

# Dynamic radiation force of acoustic waves on absorbing spheres

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We present a theoretical study on how dynamic radiation force of acoustic waves actuates on absorbing spheres. We consider the radiation force is generated by the difference-frequency component of a bichromatic plane wave in a lossless fluid. We analyze the spectrum of the dynamic radiation force for polymethylmethacrylate (lucite) and polyethylene spheres. Results reveal that absorption cause the appearance of resonances in the spectrum of the force. The information associated to the resonances might be useful in radiation force techniques for nondestructive material evaluation and biological tissue characterization.

Keywords: Acoustic Radiation Force, Viscoelasticity, Scattering

## 1. INTRODUCTION

It is of particular interest of material sciences to determine viscoelastic constants of materials. These constants are related to thermodynamic properties of the analyzed material [1]. Particularly, evaluation of elastic constants of biological tissue is a valuable aid in medical diagnosis [2]. A well established method to inquire the viscoelastic constants of solids is the Resonant Ultrasound Spectroscopy [3]. Another technique called “Ultrasound-stimulated Vibro-acoustic Spectrography” probes mechanical parameters of a specimen by means of acoustic radiation force yielded by modulated waves [4]. In this case, the induced vibration on the object is measured by a laser vibrometer. Properties of magnetic objects can be probed by measuring the induced oscillatory magnetic field with a sensitive magnetic detector [5]. Vibroacoustography technique has motivated the author to investigate how dynamic radiation force acts on viscoelastic objects. Particularly, we analyze this force on polymeric spheres.

Acoustic radiation force is a phenomenon caused by the transferring of wave momentum to an object. It can be either static or dynamic with respect to its time dependence. Commonly, acoustic radiation force has been associated to the static force [6]. This force is produced on an object by steady-state acoustic waves. The static force phenomenon occurs because the nonlinear nature of wave propagation brings up a dc component in the spectrum of the wave pressure, which is associated to the static radiation force. The dc component can be also interpreted as the time-average of the excess pressure on the object in, at least, second-order approximation. On the other hand, dynamic radiation force has a broader meaning. It is a time dependent force produced on an object by the momentum flux of any acoustic wave or pulse. In fact, dynamic radiation force is a nonlinear effect as the static radiation force is. We consider that dynamic radiation force is produced by bichromatic waves. In this case, the force exerted on an object is caused by the subharmonic component at the difference frequency of the incident pressure. Dynamic radiation force generated by modulated waves in time has been applied to different branches of science and technology. For example, in measuring ultrasound power of ultrasonic transducers [7] and inducing oscillation in bubbles and liquid drops [8].

The problem of dynamic radiation force exerted on solid

elastic spheres by a plane traveling wave has been theoretically investigated in Ref. [9]. Experimental validation of this theory was presented in Ref. [10]. In this theoretical description, the difference frequency was assumed to be very narrow. On that account the magnitude of dynamic and static radiation forces are alike because they both depend on local acoustic fields. When the difference frequency is not narrow, nonlinear cumulative effects dominate the subharmonic component. This effect is known as parametric amplification of acoustic waves [11]. It is also observed in the dynamic radiation force exerted on objects [12]. A study on dynamic radiation force acting on non-absorptive spheres taking into account parametric amplification can be found in [13].

In this paper, we obtain the dynamic radiation force on a viscoelastic sphere by solving the acoustic scattering for the sphere in the quasilinear approximation (second-order). The absorption of the sphere is considered through a frequency power-law model [14]. The pressure field obtained from the scattering problem is then integrated on the surface of the sphere yielding the dynamic radiation force. Numerical evaluation of the spectrum of this force varying with the different-frequency is presented for the lucite and the polyethylene spheres. Results show that the spectrum of dynamic radiation force exhibit resonances, which are related to the mechanical parameters of the sphere. The position of the resonances is mostly related to the characteristic impedance sphere. Other viscoelastic properties of the sphere might as well be investigated determining the position and the width of the resonances.

## 2. PHYSICAL MODEL

Consider a homogeneous and isotropic fluid with adiabatic speed of sound  $c_0$ , in which thermal conductivity and viscosity are neglected. In this case, the acoustic fields are governed by the dynamic equations of ideal fluids. By using the regular perturbation technique, one can expand velocity potential in terms of the Mach number  $\epsilon \ll 1$ , the excess of pressure in the fluid is given by  $p - p_0 = p^{(1)} + p^{(2)} + O(\epsilon^3)$ , where

$$p^{(1)} = \epsilon \rho_0 \frac{\partial \phi^{(1)}}{\partial t}, \quad (1)$$

is the linear pressure and

$$p^{(2)} = \epsilon^2 \rho_0 \frac{\partial \phi^{(2)}}{\partial t} - \mathcal{L}. \quad (2)$$

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is the second-order pressure. The function  $\mathcal{L} = (\rho_0 \mathbf{v}^{(1)} \cdot \mathbf{v}^{(1)})/2 - p^{(1)2}/(2\rho_0 c_0^2)$  is the second-order Lagrangian density of the wave with  $\mathbf{v}^{(1)}$  being the linear particle velocity. Any analysis of dynamic radiation force has to be done in at least in the second-order approximation.

The wave propagation inside viscoelastic material can be formulated in terms of boundary-value problem using complex wavenumbers for the compressional and shear waves [16]. The dependency of attenuation with frequency is general determined from experiments. Here, we assumed that both compressional and shear absorption depend linearly with the vibration frequency (hysteresis absorption).

Consider an incident plane wave propagating toward a viscoelastic object with angular frequency  $\omega$  and wavenumber  $k = \omega/c_0$ . In the hysteresis absorption model the compressional and shear wavenumbers inside the material are given, respectively, by

$$\kappa_c = \frac{c_0}{c_c} k(1 + i\alpha_c), \quad (3)$$

$$\kappa_s = \frac{c_0}{c_s} k(1 + i\alpha_s), \quad (4)$$

where  $i$  is the imaginary unit,  $\alpha_c$  and  $\alpha_s$  are the absorption coefficients.

We now describe the second-order scattering for a viscoelastic sphere of radius  $a$  placed in the  $z$ -axis at the distance  $z_0$  from the acoustic source. The sphere has density  $\rho_1$ , compressional and shear speed of sound denoted, respectively, by  $c_c$  and  $c_s$ . To produce a bichromatic plane wave with primary frequencies  $\omega_1$  and  $\omega_2$  along the  $z$  direction, we consider a biharmonic excitation at acoustic source. Our analysis is limited to the scattering of the nonlinear wave at the difference frequency  $\omega_{21} = \omega_2 - \omega_1$ . To simplify the problem we consider  $a \ll z_0$ . The boundary-value problem for the second-order acoustic scattering was solved for spherical coordinates  $(r, \theta, \phi)$  in Ref. [12]. Accordingly, the total pressure at the difference frequency in the second-order approximation is given by

$$p_{21} = E_0 \frac{\gamma k_{21} z_0}{2i} \sum_{n=0}^{+\infty} (2n+1) i^n \left[ h_n^{(2)}(k_{21} r) + S_n h_n^{(1)}(k_{21} r) \right] \times P_n(\cos \theta) e^{-i(\omega_{21} t - k_{21} z_0)}, \quad (5)$$

where  $k_{21} = \omega_{21}/c_0$ ,  $\gamma$  is the nonlinear parameter of the fluid,  $E_0 = \varepsilon^2 \rho_0 c_0^2 / 2$  is the energy density at the acoustic source. The functions  $h_n^{(1)}$  and  $h_n^{(2)}$  are, respectively, the first- and second-kind spherical Hankel functions of  $n$ th-order, while  $P_n$  is the Legendre polynomial of  $n$ th-order. The quantity  $S_n$  is the modal scattering function which depends on the difference frequency and the mechanical parameters of the material. The pressure  $p_{21}$  increases with the nonlinear parameter  $\gamma$ , the difference frequency  $\omega_{21}$ , and the distance to the acoustic source  $z_0$ . This effect is cumulative as the wave propagates.

For absorptive sphere with complex compressional and shear wave numbers the modal scattering function is given by [16]

$$S_n = \det \begin{bmatrix} a_{11} & d_{12} & d_{13} \\ a_{21} & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} \det^{-1} \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}. \quad (6)$$

The matrix elements of this equation are given in the Appendix. The modal scattering function  $S_n$  contains the elastic and absorbing properties of the sphere.

Assume that the object is insonified by a bichromatic acoustic wave with primary frequencies  $\omega_1 < \omega_2$ . The difference frequency  $\omega_{21}$  is supposed to be smaller than  $\omega_1$ . Let  $a$  be a characteristic length of the object. For example, in the case of a sphere,  $a$  is its radius. The size factors of the object for the primary and difference frequencies are, respectively,  $x_m = k_m a$  ( $m = 1, 2$ ) and  $x_{21} = (k_2 - k_1) a$ . We focus on the situation where  $x_1, x_2 > 1$  and  $x_{21} > 10^{-2}$ . In this case, the dynamic radiation force function is caused by parametric amplification. Further discussion on this issue can be found in Ref. [17]. Therefore, integrating the pressure in Eq. (5) over the surface of the sphere gives

$$\mathbf{f}_{21}(t) = \pi a^2 E_0 \hat{Y}_{21} e^{-i(\omega_{21} t - k_{21} z_0)} \mathbf{e}_z, \quad (7)$$

where the radiation force function  $\hat{Y}_{21}$  is given by

$$\hat{Y}_{21} = \frac{\gamma z_0}{a} x_{21} [h_1^{(2)}(x_{21}) + S_1(x_{21}) h_1^{(1)}(x_{21})]. \quad (8)$$

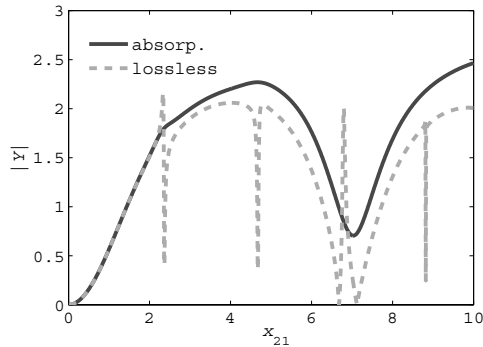
Note only the dipole mode in Eq. (5) contributes to the radiation force. This is due to the azimuthal symmetry between the incident plane wave and the sphere. The resonances are expected in the radiation force because the scattering function  $S_1$  may undergo to resonances as demonstrated in Ref. [16]. The presence of resonances in the scattering function is analogous to resonance effect in forced damped oscillators and RLC-circuits. This phenomenon in the radiation force is related to the eigenvibration modes of the sphere excited by the difference frequency wave.

### 3. NUMERICAL RESULTS

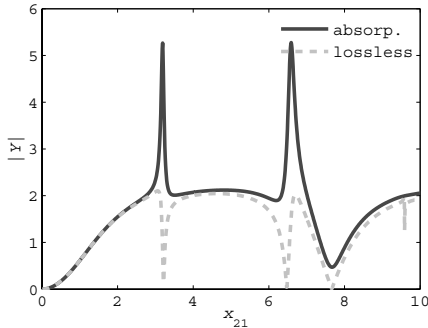
The frequency response of the dynamic radiation force on viscoelastic spheres in water is analyzed here. Two polymeric spheres are chosen, namely, lucite and polyethylene. The physical parameters of the spheres after Ref. [15] are, respectively,  $\rho_1 = 1191, 957 \text{ kg/m}^3$ ,  $c_c = 2690, 2430 \text{ m/s}$ ,  $c_s = 1340, 950 \text{ m/s}$ ,  $\alpha_c = 0.19, 0.40 \text{ dB}$ , and  $\alpha_s = 0.29, 1.20 \text{ dB}$ . The parameters for water are  $c_0 = 1500 \text{ m/s}$ ,  $\rho_0 = 1000 \text{ kg/m}^3$ , and  $\gamma = 6$ .

In Fig. 1, we plot the dynamic radiation force function varying with the difference-frequency size factor  $x_{21}$ . The results for both absorbing and lossless spheres are shown. Note that the lossless material have both shear and compressional absorbing parameters set to zero, i.e.  $\alpha_c, \alpha_s = 0$ . Thus, the complex wavenumbers  $\kappa_c$  and  $\kappa_s$  become real in Eq. (8). The radiation force for the polyethylene sphere presents dips at  $x_{21} = 2.2, 4.6, 8.7$  in the lossless case. None of them are present in the absorbing sphere. The force on the lossless lucite sphere has dips at  $x_{21} = 3.1, 6.5, 7.8$ , while on the absorptive sphere they turn into resonances.

The influence of the compressional ( $\alpha_c$ ) and shear ( $\alpha_s$ ) absorption parameters are depicted in Fig. 2. We use the physical parameters of polyethylene. Two size factors  $x_{21} = 4$  and  $6$  are chosen, which do not correspond to resonances as seen in Fig. 1.a. The radiation force exhibits a peak in the region  $0 \leq \alpha_s \leq 0.6$ . The radiation force remains constant as a function of  $\alpha_c$  in the region  $\alpha_s > 0.6$ . It decays as  $\alpha_s$  is increased.



(a) Polyethylene sphere.



(b) Lucite sphere.

FIG. 1: The dynamic radiation force as a function of the difference-frequency for polyethylene and lucite spheres.

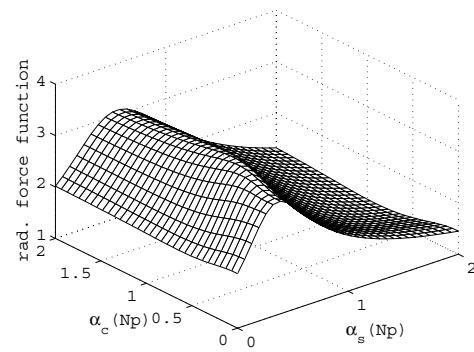
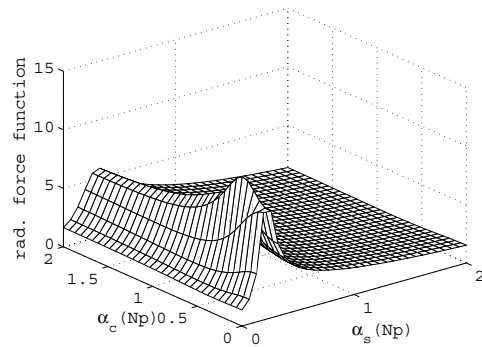
#### 4. DISCUSSION AND CONCLUSION

The dynamic radiation force acting on viscoelastic spheres has been studied. A simple formula for the radiation force function was obtained in terms of the modal scattering function. The absorption inside the sphere was accounted through the frequency power-law description. It has been shown that dynamic radiation force is due to the dipole term of the incident and scattered waves. Furthermore, the force can significantly change its profile due to absorption effects inside the sphere. Some dips and resonances in the lossless sphere has been smoothed out when absorption is taken into account. This is in agreement to the resonance scattering theory [16] applied to scattering problems by absorbing spheres.

In conclusion, the information present in the resonances might be useful assess mechanical properties of polymers and soft biological tissue. For instance, resonances may significantly change the contrast of inclusions such as microcalcification in vibro-acoustography images. A better image contrast may be achieved by sweeping the difference-frequency in order to find the inclusions' resonances in the imaged region.

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(a) Size factor  $x_{21} = 4$ .(b) Size factor  $x_{21} = 6$ .FIG. 2: Dependence of the radiation force function for the polyethylene sphere with the compressional ( $\alpha_c$ ) and shear ( $\alpha_s$ ) absorption coefficients given in Nepers.

#### APPENDIX

The elements of the matrices in Eq. (6) are:

$$\begin{aligned}
 a_{11} &= -(\rho_0/\rho_1)x_s^2 h_n^{(2)}(x_{21}), \\
 a_{21} &= x_{21} h_n^{(2)'}(x_{21}), \\
 d_{11} &= (\rho_0/\rho_1)x_s^2 h_n^{(1)}(x_{21}), \\
 d_{12} &= [2n(n+1) - x_s^2]j_n(x_c) - 4x_c j_n'(x_c), \\
 d_{13} &= 2n(n+1)[x_s j_n'(x_s) - j_n(x_s)], \\
 d_{21} &= -x_{21} h_n^{(1)'}(x_{21}), \\
 d_{22} &= x_c j_n'(x_c), \\
 d_{23} &= n(n+1)j_n(x_s), \\
 d_{32} &= 2[j_n(x_c) - x_c j_n'(x_c)], \\
 d_{33} &= 2x_s j_n'(x_s) + [x_s^2 - 2n(n+1) + 2]j_n(x_s),
 \end{aligned}$$

where  $x_c = \kappa_c a$ ,  $x_s = \kappa_s a$ , and  $j_n(\cdot)$  is the spherical Bessel function of  $n$ th-order.

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