

# Stability of Haptic Obstacle Avoidance and Force Interaction

Rolf Johansson, Magnus Annerstedt, Anders Robertsson

**Abstract**—Stability problems associated with haptics and robot control with obstacle avoidance are analyzed. Obstacle avoidance algorithms are revised to accomplish stable redesign using absolute stability and passivity theory. A modification of potential functions for haptic rendering and obstacle avoidance allowing stable operation for high stiffness is proposed. The modification leads to velocity-dependent potential-like repulsive stable haptic force interaction with obstacles. Using strictly positive real re-design, stable force interaction can be provided also for high stiffness of manipulated objects or obstacles.

**Keywords:** Force Control, Haptics, Obstacle avoidance, Passivity, Potential Functions, Robot, Stability

## I. INTRODUCTION

Recent progress in haptics aims to provide an artificial sense of touch to a human operator interaction in virtual environments, teleoperation, robot-assisted surgery. Beside the desired mechanical characteristics such as stiffness, damping, bandwidth and other aspects of impedance matching and haptic fidelity, there are important stability problems in the bi-directional force propagation and the control loops providing stable and safe interaction for the human user [1], [2], [3], [4]. An important part of the stability analysis is the physical dissipation provided by the haptic device and the control, as it plays a key role in the design process. Among various means to characterize stability limitation in analysis and design, impedance control has received significant attention [5], [6]. In order to characterize the dynamic range of achievable impedances, Colgate and Brown introduced the notion 'z-width' [7]. Research directed towards passivity analysis has pointed out various stability limitations and sources thereof such as discretization, teleoperation latencies and stiff interaction with environmental forces [8], [1], [2], [3], [4], [9]. A survey paper on bilateral teleoperation applicable to master-slave haptics was published by Hokayem and Spong [10] and stability properties of bilateral teleoperation with constant time delays were considered in [11].

Early contributions to robotic obstacle avoidance using artificial potential function was made by Khatib [12], and Brock and Khatib [13]. In the application of stable trajectory-controlled robots in work-space interactive dynamics such

The work was partly supported by the Swedish Research Council under the grants: 2005-4182, 2006-5243.

Rolf Johansson and Anders Robertsson are with the Department of Automatic Control, Lund University, PO Box 118, SE-221 00 Lund, Sweden; E-mail: Rolf.Johansson@control.lth.se; Phone +46 46 222 8791; Fax +4646 138118

Magnus Annerstedt is with the Department of Urology, Herlev University Hospital, Herlev Ringvej 2730, Herlev, Denmark and with the Department of Urology, Lund University Hospital, Lund University, SE-22185 Lund, Sweden

as force control and obstacle avoidance, new stability problems appear. Potential functions  $U_o(x)$  resulting in repulse action forces  $F_o$  with high stiffness may result in oscillating responses or unstable robot-obstacle interaction.

$$U_o(x), \quad F_o = -\frac{\partial U_o}{\partial x} \quad (1)$$

Robot control may be decomposed into kinematics, trajectory planning, trajectory control and work-space control involving force interaction and obstacle avoidance. A large body of literature exists on robot kinematics, dynamics and control, see the books of Spong and Vidyasagar [14]; Sciavicco and Siciliano [15]; Murray *et al.* [16]; Craig [17] for further references. Haptic devices consisting of an m-DOF master and an m-DOF slave device may be modeled as

$$\begin{aligned} \rho(M_1(q_1)\dot{q}_1 + C_1(q_1, \dot{q}_1)\dot{q}_1 + G_1(q_1)) &= \rho(\tau_1 - J_1^T(q_1)F_1) \\ M_2(q_2)\dot{q}_2 + C_2(q_2, \dot{q}_2)\dot{q}_2 + G_2(q_2) &= \tau_2 - J_2^T(q_2)F_2 \end{aligned} \quad (2)$$

where  $\rho$  is a user-specified power-scaling factor,  $(q_1, q_2)$ ,  $(\tau_1, \tau_2)$ ,  $(F_1, F_2)$  are the configuration coordinates, the applied forces or control variables, and the environmental forces, respectively [1].

A configuration-space dynamic model summarizing Eqs. (2) for  $n = 2m$  and also relevant for a serial robot is

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau - J^T(q)F, \quad \tau, q \in \mathbb{R}^n \quad (3)$$

where the position coordinates  $q \in \mathbb{R}^n$  with associated velocities  $\dot{q} \in \mathbb{R}^n$  are controlled with the applied torques  $\tau \in \mathbb{R}^n$ ;  $M(q)$  being the inertia matrix;  $C(q, \dot{q})\dot{q}$  the Coriolis and centripetal forces;  $G(q)$  gravitation forces and  $\tau$  the vector of joint torques.

In this paper, a stability-oriented re-design of obstacle avoidance using potential action will be made. We will derive the condition that the robot or haptic device must satisfy in order to achieve passivity of the haptic display in force interaction with an obstacle. After the modeling preliminaries providing scope for stability analysis, a precise problem formulation follows.

## II. MODELING—PRELIMINARIES

Introduce the potential energy  $\mathcal{U}(q)$  of the haptic device (or robot), the kinetic energy  $\mathcal{T}(q, \dot{q})$  and the Lagrangian

$$\mathcal{L}(q, \dot{q}) = \mathcal{T}(q, \dot{q}) - \mathcal{U}(q), \quad (4)$$

Equations of motions are provided by the Euler-Lagrange equations [18]

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}}{\partial q} = \tau \quad (5)$$

A configuration-space dynamic model of for a serial robot

$$M(q)\dot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau - J^T(q)F, \quad \tau, q \in \mathbb{R}^n \quad (6)$$

where the position coordinates  $q \in \mathbb{R}^n$  with associated velocities  $\dot{q} \in \mathbb{R}^n$  are controlled with the applied torques  $\tau \in \mathbb{R}^n$ ;  $M(q)$  being the inertia matrix;  $C(q, \dot{q})\dot{q}$  the Coriolis and centripetal forces;  $G(q)$  gravitation forces and  $\tau$  the vector of joint torques.

The end-effector coordinates  $x$  are a function of the robot configuration coordinates  $q$

$$x(t) = f(q), \quad \dot{x} = J(q)\dot{q}, \quad J(q) = \frac{\partial f(q)}{\partial q} \quad (7)$$

One purpose of robot motion control is to maintain a prescribed motion by means of compensating corrective torques  $\tau$ . The applied forces  $\tau$  may be decomposed into the trajectory control torques  $\tau_R$ , force actuation  $\tau_F$ , and obstacle avoidance  $\tau_o$

$$\tau = \tau_R + \tau_F + \tau_o \quad (8)$$

Control of the robot manipulator in operational space with a force  $F$  requires actuation of the

$$\tau_F = J^T(q)F, \quad x(t) = f(q), \quad \dot{x} = J(q)\dot{q}, \quad J(q) = \frac{\partial f(q)}{\partial q} \quad (9)$$

#### A. Obstacle Avoidance

Using an artificial potential, Khatib [12] demonstrated that robot end-effector collision avoidance can be accomplished by means of

$$U_o(x) = U_o(f(q)), \quad F_o = -\frac{\partial U_o}{\partial x} \quad (10)$$

where  $F_o$  represents a repulsive force created by the artificial potential  $U_o$ .

#### B. Stability of Trajectory Control

In order to maintain a trajectory  $q_r$ , a variety of stable trajectory control exists including 'computed torque' state feedback of the error  $\tilde{q}$

$$\tau_R = C(q, \dot{q})\dot{q} + G(q) - M(q)K_V\dot{\tilde{q}} - M(q)K_P\tilde{q} + M(q)\dot{q}_r \quad (11)$$

In the case of trajectory control without force interaction with the environment—i.e.,  $F = 0$ —the closed-loop system of Eqs. (11) and (6) exhibits the error dynamics

$$M(q)(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) = 0 \quad (12)$$

$$(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) = 0, \quad M(q) > 0, \quad \forall q \quad (13)$$

with error dynamics on state-space form

$$\frac{d}{dt} \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} -K_V & -K_P \\ I & 0 \end{bmatrix} \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} \quad (14)$$

Introduction of state-space notation for error dynamics

$$\xi = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} \in \mathbb{R}^{2n}, \quad \dot{\xi} = A\xi, \quad A = \begin{bmatrix} -K_V & -K_P \\ I & 0 \end{bmatrix} \quad (15)$$

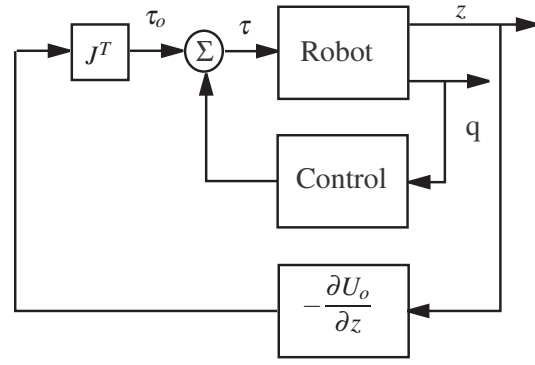


Fig. 1. Robot force control and error dynamics

Assuming a Lyapunov function  $V(\xi)$  proving stability for unconstrained motion of the computed torque solutions

$$V_0(\xi) = \xi^T P \xi, \quad \frac{dV_0(\xi)}{d\xi} = -\xi^T Q \xi < 0, \quad \|\xi\| \neq 0 \quad (16)$$

$$-Q = PA + A^T P, \quad (17)$$

Other Lyapunov-based designs with Lyapunov functions and proof of asymptotic stability can be found in [19], [20], [21].

#### C. Passivity

Using the Hamiltonian

$$\mathcal{H} = \mathcal{T}(q, \dot{q}) + \mathcal{U}(q) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \mathcal{U}(q) \quad (18)$$

Ortega and Spong (1989) demonstrated passivity to hold for the robot dynamics from input  $\tau$  to velocity  $\dot{q}$  [21], [22], [23], [24].

$$\int_{t_0}^{t_f} \dot{q}^T \tau dt = \int_{t_0}^{t_f} d\mathcal{H} = \mathcal{H}(t_f) - \mathcal{H}(t_0) \geq -\mathcal{H}(t_0) \quad (19)$$

No similar property holds for the map from input  $\tau$  to position  $q$ .

#### D. Stability of Force Control and Obstacle Avoidance

In order to maintain a trajectory in the case of obstacle avoidance where the end-effector will fail to reach  $f(q_r)$ , yet with influence from dynamics and trajectory control

$$U_o(x) = U_o(f(q)), \quad F_o = -\frac{\partial U_o}{\partial x} \quad (20)$$

with error dynamics

$$M(q)(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) = \tau_o \quad (21)$$

$$(\ddot{\tilde{q}} + K_V\dot{\tilde{q}} + K_P\tilde{q}) = M^{-1}(q)\tau_o, \quad M(q) > 0 \quad (22)$$

or

$$\frac{d}{dt} \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} -K_V & -K_P \\ I & 0 \end{bmatrix} \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} + \begin{bmatrix} M^{-1}(q) \\ 0 \end{bmatrix} \tau_o \quad (23)$$

$$\tau_o = -J^T F_o = -J^T(q) \frac{\partial U_o}{\partial x} \quad (24)$$

As Eq. (24) represents a feedback loop applied to the unconstrained system, the artificial potential action may

cause high-gain feedback exceeding the gain margin of the unconstrained system with impending instability. Unless the subsystem of Eq. (23) is strictly positive real (SPR), however, the gain margin has a finite upper limit [26], [27], [28], [29].

### E. Problem Formulation

The stability problem in case of an artificial potential field

$$U_o(x) = \frac{1}{2}(x - x_o)^T K_o(x - x_o) \quad (25)$$

located at an obstacle centered at  $x_o$  (or  $q_o$  in configuration-space coordinates) will give the repulsive force

$$F_o = -\frac{\partial U_o}{\partial x} = -K_o(x - x_o) \quad (26)$$

Stability analysis will exhibit closed-loop system properties with a stability problem reducing to the 'absolute stability' problem—*cf.*, as proposed by Lur'e and Postnikov [31], Popov [28], [32], Yakubovich [27], Kalman [26]; the circle theorem [30], [29], [25]—analyzed using the circle criterion or the Popov criterion by means of the Lur'e-Lyapunov function

$$V(x, z) = x^T P x + 2\eta \int_0^z \psi^T(\zeta) \kappa d\zeta, \quad P > 0, \quad \eta > 0 \quad (27)$$

with

$$\kappa \psi(z) = \frac{\partial U_o}{\partial x} = K(x - x_o), \quad z = (x - x_o) = f(q) - f(q_o) \quad (28)$$

A standard result from absolute stability theory is that a stability margin to the interaction stiffness  $K$  of arbitrary magnitude is accomplished only for *positive real* input-output error dynamics of Eq. (22)—*cf.*, [33], [34]. Apart from divergent instability, stability problems such as limit cycles may result from choices of too large repulsive stiffness matrices  $K$ . Thus, a potential function for a stiff rigid-body obstacle may violate the limited stability margin resulting from non-SPR error dynamics (Fig. 1).

As a result, it remains an open problem how to design or modify a repulsive artificial potential and its algorithmic organization such that a high stability margin will result, a problem which now is to be addressed.

### III. MAIN RESULT

As the artificial potential may be designed without explicit physical interpretation, we can modify the input as well as the potential function. A straightforward modification is to choose a variable  $z$  permitting a block decomposition into one strictly positive real (SPR) block which is strictly passive and one passive high-gain block (Fig. 2) permitting stable interconnection.

$$U_o(z) = \frac{1}{2}(z - z_o)^T K_o(z - z_o) \quad (29)$$

$$z = D\dot{\tilde{q}} + K\tilde{q} \quad (30)$$

$$Z(s) = (s^2 I + sK_V + K_P)^{-1}(Ds + K) \quad (31)$$

$$F_o = -\frac{\partial U_o(z)}{\partial z} = -K_o(z - z_o) \quad (32)$$

with  $D$  chosen in such a way that the transfer function  $Z(s)$  be strictly positive real (SPR)—*i.e.*, in such a way that the

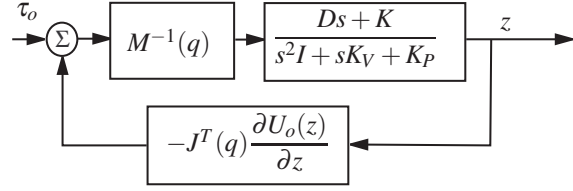


Fig. 2. Haptic force control and positive real error dynamics

Nyquist curve is to the right of the imaginary axis in the complex  $s$ -plane. For values  $K_V$ ,  $K_P$  such that  $Z(s)$  is stable, it is always possible to choose  $K > 0$  and a sufficiently large  $D > 0$  such that  $Z(s)$  be SPR. In the context of control design for high gain margin, Molander and Willems [34] showed that one suitable SPR choice is

$$0 = PA_0 + A_0^T P + Q - PBB^T P, \quad (33)$$

$$A = A_0 - BB^T P \quad (34)$$

$$PA + A^T P = -Q - PBB^T P, \quad C = B^T P \quad (35)$$

$$Q = Q^T > 0, \quad P = P^T > 0 \quad (36)$$

As a result, the stability margin is increased and larger values of the repulsive stiffness  $K_o$  may be chosen.

Introduce the haptic interaction acceleration

$$\alpha_o = M^{-1}(q)\tau_o = M^{-1}(q)J^T(q)\frac{\partial U_o(z)}{\partial z} \quad (37)$$

Consider the Lyapunov function (or storage function)

$$V(\xi, z) = \frac{1}{2}\xi^T P \xi + \eta U_o(z), \quad \eta = \text{constant} \quad (38)$$

$$= \frac{1}{2}\xi^T P \xi + \eta \int_0^z \frac{\partial U_o(\zeta)}{\partial \zeta} d\zeta \quad (39)$$

$$PA + A^T P = -Q - PBB^T P, \quad Q > 0, \quad P = P^T > 0 \quad (40)$$

with the derivative (for proof—see Appendix)

$$\frac{dV(\xi, z)}{dt} = \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \mathcal{Q} \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} < 0, \quad \left\| \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \right\| \neq 0 \quad (41)$$

$$0 > \mathcal{Q} = \begin{bmatrix} PA + A^T P & PB - \eta A^T C^T K_o^T \\ B^T P - \eta K_o C A & -\eta(K_o C B + B^T C^T K_o^T) \end{bmatrix}$$

The SPR choice  $C = B^T P$  gives  $z = C\xi$  and asymptotically stable control with the Lyapunov function derivative

$$\frac{dV(\xi, z)}{dt} = \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \mathcal{Q}_1 \begin{bmatrix} \xi \\ \alpha_o \end{bmatrix} \leq 0 \quad (42)$$

$$\mathcal{Q}_1 = \begin{bmatrix} -Q - PBB^T P & C^T - \eta A^T C^T K_o^T \\ C - \eta K_o C A & -2B^T P B \end{bmatrix} < 0 \quad (43)$$

for  $Q > 0$  chosen sufficiently large.

#### Design of Strictly Positive Real Obstacle Avoidance

Let the computed-torque open-loop dynamics be described by

$$A_0 = \begin{bmatrix} 0 & 0 \\ I_n & 0 \end{bmatrix}, \quad B = \begin{bmatrix} I_n \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ I_n \end{bmatrix}, \quad (44)$$

and solve the Riccati equation

$$0 = PA_0 + A_0^T P + Q - PBB^T P, \quad C = B^T P \quad (45)$$

$$D = K_V = B^T P B, \quad (46)$$

$$K = K_P = B^T P E, \quad (47)$$

#### Example and Simulation Experiments

Consider the 1DOF mechanical system

$$m\dot{q} = \tau + F, \quad F = 0 \quad (48)$$

$$\tau_R = -mK_V \dot{q} - mK_P \tilde{q} + m\dot{q}_r \quad (49)$$

$$\tau_o = -\frac{\partial U_o(z)}{\partial z} \quad (50)$$

$$\tau = \tau_R + \tau_o \quad (51)$$

with the state-space representation

$$\frac{d\xi}{dt} = \begin{bmatrix} -K_V & -K_P \\ I & 0 \end{bmatrix} \xi, \quad \xi = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} \in \mathbb{R}^2 \quad (52)$$

$$z = C\xi = [D \quad K] \xi \quad (53)$$

Consider the Lyapunov function (or storage function)

$$V(\xi, z) = \frac{1}{2} \xi^T P \xi + \eta U_o(z) \quad (54)$$

$$= \frac{1}{2} \xi^T P \xi + \eta \int_0^z \frac{\partial U_o(\zeta)}{\partial \zeta} d\zeta \quad (55)$$

$$PA + A^T P = -Q - PBB^T P, \quad Q > 0, P > 0 \quad (56)$$

with the derivative of Eq. (41). For  $\eta K_o = I$ , the SPR choice  $C = B^T P$  gives  $z = C\xi$  and asymptotically stable control with the Lyapunov function derivative

$$\frac{dV(\xi, z)}{dt} = \frac{1}{2} \begin{bmatrix} \xi \\ \tau_o \end{bmatrix} \begin{bmatrix} -Q - PBB^T P & (I - A^T)C^T \\ C(I - A) & -2B^T P B \end{bmatrix} \begin{bmatrix} \xi \\ \tau_o \end{bmatrix} \leq 0$$

for  $Q > 0$  chosen sufficient large.

Simulation examples with control force and obstacle avoidance force  $\tau_o$  for  $m = 1$ ,  $K_o = 1000$ ,  $K_V = 4$ ,  $K_P = 4$ ,  $K = 1$  are found in Fig. 3 ( $D = 0$ ) and Fig. 4 ( $D = 0.2$ ), respectively. Note that the elimination of the oscillating behavior as a result of the modified artificial potential using an impedance variable. For SPR haptic potential design, simulation examples of with control force and obstacle avoidance force  $\tau_o$  for  $m = 1$ ,  $Q = 4I$ ,  $K_o = 10000$ ,  $\eta K_o = 1$ ,  $D = K_V = 5.2362$ ,  $K = K_P = 3.2362$ , are found in Fig. 5. Note the elimination of the oscillating behavior as a result of the modified artificial potential using an SPR impedance variable with solution

$$P = \begin{bmatrix} 5.2360 & 3.2360 \\ 3.2360 & 6.4721 \end{bmatrix} > 0, \quad (57)$$

$$C = B^T P = [5.2360 \quad 3.2360] \quad (58)$$

$$\mathcal{Q} = \begin{bmatrix} PA + A^T P & PB - A^T C^T \\ B^T P - CA & -(CB + B^T C^T) \end{bmatrix} \quad (59)$$

$$= \begin{bmatrix} -48.36 & -27.42 & 2.00 \\ -27.42 & -20.94 & 3.24 \\ 2.00 & 3.24 & -10.47 \end{bmatrix} < 0,$$

$$\sigma(\mathcal{Q}) = [-65.5143 \quad -10.6925 \quad -3.5690] \quad (60)$$

## IV. DISCUSSION

Design of haptic systems with real or virtual haptic obstacle avoidance that guarantee stable interaction with stiff environments is a challenging task. Virtual environments are typically highly nonlinear resulting in nonpassive closed-loop systems. In this paper, we investigate how nonlinear mass-spring-damper virtual environments can be designed to guarantee the absence of oscillations and other undesired behavior for the human operator. In particular, implementation of the mass-spring-damper virtual environment is considered, revealing new results with regard to the allowable local stiffness.

In the context of haptics, there is a large literature recognizing the relevance of passivity and dissipation as a means of stability analysis and design [8], [3], [1], [4], [2]. Passivity theory also permits structural analytical approaches such as port-Hamiltonian analysis [24]. Complete stability analysis can be approached in operational space or in configuration space, using discrete-time or continuous-time analysis [9].

To the purpose of stable re-design of robotic obstacle avoidance and using a strictly positive real re-design, a successful revision of the artificial potential function was made. A major modification is that the potential function is a function of an impedance variable instead of a position variable, thus allowing strictly positive real (SPR) error dynamics. In turn, the Popov criterion (or the circle criterion) permits 'absolute stability'—*i.e.*, stability assured by the storage function (or Lyapunov function) of Eq. (38) with passivity of the interconnected haptic system. The significance of the SPR modification is demonstrated in the example with simulation experiments in Figs. 3-4.

As the modified potential function involving an impedance variable prompts state feedback, both velocity and position coordinates should be available to measurement. In cases without full state measurement, state estimation is necessary and for stable operation such estimation design must be included in the stability analysis. Only special cases permit separation properties—*i.e.*, where separate designs of state estimation and control do not challenge stability [37]. State estimation applied to the absolute stability problem and compatible with passivity design was provided in [29], [25].

The modification based on the circle criterion or Popov criterion leads to a velocity-dependent potential-like repulse force action. In Eq. (30), the linear combination  $z$  of signals is reminiscent of impedance control as introduced by Hogan [5], [6].

In addition to the scope for better and stable haptics, we expect that this contribution will be used in general-purpose force control combining with sensor fusion and motion control [38], [39], [40], [41], [42], [43], [44].

An aspect not elaborated here is the stability in the context of communication delays—*e.g.*, communication delays in teleoperation [10].

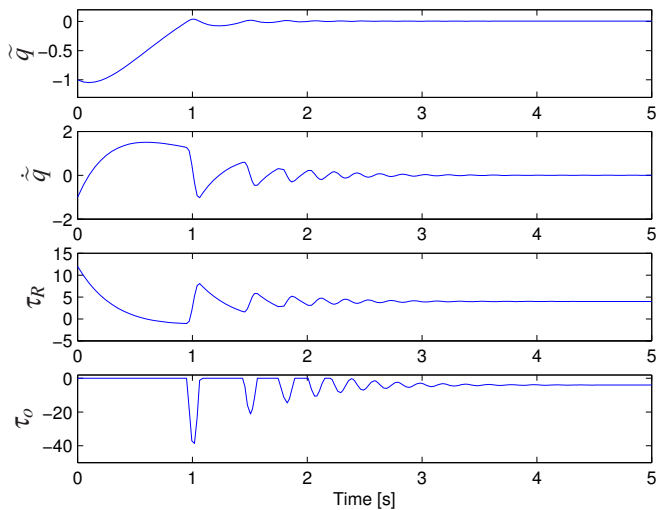


Fig. 3. Haptic obstacle repulsion using artificial potential based on position (deflection) only with stiffness  $K_o = 1000$  and  $D = 0$ . Note the oscillatory force behavior.

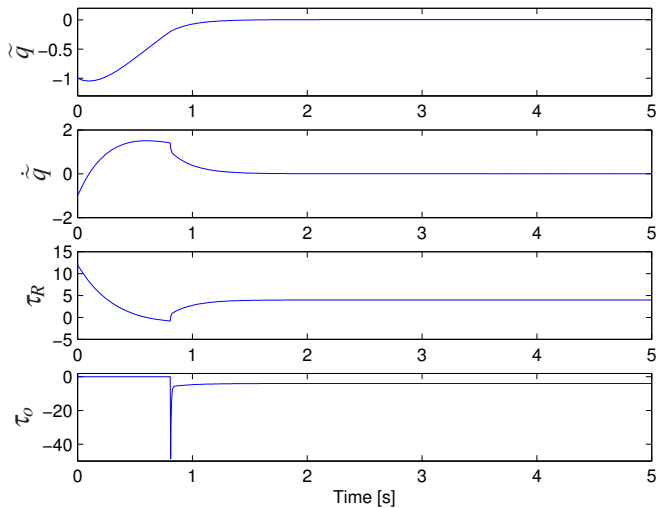


Fig. 4. Haptic obstacle repulsion using artificial potential based on impedance variable with stiffness  $K_o = 1000$  and  $D = 0.2$ . Note the stable force and velocity behaviors.

A potential application can be found in surgical robotics with haptic interfaces including both interaction of obstacle avoidance and virtual obstacle repulsion.

## V. CONCLUSIONS

To the purpose of stable re-design of haptic obstacle avoidance and using a strictly positive real (SPR) re-design, a successful passivity-based revision of the artificial potential function was made.

### Acknowledgements

The work was partly supported by the Swedish Research Council under the grants: 2005-4182, 2006-5243.

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#### APPENDIX—STABILITY

Let the obstacle avoidance acceleration be

$$\alpha(z, q) = M^{-1}(q)J^T(q)\frac{\partial U_o(z)}{\partial z} \quad (61)$$

The closed-loop system represented by Fig. 2 for  $\tau_R = \tau_F = 0$

$$\dot{x} = Ax - B\alpha_o(z, q), \quad B = \begin{bmatrix} I_n \\ 0 \end{bmatrix} \quad (62)$$

$$z = Cx = [D \quad K]x, \quad CB = D \quad (63)$$

According to the conditions of the Popov criterion [28], [36], [25], the closed-loop system dynamics with feedback interconnection are described by

$$\begin{bmatrix} \dot{x} \\ -\dot{z} \end{bmatrix} = \begin{bmatrix} A & B \\ -CA & -CB \end{bmatrix} \begin{bmatrix} x \\ -\alpha_o(z) \end{bmatrix}, \quad (64)$$

$$0 \leq -\alpha_o^T(z)(\alpha_o(z) - \kappa z) \quad (65)$$

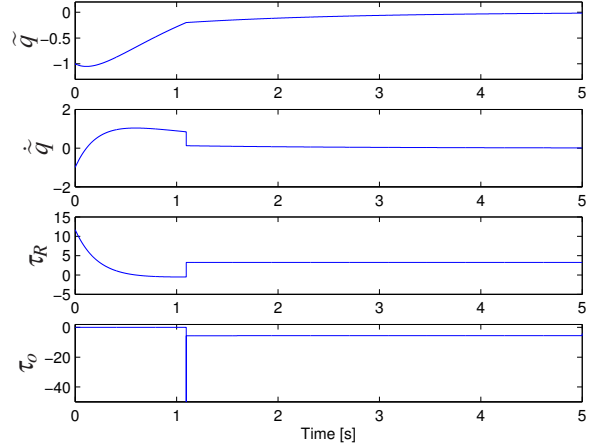


Fig. 5. Haptic repulsion using an artificial potential of an obstacle based on strictly positive real position design with stiffness  $K_o = 10000$  and  $D = 5.2362$ ,  $K = 3.2362$ . Note the stable velocity, impact and force behaviors.

The Lyapunov function candidate  $V$  and its derivative are

$$V = V(x, z) = x^T P x + 2\eta \int_0^z \alpha_o^T(\zeta) \kappa d\zeta, \quad (66)$$

$$\eta \geq 0, \quad \kappa > 0, \quad \kappa \in \mathbb{R}^{m \times m} \quad (67)$$

$$\frac{dV}{dt} = x^T P \dot{x} + \dot{x}^T P x + 2\eta \alpha_o^T(z) \kappa \dot{z} \quad (68)$$

$$= x^T (PA + A^T P)x - 2x^T P B \alpha_o(z) \quad (69)$$

$$+ 2\eta \alpha_o^T(z) \kappa C (Ax - B \alpha_o(z)) \quad (70)$$

$$\frac{dV}{dt} = \begin{bmatrix} x \\ -\alpha_o(z) \end{bmatrix}^T \mathcal{Q} \begin{bmatrix} x \\ -\alpha_o(z) \end{bmatrix} \quad (71)$$

$$\mathcal{Q} = \begin{bmatrix} PA + A^T P & PB - \eta A^T C^T \kappa^T \\ B^T P - \eta \kappa C A & -\eta (\kappa C B + B^T C^T \kappa^T) \end{bmatrix} < 0 \quad (72)$$

Under the choice

$$\kappa > M^{-1}(q)J^T(q)K_o \quad (73)$$

a quadratic constraint holds so that

$$\alpha_o^T(z)(\alpha_o(z) - \kappa z) \leq 0 \quad (74)$$

A sufficient condition for asymptotic stability is the existence of a solution to  $\mathcal{P}_o \mathcal{A}_P + \mathcal{A}_P^T \mathcal{P}_o = -\mathcal{Q} \leq 0$ ,  $\mathcal{Q}_{11} > 0$  which renders  $dV/dt$  negative definite for  $\|x\| \neq 0$  and guarantees existence of a Lyapunov function  $V$  [28], [36], [25]. In summary, for

$$\mathcal{A}_P = \begin{bmatrix} A & B \\ -\eta \kappa C A - \kappa C & -\eta \kappa C B - I_m \end{bmatrix} \quad (75)$$

a sufficient condition for asymptotic stability is the existence of a solution to the Lyapunov equation

$$\mathcal{P}_o \mathcal{A}_P + \mathcal{A}_P^T \mathcal{P}_o = -\mathcal{Q} \leq 0, \quad (76)$$

$$\mathcal{P}_o > 0, \quad \mathcal{Q} = \begin{bmatrix} \mathcal{Q}_{11} & \mathcal{Q}_{12} \\ \mathcal{Q}_{12}^T & \mathcal{Q}_{22} \end{bmatrix} \geq 0, \quad \mathcal{Q}_{11} > 0 \quad (77)$$

which renders  $dV/dt < 0$  for  $\|x\| \neq 0$  and guarantees existence of a Lyapunov function  $V$  (storage function) showing asymptotic stability and passivity.  $\square$