Kinematic Control System for Car-Like Vehicles

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Abstract. In this paper, we have highlighted the importance of the WMR model for designing control strategies. In this sense, the differential model has been used as reference model in order to design the control algorithm. After the control has been design, new actions will be generated for each additional wheel of the real vehicle (non-differential model). This new approach simplifies the overall control systems design procedure. The examples included in the paper, illustrate the more outstanding issues of the designed control. Moreover, we have particularized this control for the line tracking based on a vision system. A velocity control in the longitudinal coordinate has been implemented instead of a position control because we have no longitudinal information. Also, we have simulated and validated this control, studying the effect of the sampling time on the WMR behavior.

1 Introduction

The automation of industrial processes is frequently based on the use of wheeled mobile robots (WMRs). In particular, WMR are mainly involved in the tracking of references. In this sense, the kinematic control of a WMR is very useful for getting this reference tracking. We have focused our research in a car-like vehicle and considered the differential model as reference model in order to design the control.

Section II describes several alternatives for controlling the WMRs. Kinematic models are discussed in Section III. The kinematic control is obtained in Section IV and its restrictions are checked through a complete simulation included in Section V.

Moreover, we particularize in Section VI the achieved control to a specific application: the line tracking based on a vision system. The simulation results of line tracking, obtained in Section VII, validate the kinematic control developed.

Finally, Section VIII remarks the most important contributions and more outstanding issues of the present research.

2 Control of Wheeled Mobile Robots

2.1 Kinematic Control versus Dynamic Control

For WMR, kinematic control calculates the wheel velocities (in the rotation and orientation axles) for tracking the reference, while the dynamic control calculates the wheel accelerations (or torques). These control actions (velocities or accelerations / torques) are used as references for the low-level control loops of the motors.

The WMR dynamic control has the following drawbacks: 1) the required analysis and computation become very complex; 2) it is very sensitive to uncertainties in model parameters; 3) inertial sensors are robustless, inaccurate and expensive and 4) estimators are also inaccurate and expensive.

On the contrary, the WMR kinematic control is simpler and valid as long as the WMR linear and angular velocities have low values (no sliding) as usual in industrial environments.

2.2 WMR Control Based on Geometric Methods

This control consists on applying control actions in such a way that the WMR follows a curve that connects its present with an objective position along the reference. For instance, in [7] and [9] circulars arcs and 5^{th}-order polynoms are used, respectively.

Note that this control forces a point-to-point trajectory tracking (pursuit), so it is difficult to guarantee the stability for a particular trajectory. In fact, the main drawback of this control is to find the optimum adjustment, depending on the trajectory, of some *pursuit* parameters for a good reference tracking.

2.3 WMR Control Based on Linear Approximations

Another possibility is to obtain a linear approximation model of the WMR around an equilibrium point and, then, design a classical linear control. Continuous-time as well as discrete-time model can be generated for this linear approximation.

The drawback of this approach is the robustless of the closed-loop control systems when we get away of the equilibrium point, even leading to instability. From the experience on WMR, we can see that the validity range is very small. As an example, [2] develops a WMR continuous direction control based on a linear approximation.

2.4 WMR Control Based on Non-linear Techniques

If we use the WMR non-linear model (either the continuous or the discrete model), we have to find the non-linear control action that guarantees the system stability. For instance, [5] applies a WMR kinematic adaptative discrete control where several parameters of the algorithm are experimental adjusted.

Moreover, stability may be guaranteed through the existence of a *Lyapunov* function, based on the designed control algorithm, that fulfils the *Lyapunov* theorem for stability. For instance, [6] applies a WMR dynamic continuous control where exists a *Lyapunov* function.

2.5 WMR Control Based on State Feedback Linearization

Also, a state feedback linearization of the WMR non-linear model (either the continuous or the discrete model) allows us to apply, in a second stage, a classical control for a reference tracking: point-to-point tracking, trajectory tracking, etc.

Nevertheless, this control has the drawback of singularities, which invalidate the linearization. However, if there are no singularities or they are never achieved, this control is very suitable. So, we will consider this option.

3 WMR Kinematic Model

We focus our research in a car-like vehicle, very common in many WMR applications. From a practical point of view, a unique steerable equivalent wheel (Fig. 1) is used instead the two steerable wheels which are related through the *Ackerman* mechanism. Therefore, we consider the car-like vehicle as a tricycle vehicle.

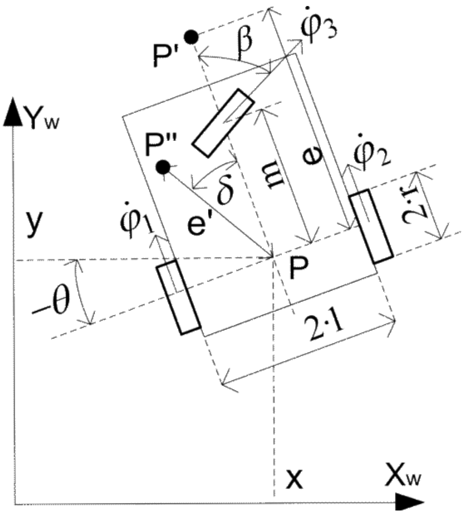

Fig. 1. Equivalent of a car-like vehicle (tricycle)

The meaning of the variables in Fig. 1 is: **P**: Midpoint of the fixed wheels axle; (X_w, Y_w): World coordinate system; (x, y): Position of the point **P** with respect to the world coordinate system; θ: Vehicle orientation with respect to the world coordinate system; (x, y, θ): Vehicle posture; **P'**: Point attached to the WMR that tracks the reference; e: Distance between **P** and **P'**; **P''**: Generic point of the vehicle, characterized by δ and e'; r: radius of the wheels; $2\cdot l$: Fixed wheels separation; m: Distance between **P** and the center of the equivalent steerable wheel; $\dot\varphi_1, \dot\varphi_2$: Rotation velocities of the fixed wheels; β: Angle of the equivalent steerable wheel; $\dot\varphi_3$: Rotation velocity of the equivalent steerable wheel.

3.1 Differential Model (Fixed Wheels Driven)

In this case, according to [3], the kinematic model of the vehicle posture is:

$$\begin{pmatrix}\dot x\\ \dot y\\ \dot\theta\end{pmatrix} = \frac{r}{2\cdot l}\cdot\begin{pmatrix} l\cdot\sin(\theta) & l\cdot\sin(\theta)\\ l\cdot\cos(\theta) & l\cdot\cos(\theta)\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}\dot\varphi_1\\ \dot\varphi_2\end{pmatrix},\ \beta = \arctan\left(\frac{(m/l)\cdot(\dot\varphi_1-\dot\varphi_2)}{(\dot\varphi_1+\dot\varphi_2)}\right),\dot\varphi_3 = \frac{m}{2\cdot l}\cdot\frac{(\dot\varphi_1-\dot\varphi_2)}{\sin(\beta)}\ . \qquad \begin{matrix}(1a)\\(1b)\\(1c)\end{matrix}$$

The rotation velocity $\dot{\varphi}_3$ can auto adjust, supposing that exists enough friction between the floor and the wheel, without sliding. Nevertheless, the angle b cannot auto adjust without sliding, since it is necessary to adjust it through the value of $\dot{\beta}$, which is obtained in (2) derivating (1b). Then, we control $\dot{\varphi}_1$ and $\dot{\varphi}_2$, and $\dot{\beta}$ for no sliding.

$$\dot{\beta} = \frac{2 \cdot m}{l} \cdot \cos^2(\beta) \cdot \frac{\ddot{\varphi}_1 \cdot \dot{\varphi}_2 - \ddot{\varphi}_2 \cdot \dot{\varphi}_1}{(\dot{\varphi}_1 + \dot{\varphi}_2)^2} \ . \tag{2}$$

3.2 Tricycle Model (Steerable Wheel Driven)

In this case, according to [3], the kinematic model of the vehicle posture is:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \frac{r}{m} \cdot \begin{pmatrix} m \cdot \sin(\theta) \cdot \cos(\beta) \\ m \cdot \cos(\theta) \cdot \cos(\beta) \\ \sin(\beta) \end{pmatrix} \cdot (\dot{\varphi}_3), \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix} = \begin{pmatrix} \cos(\beta) + \frac{l}{m} \cdot \sin(\beta) \\ \cos(\beta) - \frac{l}{m} \cdot \sin(\beta) \end{pmatrix} \cdot \dot{\varphi}_3 \ . \qquad \begin{matrix} (3a) \\ (3b) \end{matrix}$$

The rotation velocities $\dot{\varphi}_1$ and $\dot{\varphi}_2$ can auto adjust, supposing enough friction between the floor and the wheel, with no sliding. Then, we control $\dot{\varphi}_3$ and $\dot{\beta}$.

3.3 Practical Kinematic Model

Both above models relate $\dot{\varphi}_1$ and $\dot{\varphi}_2$ with $\dot{\varphi}_3$ through (2b) and (3b). So we can use (1a) or (3a) without distinction, and finally obtain the rotation/s velocity/es of the real driven wheel/s. If we name **X** to the posture (state) and **u** / **u'** to the inputs, we can rewrite (1a) and (3a) as:

$$\dot{\mathbf{X}} = B(\mathbf{X}) \cdot \mathbf{u} \tag{4a}$$

$$\dot{\mathbf{X}} = B'\left(\mathbf{X}, \int \mathbf{u'}\right) \cdot \mathbf{u'} \ . \tag{4b}$$

Note that (4a) depends linearly on the inputs, which makes easier a state feedback linearization. Therefore, we use as practical model (1a). In any case, when controlling the WMR, we must act over $\dot{\beta}$, as describes (2), for no sliding. Nevertheless, if the velocities of the fixed wheels vary discontinuously there will be sliding, since we cannot apply an infinite control action.

4 State Feedback Linearization and Control for a Car-like Vehicle

4.1 State Feedback Linearization

According to [1], we can linearize as many states as inputs with a static state feedback. In particular, as we have two inputs in (1a) we can linearize two states. Therefore, we would be able to control two state variables. If we consider as reference a trajectory in a two-dimensional space, it is enough to control two state variables, which would correspond to the WMR point coordinates that track the reference. Note that the WMR orientation is not completely free, since if we specify a path we indirectly obligate a WMR orientation.

Now, we develop a static state feedback linearization for the generic system (4a).

First, we make the state transformation (5), where $\mathbf{z_1}$ are the new states to be linearized, and $\mathbf{z_2}$ completes the difeomorphism transformation.

$$\mathbf{Z} = \begin{pmatrix} \mathbf{z_1} \\ \mathbf{z_2} \end{pmatrix} = \begin{pmatrix} \mathbf{h}(\mathbf{X}) \\ \mathbf{k}(\mathbf{X}) \end{pmatrix} \quad . \tag{5}$$

Then, the new state equation is:

$$\dot{\mathbf{Z}} = \begin{pmatrix} \dot{\mathbf{z}}_1 \\ \dot{\mathbf{z}}_2 \end{pmatrix} = \begin{pmatrix} \dfrac{\partial(\mathbf{h}(\mathbf{X}))}{\partial \mathbf{X}} \cdot \mathbf{B}(\mathbf{X}) \\ \dfrac{\partial(\mathbf{k}(\mathbf{X}))}{\partial \mathbf{X}} \cdot \mathbf{B}(\mathbf{X}) \end{pmatrix} \cdot \mathbf{u} = \begin{pmatrix} \tilde{\mathbf{h}}(\mathbf{X}) \\ \tilde{\mathbf{k}}(\mathbf{X}) \end{pmatrix} \cdot \mathbf{u} = \begin{pmatrix} \breve{\mathbf{h}}(\mathbf{Z}) \\ \breve{\mathbf{k}}(\mathbf{Z}) \end{pmatrix} \cdot \mathbf{u} \quad . \tag{6}$$

Therefore, the actions (7) linearize $\mathbf{z_1}$ so that $\mathbf{w}$ assigns its dynamic behavior.

$$\mathbf{u} = \tilde{\mathbf{h}}(\mathbf{X})^{-1} \cdot \mathbf{w} \quad . \tag{7}$$

In order to get a stable control: a) we have to assign a stable dynamics; b) the non-linearized states must be bounded (what is fulfilled always for the model (1a)); and c) singularity conditions, which depend on the singularity of $\tilde{\mathbf{h}}(\mathbf{X})$, must not arise.

4.2 Linear Control for Trajectory Tracking

We can assign, with $\mathbf{w}$, different kind of controls (8), where $\tilde{\mathbf{z}}_1 = \mathbf{z}_{1\mathbf{ref}} - \mathbf{z}_1$ is the error.

$$\mathbf{w}_a = \mathbf{A} \cdot \tilde{\mathbf{z}}_1 \ , \mathbf{w}_b = \dot{\mathbf{z}}_{1\mathbf{ref}} + \mathbf{A} \cdot \tilde{\mathbf{z}}_1 \ , \mathbf{w}_c = \dot{\mathbf{z}}_{1\mathbf{ref}} + \mathbf{A} \cdot \tilde{\mathbf{z}}_1 + \mathbf{B} \cdot \int \tilde{\mathbf{z}}_1 \quad . \tag{8a,b,c}$$

The option: (8a) is a point-to-point control where we close the loop with a proportional feedback; while (8b) is a trajectory control where we close the loop with a proportional feedback plus a derivative feedforward; and (8c) is an integral trajectory control where we close the loop with a proportional and integral feedback plus a derivative feedforward. So, considering (6) and (7) in (8a,b,c), we have:

$$\dot{\mathbf{z}}_1 + \mathbf{A} \cdot \tilde{\mathbf{z}}_1 = 0 \ , \ \dot{\tilde{\mathbf{z}}}_1 + \mathbf{A} \cdot \tilde{\mathbf{z}}_1 = 0 \ , \ \ddot{\tilde{\mathbf{z}}}_1 + \mathbf{A} \cdot \dot{\tilde{\mathbf{z}}}_1 + \mathbf{B} \cdot \tilde{\mathbf{z}}_1 = 0 \quad . \tag{9a,b,c}$$

Then, the point-to-point control (8a) has, according to (9a), a non-null velocity error (permanent error for a ramp reference) and an infinite acceleration error (for a parabola reference), so it is not acceptable.

Nevertheless, the trajectory controls (8b,c) have, according to (9b,c), null permanent error for any continuous reference. It is interesting to remark that (8c) allows us to assign an oscillating dynamics, while with (8b) there is not any oscillation. The non-oscillating behavior is selected for WMRs.

4.3 Particularization for a car-like vehicle

First, we apply a transformation **h(X)** to obtain a generic point **P''** of the WMR:

$$\mathbf{z_1} = \mathbf{h}(\mathbf{X}) = \mathbf{P''} = \begin{pmatrix} P''_x \\ P''_y \end{pmatrix} = \begin{pmatrix} x + e' \cdot \sin(\theta - \delta) \\ y + e' \cdot \cos(\theta - \delta) \end{pmatrix} . \tag{10}$$

Then, operating the matrix $\widetilde{\mathbf{h}}(\mathbf{X})$ and its singularity are:

$$\widetilde{\mathbf{h}}(\mathbf{X}) = \frac{r}{2 \cdot l} \cdot \begin{pmatrix} l \cdot \sin(\theta) & l \cdot \sin(\theta) \\ +e' \cdot \cos(\theta - \delta) & -e' \cdot \cos(\theta - \delta) \\ l \cdot \cos(\theta) & l \cdot \cos(\theta) \\ -e' \cdot \sin(\theta - \delta) & +e' \cdot \sin(\theta - \delta) \end{pmatrix} , \quad \left| \widetilde{\mathbf{h}}(\mathbf{X}) \right| = r \cdot e' \cdot \cos(\delta) = 0 \rightarrow \begin{cases} e' = 0 \\ \delta = \pm \pi / 2 \end{cases} . \quad \begin{matrix} (11a) \\ (11b) \end{matrix}$$

According to (11b), the dynamics of the WMR points on the axle of the fixed wheels cannot be linearized. Then, we can choose any WMR point as a tracking point as long as it does not belong to the axle of the fixed wheels. So, taking as a tracking point **P'**, which has $\{e' = e, \delta = 0\}$, the kinematic control results as follows,

$$\mathbf{u} = \widetilde{h}(\mathbf{X})^{-1} \cdot \mathbf{w} = \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix} = \frac{1}{r \cdot e} \cdot \begin{pmatrix} l \cdot \cos(\theta) & e \cdot \cos(\theta) \\ +e \cdot \sin(\theta) & -l \cdot \sin(\theta) \\ -l \cdot \cos(\theta) & e \cdot \cos(\theta) \\ +e \cdot \sin(\theta) & +l \cdot \sin(\theta) \end{pmatrix} \cdot \left(\begin{pmatrix} \dot{P}'_{x\_ref} \\ \dot{P}'_{y\_ref} \end{pmatrix} - \begin{pmatrix} a_x & 0 \\ 0 & a_y \end{pmatrix} \cdot \begin{pmatrix} (x + e \cdot \sin(\theta)) \\ -P'_{x\_ref} \\ (y + e \cdot \cos(\theta)) \\ -P'_{y\_ref} \end{pmatrix} \right) \cdot \tag{12}$$

where a_x and a_y state the dynamics (poles) of the tracking error in the X and Y axis.

In the designed control (12), the rotation velocities of the fixed wheels are continuous (restriction for no sliding in the steerable wheel) if the reference trajectory varies smoothly along the time, what means a smooth path.

Note that the kinematic control designed produces, provided that **P'** tracks a smooth paths, continuous curvature paths in **P** (since the angle of the steerable wheel varies continuously) without explicit them like in [8].

5 Simulation of the Kinematic Control Designed

We show two examples for the kinematic control designed in a simulation environment. Both of them have r = 0.2m, e = 2m, l = 0.8m, $a_x = a_y = 2\ s^{-1}$. Moreover, the first example has m = 4m, $x_0 = 0$, y_0 = 4m, θ_0 = 180º, and the second example m = 2m, $x_0 = y_0 = \theta_0 = 0$. {m → meters, s → seconds, º → degrees }.

The reference of the first example is a semi-circumference followed by a straight line. We can see that the designed control works properly: **P'** tracks the reference (Fig. 2a) and the constant rate of the tracking error is 2s (Fig. 2b). Note that the angle β required for no sliding varies continuously, so exists a control action (although it varies instantaneously in the non-differentiable points of β, e.g. t ≅ 6.5 s) that produces this evolution. Nevertheless, we must initialize the angle β as it corresponds.

The reference of the second example has a non-smooth point. Apparently the designed control works properly: **P'** tracks the reference (Fig. 3a) and the constant rate of the tracking error is 2s (Fig. 3b). But the angle β for no sliding undergoes two discontinuities (Fig. 3b). The value of the first discontinuity is p, and it is avoided calculating the *arctan* of (1b) in four quadrants.

Nevertheless, the second discontinuity, produced when **P’** is on the non-smooth point of the reference, means an unrealizable control action. For preventing the sliding in this situation, we should stop the WMR and reorient the steerable wheel.

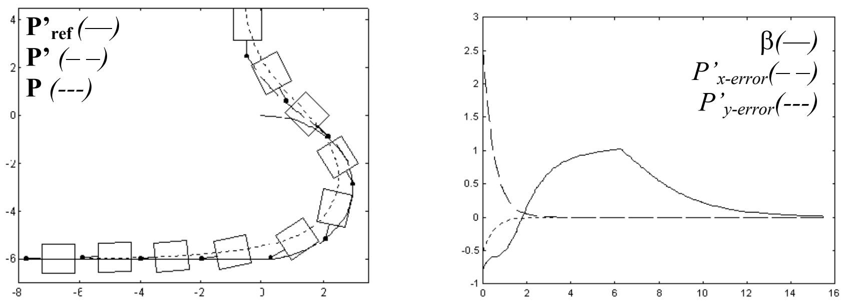

Fig. 2. First example: a) paths described b) signals evolution

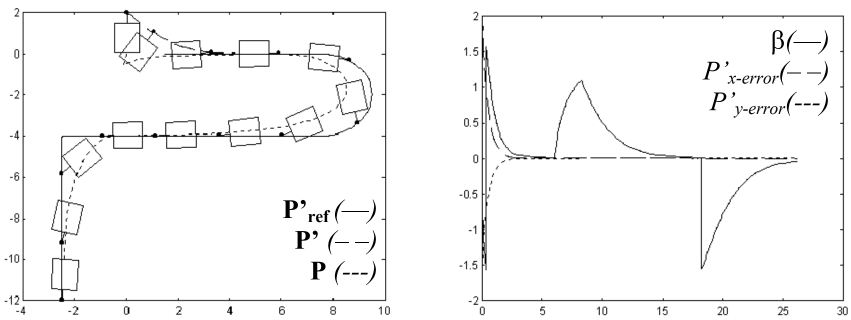

Fig. 3. Second example: a) paths described b) signals evolution

Note that **P** describes a smooth path, although it has a curvature discontinuity when **P’** is on the non-smooth point of the reference (Fig. 3a). So, a differential WMR (i.e. without steerable wheel) is able to track with **P’** non-smooth references.

Moreover, there is a maneuver (change in the direction of the movement) in the beginning of the tracking (Fig.3a). This denotes that the kinematic control designed does not distinguish between forward or backward tracking. So, we should orientate the WMR to the reference, to avoid a backward tracking, with a previous trajectory.

6 Kinematic Control for the Line Tracking with a Vision System

The WMR positioning {y, *h*} with respect to a line (Fig. 4), obtained with the vision system of [4], is related with the WMR posture as follows:

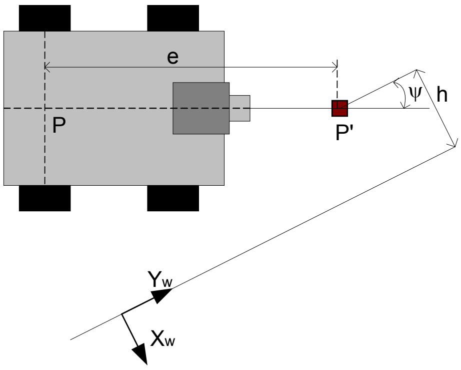

Fig. 4. WMR positioning with respect to a line with a vision system

$$\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} -(e \cdot \sin(\psi) + h) \\ \text{Indeterminated} \\ \psi \end{pmatrix} . \tag{13}$$

Then, x and q are directly observables with {y, h} and y is not. Also, from (1a):

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \mathbf{B}(\theta) \cdot \mathbf{u} . \tag{14}$$

Therefore, we cannot either observe y indirectly, since it does not affect x or q.
On the other hand, we can rewrite the control action (17) as:

$$\mathbf{u} = \tilde{h}^{-1}(\theta) \cdot \left(\dot{\mathbf{P}}'_{\mathbf{ref}} - \mathbf{A} \cdot \begin{pmatrix} P'_x(\theta, x) - P'_{x\_ref} \\ P'_x(\theta, y) - P'_{y\_ref} \end{pmatrix} \right) . \tag{15}$$

According to (15), the state feedback linearization depends on q, while the dynamics assignation w uses (in general) x, y and q. Then, as we have no y information (due to a vision limitation) we particularize the dynamics assignation so that we apply a velocity control (instead of a position control) in the y direction. Then, (15) results:

$$\mathbf{u} = \tilde{\mathbf{h}}^{-1}(\theta) \cdot \left(\dot{\mathbf{P}}'_{\mathbf{ref}} - \begin{pmatrix} a_x \cdot \left(P'_x(\theta, x) - P'_{x\_ref} \right) \\ 0 \end{pmatrix} \right) = \frac{1}{r \cdot e} \cdot \begin{pmatrix} l \cdot \cos(\theta) + e \cdot \sin(\theta) & e \cdot \cos(\theta) - l \cdot \sin(\theta) \\ -l \cdot \cos(\theta) + e \cdot \sin(\theta) & e \cdot \cos(\theta) + l \cdot \sin(\theta) \end{pmatrix} \cdot \begin{pmatrix} -a_x \cdot (x + e \cdot \sin(\theta)) \\ \dot{P}'_{y\_ref} \end{pmatrix} . \tag{16}$$

This control means to track the line with a velocity v_{ref}. Using (13) in (16) we get:

$$\mathbf{u} = \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix} = \frac{1}{r \cdot e} \cdot \begin{pmatrix} l \cdot \cos(\psi) + e \cdot \sin(\psi) & e \cdot \cos(\psi) - l \cdot \sin(\psi) \\ -l \cdot \cos(\psi) + e \cdot \sin(\psi) & e \cdot \cos(\psi) + l \cdot \sin(\psi) \end{pmatrix} \cdot \begin{pmatrix} a_x \cdot h \\ v_{ref} \end{pmatrix} . \tag{17}$$

Moreover, taking into account the processing time of the vision system, the kinematic control de control (17) is implemented in a discrete way. Then, we must guarantee the stability for discrete control actions. Then, using the rectangle approximation in (9) and considering constant the WMR orientation between samples (T):

$$\frac{d(\mathbf{z_1}(t))}{dt} \approx \frac{\mathbf{z_1}(k+1)-\mathbf{z_1}(k)}{T}, \tilde{\mathbf{h}}(\psi(t)) \approx \tilde{\mathbf{h}}(\psi(k)) \ . \tag{18a}$$

(18b)

Then, it is straightforward that the dynamics of the discrete error is:

$$\tilde{\mathbf{z}}_{\mathbf{k+1}} - (\mathbf{I} - T \cdot \mathbf{A}) \cdot \tilde{\mathbf{z}}_{\mathbf{k}} = 0 \ . \tag{19}$$

Therefore, the discrete poles a_d related to the continuous poles a_c are:

$$a_d = 1 - T \cdot a_c \ . \tag{20}$$

The above expression is useful to assign an adequate dynamics. Moreover, in order to validate the approximations of (18), overcoat (18b), we have to assign a dynamics slower than the sampling time.

Other posing is to use the kinematic model (1a) for positioning the WMR when we have no positioning from the vision system so that we use a so fast sampling time as we need.

7 Simulation of the Kinematic Control Designed for Line Tracking

Next, we show several simulated examples for the control designed in the previous section. All of them have: r = 0.05m, e = 0.39m, l = 0.17m, m = 0.3m, v_{ref} = 0.1m/s, x_0 = -0.3m, y_0 = q_0 = 0. Also, we have estimated a sampling time of T = 0.5s for the image processing.

The first and second examples have a_x=0.3s^{-1}, which means that we assign a dynamics around 7 times slower than sampling time of the control, what is acceptable. Their simulations ratify this, since the tracking is well performed (Fig. 5a,b).

Moreover, to prove robustness control, we have introduced in the second example a random noise (bounded to 2cm in WMR separation and 3º in WMR orientation), giving weak oscillations on the control action (Fig. 5e).

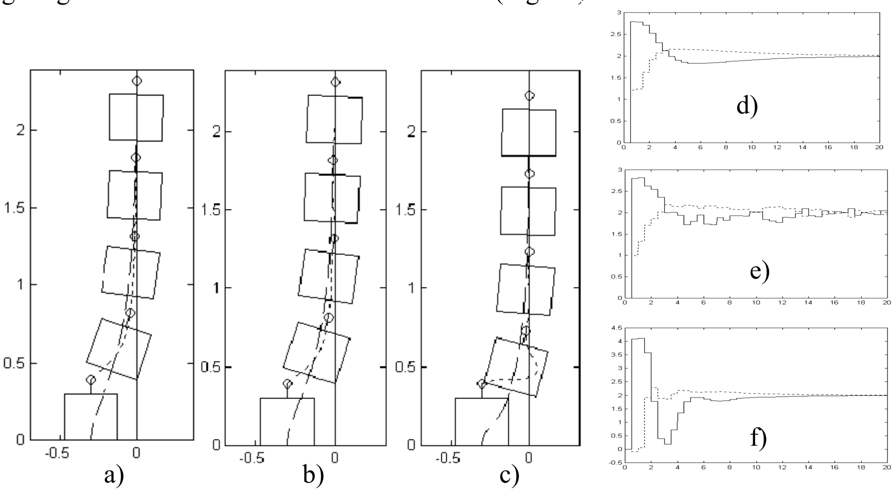

Fig. 5. Simulations of the control designed for line tracking

The third example has $a_x = 0.8s^{-1}$, which means that we assign a tracking dynamics around 2.5 times slower that the sampling time of the control, what is in the limit of an admissible value. In fact, the tracking error describes (Fig. 5c) a small oscillation and the control actions (Fig. 5f) are more drastic, so that we watch the beginning of a non-stable dynamics.

Moreover, another problem arises when we assign a fast dynamics: the reference line may disappear from the plane image of the vision system, especially if the tracking point does not match the cross between the axis of the camera and the floor.

8 Conclusions

In this paper, we have tried to highlight the importance of the WMR model for designing control strategies. In this sense, the differential model has been used as reference model in order to design the control algorithm. After the control has been design, new action will be generated for the additional wheels of the real vehicle (non-differential model). This new approach simplifies the overall control design procedure. The simulated examples, illustrate the more outstanding issues of the control.

Moreover, we have particularized this control for the line tracking based on a vision system. A velocity control in the longitudinal coordinate has been implemented instead of a position control, as we have no longitudinal information. Also, we have simulated and validated this control, studying the effect of the sampling time on the WMR behavior.

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