Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics and Chemistry of the Earth 30 (2005) 420–425
PHYSICS and CHEMISTRY of the EARTH
www.elsevier.com/locate/pce

Assessment of serious water shortage in the Icelandic water resource system

H. Jonsdottir [a,*], J. Eliasson [b], H. Madsen [a]

[a] Department of Informatics and Mathematical Modelling, Bldg. 321 DTU, DK-2800 Lyngby, Denmark
[b] Department of Civil and Environmental Engineering, University of Iceland, Hjarðarhaga 2-6, 107 Reykjavik, Iceland

Abstract

Water resources are economically important and environmentally extremely vulnerable. The electrical power system in Iceland is hydropower based and due to the country's isolation, power import is not an option as elsewhere in Europe. In the hydropower system, a water shortage is met by flow augmentation from reservoirs. The management of these reservoirs are a human intervention in a natural flow and therefore necessarily limited by environmental regulations. During a heavy drought, the available water storage in the reservoir may not be sufficient to cater for the demand and consequently there will be a shortage of electrical power. This is politically acceptable as long as it only touches heavy industries but not power deliveries to the common market. Empty or near empty reservoirs cause power shortage that will be felt by homeowners and businesses, until spring thaw sets in and inflow to the reservoirs begins. If such a power shortage event occurs, it will cause heavy social problems and a political decision making will follow. It is commonly agreed, that management methods leading to such a disastrous event as a general power shortage in the whole country, are not acceptable. It is therefore very important to have mathematical tools to estimate the risk of water shortage in the system when searching for the best management method. In view of the fact that the subject is to estimate the risk of events that have to be very rare, i.e. with large recurrence time, stochastic simulation is used to produce synthetically run-off records with adequate length, in order to estimate very rare droughts. The method chosen is to make the run-off series stationary in the mean and the variance and simulating the resulting stationary process. When this method is chosen, future trends in the run-off from climate change and glacier reduction can easily be incorporated in the model. The probabilities of extreme droughts are calculated and their frequencies are compared to theoretical distributions.

Keywords: Droughts; Stochastic simulation; Hydropower plant

1. Introduction

Computer simulations have been used to analyze the capacity of the Icelandic power system since about 1970. The simulation system has steadily been upgraded and extended to meet the various requirements for specified information on risks and capacity figures. However, simulations with stochastic flow models have not been much used so far, except for a few attempts in the years 1970–1990. One of the main questions is the risk of emptying the main reservoir and the magnitude of the following drought. Such a drought will inevitably cause a major power shortage. If this power shortage is long enough (more than a few days) it will cause serious social and economic problems such as degradation of food stocks in cold storage, operational failure of large district heating systems and immense difficulty in communication and telecommunication.

Stochastic methods have been known in hydraulic design for quite some time, e.g., Plate (1992) but they are still not extensively used in risk assessment. One of the major questions in the simulation analysis of

* Corresponding author. Tel.: +354 5696051; fax: +354 5688896.
E-mail address: hj@os.is (H. Jonsdottir).

1474-7065/$ - see front matter
doi:10.1016/j.pce.2005.06.007

the Icelandic power system is the performance of the reservoirs and the magnitude and the energy shortage if they run dry (Johannsson and Eliasson, 2002, 2003).

The method of using all available flow series in order to design a reservoir large enough to sustain a predefined flow output is well known in hydraulic engineering. The non-computerized graphical version can be seen e.g., in Linsley and Franzini (1964). This method is still largely used by engineers in reservoir capacity planning, but the method cannot predict the risk of water shortage. It is however, evident that the longer the inflow series is, the more reliable is the resulting volume capacity, but there is no way of presenting this knowledge in an explicit form. However simulation with stochastic flow models can provide that. To demonstrate this principle we have selected a reservoir in the river Tungnaá in Iceland, originally proposed in 1960 but not yet built.

2. Regulated flow and volume of reservoir

The major decision of a hydropower construction is how much power is to be produced. The power produced is linearly dependent on the flow. To supply the power net a flow is needed that is very constant compared to the natural flow; this constant outflow is known as the regulated flow. These decisions are made on a basis of discharge time series. Figs. 1 and 2 show the discharge in the river Tungnaá.

A diagram of a simple hydropower plant with one reservoir is shown in Fig. 3. Such a model is used by the National Power Company of Iceland in order to calculate water values in their system simulation studies (Johannsson and Eliasson, 2002).

It is clear that the maximum regulated flow is the average flow Q_{mean} of the whole series, shown in Fig. 1, and then no flow is bypassed at any point in time.

The reservoir is high up in the mountains. Its purpose is flow augmentation for a series of power stations, at lower elevations, downstream in the river basin. The

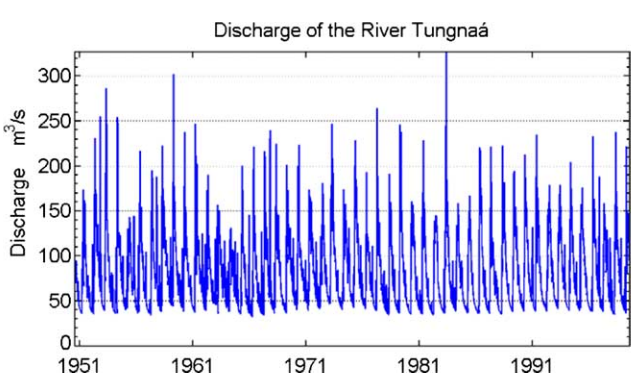

Fig. 1. Natural flow in the river Tungnaá September 1st 1951–August 31st 2001 (Nat. Energy Authority of Iceland).

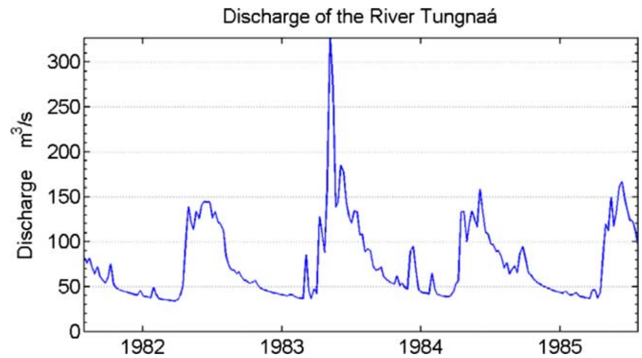

Fig. 2. Natural flow in the river Tungnaá September 1st 1981–August 31st 1985 (Nat. Energy Authority of Iceland).

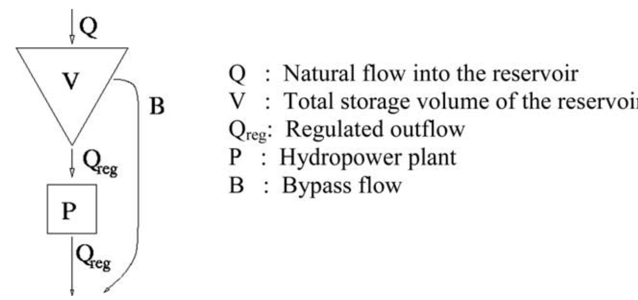

Fig. 3. Schematic drawing of reservoir (V), and power station (P).

water level in the reservoir does not affect the power capacity of any of these stations. In the following analysis it will be assumed for the sake of simplicity, that Q_{reg} is constant in time which implies that P is constant and the outflow from the power station is constant and equal to Q_{reg}. In practice Q_{reg} will be somewhat larger in wintertime than in summertime; this will increase the storage volume requirement somewhat, so the V values discussed in this article can be regarded as minimum values.

In general the water balance is calculated as the total inflow minus the total regulated flow at any given time step i.e.

$$\text{Balance}(i) = \int_0^i Q(\tau)\,\mathrm{d}\tau - i \cdot Q_{\text{reg}}. \tag{1}$$

Fig. 4 shows the water balance for the data in Fig. 1 with regulated flow as the average flow, i.e. $Q_{\text{reg}} = Q_{\text{mean}}$. The water balance means the balance between total inflow and total outflow. The only evaporation and rainfall to be considered is on the reservoirs surface itself and that water amount is negligible.

As mentioned the largest possible flow which can be regulated is the average flow Q_{mean}. Define $V_{\max}$ as the smallest reservoir which can serve the maximum regulated flow, Q_{mean}. Considering the time series of the water balance, Balance(i), defined by Eq. (1) and shown in Fig. 4, it is clear that the volume $V_{\max} =$

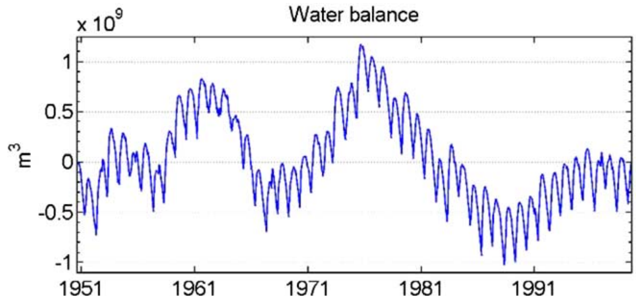

Fig. 4. The water balance.

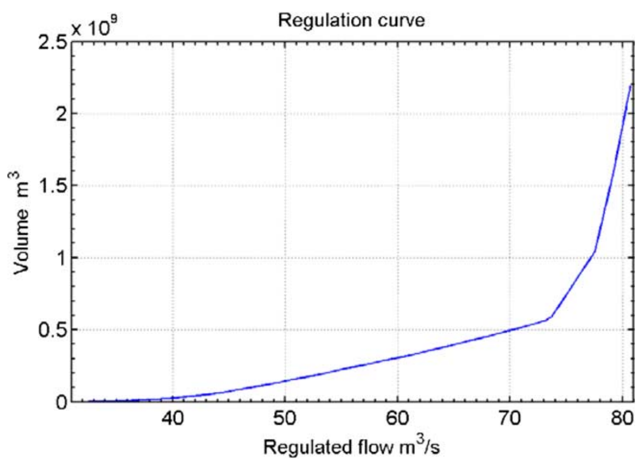

Fig. 5. Regulation curve for the Tungnaá river.

max[Balance(i)] − min[Balance(i)]. This volume serves the purpose of securing zero bypass and thus maintains the average flow as regulated flow, i.e. $Q_{\text{mean}} = Q_{\text{reg}}$. In general for a given regulated flow Q_{reg} there is a corresponding volume V which is the smallest reservoir volume that can secure Q_{reg} without any water shortage occurring according to the flow series in Fig. 1. In other words, for a given V, Q_{reg} can be calculated as the maximum regulated flow such that no water shortage occurs. Fig. 5 shows this curve for the Tungnaá data in Fig. 1, note the point $(Q_{\text{mean}}, V_{\text{max}}) = (80.7\ \text{m}^3/\text{s},\ 2192.8 \times 10^6\ \text{m}^3)$.

The curve (Q_{reg}, V) is completely based on the time series for Q and from a deterministic point of view the risk of water shortage is zero, when using a point (Q_{reg}, V) from the curve. In order to estimate the risk of water shortage a stochastic model is required to perform simulation studies.

3. Stochastic model

The model chosen is a stochastic periodic model as suggested by Yevjevich (1976). The length of the period is denoted as T and number of periods is denoted as n. Let Q denote the matrix of discharge data

$$Q = \begin{pmatrix} Q(1,1) & \cdots & Q(1,T) \\ \vdots & \ddots & \vdots \\ Q(n,1) & \cdots & Q(n,T) \end{pmatrix}.$$

Define $P(t)$ as a periodic mean, and $S(t)$ as a periodic standard deviation. $P(t)$ is estimated as

$$P(t) = \frac{1}{n}\sum_{j=1}^{n} Q(j,t) \quad t = 1,\ldots,T \tag{2}$$

and similarly $S(t)$ is estimated as

$$S(t) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(Q(j,t) - P(t))^2}. \tag{3}$$

The standardized residuals are calculated as

$$Y(j,t) = \frac{Q(j,t) - P(t)}{S(t)} \quad j = 1,\ldots,n \text{ and } t = 1,\ldots,T. \tag{4}$$

The matrix of standardized residuals is reorganized as a row vector by $Y_{\text{vect}} = Y(1,1),\ldots, Y(n,1), Y(2,1),\ldots, Y(2,n),\ldots, Y(T,n)$ and fitted to a seasonal AR model $\phi(B)\Phi(B^{\text{T}})Y(i) = e(i),\ i = 1,\ldots, T\cdot n$. The operator ϕ is a polynomial of degree p in the backward shift operator B, i.e. $\phi(B)Y(i) = (1 - a_1B - \cdots - a_pB^p)Y(i) = Y(i) - a_1Y(i-1) - \cdots - a_p\ Y(i-p)$. Similarly the operator Φ is a polynomial of degree p in the seasonal backward shift operator B^{T}, thus representing the seasonal component of the AR model, if needed. Then the stochastic periodic model (Yevjevich, 1976) is written as

$$\begin{aligned} \tilde{Q}(t) &= P(t) + S(t)\tilde{Y}(t) \quad t = 1,\ldots,T \\ \varphi(B)\Phi(B^{\text{T}})\tilde{Y}(t) &= e(t) \quad e(t) \sim N(0,\sigma^2) \end{aligned} \tag{5}$$

where $\tilde{Y}(t)$ is simulated by using the seasonal AR model as in Eq. (5) and $\tilde{Q}(t)$ is the periodic discharge, simulated by using the $\tilde{Y}(t)$.

In this project a sampling time of one week is chosen. The daily discharge data are low pass filtered with a seven day average in order to decrease the variance of the data, but yet the weekly sampling time is small enough for decision making. Thus the length of the period is 52 time steps, and the data available span 49 seasonal periods.

To ensure that physical laws are conserved, such as nonnegative flow it was chosen to transform the data by using the logarithm base ($\log_e$) of the data. For the transformed data the appropriate model was found i.e.

$$\begin{aligned} \tilde{Q}_{\log}(t) &= P_{\log}(t) + S_{\log}(t)\tilde{Y}_{\log}(t) \quad t = 1,\ldots,\text{T} \\ \varphi(B)\Phi(B^{\text{T}})\tilde{Y}_{\log}(t) &= \varepsilon(t) \quad \varepsilon(t) \sim N(0,\sigma^2) \end{aligned} \tag{6}$$

where $P_{\log}(t)$ and $S_{\log}(t)$ are calculated from the log-transformed data. Afterwards the residuals $\text{Y}_{\log}(t) = (Q_{\log}(t) - P_{\log}(t))/S_{\log}(t)$ are modelled. This transformation implies that the residuals in the original model $e(t)$

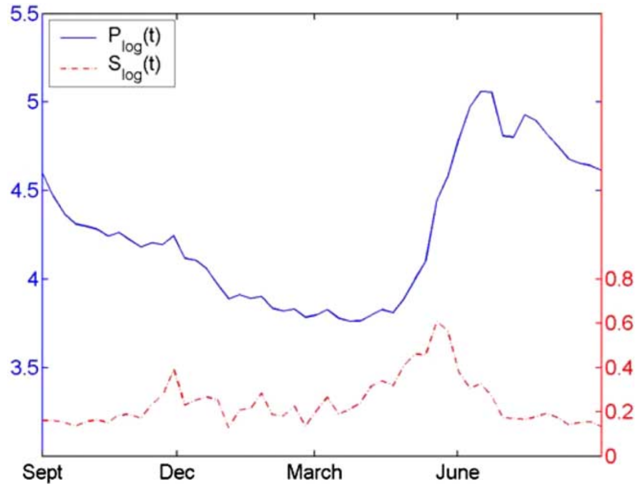

Fig. 6. The periodic average $P_{\log}(t)$ and the periodic standard deviation $S_{\log}(t)$ of the log-transformed data.

as defined in Eq. (5) are lognormal distributed, which indeed has a physical meaning. The calculations are performed in the program Splus and the result is as follows:

$$(1-0.62B)Y_{\log}(t)=\varepsilon(t) \quad \varepsilon(t)\sim N(0,0.06) \tag{7}$$

i.e. an AR(1) model without a seasonal component. Fig. 6 shows the curves $P_{\log}(t)$ and $S_{\log}(t)$. Note the similarity between the derivative of the flow (i.e. $\nabla P_{\log}(t)$) and the standard deviation (i.e. $S_{\log}(t)$).

4. The simulation study

The simulation was performed using the theoretical model in Eq. (6) where the process $Y_{\log}(t)$ is an AR(1) model without a seasonal component as estimated in Eq. (7), the resulting simulated series is denoted $Q_{\text{sim}}(t)$. Two droughts are defined as independent if they either occur in two different years or if they occur in the same year and the reservoir is refilled between the two droughts, two or more dependent droughts are grouped together in single independent droughts. The water shortage is calculated as the total shortage of water within a single independent drought. Let X denote the total water shortage within a drought, then the required probability is the probability of a water shortage larger than x, i.e. $P(X\geqslant x)$ where X denotes the random variable of water shortage. Simulations were performed for several pairs (Q_{reg}, V) on the regulation curve shown in Fig. 5. The goal is to estimate probabilities of events that are very rare and it was found necessary to simulate for 50,000 years in order to achieve a stable estimate of the water shortage probabilities. Note that the simulations of 50,000 years does not imply prediction 50,000 years into the future but a stochastic generation for 50,000 years given that the weather condition will be like the past 50 years which were used for parameter estimation in the stochastic model. However, since the deviation series $Y(t)$ is stationary in mean and variance, climate change predictions for future trends in runoff series such as glacier melt can be taken into account either as deterministic or stochastic variables depending on the climate change model output.

None of the simulation results included events with two independent droughts within the same year thus the probabilities of water shortage is estimated as

$$P(X\geqslant x) = \frac{\text{Number of years with water shortage larger or equal than } x}{\text{Number of years in simulation}} \tag{8}$$

Thus there are estimated n probability values $p_1,\ldots,p_n$, where n is the total number of droughts which occurred in the simulation and

$$p_j = P(X\geqslant x_j) \tag{9}$$

where x_j is the jth largest water shortage, thus $p_1 = 1/50{,}000$, $p_2 = 2/50{,}000,\ldots$, $p_n = n/50{,}000$.

Note that $P(X\geqslant x)\approx P(X>x)=1-F(x)$, where $F(x)$ is the probability distribution function. The presented results are from simulations where the reservoir is 1315.7×10^6 m^3 and regulated flow is 78.36 m^3/s, which is a point on the regulation curve illustrated in Fig. 5. The simulations were repeated 100 times in order to obtain information about the variations. Consequently for each probability p_j there correspond 100 different x values $x(j,1),x(j,2),\ldots,x(j,100)$, which yielded the estimate $p_j = P(X\geqslant x(j,i))$, $i=1,\ldots,100$. The conditional distribution of the random variable $\{x(j,i)|p(j)\}$ is assumed to be a normal distribution $N(\mu(j),\sigma^2(j))$. The mean, $\mu(j)$, and the variance $\sigma^2(j)$ are estimated (using 100 observations) with the maximum likelihood method and the estimated mean $\hat{\mu}(j)$ will be denoted as x_{mean} and the pairs $(x_{\text{mean}}(i),p(i))$ will be referred to as the simulation result. The generalized extreme value distribution (GEV), is fitted to the accumulated probabilities $(1-p)$. The GEV distribution can be parameterized as (Reiss and Thomas, 1997)

$$P(X\geqslant x) = \text{GEV}_{\alpha,\beta,\xi}(x) = \exp\left[-\left(1+\xi\frac{x-\alpha}{\beta}\right)^{\frac{-1}{\xi}}\right]. \tag{10}$$

The parameters are estimated using the least square method and the result is shown in Table 1.

Fig. 7 shows the (x_{mean},p), the 2.5% quantile, $(x_{0.025},p)$, the 97.5% quantile, $(x_{0.975},p)$ and the fitted distribution $(x_{\text{mean}},p_{\text{fit}})$, i.e. $1-\text{GEV}(x)$.

The difference of the simulation result and the fitted probabilities can hardly be detected, but from probabilities around 0.1% (i.e. recurrence time 1000 years) the fitted distribution converges to zero faster then the simulated probabilities. This can better be detected in a

Table 1
Estimated parameters in the GEV distribution using the least square method

	α	β	ξ
Estimation	−897.95	376.86	−0.2744

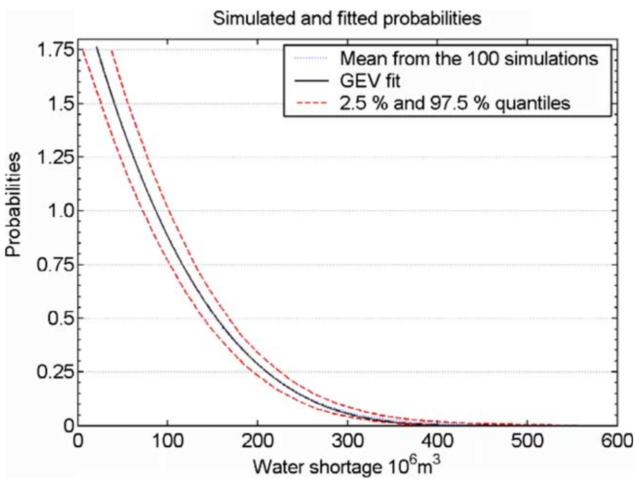

Fig. 7. The estimated mean value of the repeated simulations, approximated 2.5% and approximated 97.5% quantiles and the fitted GEV distribution.

quantile diagram. There is an interest in the tail of the distribution and consequently it was chosen to divide the data into as many intervals as reasonable. Hence, 160 intervals with equal probabilities are generated, with the expected number of observations in each interval as 5.48. Fig. 8 shows a quantile diagram for the GEV(x) distribution, with $(\alpha, \beta, \xi) = (-897.95, 376.86, -0.2744)$ and estimated probabilities from histogram with the 160 intervals. The chi-square test statistics for test of distribution using the same intervals is $z = 12.0228$, and the

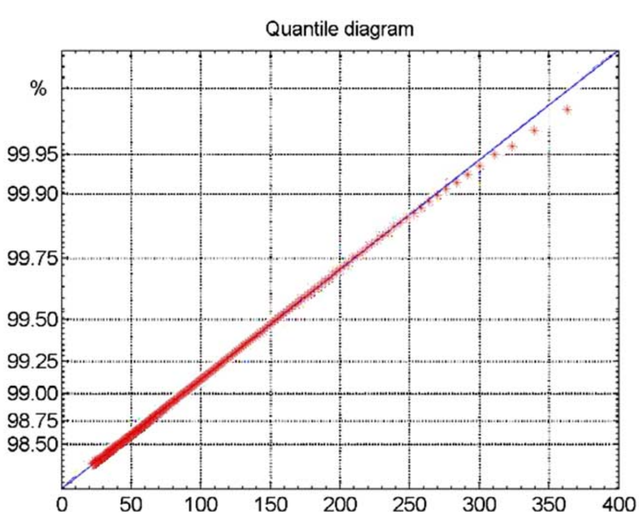

Fig. 8. Quantile diagram for the data compared to the GEV distribution with the parameter estimate according to Table 1 $(\alpha, \beta, \xi) = (-897.95, 376.86, -0.2744)$.

90% quantile is $\chi(156)^2_{0.9} = 179$ and it follows that a hypotheses that the water shortage is GEV distributed with the estimated parameters as shown in Table 1, $(\alpha, \beta, \xi) = (-897.95, 376.86, -0.2744)$ is accepted. However, it must be kept in mind that the simulations are necessary in order to estimate the parameters.

The shape parameter ξ in the generalized extreme value distribution has been estimated as negative, thus the extreme value distribution is identified as the Weibull distribution (Reiss and Thomas, 1997).

The domain the of distribution is the interval $]-\infty, -\beta/\xi + \alpha] =]-\infty, 475.23]$. Note that about 98% of the probability mass is below zero, i.e. with negative x values. The mean and the standard deviation of the distribution are (Reiss and Thomas, 1997):

$$E(x) = \mu = \alpha - \frac{\beta}{\xi} + \frac{\beta}{\xi}\Gamma(1-\xi) = -762.96 \quad (11)$$

$$V(x) = \sigma^2 = \left(\frac{-\beta}{\xi}\right)^2 \Gamma(1-2\xi) + \Gamma^2(1-\xi) = 1791.39^2 \quad (12)$$

The Weibull distribution can be re-parameterized with domain $]-475.27, \infty[$. Then the distribution function is reflected about the y-axis and the random variables multiplied by -1. Hence, the distribution function becomes

$$1 - \exp\left(-\left(1 + \xi\frac{-x-\alpha}{\beta}\right)^{-1/\xi}\right) \quad \text{with } x \geqslant -(-\beta/\xi + \alpha) \quad (13)$$

setting $k = -1/\xi > 0$, $b = k\beta = -\beta/\xi$ and $a = -(\alpha + k\beta) = -\alpha + \beta/\xi$ this becomes

$$1 - \exp\left(-\left(\frac{x-a}{b}\right)^k\right) \quad \text{with } x \geqslant a \quad (14)$$

which is a more commonly used parameterization in hydrology. Using this interpretation the random variable is interpreted as the result of the water balance equation

$$X(i) = \int_0^i Q_{\text{sim}}(\tau)\,\mathrm{d}\tau - i \cdot Q_{\text{reg}} \quad (15)$$

and water shortage will occur if $X(i)$ is negative. The stochastic formulation of a water shortage, using this interpretation, is a peak below threshold study, with threshold zero, see e.g. Medova and Kyriacou (2000). On the other hand for practical purposes it is more convenient to work with water shortages as positive variables with decreasing probabilities. (Note that the dynamic variable $X(t)$ in Eq. (15) is an unstable time series i.e. with pole equal to one, since $(1-B)X(t) = u(t)$ with $u(t) = Q_{\text{sim}}(t) - Q_{\text{reg}}$).

Using the quantile diagram in Fig. 8, the probability of water shortage of 155 million m^3 is 0.5% and thus the recurrence time for a water shortage of 155 million m^3 is

200 years or larger. A water shortage of 155 million m^3 means that the power station is out of operation for about three weeks. There is a 15–30% probability that a large drought like that will occur in the economical lifetime of the project, which is 30–60 years for hydropower stations in Iceland.

5. Conclusions

The risk of water shortage in a hydropower plant has been estimated through stochastic modeling and simulations. In general the available data are used for design of a hydropower plant. Thus the recurrence time of drought is large and therefore a very long time series is needed in order to estimate the drought risk. The stochastic simulations produce a time series long enough for achieving an estimate of the probability distribution function of a water shortage. Furthermore the repeated simulations provide an estimate of uncertainty of the probability function estimation. The simulation study in this project was performed for a simple system with one hydropower plant and one reservoir, but using the tools already developed for power system studies in Iceland, it is straightforward to extend the model for more complicated systems.

As mentioned the recurrence time for water shortage of 155 million m^3 is estimated to be 200 years or larger, which means the power station is out of operation for about three weeks. For a water power station with a life time of 50 years, there is a probability of 25% that water shortage of this magnitude will occur in the economical lifetime. On top of that, there is a great probability that water shortage will occur in other reservoirs as well due to spatial correlation in Icelandic run-off data. A power failure of this magnitude will most likely be considered socially and politically unacceptable with disastrous consequences for power system management practices.

Acknowledgment

The authors wish to thank The National Energy Authority in Iceland for providing the discharge data used in this study.

References

Johannsson, S., Eliasson, E.B., 2002. Simulation model of the hydrothermal power system in Iceland. Technical Report, Annad veldi ehf. Available from: <www.veldi.is>.

Johannsson, S., Eliasson, E.B., 2003. Simulation model of transmission constrained hydrothermal power system. Technical Report, Annad veldi ehf. Available from: <www.veldi.is>.

Linsley, R.K., Franzini, J., 1964. Water Resources Engineering. McGraw-Hill, New York.

Medova, E., Kyriacou, M., 2000. Extreme values and the measurement of operational risk. Oper. Risk 1 (8), 12–15.

Plate, E.J., 1992. Stochastic design in hydraulics. In: Proceedings of the International Symposium on Stochastic Hydraulics.

Reiss, R.-D., Thomas, M., 1997. Statistical Analysis of Extreme Values. Birkhäuser Verlag, Switzerland.

Yevjevich, V., 1976. Structure of natural hydrologic time series. In: Shen, H.W. (Ed.), Stochastic Approaches to Water Resources, Colorado.