent of the total energy.

powerful tool for decomposing time series data into orthogonal components with different frequencies. Each frequency is localized in the time domain, which makes it possible to quantify correlations between time series at different time horizons.

In this article, we focus on the estimation of the CAPM at different time scales for Chile's stock market. Our sample is comprised of twenty four stocks that were actively traded on the Santiago Stock Exchange over 1997–2002. We find evidence in support of the CAPM at a medium–term horizon. We extend the literature in this area to analyze the impact of time scaling on the computation of value at risk. We conclude that risk is concentrated at the higher frequencies of the data.

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Tables

Table 1 Descriptive statistics of excess returns

Stock	Trading days	Average	Median	Std. Dev.	1st quartile	3rd-quartile	Excess Kurtosis
BESALCO	89%	–0.001	0.000	0.023	–0.007	0.000	16.8
CAP	95%	–0.001	0.000	0.020	–0.011	0.008	3.9
CERVEZAS	88%	0.000	0.000	0.022	–0.007	0.007	13.0
CGE	93%	–0.001	0.000	0.029	–0.015	0.010	2.8
CMPC	100%	0.000	0.000	0.015	–0.008	0.006	2.3
COLBUN	99%	0.000	0.000	0.022	–0.001	0.000	3.0
COPEC	100%	0.000	0.000	0.018	–0.009	0.009	2.8
CTC–A	100%	–0.001	0.000	0.021	–0.012	0.009	5.7
CUPRUM	96%	0.000	0.000	0.019	–0.006	0.006	7.2
CHILECTRA	92%	–0.001	0.000	0.017	–0.008	0.005	7.0
D&S	98%	0.000	0.000	0.024	–0.010	0.010	12.7
ENDESA	100%	–0.001	0.000	0.019	–0.010	0.009	8.5
ENERSIS	100%	–0.001	0.000	0.021	–0.011	0.009	4.1
ENTEL	100%	0.000	0.000	0.020	–0.011	0.009	3.1
FALABELLA	99%	0.000	0.000	0.019	–0.010	0.009	4.9
GASCO	89%	0.000	0.000	0.017	–0.002	0.003	5.1
IANSA	99%	–0.001	0.000	0.025	–0.015	0.003	3.5
LAN	86%	0.000	–0.001	0.014	–0.007	0.006	6.3
MASISA	94%	–0.001	0.000	0.022	–0.010	0.009	3.9
ORO BLANCO	93%	–0.001	0.000	0.029	–0.015	0.010	2.8
PARIS	93%	0.000	0.000	0.019	–0.010	0.007	6.4
SAN PEDRO	98%	0.000	0.000	0.016	–0.006	0.007	5.1
SM–CHILE B	97%	0.000	0.000	0.022	–0.001	0.000	14.1
SQM–B	89%	–0.001	0.000	0.023	–0.010	0.009	34.8
IPSA (Market)	100%	0.000	–0.001	0.013	–0.007	0.006	6.2

Notes: (1) The data source is the Santiago Stock Exchange. The sample period is January 1997-September 2002, and returns are daily. (2) Trading days represent the percentage of business days over the sample period on which the stock was traded. (3) IPSA is a proxy for the market portfolio. It gathers the forty stocks that were most actively traded over the past year. (4) The proxy for the risk-free rate is the interest rate paid on 30-day deposits.

Table 2 Individual Excess Returns on the Recomposed Crystals of the Market Portfolio

	Beta						R^2					
Stock	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**
BESALCO	0.318	0.228	0.225	0.712	0.311	1.615	0.019	0.005	0.002	0.011	0.002	0.033
CAP	0.592	0.560	0.747	0.515	0.681	0.690	0.054	0.037	0.054	0.009	0.015	0.005
CERVEZAS	0.245	0.679	0.992	1.014	0.832	0.642	0.009	0.045	0.064	0.035	0.025	0.003
CGE	0.153	0.141	0.246	0.323	0.694	0.530	0.010	0.004	0.012	0.009	0.028	0.012
CMPC	0.525	0.558	0.814	0.488	0.510	1.084	0.067	0.062	0.068	0.016	0.009	0.030
COLBUN	0.598	0.497	0.515	0.412	0.450	0.773	0.053	0.021	0.014	0.006	0.005	0.007
COPEC	0.924	0.826	0.955	0.709	0.734	1.112	0.155	0.100	0.069	0.024	0.014	0.026
CTC–A	1.208	1.383	1.383	1.267	1.236	1.033	0.192	0.203	0.105	0.056	0.028	0.016
CUPRUM	0.251	0.515	0.427	0.837	1.005	0.667	0.010	0.036	0.015	0.031	0.034	0.006
CHILECTRA	0.723	0.734	0.928	0.799	0.938	0.762	0.123	0.098	0.073	0.043	0.024	0.013
D&S	1.008	0.826	0.884	1.118	1.328	1.186	0.114	0.054	0.037	0.031	0.033	0.015
ENDESA	1.200	1.227	1.062	0.975	1.183	1.089	0.212	0.204	0.078	0.037	0.033	0.020
ENERSIS	1.276	1.200	1.135	1.172	1.206	0.943	0.213	0.173	0.079	0.047	0.030	0.013
ENTEL	0.741	0.769	0.750	0.842	0.717	0.871	0.081	0.072	0.036	0.026	0.013	0.010
FALABELLA	0.758	0.733	0.738	0.721	0.960	1.063	0.099	0.065	0.037	0.020	0.031	0.014
GASCO	0.160	0.267	0.284	0.422	0.771	0.604	0.006	0.013	0.009	0.009	0.028	0.008
IANSA	0.881	0.966	0.976	1.061	0.656	1.001	0.076	0.067	0.036	0.032	0.007	0.009
LAN	0.200	0.266	0.739	0.755	0.934	2.100	0.004	0.004	0.019	0.014	0.013	0.034
MASISA	0.519	0.460	0.727	1.065	1.095	1.444	0.043	0.019	0.032	0.031	0.033	0.015
ORO BLANCO	0.585	0.711	0.766	0.799	0.478	0.980	0.028	0.031	0.021	0.009	0.003	0.006
PARIS	0.513	0.743	0.783	0.669	1.088	0.999	0.048	0.073	0.044	0.022	0.043	0.011
SAN PEDRO	0.243	0.425	0.543	0.293	0.590	0.855	0.016	0.032	0.037	0.004	0.014	0.017
SM–CHILE B	0.361	0.171	0.103	0.445	0.576	0.329	0.018	0.003	0.001	0.006	0.008	0.001
SQM–B	0.827	0.978	1.052	0.902	0.994	1.078	0.079	0.100	0.057	0.022	0.020	0.017
Mean	0.617	0.661	0.741	0.763	0.832	0.977	0.072	0.063	0.042	0.023	0.021	0.014
Std. Dev	0.344	0.335	0.317	0.278	0.277	0.371	0.066	0.059	0.028	0.014	0.012	0.009

Notes: (1) Scale 1: 2-4 days, scale 2: 4-8 days, scale 3: 8-16 days, scale 4: 16-32 days, scale 5: 32-64 days, and scale 6: 64-128 days.(2) D1 is the recomposed crystal of the market portfolio at scale 1, etc. The betas are obtained by running a regression of the individual stock excess return on the recomposed crystal of the market portfolio.

Table 3 Individual Excess Returns on the Recomposed Residues of the Market Portfolio

	Beta						R^2					
Stock	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**
BESALCO	0.414	0.577	0.816	0.865	1.506	1.341	0.036	0.038	0.045	0.034	0.048	0.015
CAP	0.657	0.721	0.693	0.803	1.006	1.359	0.129	0.094	0.040	0.033	0.020	0.017
CERVEZAS	0.839	0.945	0.908	0.836	0.846	1.041	0.173	0.132	0.069	0.035	0.009	0.007
CGE	0.298	0.384	0.507	0.629	0.577	0.654	0.056	0.060	0.055	0.051	0.024	0.011
CMPC	0.647	0.718	0.651	0.795	1.039	0.968	0.186	0.127	0.061	0.048	0.044	0.015
COLBUN	0.506	0.513	0.511	0.597	0.787	0.809	0.055	0.034	0.020	0.014	0.011	0.004
COPEC	0.857	0.883	0.830	0.933	1.097	1.068	0.240	0.141	0.072	0.049	0.037	0.012
CTC–A	1.312	1.255	1.162	1.073	0.938	0.743	0.408	0.206	0.102	0.047	0.020	0.004
CUPRUM	0.615	0.690	0.902	0.953	0.873	1.153	0.119	0.085	0.081	0.051	0.017	0.012
CHILECTRA	0.803	0.861	0.817	0.838	0.760	0.757	0.255	0.159	0.086	0.043	0.020	0.006
D&S	0.960	1.059	1.197	1.254	1.166	1.125	0.171	0.119	0.085	0.055	0.022	0.006
ENDESA	1.138	1.061	1.061	1.130	1.081	1.065	0.378	0.176	0.098	0.062	0.029	0.010
ENERSIS	1.158	1.122	1.112	1.063	0.931	0.908	0.348	0.175	0.096	0.049	0.019	0.006
ENTEL	0.804	0.833	0.898	0.942	1.211	1.806	0.174	0.103	0.068	0.042	0.031	0.025
FALABELLA	0.796	0.844	0.918	1.056	1.194	1.344	0.180	0.116	0.081	0.063	0.033	0.019
GASCO	0.375	0.455	0.603	0.718	0.648	0.731	0.062	0.052	0.049	0.043	0.015	0.007
IANSA	0.957	0.950	0.932	0.811	0.990	0.969	0.153	0.086	0.050	0.019	0.013	0.005
LAN	0.694	0.976	1.140	1.485	2.118	2.148	0.069	0.082	0.066	0.059	0.056	0.022
MASISA	0.757	0.959	1.158	1.220	1.448	1.451	0.126	0.120	0.094	0.063	0.031	0.016
ORO BLANCO	0.727	0.739	0.716	0.668	0.936	0.863	0.073	0.042	0.021	0.012	0.009	0.003
PARIS	0.802	0.846	0.889	1.077	1.059	1.129	0.200	0.128	0.085	0.067	0.024	0.012
SAN PEDRO	0.497	0.548	0.553	0.714	0.854	0.850	0.105	0.075	0.037	0.038	0.026	0.009
SM–CHILE B	0.249	0.316	0.488	0.517	0.447	0.654	0.016	0.013	0.018	0.012	0.004	0.003
SQM–B	0.990	1.000	0.961	0.999	1.004	0.861	0.221	0.120	0.063	0.041	0.022	0.005
Mean	**0.744**	**0.802**	**0.851**	**0.916**	**1.022**	**1.075**	**0.164**	**0.103**	**0.064**	**0.043**	**0.024**	**0.011**
Std. Dev	**0.270**	**0.242**	**0.223**	**0.230**	**0.337**	**0.362**	**0.106**	**0.049**	**0.025**	**0.016**	**0.013**	**0.006**

Notes: (1) Scale 1: 2-4 days, scale 2: 4-8 days, scale 3: 8-16 days, scale 4: 16-32 days, scale 5: 32-64 days, and scale 6: 64-128 days. (2) The recomposed residues are computed as $S_m^j = R_m - \sum_{i=1}^{j} D_m^i$, where D_m^j is the recomposed crystal of the market portfolio at scale j.

Table 4 Beta computed from Recomposed Crystals of Individual Stocks and the Market Portfolio

	Beta for each scale						R^2 for each scale					
Stock	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
BESALCO	0.281	0.230	0.400	0.535	0.684	1.494	0.036	0.025	0.049	0.075	0.200	0.364
CAP	0.554	0.607	0.773	0.736	0.577	0.721	0.121	0.170	0.316	0.277	0.284	0.296
CERVEZAS	0.283	0.628	0.931	1.074	0.926	1.058	0.032	0.151	0.328	0.346	0.553	0.565
CGE	0.116	0.222	0.231	0.282	0.612	0.486	0.012	0.058	0.077	0.080	0.354	0.288
CMPC	0.516	0.577	0.733	0.549	0.754	0.841	0.168	0.246	0.356	0.246	0.400	0.560
COLBUN	0.600	0.480	0.574	0.459	0.471	0.508	0.087	0.089	0.182	0.147	0.185	0.464
COPEC	0.894	0.846	0.937	0.779	0.872	1.037	0.354	0.388	0.423	0.392	0.513	0.670
CTC–A	1.259	1.358	1.355	1.209	1.330	1.084	0.551	0.613	0.649	0.663	0.802	0.781
CUPRUM	0.298	0.417	0.562	0.893	0.914	1.019	0.038	0.105	0.151	0.264	0.373	0.566
CHILECTRA	0.739	0.726	0.925	0.837	0.803	1.031	0.307	0.339	0.513	0.503	0.488	0.661
D&S	0.899	0.913	0.945	1.335	1.208	1.331	0.204	0.265	0.295	0.498	0.667	0.741
ENDESA	1.192	1.197	1.097	1.060	0.987	1.095	0.533	0.655	0.553	0.593	0.625	0.857
ENERSIS	1.223	1.218	1.180	1.177	1.022	0.937	0.527	0.572	0.586	0.636	0.601	0.579
ENTEL	0.734	0.763	0.744	0.780	0.750	0.936	0.209	0.263	0.223	0.295	0.352	0.618
FALABELLA	0.701	0.845	0.761	0.792	0.904	1.106	0.187	0.319	0.331	0.350	0.476	0.680
GASCO	0.142	0.311	0.313	0.232	0.649	0.674	0.012	0.062	0.093	0.036	0.281	0.385
IANSA	0.914	0.988	0.968	0.848	0.856	0.956	0.170	0.275	0.292	0.249	0.288	0.563
LAN	0.151	0.325	0.553	1.023	0.963	1.347	0.006	0.031	0.064	0.250	0.275	0.434
MASISA	0.466	0.560	0.707	0.944	0.975	1.317	0.074	0.144	0.192	0.352	0.476	0.673
ORO BLANCO	0.572	0.750	0.813	0.626	0.534	0.760	0.054	0.120	0.161	0.148	0.163	0.386
PARIS	0.522	0.699	0.757	0.795	1.032	1.202	0.122	0.242	0.312	0.366	0.571	0.587
SAN PEDRO	0.203	0.486	0.540	0.408	0.675	0.871	0.023	0.168	0.243	0.182	0.450	0.671
SM–CHILE B	0.314	0.194	0.201	0.327	0.380	0.333	0.027	0.016	0.021	0.064	0.121	0.156
SQM–B	0.860	0.988	0.992	0.920	1.179	1.088	0.211	0.339	0.339	0.370	0.549	0.589
Mean	**0.601**	**0.680**	**0.750**	**0.776**	**0.836**	**0.968**	**0.169**	**0.236**	**0.281**	**0.308**	**0.419**	**0.547**
Std. Dev.	**0.347**	**0.324**	**0.291**	**0.300**	**0.238**	**0.286**	**0.171**	**0.182**	**0.173**	**0.178**	**0.175**	**0.170**

Notes: Scale 1: 2-4 days, scale 2: 4-8 days, scale 3: 8-16 days, scale 4: 16-32 days, scale 5: 32-64 days, and scale 6: 64-128 days. (2) The wavelet beta estimator for asset i, at scale j, is computed as $\hat{\beta}_i(\tau_j) = \frac{\hat{\upsilon}^2_{R_i R_m}(\tau_j)}{\hat{\upsilon}^2_{R_m}(\tau_j)}$, whereas the corresponding R^2 is calculated as $R^2_i(\tau_j) = \hat{\beta}_i(\tau_j)^2 \frac{\hat{\upsilon}^2_{R_m}(\tau_j)}{\hat{\upsilon}^2_{R_i}(\tau_j)}$.

Table 5 Market Risk Premium Estimates at Different Scales

	Constant	p-value	Slope	p-value	R^2
Raw data	–0.002	0.948	–0.059	0.083	0.13
Scale 1	–0.010	0.597	–0.054	0.050	0.16
Scale 2	–0.009	0.682	–0.049	0.103	0.12
Scale 3	–0.002	0.952	–0.054	0.104	0.12
Scale 4	–0.018	0.506	–0.031	0.348	0.04
Scale 5	–0.042	0.263	–0.001	0.985	0.00
Scale 6	–0.024	0.496	–0.019	0.591	0.01

Notes: (1) The parameter estimates are obtained from a linear regression of the average stock excess return on the stock beta at each scale. (2) Scale 1: 2-4 days, scale 2: 4-8 days, scale 3: 8-16 days, scale 4: 16-32 days, scale 5: 32-64 days, and scale 6: 64-128 days. (3) The market risk premium is a daily average, and it is expressed in percentages. For instance, at scale 1, the risk premium is –0.054 percent per day, or –12.6 percent with annual compound.

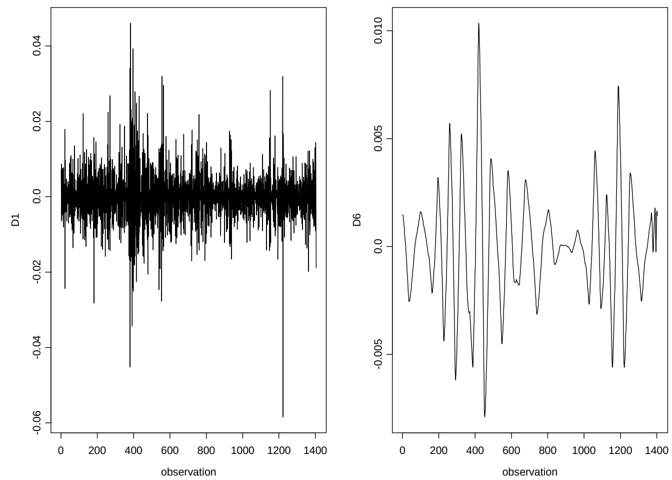
Table 6 Value at Risk (VaR) at different time scales

	95%-VaR	Contribution to total VaR (%)
Scale 1	0.010	30.04
Scale 2	0.009	25.73
Scale 3	0.008	17.89
Scale 4	0.006	10.68
Scale 5	0.005	7.20
Scale 6	0.005	8.45
Total		99.99
Raw data	0.01825	
Recomposed data	0.01827	

Notes: (1) The VaR represents the potential loss, per peso invested, in 1-day horizon at the 95 percent confidence level. (2) The VaR at scale j is computed according to equation (19), where scale 1: 2-4 days, scale 2: 4-8 days, scale 3: 8-16 days, scale 4: 16-32 days, scale 5: 32-64 days, and scale 6: 64-128 days. For simplicity, we use the quantiles of standard normal distribution. (3) The contribution to VaR of scale j is computed according to expression (22). (5) The VaR for the recomposed data is calculated according to (21).

Figures

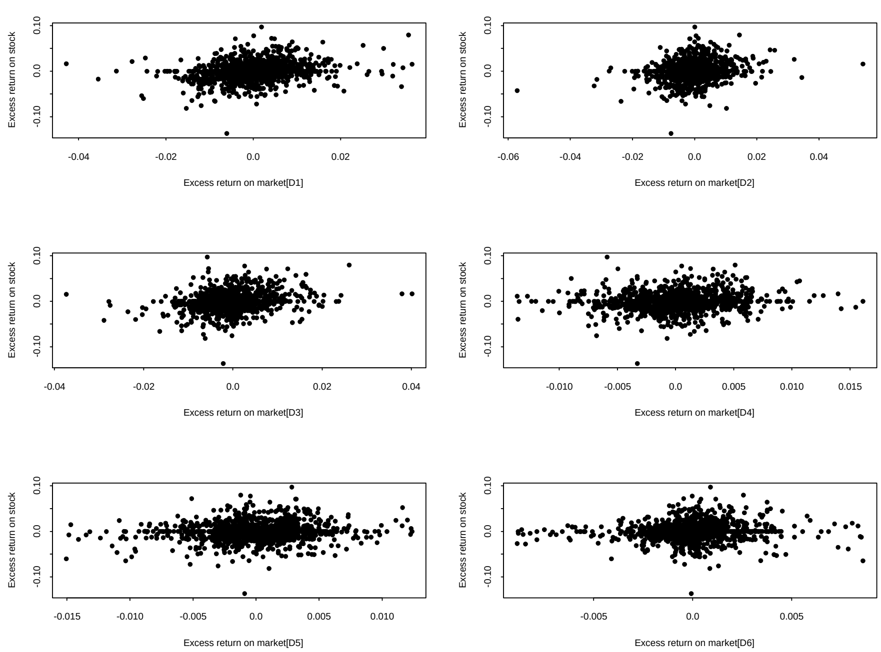
Figure 1 Recomposed Crystals D1 and D6 of the Market Portfolio



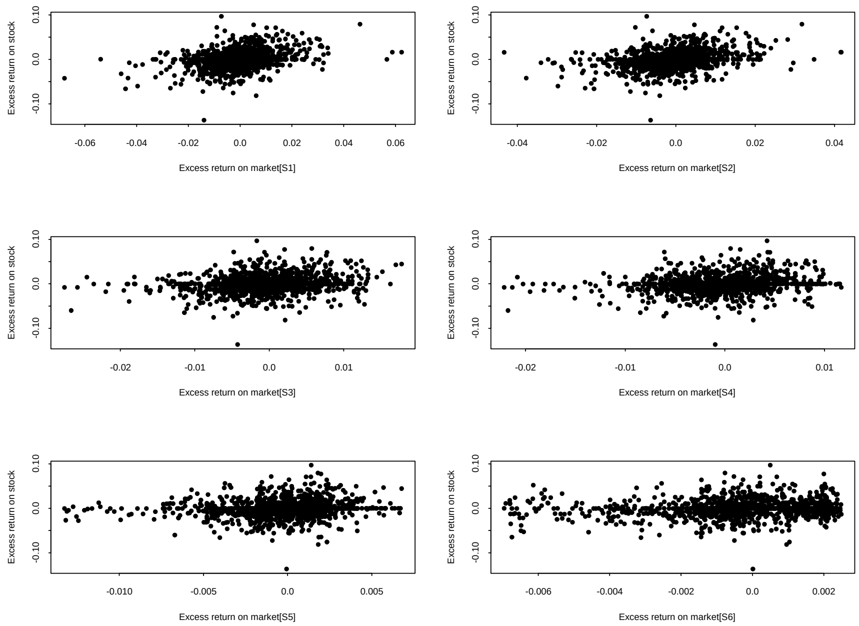
Note: The wavelet function is a *symmmlet*, s8. The number is related to the width and smoothness of the wavelet function (see Bruce and Gao, 1996). The market portfolio is approximated by the Price Index of Selected Stocks (IPSA). The excess return on the IPSA is daily, and the sample period covers January 1997-September 2002.

Figure 2 CAP Stock and Time-decomposition of the Market Portfolio

(a) CAP excess return on different scales of the market portfolio

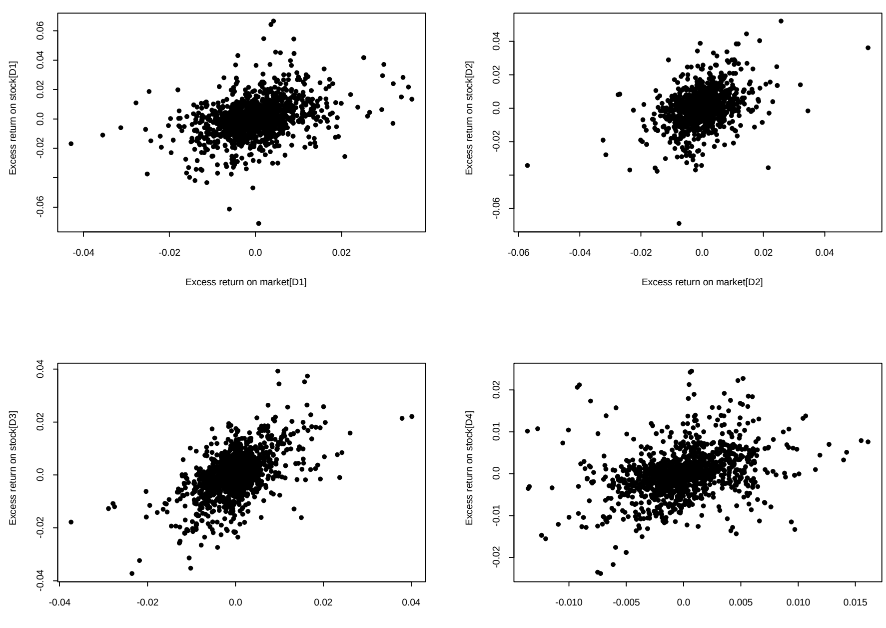


(b) CAP excess return on market portfolio residues at different scales



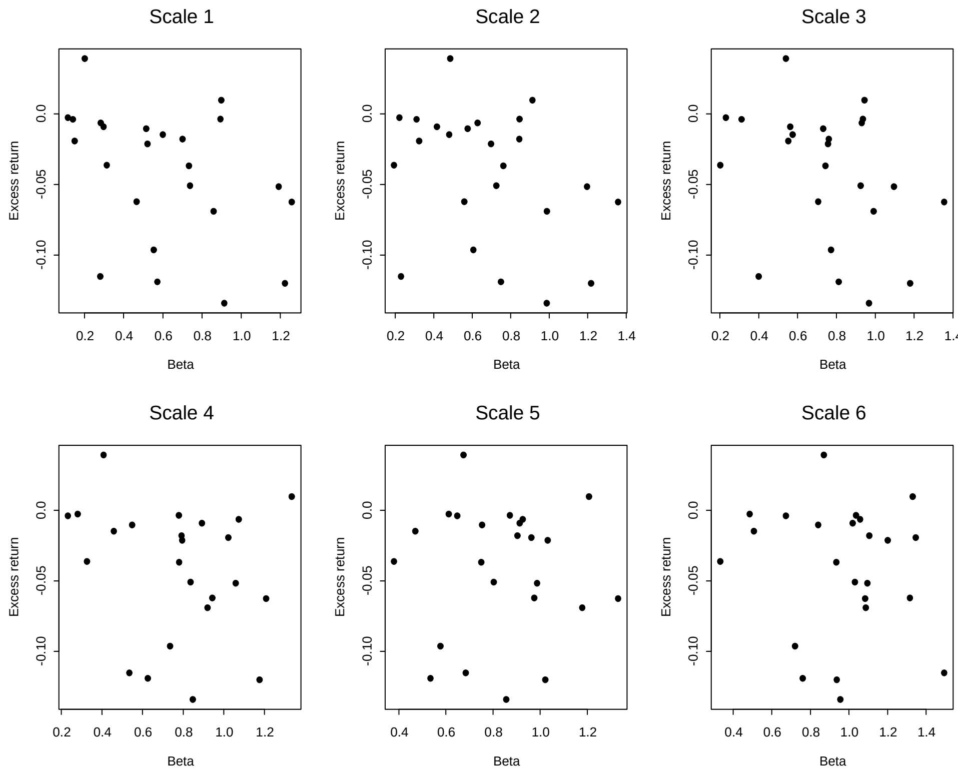
Note: The wavelet function is a *symmmlet*, s8.

Figure 3 Recomposed Crystals of CAP on Recomposed Crystals of the Market Portfolio



Note: The wavelet function is a *symmmlet*, s8.

Figure 4 Average risk premium at different scales



Notes: (1) The average excess return on each individual stock is plotted on its beta at different scales. (2) Scale 1: 2-4 day period, scale 2: 4-8 day period, scale 3: 8-16 period, scale 4: 16-32 day, scale 5: 32-64 days, and scale 6: 64-128 days.