# BLIND SYMBOL IDENTIFIABILITY OF ORTHOGONAL SPACE-TIME BLOCK CODES 

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#### Abstract

This paper addresses the blind symbol identifiability of the orthogonal space-time block code (OSTBC) scheme. That is, the conditions under which OSTBC symbols can be identified without ambiguity when channel state information is not available. In many space-time communication schemes, achieving unique blind symbol identification requires certain assumptions on the number of receiver antennas and the rank of the channel matrix. In this paper we show that unique blind symbol identification of OSTBCs is possible for any number of receiver antennas and for any (nonzero) channel matrix. This attractive unique identifiability result is shown to be achieved by a class of OSTBCs that exhibit certain matrix non-rotational properties. Using these properties, we validate the identifiability of a number of commonly used OSTBCs.


## 1. INTRODUCTION

Orthogonal space-time block coding [1-3] is a multiple-input-multiple-output (MIMO) communication scheme that uses a simple code structure to achieve the maximum spatial diversity gain. Assuming that channel state information (CSI) is available at the receiver, the maximum-likelihood (ML) detection of orthogonal space-time block codes (OSTBCs) requires only simple linear processing. This feature is attractive from a detection standpoint because in many other MIMO schemes, achieving ML detection with CSI often turns out to be a computationally challenging optimization problem [4-6]. ML detection without CSI, commonly referred to as blind ML detection, has recently been investigated for the OSTBC scheme [7, 8]. Like the coherent ML detection case, the OSTBC scheme is shown to exhibit a much simpler blind ML detection problem compared to that of other linear space-time schemes such as [9-12]. This simplicity has led to an effective blind receiver [8] that accurately approximates blind ML detection with a worse-case complexity polynomial in the number of symbols processed.

A key aspect of blind detection techniques is blind symbol identifiability. That is, the conditions under which the information symbols can be uniquely detected, up to mild effects such as a sign. In many linear MIMO schemes such as spatial multiplexing (SM) [9-11] and linear dispersion based block coding [12,13], unique symbol identifiability relies on several factors such as the rank of the channel matrix, the number of receiver antennas, and the data frame length. In the SM scheme, for instance, the transmitted symbols are uniquely identifiable when the channel matrix is of full rank, the number of receiver antennas is no less than

[^0]that of transmitter antennas, and the frame length is very large relative to the number of transmitter antennas [11]. We will show that in the OSTBC scheme, unique symbol identifiability depends solely on the code structure. In other words, unique blind symbol identification of OSTBCs is possible for any number of receiver antennas and for any (nonzero) channel matrix. In the following, we will illustrate how the structure of the OSTBC guarantees such an attractive unique symbol identifiability result. Some examples of uniquely identifiable OSTBCs will also be given.

## 2. BACKGROUND

In this section we review the orthogonal space-time block coding scheme, and describe its blind symbol identifiability problem.

### 2.1. Signal Model for the OSTBC Scheme

Consider the transmission of a space-time block code (STBC) over a flat fading channel using $M_{t}$ transmitter antennas. Let $T$ define the time block length of the STBC, and assume that the receiver is equipped with $M_{r}$ antennas. The received STBC signal can be modeled as

$$
\begin{equation*}
\mathbf{Y}=\mathbf{H C}(\mathbf{s})+\mathbf{V}, \quad \mathbf{s} \in \mathfrak{A}^{K} \tag{1}
\end{equation*}
$$

where

s
$\mathbf{C}(\mathbf{s}) \in \mathbb{C}^{M_{t} \times T}$
code matrix transmitted by the $M_{t}$ transmitter antennas;
$\mathbf{Y} \in \mathbb{C}^{M_{r} \times T} \quad$ received code matrix;
$\mathcal{A}$ symbol constellation set;
$K \quad$ number of symbols per block;
$\mathbf{V} \in \mathbb{C}^{M_{r} \times T} \quad$ additive white Gaussian noise (AWGN) matrix.
In the orthogonal STBC (OSTBC) scheme with real symbol constellations, $\mathbf{C}($.$) takes on the following linear structure [2,3]$

$$
\begin{equation*}
\mathbf{C}(\mathbf{s})=\sum_{k=1}^{K} \mathbf{X}_{k} s_{k}, \tag{2}
\end{equation*}
$$

where $s_{k}$ is the $k$ th element of $\mathbf{s}$, and $\mathbf{X}_{k} \in \mathbb{R}^{M_{t} \times T}$ are constituent matrices of the code. The set $\left\{\mathbf{X}_{k}\right\}_{k=1}^{K}$ satisfies $M_{t} \leq T, K \leq T$, and

$$
\begin{equation*}
\mathbf{X}_{k} \mathbf{X}_{k}^{T}=\mathbf{I}, \quad \mathbf{X}_{k} \mathbf{X}_{\ell}^{T}=-\mathbf{X}_{\ell} \mathbf{X}_{k}^{T}, \quad k \neq \ell \tag{3}
\end{equation*}
$$

An OSTBC is said to be of full rate if $K=T$. In this work we focus on binary phase shift keying (BPSK) constellations; i.e., $\mathcal{A}=$ $\{-1,+1\}$, and we assume that the symbols are independent of one another. It can be verified from (3) that

$$
\begin{equation*}
\mathbf{C}(\mathbf{s}) \mathbf{C}^{T}(\mathbf{s})=K \mathbf{I}, \quad \text { for any } \mathbf{s} \in\{-1,+1\}^{K} . \tag{4}
\end{equation*}
$$

This semi-orthogonality property leads to two important advantages: i) the maximum spatial diversity gain is achieved; and ii) with channel state information (CSI), maximum-likelihood (ML) detection requires only simple linear processing. It was shown [8] that the ML detector for unknown CSI can also be much simplified by exploiting the structure in (4). As a tradeoff for the full diversity gain, the data rate of the OSTBC scheme (i.e., $K / T$ symbols $/ \mathrm{s} / \mathrm{Hz}$ ) is inherently restricted to be no greater than $1 \mathrm{symbol} / \mathrm{s} / \mathrm{Hz}$, which is lower than that of some MIMO schemes such as spatial multiplexing [9,10] and linear dispersion block coding [12, 14].

### 2.2. Blind Detection and Identifiability

In the scenario of blind detection or detection without CSI, the ML detector structure depends on the MIMO fading channel model. Here we apply two usual assumptions that lead to the so-called deterministic blind ML detector [7]. First, we assume that the MIMO fading coefficients are slowly time varying such that $\mathbf{H}$ remains fixed over $P$ consecutive code blocks. In this case it is appropriate to add a block index, $p$, to the OSTBC signal model in (1):

$$
\begin{equation*}
\mathbf{Y}_{p}=\mathbf{H C}\left(\mathbf{s}_{p}\right)+\mathbf{V}_{p}, \quad p=1, \ldots, P \tag{5}
\end{equation*}
$$

where $\mathbf{Y}_{p}$ is the $p$ th received signal block, $\mathbf{s}_{p} \in\{-1,+1\}^{K}$ is the $p$ th symbol block, and $\mathbf{V}_{p}$ is again AWGN. Second, we assume $\mathbf{H}$ to be deteministically unknown. For notational convenience, we collect all information symbol blocks to form a single matrix

$$
\begin{equation*}
\mathbf{S}=\left[\mathbf{s}_{1}, \ldots, \mathbf{s}_{P}\right] \in\{-1,+1\}^{K \times P} . \tag{6}
\end{equation*}
$$

Under the above two assumptions, the blind ML receiver is shown to take on the form [7,8,11]:

$$
\begin{equation*}
\{\hat{\mathbf{H}}, \hat{\mathbf{S}}\}=\arg \min _{\substack{\overline{\mathbf{S}} \in\left\{\mathbb{C}^{\mathbb{M}_{r} \times M_{t}},-1,+1\right\}^{K \times p}}} \sum_{p=1}^{P}\left\|\mathbf{Y}_{p}-\overline{\mathbf{H}} \mathbf{C}\left(\overline{\mathbf{s}}_{p}\right)\right\|_{F}^{2} \tag{7}
\end{equation*}
$$

where $\|\cdot\|_{F}$ is the Frobenius norm. Note from (7) that the blind ML detector jointly detects $\mathbf{S}$ from the whole received signal frame [ $\mathbf{Y}_{1}, \ldots, \mathbf{Y}_{P}$ ], unlike the known CSI case where it is sufficient to detect each $\mathbf{s}_{p}$ from its respective received code block $\mathbf{Y}_{p}$.

In this work we are interested in the blind identifiability problem. Other issues such as realization of (7) can be found elsewhere $[7,8]$. To understand blind identifiability, consider that noise is absent (i.e., $\mathbf{V}_{p}=\mathbf{0}$ for all $p$ ). Then $\{\hat{\mathbf{H}}, \hat{\mathbf{S}}\}=\{\mathbf{H}, \mathbf{S}\}$ is an ML solution minimizing the objective function in (7). But $\{\mathbf{H}, \mathbf{S}\}$ is the unique ML solution only when we cannot find another solution, denoted by $\{\tilde{\mathbf{H}}, \tilde{\mathbf{S}}\}$, such that

$$
\begin{equation*}
\mathbf{H C}\left(\mathbf{s}_{p}\right)=\tilde{\mathbf{H}} \mathbf{C}\left(\tilde{\mathbf{s}}_{p}\right), \quad p=1, \ldots, P . \tag{8}
\end{equation*}
$$

One immediately obvious situation leading to (8) is when $\{\tilde{\mathbf{H}}, \tilde{\mathbf{S}}\}=$ $\{-\mathbf{H},-\mathbf{S}\}$. Fortunately, this particular ambiguity can be easily resolved by other means [11]. In the subsequent section, we will place our emphasis on examining the existence and non-existence of the other code ambiguity possibilities. This aspect is equally
important to the nonzero noise case, since an ML solution $\{\hat{\mathbf{H}}, \hat{\mathbf{C}}\}$ could be subjected to a code ambiguity effect similar to (8).

In order to place in context the blind identifiability properties of OSTBCs that we will derive below, we briefly review the blind identifiability properties of the spatial multiplexing (SM) scheme. In the SM scheme, every transmitter antenna transmits its own independent stream of information symbols. The model in (5) represents the SM received signal if we set $T=1, K=M_{t}$, and $\mathbf{C}\left(\mathbf{s}_{p}\right)=\mathbf{s}_{p}$. The blind identifiability of SM is as follows:

Theorem 1 (Talwar et. al [11]) Consider the SM scheme with BPSK symbols. Given $\left[\mathbf{Y}_{1}, \ldots, \mathbf{Y}_{P}\right]=\mathbf{H S}$, the symbol matrix $\mathbf{S}$ can be uniquely identified up to signs and permutations in the rows of $\mathbf{S}$ if
i) $\mathbf{H}$ is of full column rank; and
ii) the columns of $\mathbf{S}$ contain at least $2^{M_{t}-1}$ distinct ${ }^{1} M_{t}$-tuples in $\{-1,+1\}^{M_{t}}$.

The (sufficient) conditions in this theorem place two restrictions on the applicability of blind SM detection. First, to satisfy Condition i) of Theorem 1 , it is necessary that $M_{r} \geq M_{t}$. Second, we need $P \gg 2^{M_{t}-1}$ such that Condition ii) of Theorem 1 occurs with a high probability.

In the next section we will show that the blind identifiability conditions in the OSTBC scheme are much less restrictive than those in the SM. In particular, there is no restriction on the channel matrix $\mathbf{H}$ and the number of receiver antennas.

## 3. BLIND IDENTIFIABILITY IN OSTBCS

In this section we examine the blind symbol identifiability aspects of the OSTBC scheme. Due to the lack of space, some of the theorems will be stated without proof. Details for the proofs will be given elsewhere [15].

### 3.1. The Code Rotation Problem

We consider a class of OSTBCs, which we call rotatable OSTBCs. In blind detection, rotatable OSTBCs always result in non-unique symbol identification and hence should be avoided. The definition of rotatable OSTBCs is as follows:

Definition 1 An OSTBC matrix $\mathbf{C}($.$) is said to be rotatable if$ there exists a matrix $\mathbf{Q} \in \mathbb{R}^{M_{t} \times M_{t}}, \mathbf{Q} \neq \pm \mathbf{I}$, such that for any $\mathbf{s} \in\{-1,+1\}^{K}$,

$$
\begin{equation*}
\mathbf{Q C}(\mathbf{s})=\mathbf{C}(\tilde{\mathbf{s}}) \tag{9}
\end{equation*}
$$

for some $\tilde{\mathbf{s}} \neq \pm \mathbf{s}, \tilde{\mathbf{s}} \in\{-1,+1\}^{K}$. Otherwise, $\mathbf{C}($.$) is said to be$ non-rotatable. Such a $\mathbf{Q}$, if exists, is called a code rotation matrix.

It immediately follows from Definition 1 and the orthogonality property in (4) that any code rotation matrix $\mathbf{Q}$ is orthonormal. To demonstrate the problem with rotatable OSTBCs, consider again the noise free received signal matrices $\mathbf{Y}_{p}=\mathbf{H C}\left(\mathbf{s}_{p}\right)$, where $p=1, \ldots, P$. Suppose that $\mathbf{C}($.$) is rotatable, and let \mathbf{Q}$ be a code rotation matrix of $\mathbf{C}($.$) . Then each \mathbf{Y}_{p}$ can be alternatively expressed as

$$
\begin{align*}
\mathbf{Y}_{p} & =\mathbf{H} \mathbf{Q}^{T} \mathbf{Q C}\left(\mathbf{s}_{p}\right) \\
& =\left(\mathbf{H Q}^{T}\right) \mathbf{C}\left(\tilde{\mathbf{s}}_{p}\right) \tag{10}
\end{align*}
$$

[^1]for some $\tilde{\mathbf{s}}_{p} \neq \pm \mathbf{s}_{p}, \tilde{\mathbf{s}}_{p} \in\{-1,+1\}^{K}$. It is clear from (10) that the rotatable code results in code ambiguity for any $\mathbf{S} \in\{-1,+1\}^{K \times P}$.

The following is an example of a rotatable OSTBC:
Example 1 Consider a code with $M_{t}=T=K=2$ [2], given by

$$
\mathbf{X}_{1}=\left[\begin{array}{ll}
1 & 0  \tag{11}\\
0 & 1
\end{array}\right], \quad \mathbf{X}_{2}=\left[\begin{array}{cc}
0 & -1 \\
1 & 0
\end{array}\right]
$$

Define a matrix $\mathbf{Q}=\mathbf{X}_{2}$. Since $\mathbf{Q} \mathbf{X}_{1}=\mathbf{X}_{2}$ and $\mathbf{Q} \mathbf{X}_{2}=-\mathbf{X}_{1}$,

$$
\begin{equation*}
\mathbf{Q C}\left(\left[s_{1} s_{2}\right]^{T}\right)=\mathbf{C}\left(\left[-s_{2} s_{1}\right]^{T}\right) \tag{12}
\end{equation*}
$$

in which the symbols $s_{1}$ and $s_{2}$ are interchanged and sign altered after the matrix multiplication process. Eq. (12) indicates that this OSTBC is rotatable.

Inspired by Example 1, we consider a particular code rotation case as follows:

Lemma 1 Suppose that there is a matrix $\mathbf{Q} \in \mathbb{R}^{M_{t} \times M_{t}}$ satisfying

$$
\begin{equation*}
\mathbf{Q} \mathbf{X}_{k}=d_{k} \mathbf{X}_{i_{k}} \tag{13}
\end{equation*}
$$

for $k=1, \ldots, K$, where $d_{k} \in\{-1,+1\}, i_{k} \in\{1,2, \ldots, K\}$ is an index with $i_{k} \neq k$ and $i_{k} \neq i_{\ell}$ for $k \neq \ell$. In this case $\mathbf{C}($.$) is rotatable.$

Lemma 1 describes the case where code rotation results in reordering and sign altering of each $\mathbf{s}_{p}$. In fact, all rotatable codes fall into the code rotation case in Lemma 1, as we show in the following theorem:

Theorem 2 If $\mathbf{C}($.$) is rotatable and \mathbf{Q}$ is its associated code rotation matrix, then the condition in (13) holds.

Using Theorem 2, we can identify a class of OSTBCs that are non-rotatable. Let $\mathcal{R}(\mathbf{A})$ denote the range space of $\mathbf{A}$. Consider the following corollary:

Corollary 1 If the set of OSTBC constituent matrices $\left\{\mathbf{X}_{k}\right\}_{k=1}^{K}$ is such that for any $k \neq \ell$,

$$
\begin{equation*}
\mathcal{R}\left(\mathbf{X}_{k}^{T}\right) \neq \mathcal{R}\left(\mathbf{X}_{\ell}^{T}\right) \tag{14}
\end{equation*}
$$

then the respective OSTBC is non-rotatable.
The proof of Corollary 1 is as follows. To satisfy (13), it is necessary that some linear combinations of the rows of $\mathbf{X}_{k}$ form the rows of $\mathbf{X}_{i_{k}}$. This condition is impossible if (14) holds.

The following is an example of a non-rotatable OSTBC that satisfies the sufficient non-rotatability condition in Corollary 1.

Example 2 Consider the $3 \times 4$ full rate OSTBC with constituent matrices [16]

$$
\mathbf{X}_{1}=\left[\begin{array}{c}
\mathbf{e}_{1}^{T}  \tag{15}\\
\mathbf{e}_{2}^{T} \\
\mathbf{e}_{3}^{T}
\end{array}\right], \mathbf{X}_{2}=\left[\begin{array}{c}
\mathbf{e}_{2}^{T} \\
-\mathbf{e}_{1}^{T} \\
\mathbf{e}_{4}^{T}
\end{array}\right], \mathbf{X}_{3}=\left[\begin{array}{c}
\mathbf{e}_{3}^{T} \\
-\mathbf{e}_{4}^{T} \\
-\mathbf{e}_{1}^{T}
\end{array}\right], \mathbf{X}_{4}=\left[\begin{array}{c}
\mathbf{e}_{4}^{T} \\
\mathbf{e}_{3}^{T} \\
-\mathbf{e}_{2}
\end{array}\right]
$$

where $\mathbf{e}_{i} \in \mathbb{R}^{4}$ is a unit vector with the nonzero element at the $i$ entry. Let us compare $\mathbf{X}_{1}$ and $\mathbf{X}_{2}$. We see that no linear combination of the rows of $\mathbf{X}_{2}$ will form the 3rd row of $\mathbf{X}_{1}$; i.e., $\mathbf{e}_{3}$. In other words, we have $\mathcal{R}\left(\mathbf{X}_{1}^{T}\right) \neq \mathcal{R}\left(\mathbf{X}_{2}^{T}\right)$. Likewise it is verified that $\mathcal{R}\left(\mathbf{X}_{k}^{T}\right) \neq \mathcal{R}\left(\mathbf{X}_{\ell}^{T}\right)$ for any $k \neq \ell$. Hence this OSTBC is nonrotatable by Corollary 1 . The non-rotatability of this OSTBC was also confirmed by numerical inspection.

We should point out that Corollary 1 is not applicable to the case of $M_{t}=T$. In this square OSTBC case, the code constituent matrices $\mathbf{X}_{k}$ are orthogonal matrices due to the property in (3). Subsequently we have that $\mathcal{R}\left(\mathbf{X}_{k}^{T}\right)=\mathbb{R}^{T}$ for all $k$, which violates (14). In fact, the following subsection will show that square OSTBCs do not generally guarantee unique blind symbol identification.

### 3.2. Strict Code Non-Rotatability

The previous subsection has illustrated that for unique blind symbol identification, the OSTBC has to be non-rotatable. However, using non-rotatable codes is not sufficient to guarantee unique symbol identification. We notice from Definition 1 that for a nonrotatable code, there does not exist a matrix $\mathbf{Q}$ such that the code ambiguity situation

$$
\begin{equation*}
\mathbf{Q C}\left(\mathbf{s}^{\prime}\right)=\mathbf{C}\left(\mathbf{s}^{\prime \prime}\right), \quad \mathbf{s}^{\prime \prime} \in\{-1,+1\}^{K}-\left\{ \pm \mathbf{s}^{\prime}\right\} \tag{16}
\end{equation*}
$$

can be satisfied for every $\mathbf{s}^{\prime} \in\{-1,+1\}^{K}$. The non-rotatable code property, however, does not rule out the possibility that (16) can be satisfied for some $\mathbf{s}^{\prime} \in\{-1,+1\}^{K}$. Suppose that $\mathbf{C}($.$) is non-$ rotatable but satisfies (16) for two particular distinct symbol vectors $\mathbf{s}^{\prime}, \mathbf{s}^{\prime \prime} \in\{-1,+1\}^{K}$. Then, one can verify from (10) that the previously shown code ambiguity problem may not occur for every $\mathbf{S}$, but will occur when $\mathbf{S}=\left[ \pm \mathbf{s}^{\prime}, \pm \mathbf{s}^{\prime}, \ldots, \pm \mathbf{s}^{\prime}\right]$ or when $\mathbf{S}=\left[ \pm \mathbf{s}^{\prime \prime}, \pm \mathbf{s}^{\prime \prime}, \ldots, \pm \mathbf{s}^{\prime \prime}\right]$. This problem motivates us to consider a stronger version of code non-rotatability:

Definition 2 An OSTBC matrix $\mathbf{C}($.$) is said to be strictly non-$ rotatable if there does not exist a matrix $\mathbf{Q} \in \mathbb{R}^{M_{t} \times M_{t}}$ such that

$$
\begin{equation*}
\mathbf{Q C}\left(\mathbf{s}^{\prime}\right)=\mathbf{C}\left(\mathbf{s}^{\prime \prime}\right) \tag{17}
\end{equation*}
$$

for any $\mathbf{s}^{\prime}, \mathbf{s}^{\prime \prime} \in\{-1,+1\}^{K}, \mathbf{s}^{\prime} \neq \pm \mathbf{s}^{\prime \prime}$; or equivalently if

$$
\begin{equation*}
\mathcal{R}\left(\mathbf{C}^{T}\left(\mathbf{s}^{\prime}\right)\right) \neq \mathcal{R}\left(\mathbf{C}^{T}\left(\mathbf{s}^{\prime \prime}\right)\right) \tag{18}
\end{equation*}
$$

for any $\mathbf{s}^{\prime}, \mathbf{s}^{\prime \prime} \in\{-1,+1\}^{K}, \mathbf{s}^{\prime} \neq \pm \mathbf{s}^{\prime \prime}$.
From (18) we obtain the following result:
Lemma 2 No OSTBC with $M_{t}=T$ can be strictly non-rotatable.
Lemma 2 is due to the fact that for $M_{t}=T, \mathbf{C}($.$) is a square orthog-$ onal matrix [cf., the property in (4)] and therefore $\mathcal{R}\left(\mathbf{C}^{T}\left(\mathbf{s}^{\prime}\right)\right)=$ $\mathbb{R}^{T}$ for any $\mathbf{s}^{\prime}$.

We conclude from the above discussion that for arbitrary $\mathbf{S}$, using strictly non-rotatable codes is a necessary condition for unique symbol identification. In fact, we can also show that the strictly non-rotatable code condition is also a sufficient condition for unique symbol identification, as described in the following theorem:

Theorem 3 Consider a non-trivial case where $\mathbf{H} \neq \mathbf{0}, \quad \mathbf{S} \in$ $\{-1,+1\}^{K \times P}$, and $P \geq 1$ are arbitrary. Given $\mathbf{Y}_{p}=\mathbf{H C}\left(\mathbf{s}_{p}\right)$ for $p=1, \ldots, P, \mathbf{S}$ is uniquely identifiable up to a sign if and only if $\mathbf{C}($.$) is strictly non-rotatable.$

A comparison of the blind symbol identifiability of the OSTBC and SM schemes (cf., Theorems 3 and 1, respectively) reveals an attractive benefit of using the OSTBC scheme. To guarantee unique symbol identification in the SM scheme, we need the MIMO channel matrix $\mathbf{H}$ to be of full column rank. This implies

| full rate code |  | non-rotatable? | strictly <br> non-rotatable? |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{M}_{t}$ | $T$ |  | no |
| 2 | 2 | no | yes |
| 3 | 4 | yes | no |
| 4 | 4 | no | yes |
| 5 | 8 | yes | no |
| 6 | 8 | yes | yes |
| 7 | 8 | yes | no |

Table 1: Rotatability of various full rate OSTBCs. The $M_{t} \times 8$ codes with $M_{t}<8$ are obtained by keeping the first $M_{t}$ rows of the $8 \times 8$ full rate code in [2].
that blind SM detection is not applicable to certain MIMO channel scenarios such as a single receiver antenna and a rank deficient $\mathbf{H}$. Since Theorem 3 indicates that unique OSTBC symbol identification does not depend on $\mathbf{H}$, blind OSTBC detection can be used effectively for any nonzero $\mathbf{H}$, including the two harsh channels mentioned above.

To further illustrate the blind identifiability advantage of the strictly non-rotatable OSTBCs, we further consider sufficient conditions under which OSTBC symbols can be uniquely identified without the strictly non-rotatable code property. One such situation is shown in the following theorem:

Theorem 4 Consider a case where $\mathbf{H} \neq \mathbf{0}$ is arbitrary. Given $\mathbf{Y}_{p}=\mathbf{H C}\left(\mathbf{s}_{p}\right)$ for $p=1, \ldots, P, \mathbf{S}$ is uniquely identifiable up to a sign if
i) $\mathbf{C}($.$) is non-rotatable; and$
ii) $\mathbf{S}$ contains at least $2^{K-1}$ distinct $K$-tuples in $\{-1,+1\}^{K}$.

Theorem 4 shows that non-rotatable codes can lead to unique blind symbol identification for any (nonzero) channel matrix. To guarantee unique blind identification with non-rotatable codes, however, we need $P \gg 2^{K-1}$ such that Condition ii) of Theorem 4 occurs with a high probability. This restriction on the data frame length is not needed for the case of strictly non-rotatable codes in Theorem 3.

Since code non-rotatability plays in a key role in blind symbol identifiability, we numerically checked the non-rotatability of a number of commonly encountered full rate OSTBCs $[2,3]$. The results are shown in Table 1. We see that only the $3 \times 4,5 \times 8$, and $7 \times 8$ full rate codes are strictly non-rotatable. Moreover, none of the square codes is strictly non-rotatable, which verifies the result in Lemma 2. Table 1 also indicates that a strictly non-rotatable code is non-rotatable. However the converse is not necessarily true; e.g., the $8 \times 8$ full rate code.

## 4. CONCLUSION AND DISCUSSION

In this paper, the blind symbol identifiability of the OSTBC scheme has been examined. We have shown that the OSTBC symbols are uniquely identifiable up to a sign if and only if the OSTBC matrix exhibits a so-called strictly non-rotatable property. This property does not depend on factors such as the number of receiver antennas, and the rank of the channel matrix, all of which play a crucial role in the blind symbol identifiability of several other linear MIMO schemes [9, 10, 12-14]. This implies that blind

OSTBC detection is applicable to any (flat fading) MIMO channel, an attractive feature that the other MIMO schemes may not easily achieve.

As strictly non-rotatable OSTBCs guarantee unique symbol identification, it will be useful to further study their properties. Moreover, it will be interesting to extend the present ideas to the complex OSTBC case. These future directions are currently being explored.

## 5. REFERENCES

[1] S.M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE J. Select. Areas Commun., vol. 16, no. 8, pp. 1451-1458, 1998.
[2] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, "Space-time block codes from orthogonal designs," IEEE Trans. Inform. Theory, vol. 45, no. 5, pp. 1456-1467, 1999.
[3] P. Stoica and G. Ganesan, "Space-time block codes: a maximum SNR approach," IEEE Trans. Inform. Theory, vol. 47, no. 4, pp. 1650-1656, 2001.
[4] D. Gesbert, M. Shafi, D.-S. Shiu, P.J. Smith, and Ayman Naguib, "From theory to practice: an overview of MIMO space-time coded wireless systems," IEEE J. Select. Areas Commun., vol. 21, no. 3, pp. 281-302, 2003.
[5] O. Damen, A. Chkeif, and J.-C. Belfiore, "Lattice code decoder for space-time codes," IEEE Commun. Lett., vol. 4, no. 5, pp. 161-163, 2000.
[6] J. Jaldén, C. Martin, and B. Ottersten, "Semidefinite programming for detection in linear systems- optimality conditions and spacetime decoding," Proc. 2003 Int. Conf. Acoust., Speech and Signal Processing, vol. 4, pp. 9-12, 2003.
[7] E.G. Larsson, P. Stoica, and J. Li, "Orthogonal space-time block codes: maximum likelihood detection for unknown channels and unstructured interferences," IEEE Trans. Signal Processing, vol. 51, no. 2, pp. 362-372, 2003.
[8] W.-K. Ma, P.C. Ching, T.N. Davidson, and X.-G. Xia, "Blind maximum-likelihood decoding for orthogonal space-time block codes: a semidefinite relaxation approach," Proc. IEEE 2003 Global Commun. Conf., December 2003.
[9] G.J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multielement antennas," Bell Labs Tech. J., vol. 1, no. 2, pp. 41-59, Autumn 1996.
[10] G.D. Golden, C.J. Foschini, R.A. Valenzuela, and P.W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," Electron. Lett., vol. 35, no. 1, pp. 14-15, 1999.
[11] S. Talwar, M. Viberg, and A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array- part I: algorithms," IEEE Trans. Signal Processing, vol. 44, no. 5, pp. 1184-1197, 1996.
[12] N.D. Sidiropoulos and R.S. Budampati, "Khatri-Rao space-time codes," IEEE Trans. Signal Processing, vol. 50, no. 10, pp. 23962407, 2002.
[13] A.L. Swindlehurst and G. Leus, "Blind and semi-blind equalization for generalized space-time block codes," IEEE Trans. Signal Processing, vol. 50, no. 10, pp. 2589-2498, 2002.
[14] B. Hassibi and B.M. Hochwald, "High-rate codes that are linear in space and time," IEEE Trans. Inform. Theory, vol. 48, no. 7, pp. 1804-1824, 2002.
[15] W.-K. Ma, "Blind and semiblind maximum-likelihood decoding for orthogonal space-time block codes," in preparation, 2003.
[16] G. Ganesan and P. Stoica, "Space-time diversity using orthogonal and amicable orthogonal designs," Proc. 2000 Int. Conf. Acoust., Speech and Signal Processing, vol. 5, pp. 2561-2564, 2000.


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[^1]:    ${ }^{1}$ Here, two vectors $\mathbf{a}$ and $\mathbf{b}$ are said to be distinct if $\mathbf{a} \neq \pm \mathbf{b}$.

