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LONGITUDINAL TIRE FORCE ESTIMATION WITH UNKNOWN INPUT OBSERVER

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ABSTRACT

This paper presents the development of a longitudinal tire force estimation algorithm. Rather than using highly nonlinear tire-road friction models, we propose a decentralized estimation algorithm which treats the longitudinal tire force of the individual wheel as an unknown input signal for the one-DOF wheel dynamic model. Two types of unknown input observers are designed to estimate the longitudinal tire force based on the wheel speed measurement in this study. To make the estimated tire force signals also satisfy the longitudinal dynamic model of the vehicle, the estimation results of the unknown input observer are integrated with the longitudinal acceleration measurement by using a projection method.

INTRODUCTION

Tire-road friction force is a crucial signal in various automotive active safety systems, such as anti-lock braking systems (ABS), traction control (TC) and electronic stability control (ESC) [1]. However, no commercial vehicles are equipped with sensors which can directly measure this force signal. This is due to either cost concern or technical challenge. This provides a need for an efficient and accurate estimation algorithm. In fact, the ever-increasing demand for safety and driving comfort makes it a very active research field in both academic society and auto industry. Many research results can be found in the literature. However, tire-road friction is a very complex physical phenomenon, which is represented by various complicated mathematical models, such as Magic Formula [2] and Dugoff tire model [3]. To utilize this kind of model, an online identification algorithm should be developed to detect the change of those parameters that classify road conditions. Even after considerable simplification, those models are still highly nonlinear, such as LuGre model [14], which makes the design of the controller or estimator extremely challenging [16] [17] [18]. Moreover, the coupling between longitudinal and lateral tire forces further complicates the design process and lowers the robustness of the control and estimation algorithm. Another issue is that those tire model based estimation methods in [17] [18] are allowed to be used only when the persistent excitation condition is satisfied. However, this assumption often fails in real world applications. In this paper, we present a longitudinal tire force estimation algorithm without using any tire-road friction models. Instead, the longitudinal tire force of the individual wheel is regarded as an unknown input signal for the wheel spinning dynamic model. A significant advantage of this approach is that no complex tire models are involved in the estimation algorithm which not only relieves the computation burden but also increases the robustness with respect to the large variation of the road conditions.

The paper is organized as follows. First a one DOF wheel dynamical model is given, based on which two estimator design methods are developed. The next section shows how the estimation results from the individual wheel are integrated with the longitudinal dynamical model of the whole vehicle to further improve the accuracy of the estimation. Finally, the experimental results are presented, followed by the conclusions.

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Table 1. Parameter Definition of One DOF Wheel Dynamical Model

Parameters	Explanation
I_w	rotation inertia of the wheel
T_d	drive torque acting on the individual wheel
T_b	brake torque acting on the individual wheel
b	coefficient of linear viscous damping
F_x	longitudinal tire force
r	effective radius of the individual wheel

One DOF Wheel Dynamical Model

For the rotational motion of individual wheel, it is modeled as a rigid body that rotates about its CG as shown in Fig. (1).

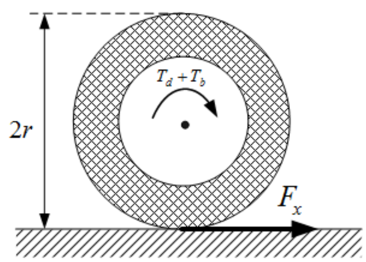

Figure 1. One DOF Wheel Dynamic Model

The dynamic equation is shown in Eq. (1) [3].

$$I_w \dot{\omega} = T_d + T_b - b\,\omega - rF_x \tag{1}$$

where ω denotes the angular velocity of the individual wheel. The definitions of other parameters can be seen in Tab (1). The effective radius r of the individual wheel is a time-varying parameter that depends on the tire pressure and maneuver conditions. The estimation of the effective radius is beyond the scope of this paper. Here, we assume that this is a known parameter and its variation can be estimated from the measured normal load [12]. Moreover, the linear damping torque $b\,\omega$ can be ignored during acceleration and hard braking due to its relative small amplitude compared with other inputs. This leads to the following simpler form of Eq. (1).

$$I_w \dot{\omega} = T_d + T_b - rF_x \tag{2}$$

As can be seen from Eq. (2), the longitudinal tire force F_x is the only unknown input signal that needs to be estimated. Compared with those tire model-based estimation algorithm, one distinct advantage of this model is that the longitudinal and lateral tire forces are completely decoupled, since the lateral tire force makes no contribution to the response of wheel speed signal.

Individual Tire Force Estimation

In this section, we will discuss how to use the wheel dynamic model and wheel speed measurement to estimate the longitudinal tire force. The same with those tire model based force estimation methods, such as [17] [18], we assume that wheel speed ω, drive torque T_d and brake torque T_b are all available. Although the two torque signals are not directly measured in the current vehicle active safety systems, they can be estimated for other sources. For example, the brake torque can be estimated from the measurement of pressure of the hydraulic cylinder in the ABS systems that are widely used today. While, the paper [19] proposed an estimation method for the drive torque. Then, a quite straightforward method is to differentiate the wheel speed signal directly and simple algebraic manipulation can give us the estimation of the longitudinal tire force. However, this method will definitely be vulnerable to the high-frequency content in the sensor noise. In the following, two types of unknown input estimators will be discussed, both of which avoid direct differentiation of the wheel speed signal.

Kalman Filter as an Unknown Input Estimator

Kalman filter is widely used as an unknown input estimator. It models the evolution of unknown input signals as a random walk [4]. Before discussing the Kalman filter, let's derive the discrete-time wheel dynamic model in Eq. (2) by using Euler's method.

$$\begin{aligned}\omega(k+1) &= \omega(k) + h\dot{\omega}(k) \\ &= \omega(k) + h\tfrac{T_d(k)+T_b(k)}{I_w} - h\tfrac{rF_x(k)}{I_w}\end{aligned} \tag{3}$$

where h is the sampling time. To be consistent with the data transmission rate (= 100Hz) in the current automotive active safety systems, h is chosen as 0.01 second. Then, the state-space representation of the wheel dynamic model for the Kalman filter is

$$\begin{aligned}F_x(k+1) &= F_x(k) + w_1(k) \\ \omega(k+1) &= \omega(k) - h\,\tfrac{r}{I_w}F_x(k) + h\,\tfrac{\tau(k)}{I_w} + w_2(k) \\ y(k) &= \omega(k) + v(k)\end{aligned} \tag{4}$$

where w_1 is the random walk signal that causes the update of tire force F_x; w_2 is the unmeasured process disturbance acting on the wheels; v_k is the measurement noise from the wheel speed sensor. $\tau(k)$ is defined as the sum of drive torque and brake torque acting on the wheel, which is treated as the measured input signal here.

$$\tau(k) = T_d(k) + T_b(k) \tag{5}$$

If we define the state vector $x(k)$ as $x(k) = \left[F_x(k)\ \omega(k)\right]^T$, the above discrete-time wheel dynamic model can be rewritten as the standard state-space form $x(k+1) = Ax(k) + B\tau(k) + w(k)$ and $y(k) = Cx(k) + v(k)$. The matrices A, B and C are

$$A = \begin{pmatrix} 1 & 0 \\ -h\frac{r}{I_w} & 1 \end{pmatrix} \; B = \begin{pmatrix} 0 \\ h\frac{1}{I_w} \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{6}$$

The observability matrix is

$$\begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -h\frac{r}{I_w} & 1 \end{pmatrix} \tag{7}$$

It is evident that this is a full row rank matrix, which implies that the state space model in Eq. (6) is an observable one. Therefore, it is reasonable to use a Kalman filter and wheel speed measurement to estimate the unmeasured tire force signal. Unlike the tire model based estimation methods presented in [17] [18], this estimator makes the persistent excitation condition unnecessary. The covariance matrices $Q(k)$ of the process noise and $R(k)$ of the measurement noise are

$$Q(k) = E\begin{pmatrix} w_1^2(k) & w_1(k)\cdot w_2(k) \\ w_2(k)\cdot w_1(k) & w_2^2(k) \end{pmatrix} \qquad R(k) = E(v^2(k)) \tag{8}$$

If we assume the process noise w_1 and w_2 are uncorrelated, $Q(k)$ becomes a diagonal matrix. The objective of the Kalman filter is to optimally estimate the unknown input signal $F_x(k)$ in terms of the minimum-variance criterion. The Kalman filter equations for each step are summarized in Eq. (9) [5].

$$\begin{aligned} P_k^- &= AP_{k-1}^+ A^T + Q_{k-1} \\ K_k &= P_k^- C^T (CP_k^- C^T + R)^{-1} \\ \hat{x}_k^- &= A\hat{x}_{k-1}^+ + Bu_{k-1} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k(y_k - C\hat{x}_k^-) \\ P_k^+ &= (I - K_k C)P_k^- \end{aligned} \tag{9}$$

Here, we regard $Q(k)$ and $R(k)$ as tuning parameters. To guarantee fast convergence, it is advisable to increase the covariance of the first process noise signal $w_1(k)$. In this case, the Kalman filter will put more confidence on the measured wheel speed signal. However, it is also commonly known that large covariance of the process noise will defy our expectation of a smooth estimation signal. Therefore, these two conflicting design objectives require us to tune $Q(k)$ and $R(k)$ in a balanced way. A heuristic trial-and-error method is usually used in a practical situation.

This Kalman filter-based tire force estimation algorithm can only be applied to the individual wheel. But we can easily extend it to one Kalman filter for all four wheels by defining the following vectors and diagonal block matrices.

$$\begin{aligned} &A = \text{diag}\left(A_{lf}, A_{rf}, A_{lr}, A_{rr}\right); \quad x = \left(x_{lf}^T, x_{rf}^T, x_{lr}^T, x_{rr}^T\right)^T \\ &B = \text{diag}\left(B_{lf}, B_{rf}, B_{lr}, B_{rr}\right); \quad u = \left(u_{lf}^T, u_{rf}^T, u_{lr}^T, u_{rr}^T\right)^T \\ &H = \text{diag}\left(H_{lf}, H_{rf}, H_{lr}, H_{rr}\right); \; y = \left(y_{lf}^T, y_{rf}^T, y_{lr}^T, y_{rr}^T\right)^T \\ &P = \text{diag}\left(P_{lf}, P_{rf}, P_{lr}, P_{rr}\right); \\ &K = \text{diag}\left(K_{lf}, K_{rf}, K_{lr}, K_{rr}\right); \end{aligned} \tag{10}$$

where the subscripts lf, rf, lr and rr represent the left-front, right-front, left-rear, right-rear wheels respectively. We can then apply these vectors and matrices to the formulas in Eq. (9) to get the longitudinal tire force estimation for all four wheels.

Feedback Controller as an Unknown Input Estimator

This kind of unknown input estimator incorporates a virtual wheel dynamic model, which is the same as Eq. (2). The virtual model is driven by the measured or estimated torque signals T_d, T_b and the output of the controller. The controller's input is the tracking error of the wheel speed, which is the difference between the measured wheel speed signal and the virtual model's output. Because the input-output relation of the one DOF wheel dynamic model is a causal one. If the virtual model's output can accurately follow the measured wheel speed signal, it implies that the controller's output or the control action indeed compensates the unmeasured input $-rF_x$, which is the torque produced by the longitudinal tire force [11]. Feedback control design methods with the aim of achieving good tracking performance can be used. In this paper, we use the PID controller as an example. The general structure of this type of unknown input estimator is shown in Fig. (2), which is indicated by the green dash line.

Before discussing the model used by the estimator, let's define the symbol of a low-pass filter signal. For example, we will use the symbol $y_T(t)$ to denote the low-pass filtered signal $y(t)$ in the subsequent formulas. This is illustrated in Fig. (3).

Then, the time-domain model for the proposed estimator is

$$I_w\dot{\hat{\omega}}_T = (T_d + T_b)_T + K_p\tilde{\omega} + K_i\int_0^t \tilde{\omega}dt + (-I_w + K_d)\dot{\tilde{\omega}}_T \tag{11}$$

where $\tilde{\omega} = \omega - \hat{\omega}$ is the deviation between measured wheel speed ω and virtual model's output $\hat{\omega}$; K_p, K_i, K_d and T are all tuning parameters. To analyze the convergence of the estimator, let's transform the above time-domain wheel dynamic model and estimator model to the s domain.

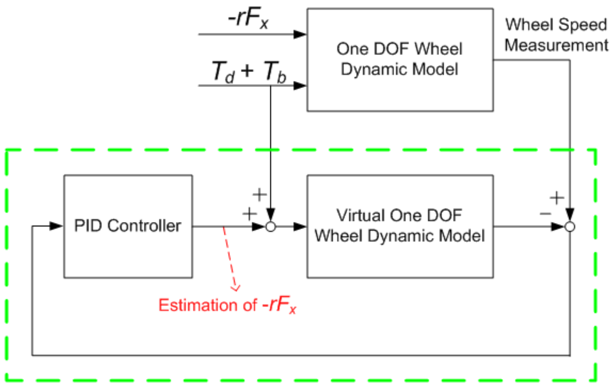

Figure 2. General Structure of Feedback Controller Based Unknown Input Estimation

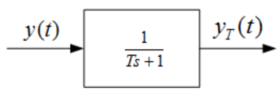

Figure 3. Illustration of low-pass filtered signal $y_T(t)$

- For wheel dynamic model in Eq. (2)

$$
\begin{aligned}
I_w\dot{\omega} &= T_d + T_b - rF_x \\
&\Downarrow Laplace\ Transformation \\
I_w s\omega &= T_d(s) + T_b(s) - rF_x(s) \\
&\Downarrow Low-pass\ Filter \\
\tfrac{1}{Ts+1} I_w s\omega &= \tfrac{1}{Ts+1}[T_d(s) + T_b(s)] - \tfrac{1}{Ts+1} rF_x(s)
\end{aligned}
\tag{12}
$$

- For the estimator in Eq. (11)

$$
\begin{aligned}
\tfrac{1}{Ts+1} I_w s\hat{\omega} = {} & \tfrac{1}{Ts+1}[T_d(s) + T_b(s)] \\
& + K_p\tilde{\omega}(s) + K_i\tfrac{1}{s}\tilde{\omega}(s) + \tfrac{-I_w+K_d}{Ts+1}s\tilde{\omega}(s)
\end{aligned}
\tag{13}
$$

If we subtract Eq. (13) from the last formula in Eq. (12), the relation between wheel speed estimation error $\tilde{\omega}$ and the longitudinal tire force F_x can be derived as

$$
-\frac{1}{Ts+1} rF_x(s) = K_p\tilde{\omega}(s) + K_i\frac{1}{s}\tilde{\omega}(s) + \frac{K_d s}{Ts+1}\tilde{\omega}(s) \tag{14}
$$

Therefore, the estimation of the tire force $\hat{F}_x(s)$ is

$$
\begin{aligned}
\hat{F}_x(s) &= \tfrac{1}{Ts+1} F_x(s) \\
&= -\tfrac{1}{r}[K_p\tilde{\omega}(s) + K_i\tfrac{1}{s}\tilde{\omega}(s) + \tfrac{K_d s}{Ts+1}\tilde{\omega}(s)]
\end{aligned}
\tag{15}
$$

It is also easy to find that the right side of Eq. (15) is the transfer function of a PID controller [9]. The tuning of the four parameters, K_p, K_i, K_d and T, can resort to the existing successful tuning rules of a PID controller [10]. The low-pass filter is included to

- Suppress the high-frequency noise in the feedback loop;
- Approximate the differentiator in the PID controller in low-frequency range;

Eq. (15) shows that the low-pass weighted filter $1/(Ts+1)$ gives us the approximation of the unmeasured friction force in the low-frequency range. It is obvious that the smaller the time constant T, the wider the frequency band of the tire force information can be extracted from $\tilde{\omega}$. We expect that the estimated force signal in Eq. (15) should not only respond fast enough to capture the change of T_d, T_b and ω but also minimizing the subsequent noise effect. Multi-objective optimization method can be used to solve this controller design problem [7] [13] [15].

From the estimator model in Eq. (11), we can find that it contains three integrators, which implies a third-order estimator. The state variables x_1, x_2, x_3 are chosen as the outputs of the virtual model, the integration part of the PID controller and the low-pass filter respectively. The state-space realization is shown as

$$
\begin{aligned}
\dot{x}_{ii} &= A_{ii}x_{ii} + B_{ii}u_{ii} \\
y_{ii} &= C_{ii}x_{ii} + D_{ii}u_{ii}
\end{aligned}
\tag{16}
$$

where ii is the index of the individual wheel, $x_{ii} = \left(x_1\ x_2\ x_3\right)^T$, $u_{ii} = \left(\omega\ \tau_T\right)^T$ and $y_{ii} = \hat{F}_x$. The matrices A_{ii}, B_{ii}, C_{ii} and D_{ii} are

$$
\begin{aligned}
A_{ii} &= \begin{pmatrix}
\frac{I_w - TK_p - K_d}{T(TK_p+K_d)} & \frac{K_i}{TK_p+K_d} & -\frac{-I_w+K_d}{T(TK_p+K_d)} \\
-\frac{I_w}{TK_p+K_d} & -\frac{TK_i}{TK_p+K_d} & \frac{-I_w+K_d}{TK_p+K_d} \\
-\frac{I_w}{T(TK_p+K_d)} & -\frac{K_i}{TK_p+K_d} & \frac{-I_w-TK_p}{T(TK_p+K_d)}
\end{pmatrix} \\
B_{ii} &= \begin{pmatrix}
-\frac{I_w - TK_p - K_d}{T(TK_p+K_d)} & \frac{1}{TK_p+K_d} \\
\frac{I_w}{TK_p+K_d} & -\frac{T}{TK_p+K_d} \\
\frac{I_w}{T(TK_p+K_d)} & -\frac{1}{TK_p+K_d}
\end{pmatrix} \\
C_{ii} &= -\tfrac{1}{r}\left(-\tfrac{I_w}{T}\ 0\ -\tfrac{I_w}{T}\right) \\
D_{ii} &= -\tfrac{1}{r}\left(\tfrac{I_w}{T}\ -1\right)
\end{aligned}
\tag{17}
$$

Similar to the Kalman filter case, we can combine the state-space models for the estimators of each wheel together as

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}
\tag{18}
$$

with the following vectors and diagonal block matrices

$$A = \text{diag}\left(A_{lf}, A_{rf}, A_{lr}, A_{rr}\right); \; x = \left(x_{lf}^T, x_{rf}^T, x_{lr}^T, x_{rr}^T\right)^T$$
$$B = \text{diag}\left(B_{lf}, B_{rf}, B_{lr}, B_{rr}\right); \; u = \left(u_{lf}^T, u_{rf}^T, u_{lr}^T, u_{rr}^T\right)^T$$
$$C = \text{diag}\left(C_{lf}, C_{rf}, C_{lr}, C_{rr}\right); \; y = \left(y_{lf}^T, y_{rf}^T, y_{lr}^T, y_{rr}^T\right)^T$$
$$D = \text{diag}\left(D_{lf}, D_{rf}, D_{lr}, D_{rr}\right);$$

where the subscripts lf, rf, lr and rr still represent the left-front, right-front, left-rear, right-rear wheels respectively as the Kalman filter case.

Adding Longitudinal Acceleration as a Constraint

In the previous two sections, we develop two types of unknown input estimation algorithm based only on the one DOF wheel dynamic model. For each wheel, the longitudinal tire force is estimated only from its own spinning speed measurement. This implies that the estimation result of the individual wheel is totally independent of the states of other wheels. Furthermore, the inaccurate information of the torque signals and effective radius can result in a biased estimation. One way to verify the accuracy of the above one DOF wheel dynamic model based estimation method is to use the longitudinal vehicle dynamic model as a constraint. From Newton's second law, we can easily derive the following dynamic model of the longitudinal motion [12].

$$M(\dot{v}_x - \psi v_y) \approx F_{lf} + F_{rf} + F_{lr} + F_{rr} \tag{19}$$

where M and $\dot{v}_x$ are the total mass of the vehicle and measured longitudinal acceleration at the CG respectively. ψ and v_y are the yaw rate of the vehicle body and lateral velocity of the CG respectively. While, F_{lf}, F_{rf}, F_{lr} and F_{rr} denote the longitudinal tire forces of the left front wheel, right front wheel, left rear wheel and right rear wheel respectively. The operator $\approx$ implies that this model includes some unmodeled small-amplitude input sources, such as air resistance and a component of gravity caused by a non-zero bank angle. If we assume the vehicle is running straightly on a flat road, which implies that the yaw rate ψ is zero, then the longitudinal dynamic model is simplified as

$$M\dot{v}_x \approx F_{lf} + F_{rf} + F_{lr} + F_{rr} \tag{20}$$

For the estimated longitudinal tire forces to satisfy both individual wheel dynamics and longitudinal dynamics of the whole vehicle, we propose to use the results in the previous section as an initial one. Then, a correction term can be added based on Eq. (20) to further improve the estimation accuracy [6].

Inequality Constraint

Inequality constraint applies to the situations that the relationship among the state variables is only roughly known or that the constraint function has some bounded uncertainty [6]. Let's consider the following inequality for the estimated tire forces and measured longitudinal acceleration signal.

$$|M\dot{v}_x - (\tilde{F}_{lf} + \tilde{F}_{rf} + \tilde{F}_{lr} + \tilde{F}_{rr})| \le \varepsilon \tag{21}$$

where $\tilde{F}_{lf}$, $\tilde{F}_{rf}$, $\tilde{F}_{lr}$ and $\tilde{F}_{rr}$ are the augmented longitudinal tire force estimation after taking the inequality constraint into account. While, ε represents the upper bound of those unknown input sources and uncertainty in the longitudinal dynamic model, such as air resistance and inertia force caused by inaccurate vehicle mass information. One common method is to regard this inequality constraint as an additional measurement equation as shown in Eq. (22), where the covariance of the measurement noise is chosen to be the same as the margin of the above inequality [6].

$$M\dot{v}_x = \tilde{F}_{lf} + \tilde{F}_{rf} + \tilde{F}_{lr} + \tilde{F}_{rr} + \varepsilon \tag{22}$$

Then, the output of the unknown input observer can be regarded as the estimation results of time-update step. Next step is to use Eq. (22) to augment this time update results by applying kalman filter algorithm again. However, the pair of the diagonal block state matrix in either Eq. (10) or Eq. (18) and this additional state output matrix $\left[1\ 1\ 1\ 1\right]$ is an unobservable one. Another method tries to minimize the cost function of [12].

$$\tilde{x}_k = \text{argmin}_x (x - \hat{x}_k)^T W (x - \hat{x}_k) \tag{23}$$

subject to

$$|M\dot{v}_x - (\tilde{F}_{lf} + \tilde{F}_{rf} + \tilde{F}_{lr} + \tilde{F}_{rr})| \le \varepsilon$$

where $\hat{x}_k$ is the estimated force signals by using either of the two estimation methods discussed in section . $\tilde{x}_k$ is the augmented estimation that satisfies the given inequality constraints and W is a positive-definite weighting matrix. This optimization problem is equivalent to the following semidefinite programming problem, which can be easily solved by the efficient convex optimization algorithm, through the Schur complement [7].

$$\min \gamma \tag{24}$$

subject to the linear matrix inequality (LMI) constraint

$$\begin{pmatrix} \gamma & (x - \hat{x}_k)^T \\ (x - \hat{x}_k) & W^{-1} \end{pmatrix} > 0$$

and the linear algebraic inequality constraint

$$-\varepsilon \leq M\dot{v}_x - (\tilde{F}_{lf} + \tilde{F}_{rf} + \tilde{F}_{lr} + \tilde{F}_{rr}) \leq \varepsilon$$

where both γ and x are decision variables. x represents the vector of four augmented estimated force signals and γ is the lowest upper bound of the cost function in Eq. (23). Unfortunately, this kind of numerical optimization algorithm is often not practical in real-world applications due to limited computation resources.

Equality Constraint

Equality constraint applies to the situations where the relationship among the state variables are well known with very good accuracy [6]. Let's consider the following equality constraint for the estimated tire force signals and measured longitudinal acceleration.

$$M\dot{v}_x = \tilde{F}_{lf} + \tilde{F}_{rf} + \tilde{F}_{lr} + \tilde{F}_{rr} \tag{25}$$

Some literature regards the equality constraint as a perfect measurement with zero covariance noise [6]. Kalman filter can be used to get the augmentation. However, the observability problem as described in the inequality case makes this method questionable. Instead, we can still use the optimization algorithm to get the augmentation, which is defined in Eq. (26) [6].

$$\tilde{x}_k = \mathrm{argmin}_x (x - \hat{x}_k)^T W (x - \hat{x}_k) \tag{26}$$

subject to

$$M\dot{v}_x = \tilde{F}_{lf} + \tilde{F}_{rf} + \tilde{F}_{lr} + \tilde{F}_{rr}$$

and

$$\tilde{F}_{lr} = \hat{F}_{lr}; \;\; \tilde{F}_{rr} = \hat{F}_{rr} \;\; \text{(optional)}$$

where $\hat{x}_k$, $\tilde{x}_k$ and W have the same definitions as those in the case for inequality constraint. The last equality constraint is an optional one, since it only applies to the situation where we believe that the estimation results for the two rear wheels from the method discussed in section have already been accurate enough and no augmentation is needed. For example, during the acceleration of a front wheel drived (FWD) vehicle, neither drive torque nor brake torque acts on the rear wheels. In this case, there is no uncertainty in the measured input signals, which makes the previous unknown input estimator very accurate. Therefore, we can use this optional equality constraint to achieve no augmentation for these signals.

Although this equality constrained optimization method neglects some uncertainty in the longitudinal vehicle dynamic model that is considered in the inequality constraint case, one advantage of this method is that it can be solved in an analytical way by using the Lagrange multiplier method [8]. If we define the matrix D and vector d as

$$D = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}; \qquad d = M\dot{v}_x$$

or

$$D = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \qquad d = \begin{pmatrix} M\dot{v}_x \\ \hat{F}_{lr} \\ \hat{F}_{rr} \end{pmatrix}$$

where all the equality constraints are abstracted to the form $D\tilde{x} = d$, then the solution to the optimization problem in Eq. (26) is [6]

$$\tilde{x}_k = \hat{x}_k - W^{-1}D^T(DW^{-1}D^T)^{-1}(D\hat{x}_k - d) \tag{27}$$

It is obvious that $W^{-1}D^T(DW^{-1}D^T)^{-1}$ acts as a linear correction gain in Eq. (27). This method is practical since it does not need online iteration to solve for the optimal solution. The only remaining problem is how to tune the weighting matrix W. In the literature, W is usually selected as [6]

$$W = I \quad \text{or} \quad W = (P_k^+)^{-1} \tag{28}$$

For $W = I$, the solution $\tilde{x}$ is the orthogonal projection of the original estimation result $\hat{x}$ into the subspace $D\tilde{x} = d$. For $W = (P_k^+)^{-1}$, where P_k^+ is the measurement updated covariance matrix at the kth step, the resulting $\tilde{x}$ is a point in the subspace $D\tilde{x} = d$ that is close to the original estimation $\hat{x}$ in a weighted least-square sense.

Experimental Results

To verify the developed longitudinal tire force estimation algorithm, both CarSim simulation and vehicle test data are utilized. The only difference between these two kinds of data is that only the brake torque is measured in the test. While, both brake torque and drive torque information are available in the CarSim simulation. For the unknown input estimator part, both the Kalman filter-based and feedback controller-based methods are simulated. For the constraint augmentation part, only equality constraint are considered.

CarSim Data: The simulated vehicle is a sedan whose mass and rotational moment of inertia of each individual wheel are

$$M = 950\,(\mathrm{kg}) \quad \text{and} \quad I_w = 0.6\,(\mathrm{kg}\cdot\mathrm{m}^2)$$

In the first case, the vehicle accelerates on a flat road whose maximum μ is 0.8. The simulation result for the left-front tire can be

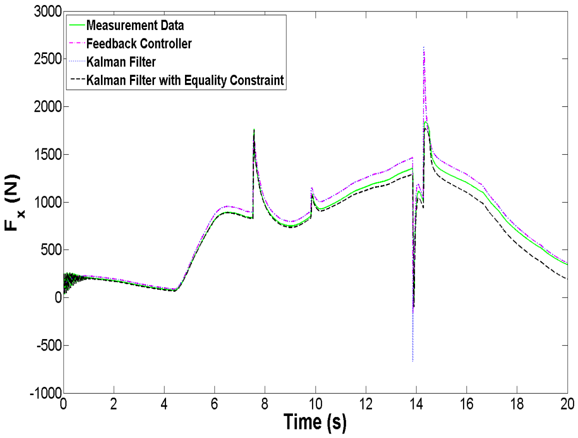

Figure 4. Estimation results from CarSim data for the case $\mu_{\max} = 0.8$

seen in Fig. (4). In the second case, the vehicle accelerates on a icy road whose maximum μ is 0.2 and a closed-loop speed regulator is utilized. The simulation result for the left-front tire can be seen in Fig. (5). We can see that both the estimation results of Kalman filter and feedback controller based methods performs very well but have an undesired impulse response at around 8ms due to a sudden jump of the drive torque at that instant. Fortunately, this kind of phenomenon can be filtered out by the projection method which includes the longitudinal acceleration as a constraint. This is because the tire-road friction force is easy to be saturated on this kind of low-friction surface. Therefore, the tire force signal cannot follow the change of the drive torque quickly and also does the longitudinal acceleration. The projection method strictly constrains the estimated tire force signals to the subspace defined by the longitudinal acceleration, which makes the estimation results insensitive to the impulsive change of the drive torque.

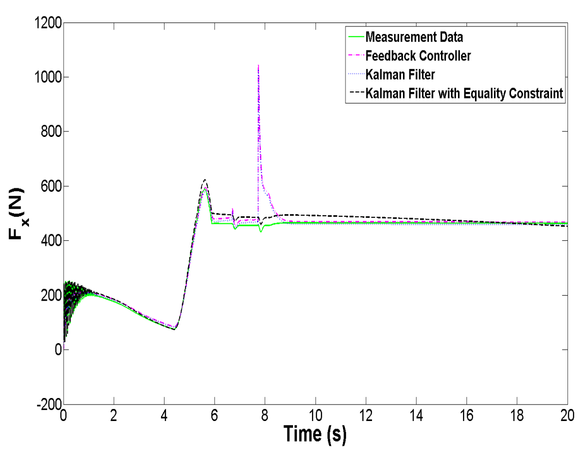

Figure 5. Estimation results from CarSim data for the case $\mu_{\max} = 0.2$

Real Experimental Data: The vehicle used in the real test is a large sedan. Its mass and rotational moment of inertia of each individual wheel are

$$M = 2083\,(\mathrm{kg}) \quad \text{and} \quad I_w = 1.2\,(\mathrm{kg}\cdot\mathrm{m}^2)$$

The vehicle accelerates on a flat road whose maximum μ is about 0.8. The estimation result for the left-front tire can be seen in Fig. (6). All three methods achieve similar acceptable performance.

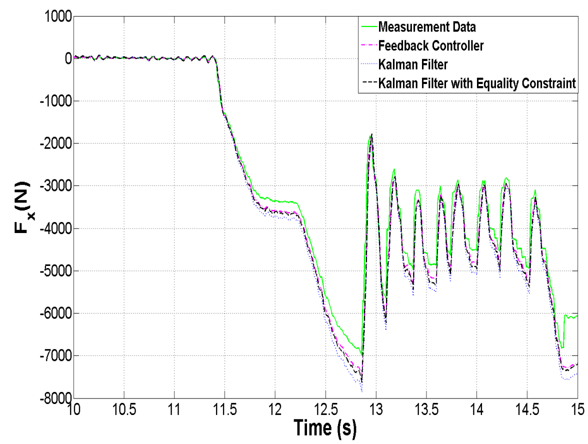

Figure 6. Estimation results from real test data

CONCLUSIONS

In this paper, we proposed a model-based algorithm to estimate the longitudinal tire force without resorting to any complex tire-road friction models. This estimator integrates the information from the dynamic model of individual wheel and the longitudinal motion of the vehicle to derive robust estimation results. The simulation results show that this estimation algorithm generally provides good results using both CarSim simulation and vehicle test data. Compared with the tire-model based algorithm, the simple structure of the proposed estimator significantly increases the computation efficiency, which makes it an acceptable solution for practical applications.

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REFERENCES

[1] Gustafsson, F., ”Statistical Signal Processing for Automotive Safety Systems”' in *2005 IEEE/SP 13th Workshop on Statistical Signal Processing*, pp 1428 - 1435, 2005.

[2] H. B. Pacejka, *Tire and Vehicle Dynamics*, Butterworth-Heinemann, 2002.

[3] Rajamani, R., *Vehicle Dynamics and Control*, Springer Verlag, New York 2005.

[4] Gustafsson, F., ”Slip-Based Tire-Road Friction Estimation”, *Automatica*, Vol. 33(6), pp 1087 - 1099, 1997.

[5] Simon, D., *Optimal State Estimation*, 1st Edition, John Wiley & Sons, 2006.

[6] Simon, D, ”Kalman Filtering with State Constraints: A Survey of Linear and Nonlinear Algorithms”', *IET Control Theory and Applications*, Vol. 4, Iss. 8 pp 1303 - 1318, 2010.

[7] S.P.Boyd, L.El Ghaoui, E.Feron and V.Balakrishan., *Linear Matrix Inequalities In Systems and Control Theory*, SIAM Studies in Applied Mathematics 15, SIAM, Philadelphia, 1994.

[8] Edwin Kah Pin Chong, Stanislaw H. Zak, *An introduction to optimization*, 3rd Edition John Wiley & Sons, 2008.

[9] Gene F. Franklin, J. David Powell, Abbas Emami-Naeini, *Feedback Control of Dynamic Systems*, 5th Edition, Pearson Prentice Hall, 2006.

[10] Kristiansson, B., Lennartson, B., ”Robust Tuning of PI and PID Controllers: using derivative action despite sensor noise”, *IEEE Control Systems Magazine*, Vol. 26, Iss. 1 pp 55 - 69, February 2006.

[11] Stotsky, A., Kolmanovsky, I., ”Application of Input Estimation Techniques to Charge Estimation and Control in Automotive Engines”, *Control System Practice*, Vol. 10, Iss. 12 pp 1371-1383, December 2006.

[12] Reza N. Jazar, *Vehicle Dynamics: Theory and Applications*, 1st Edition, Springer, 2008.

[13] Skogestad S., Postlethwaite I., *Multivariable Feedback Control: Analysis and Design*, 2nd Edition John Wiley & Sons, November 2005.

[14] Canudas-de-Wit C., Tsiotras P., Velenis E., Basset M. and Gissinger G. ”Dynamic Friction Models for Road/Tire Longitudinal Interaction”, *Vehicle Systems Dynamics*, Vol. 39, No. 3, pp 189-226, 2003.

[15] S.P.Boyd, C. H. Barratt *Linear Controller Design: Limits of Performance*, 1st Edition, Prentice Hall, January 1991.

[16] Yi J., Alvarez L., and Horowitz R. ”Adaptive Emergency Braking Control with Underestimation of Friction Coefficient”, *IEEE Trans. Control Syst. Technol*, Vol. 10, Iss. 3, pp 381-392, 2002.

[17] Canudas-de-Wit, C.; Petersen M.L., and Shiriaev A. ”A new nonlinear observer for tire/road distributed contact friction”, in *Proceedings of 42nd IEEE Conference on Decision and Control, 2003.*, Vol. 3, pp 2246 - 2251, 2003.

[18] Yan Chen, Junmin Wang, ”Vehicle-longitudinal-motion-independent real-time tire-road friction coefficient estimation”, in *Proceedings of 49th IEEE Conference on Decision and Control, 2010.*, pp 2910 - 2915, 2010.

[19] R. A. Masmoudi, J. K. Hedrick, ”Estimation of vehicle shaft torque using nonlinear observers”, in *ASME J. Dyn. Syst. Meas. Control*, Vol. 114, Iss. 3, pp. 394400, Sep. 1992.