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Three-Dimensional Magnetic Field Modeling of a Cylindrical Halbach Array

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A semi-analytical description of the 3-D magnetic field distribution of a cylindrical quasi-Halbach permanent magnet array is derived. This model avoids the necessity of time-consuming finite element analyses and allows for fast parameterization to investigate the influence of the number of segments on the magnetic flux density distribution. The segmented magnet is used to approximate an ideal radial magnetized ring in a cylindrical quasi-Halbach array. The model is obtained by solving the Maxwell equations using the magnetic scalar potential and describes the magnetic fields by a Fourier series.

Index Terms—Actuators, magnetic fields, magnetic scalar potential, modeling, permanent magnets.

I. INTRODUCTION

THE NEED FOR efficient actuators with high force density in industrial applications is rapidly growing. Permanent magnet (PM) actuators appear to be a good class of machines to fulfill these requirements. Several papers have been written on the subject of the design of PM magnet actuators with various PM configurations. For example, Halbach structures are exploited to achieve an even higher power density with an increased amount of PM material [1]. Due to the evolution of the magnetic materials and production techniques, PMs in various shapes and with different magnetization patterns emerge and the approximation of an ideal Halbach magnetization improves. In spite of all efforts, the price of these magnets at this moment is still high, and hence, the use of magnets with simple shapes and an easy magnetization is preferred. Therefore, by using multiple small magnets with a simple shape, more complex magnetization patterns are approximated.

Regarding tubular permanent magnet actuators (TPMAs), several papers discuss the application of quasi-Halbach magnetization shown in Fig. 1(a), with radial and axial magnetized PMs. However, the radial magnetized ring magnet shown in Fig. 1(b) is difficult to magnetize especially for small radii. Therefore, in practice this PM is often approximated by diametrically magnetized segments as shown in Fig. 1(c). This segmented PM results in a 3-D effect, hence, for the exact magnetic fields in the actuator, a 3-D analysis is required. Previous papers describing tubular actuators with Halbach magnetization consider the 2-D problem with perfect radial magnetized magnets [2], [3].

In this paper a 3-D model is derived which provides the magnetic field expression for segmented quasi-Halbach arrays. To calculate the magnetic fields, a semi-analytical formulation of the magnetic scalar potential in the 3-D cylindrical coordinate system is derived. Although the model is quite complex to derive, it avoids the use of time consuming 3-D Finite element analysis (FEA), and once implemented can easily be used to

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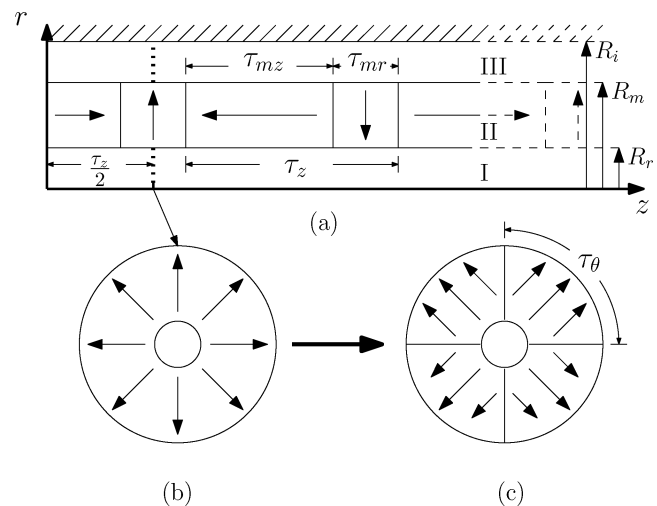


Fig. 1. (a) Cylindrical quasi-Halbach magnet array, (b) ideal radial magnetized ring magnet, (c) approximated radial magnetized ring magnet.

calculate the 3-D field effects. In Section II the derivation of the model including the analytical solution for the magnetic scalar potential and the flux density is presented. Section III compares the results from the model with finite element simulations.

II. MODELING

The aim of the model is to calculate the magnetic field in region I, II and III as depicted in Fig. 1(a). To obtain a model describing the magnetic fields, the Maxwell equations can be solved using the magnetic vector potential \vec{A} or the magnetic scalar potential φ . The magnetic vector potential results in a description of the magnetic flux density \vec{B} while the magnetic scalar potential gives the magnetic field strength \vec{H} .

The main disadvantage of φ is that it can be applied only in current free problems, however, this paper investigates only the magnetic field due to the permanent magnets, and hence, the model is current free. A major advantage of φ is the reduced complexity in 3-D problems. The magnetic vector potential will result in a vector with three components each a function of r, z, θ , while the magnetic scalar potential is a single scalar function of r, z, θ . Therefore, the magnetic scalar potential is used which is defined as

$$\vec{H} = -\nabla\varphi. \quad (1)$$

In the model, the following assumptions are made:

- 1) The soft-magnetic parts are infinitely permeable
- 2) The cylinder is infinitely long, the end-effects are not taken into account.
- 3) The permanent magnets have a linear demagnetization characteristic, and are fully magnetized in the direction of magnetization.
- 4) All regions are non-conducting and current-free.

The scalar potential has to be solved in the source free regions, I and III, and in the permanent magnet region II. In the latter region this results in the Poisson (2), and in region I and III in the Laplace (3)

$$\nabla^2 \varphi = \frac{1}{\mu_r} \nabla \cdot \vec{M} \quad (2)$$

$$\nabla^2 \varphi = 0 \quad (3)$$

where μ_r is the relative permeability of the permanent magnets and \vec{M} the magnetization vector describing the magnet array by means of a Fourier series.

A. Magnetization

The magnetization vector describing the cylindrical Halbach array has three components

$$\vec{M}(\theta, z) = M_r \vec{e}_r + M_\theta \vec{e}_\theta + M_z \vec{e}_z \quad (4)$$

where $\vec{e}_r, \vec{e}_\theta, \vec{e}_z$ are the unit vectors in the radial, angular and axial direction respectively. The position dependency is provided by the product of two Fourier series. The first Fourier series describes the magnetization as function of θ and has a fundamental period of $2\tau_\theta$. This pole pitch τ_θ is defined by the number of segments of the ring magnet, N_s , through $\tau_\theta = (2\pi)/(N_s)$. The second Fourier series describes the dependency in the z direction and has a fundamental period of $2\tau_z$.

In general, the magnetization of a quasi-Halbach array can be split in two parts, the normal magnetized magnets and the tangential magnetized magnets.

1) *Normal Magnetized Magnets:* The normal magnetized magnets are in this case the segmented diametrically magnetized ring magnet as shown in Fig. 1(c). Due to these diametrically magnetized magnets, the magnetization vector contains, besides a radial component, M_r , a component in the angular, direction, M_θ . In Fig. 2, the two components of the magnetization are shown representing a magnet which consists of four segments.

Besides the dependency of θ , M_r and M_θ are a function of z as the normal magnetized magnets are enclosed by tangential magnetized magnets in the axial, z , direction. Hence, the magnetization can be described as

$$M_r(\theta, z) = \frac{B_{rem}}{\mu_0} \sum_{n=0}^{\infty} M_{rn} \cos(w\theta) \times \sum_{k=1}^{\infty} M_{rk} \sin(mz) \quad (5)$$

$$M_\theta(\theta, z) = \frac{B_{rem}}{\mu_0} \sum_{n=0}^{\infty} M_{\theta n} \sin(w\theta) \times \sum_{k=1}^{\infty} M_{\theta k} \sin(mz) \quad (6)$$

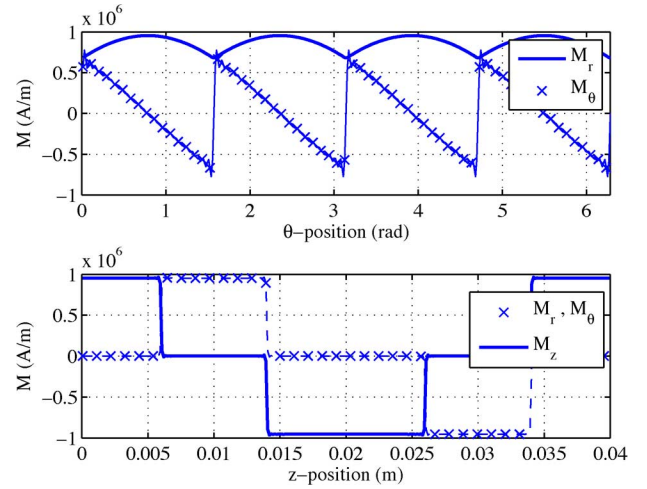


Fig. 2. Magnetization vector where the upper graph describes a ring magnet containing four segments as function of θ and the second graph describes the magnetization as function of z

where $w = (n\pi)/(\tau_\theta)$, are the spatial frequencies in the angular direction and $m = (k\pi)/(\tau_z)$, are the spatial frequencies in the axial direction. Since M_r has a DC-offset in the angular direction, as can be seen in Fig. 2, the summation of the Fourier series describing the angular dependency starts from $n = 0$. The coefficients of the Fourier series are

$$M_{rn} = \begin{cases} \frac{-8\pi \cos(\frac{n\pi}{2}) (-1)^n \sin(\frac{\pi}{N_s})}{N_s (n^2 \pi^2 - (\frac{2\pi}{N_s})^2)} & \text{for } n \neq \frac{2}{N_s} \\ 0 & \text{for } n = \frac{2}{N_s} \\ \frac{N_s \sin(\frac{\pi}{N_s})}{\pi} & \text{for } n = 0 \end{cases} \quad (7)$$

$$M_{rk} = \frac{4}{k\pi} \sin\left(\frac{(1 - \frac{\tau_{mr}}{\tau_z})k\pi}{2}\right) \sin\left(\frac{k\pi}{2}\right) \quad (8)$$

$$M_{\theta n} = \begin{cases} \frac{4n\pi \cos(\frac{n\pi}{2})^2 \sin(\frac{\pi}{N_s})}{n^2 \pi^2 - (\frac{2\pi}{N_s})^2} & \text{for } n \neq \frac{2}{N_s} \\ 0 & \text{for } n = \frac{2}{N_s} \\ 0 & \text{for } n = 0 \end{cases} \quad (9)$$

$$M_{\theta k} = M_{rk}. \quad (10)$$

2) *Tangential Magnetized Magnets:* To reduce the complexity of the expression of the magnetic scalar potential, the magnetization of the tangential (or axially) magnetized magnets, M_z , has the same form as M_r and M_θ

$$M_z(\theta, z) = \frac{B_{rem}}{\mu_0} \sum_{n=0}^{\infty} M_{zn} \cos(w\theta) \sum_{k=1}^{\infty} M_{zk} \cos(mz). \quad (11)$$

However, $M_z(\theta, z)$ is independent of the angular position, θ , as described in Fig. 2. Therefore, M_{zn} has only a nonzero value for $n = 0$, while M_{zk} describes a square wave.

$$M_{zn} = \begin{cases} 0 & \text{for } n \neq 0 \\ 1 & \text{for } n = 0 \end{cases} \quad (12)$$

$$M_{zk} = \frac{4}{k\pi} \cos\left(\frac{(1 - \alpha_{pz})k\pi}{2}\right) \sin\left(\frac{k\pi}{2}\right) \quad (13)$$

B. Magnetic Field Description

In [4] a solution for the magnetic scalar potential for a 3-D problem with radial magnetization in cylindrical coordinates is given. The solution consists of a Fourier series where the coefficients contain modified Bessel functions of the first and second kind, $\mathcal{I}_w(mr)$ and $\mathcal{K}_w(mr)$ respectively. For the problem described in this paper, a similar solution is used, however the magnetization in this problem has, besides a radial component, components in the z and θ direction.

Solving (2) using the method of separation of variables results in the magnetic scalar potential as given in (14). Inserting this expression in (1) results in expressions for the three components of the magnetic field with unknown coefficients a_{nk} and b_{nk} for each region. These equations describe the magnetic field in all regions, I, II and III, as shown in Fig. 1. In the source free regions I and III, the expression can be simplified because the functions G_r, G_z, G_θ are zero

$$\begin{aligned} \varphi(r, \theta, z) = & \sum_{n=0}^{\infty} \sum_{k=1}^{\infty} \\ & - \frac{1}{m} (a_{nk} \mathcal{I}_w(mr) + b_{nk} \mathcal{K}_w(mr)) \\ & + G_r(r) + G_z(r) + G_\theta(r) \cos(w\theta) \sin(mz) \end{aligned} \quad (14)$$

$$\begin{aligned} G_r(r) = & \frac{mM_{rk}M_{rn}B_{rem}}{\mu_0\mu_r} \\ & \times \left(\mathcal{I}_w(mr) \int_{R_r}^r \mathcal{K}_w(m\nu) d\nu \right. \\ & \left. - \mathcal{K}_w(mr) \int_{R_r}^r \mathcal{I}_w(m\nu) d\nu \right) \end{aligned} \quad (15)$$

$$\begin{aligned} G_\theta(r) = & - \frac{wmM_{\theta k}M_{\theta n}B_{rem}}{\mu_0\mu_r} \\ & \times \left(\mathcal{I}_w(mr) \int_{R_r}^r \nu \mathcal{K}_w(m\nu) d\nu \right. \\ & \left. - \mathcal{K}_w(mr) \int_{R_r}^r \nu \mathcal{I}_w(m\nu) d\nu \right) \end{aligned} \quad (16)$$

$$\begin{aligned} G_z(r) = & \frac{mmM_{zk}M_{zn}B_{rem}}{\mu_0\mu_r} \\ & \times \left(\mathcal{I}_w(mr) \int_{R_r}^r \mathcal{K}_w(m\nu) d\nu \right. \\ & \left. - \mathcal{K}_w(mr) \int_{R_r}^r \mathcal{I}_w(m\nu) d\nu \right). \end{aligned} \quad (17)$$

C. Boundary Conditions

The unknown coefficients a_{nk} and b_{nk} for each region in (14) can be found by solving boundary conditions in all regions. From the axis-symmetric coordinate system follows the first Dirichlet boundary condition. Furthermore, the magnetic scalar potential, φ , and the normal component of the magnetic flux density, B_r , should be continuous at each interface resulting in boundary condition 2 to 5 as listed below. As the iron at $r = R_i$ is assumed to be infinitely permeable, the tangential components of the magnetic field strength in region III at $r = R_i$ should be

TABLE I
MODEL PARAMETERS

Property	Value	Description
B_{rem} (T)	1.2	Remanent flux density of PM
μ_r	1.05	Relative permeability of PM
R_r (mm)	4.0	Magnet inner radius
R_m (mm)	8.0	Magnet outer radius
R_i (mm)	10.0	Iron inner radius
τ_z (mm)	20.0	Pole pitch in axial direction
α_{pz}	0.6	Magnet to pole pitch ratio
N_s	4	Number of segments of ring magnet
τ_θ (mm)	$\frac{\pi}{2}$	Pole pitch in angular direction
N	20	Number of harmonics in angular direction
K	20	Number of harmonics in axial direction

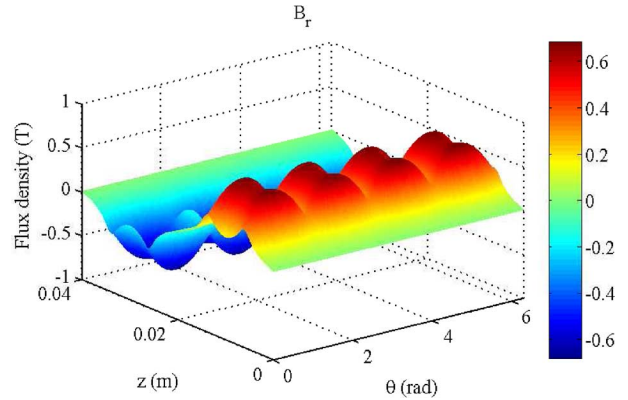


Fig. 3. A 3-D representation of the radial component of the flux density in the center of region III, calculated using the semi-analytical model.

zero. As the magnetic field strength is the derivative of the magnetic scalar potential (14), the magnetic scalar potential should be constant, c , at this radius

- 1) $\varphi_I = 0|_{r=0} \forall z, \theta$
- 2) $\varphi_I = \varphi_{II}|_{r=R_r} \forall z, \theta$
- 3) $B_{rI} = B_{rII}|_{r=R_r} \forall z, \theta$
- 4) $\varphi_{II} = \varphi_{III}|_{r=R_m} \forall z, \theta$
- 5) $B_{rII} = B_{rIII}|_{r=R_m} \forall z, \theta$
- 6) $\varphi_{III} = c|_{r=R_i} \forall z, \theta$.

III. RESULTS

As the expression of the magnetic scalar potential (14) contains twice an infinite sum, an approximation over a finite number of harmonics is needed to find a solution. Therefore, in the following sections, the number of harmonics describing the potential and magnetic fields in the axial direction is limited to K while the number of harmonics in the angular direction is limited to N . Consequently, solving the boundary conditions results in $6 \times K \times N$ equations.

This set of equations is implemented in MATLAB[®] together with the magnetization and numerical approximations for the integrals in $G_r(r), G_z(r), G_\theta(r)$. This results in a solution for the magnetic fields in the three regions as shown in Fig. 1(a). The magnetic field distribution is calculated for the geometric parameters given in Table I. In Fig. 3, the radial component of the flux density in the center of region III is shown. The four segments of the normal magnetized magnet are clearly visible.

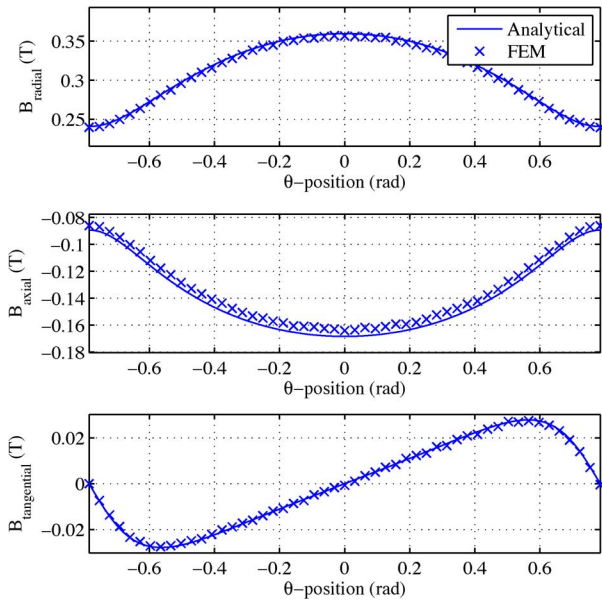


Fig. 4. Comparison of the flux density in the center of region III (at $r = 9.0$ mm) calculated by FEM model and the analytical model. The three components of the flux density are shown as function of the angular position at a constant $z = 5.0$ mm.

A. Comparison With FEM

The results of the model are verified with a 3-D FEM model created in FLUX3D with the same assumptions as listed in Section II, i.e., an infinitely long cylinder and infinitely permeable iron. The magnetic field distribution in the center of region III at $r = 9.0$ mm is depicted in Figs. 5 and 4. In Fig. 5, the three components of the flux density are calculated for a constant $z = 5.0$ mm as function of the angular position θ . The figure shows the flux density for $-(\tau_{\theta})/(2) < \theta < (\tau_{\theta})/(2)$ and shows good agreement between the FEM model and the analytical model. Fig. 4 shows the flux density components for a constant $\theta = -(\pi)/(8)$ as function of the axial position for $-(\tau_z)/(2) < z < (\tau_z)/(2)$. The deviation between the two models is in all points smaller than 3%. However, as can be seen for the tangential component of the flux density in Fig. 4, the FEM results contain some small fluctuations, which contribute to the total deviation as well.

IV. CONCLUSION

This paper concerns the 3-D modeling of a cylindrical quasi-Halbach permanent magnet array. The effects of segmentation of the normal magnetized magnets are calculated

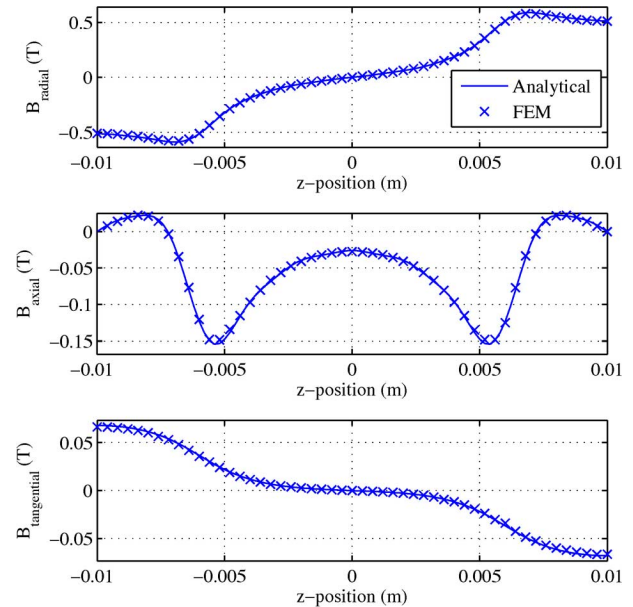


Fig. 5. Comparison of the flux density in the center of region III (at $r = 9.0$ mm) calculated by FEM model and the analytical model. The three components of the flux density are shown as function of the axial position at a constant $\theta = -(\pi)/(8)$ rad.

which were neglected in previous publications regarding 2-D modeling quasi-Halbach arrays [2], [3]. The derivation of a three dimensional semi-analytical model describing the segmentation effect in a quasi-Halbach cylinder is presented. The magnetic field description is obtained by solving the Maxwell equations using the magnetic scalar potential. The resulting model is implemented and verified with a FEM model and shows good agreement. The model can be used to calculate the effect of using segments in a quasi-Halbach array without the need for a time-consuming FEM analysis.

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