Non linear space representation - perspective with six vanishing-points

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Abstract

The objective of this paper is to give an overview of the main principles of space representation in perspective with six vanishing points.

The system built up from perspectives based on the number of the vanishing points is finished with the sixth one, in case that the vanishing points are both ideal points of each of the 3 axes of a cartesian coordinate system. Thereby besides the origin of the above mentioned cartesian coordinate system, the whole space can be represented on one image.

Processing and displaying the data of a scene helps to learn visualising the world in the new system compared to the traditional perspectives, and also shows further directions for technical development.

1. Space representation

Representation of the objects surrounding us is an issue that has been mentioned since the first objects appeared. Since the vision is the most important way of perception in our life, registering the information we can receive visually is essential.

Reviewing the history of visual arts many different solutions can be found and understood depending on the technical opportunities and the target of the actual scene to be represented. One of the most important parameters that define the method is the plane on which we create the image of the original scene. Figure 1. and 2. illustrates two solutions for this question.

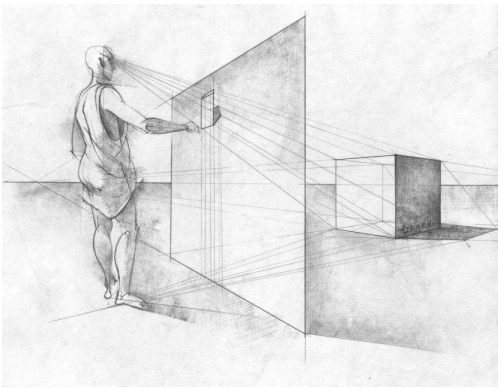

Fig.1. Drawing on plane [1]

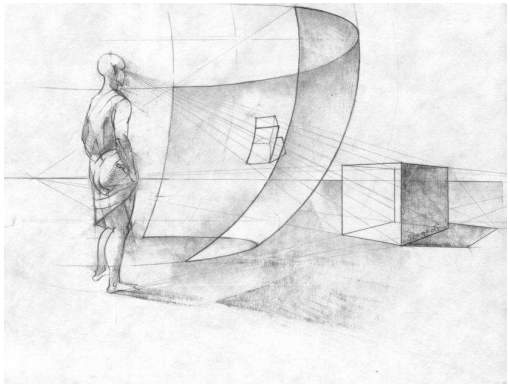

Fig.2. Drawing on sphere [1]

Figure 1 shows mapping for plane that is the method we mostly call central projection. Figure 2 shows the idea of creating an image on a spherical surface, the method is based on measuring angles. The two methods have different strengths and weaknesses and they are considered as two independent methods. Since, increasing the radius of the sphere and reducing the angle in which the whole scene can be seen, the spherical surface we use can be considered almost planar and the two images can be comparable.

2. Perspective systems

The idea to define a special point later called vanishing point, the ideal point of specified lines belongs to the renaissance arts. Brunelleshci's linear perspective is one of the first steps in perspective representation.[2]

Using one vanishing point the specified lines are most often the lines perpendicular to the plane of the mapping, so the lines parallel to the direction of view of the observer. This way a scene that fits to a 30 to 40° cone of view can be represented without significant distortion. Avoiding the possible distortions widening the scene of observation the number of the vanishing points should be increasing and the specified lines should be selected considering the composition of the actual scene.

In the system we built to show the way to the sixth vanishing points we use a fixed cartesian coordinate system, where the origin is the eye of the observer and the direction of view is parallel to the x-axis. [3]

As we extend more the field of represented view, we meet the problem: parallel lines seem to converge to different vanishing points so the picture of lines are not always linear. (see Figure 4.) This contradiction can be possible because while we use perspective with one, two or three vanishing points, only half-lines, because we ignore the three other vanishing points. The first perspective which can represent a whole line, so which can provide 180° field of view in one direction is the perspective with 4 vanishing points. Defining this vanishing point and all following ones means that we duplicate the representable part of space. In our example the 4th vanishing point is the second ideal point of z-axis (it is always optional in which direction we want to extend the view, naturally it depends on the scene).

3. The sixth vanishing point

Increasing the number of vanishing point using systematically the ideal points of the axes of our coordinate-system gives one image that gives information of the scene except the point of the observation. Figure 3. shows the image of point P in the cartesian coordinate system and in perspective with six vanishing points.

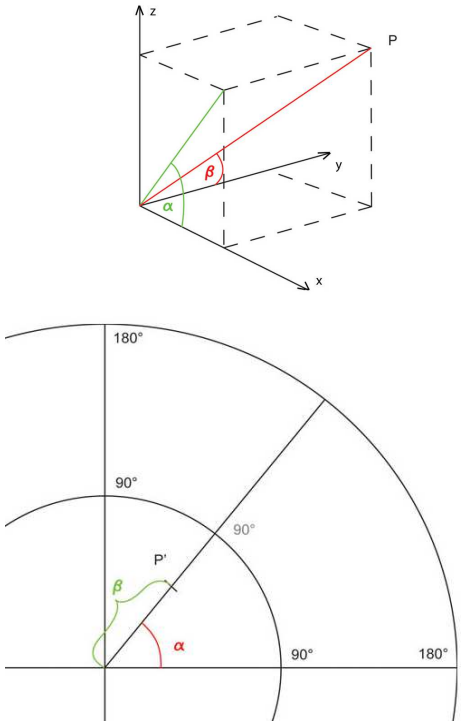

Fig. 3. Image of point P with Cartesian coordinates and in perspective with six vanishing points

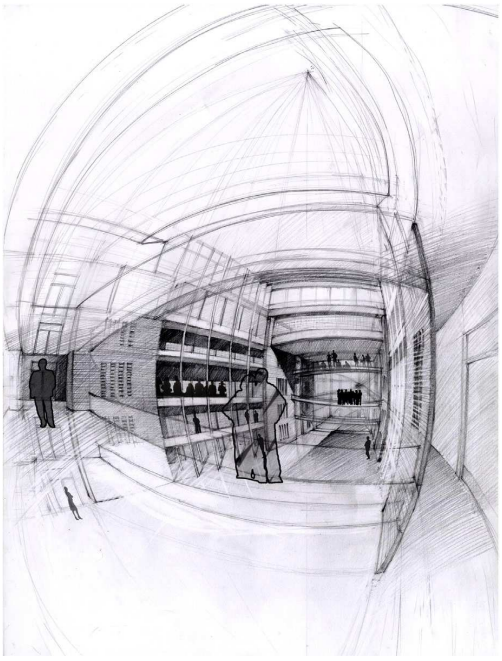

Fig.4. Drawing of an interior in spherical perspective [4]

What do we know about the sixth vanishing point?

1. We can see the straight section between the vanishing point in front of us and behind us in 180°.

2. If the angle between one point's position vector and the x-axis is 180° then that point is exactly behind us and its image coincides with the sixth vanishing point. As it follows from the above mentioned conditions the 6th vanishing point`s image is a circle which has a radius corresponding to 180° and its center point is the image of the origin.(See Figure 5.)

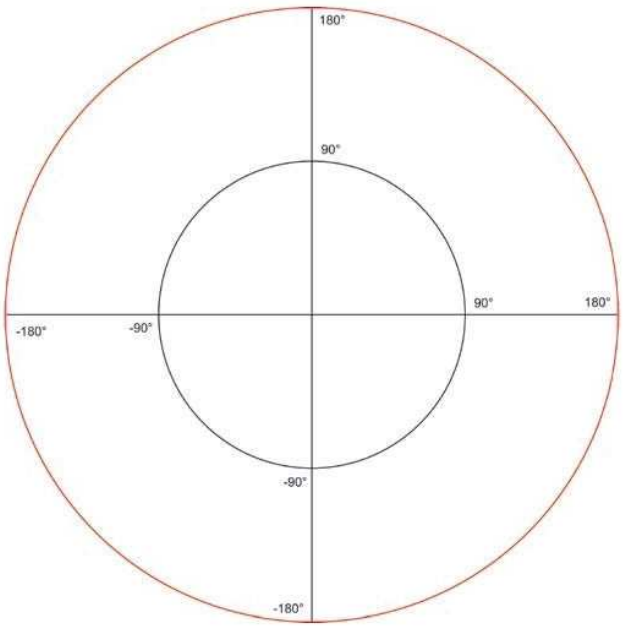

Fig. 5. The circle in which the whole space can be represented in perspective with six vanishing points

The method refers to figure 2, since we use two angles to define the position of the point. The angles can be calculated with the following equations.

$$\alpha = arctg\frac{z}{x} \quad (1)$$

$$\beta = arctg\frac{\sqrt{\left(x^2 + z^2\right)}}{y} \quad (2)$$

Using this equations the actual eighth part of space should be considered. We can carry out similar calculations, these are trivial.

4. Computer visualisation

The advantages of this system is that with the above mentioned equations is that they are easy to handle digitally. The first steps made by us are soma simple programs to show the steps of this representation.

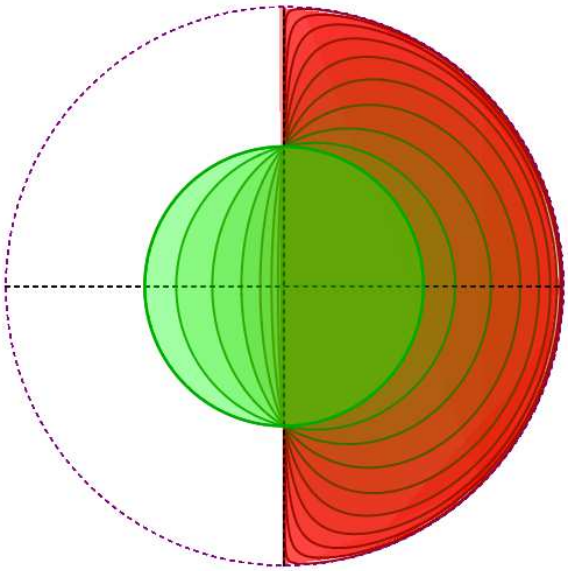

Fig.6. Steps of rotation fro plane [a,a] to [a,a]

Figure 6 shows the steps of rotation of planes from [y,z] to plane [x,z]. Figure 7 shows the steps of rotation from plane [x,y] to plane [x,z]. The image of the planes are curves here, the fill of the curves is only for the animation.

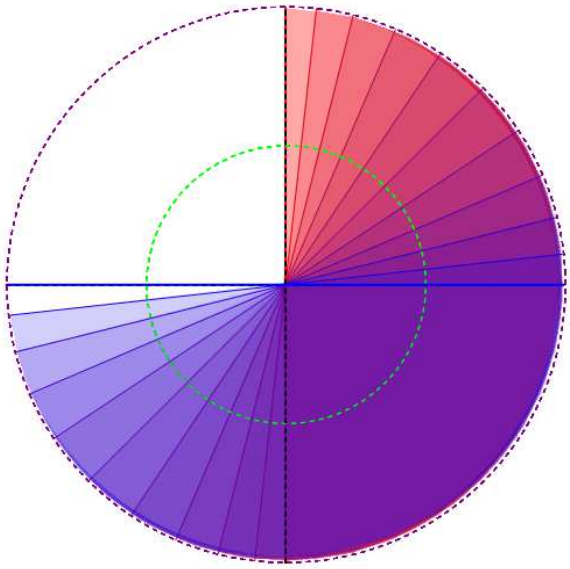

Fig.7. Steps of rotation fro plane [a,a] to [a,a]

Figure 8 illustrates a dodecaeder in axonometry and perspective with six vanishing points. As it can be seen, the image close to the direction of view is quite similar to the visual world we are used to. The part of the space that is between us is much less expressive but understanding the two angles that define the mapping, the image can be clearly interpreted

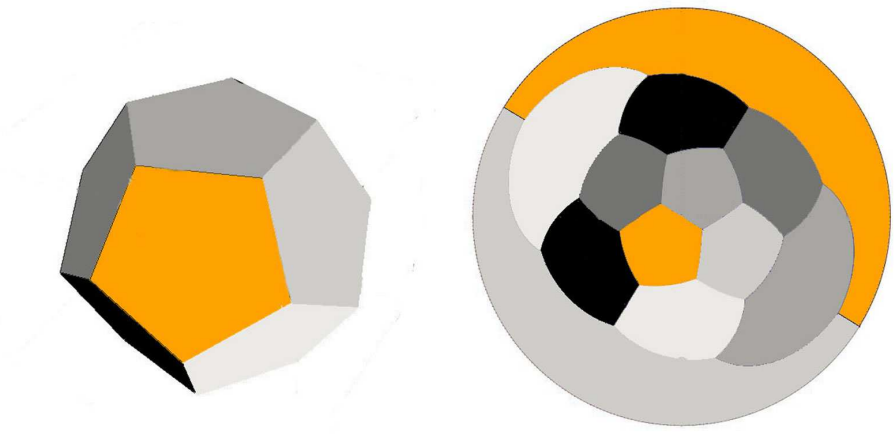

Fig.7. Dodecaeder in axonometry and in perspective with six vanishing points

5. Conclusion

Perspective with six vanishing points can give a system in which one image can contain information of the whole space around the observer. However the image is not expressive at every part, after creating an image, transformations can be done digitally. Further research topics are to observe possibilities of using this perspective in education, creating a camera that can make an image based on this method.

Bibliography

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