

Costly Capital Allocation with Credit Market Frictions as a Propagation Mechanism*

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Abstract

Evidence from firm-level data shows that capital separation and reallocation are important phenomena. Aggregate gross investment flows are thus substantially larger than the net flows reported in the national accounts. Furthermore, separation rates vary inversely with the cycle while reallocation rates are strongly procyclical. This suggests that capital stocks are more volatile and procyclical than in standard business cycle models.

We build a search-and-matching model for capital with endogenous separation due to costly state verification to capture these firm-level characteristics about capital flows. We find that compared to the frictionless counterpart but also compared to models of financial frictions without costly capital reallocation, our model generates substantial internal amplification and matches surprisingly well the persistence in U.S. output growth. This suggests that credit market frictions *in combination with* costly capital allocation have important business cycle effects.

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1 Introduction

Investment in physical capital is often firm-specific and involves a costly allocation process. This explains why the distribution of investment across firms is wide and why capital remains with the same productive unit for several years on average. At the same time, recent evidence from firm-level data shows that capital separation is an important phenomenon over and beyond depreciation and that investment in used capital represents up to one fourth of total investment. In addition, capital separation rates vary inversely with the business cycle while reallocation rates are strongly procyclical.

These findings have important implications. First, aggregate gross investment flows computed from firm-level data are substantially larger than the net investment flows reported in the national accounts (where reallocation is by definition missed) and exceed depreciation at almost all phases of the business cycle. Second, the inverse cyclical behavior of separation and reallocation suggests that productive capital stocks are more volatile and procyclical than implied by standard Dynamic General Equilibrium (DGE) models. In these models, households invest in a generic stock of capital that is rented out to identical firms on a period-by-period basis and can be reallocated costlessly. Aggregate capital stocks are therefore simply the sum of current and past investment flows net of depreciation. Since depreciation rates are assumed constant over the cycle, capital stock dynamics are smooth relative to output and contribute little to the cyclical propagation of exogenous shocks.

In this paper, we propose a new model that captures the main characteristics of firm-level evidence on investment and capital allocation, yet remains tractable and ready-to-use in a DGE context. The main objective is to examine whether a more realistic description of capital flows helps to generate internal amplification to small shocks and persistence in output growth. Our paper therefore relates to the literature on the business cycle implications of variable capital utilization such as Burnside and Eichenbaum (1996) and embodied technological progress such as Gilchrist and Williams (2000). These studies focus on the effects of the volatile and procyclical nature of *effective capital use*. Alternatively, our focus is on the effects of more volatile and procyclical *capital measures* as suggested by firm-level data.

The model we propose extends the standard real business cycle (RBC) benchmark along two dimensions. First, firms must post projects at a cost and search for available capital to under-

take investments. The probability of a match depends on how much capital is made available by lenders relative to the total number of projects posted. Second, matched capital remains with the same firm until separation occurs. Capital separation is in part endogenous due to credit market frictions. Following the literature initiated by Townsend (1979), we introduce an idiosyncratic productivity shock that capital lenders can only observe at a cost. This costly state verification assumption gives rise to an agency problem that implies a debt contract with endogenous separation of capital from firms whose productivity falls below a specified threshold. Separated capital is returned to the household, but reallocation is costly due to the aforementioned search friction.

For reasonable calibrations, our model generates substantial endogenous amplification to exogenous technology shocks and implies output dynamics that come surprisingly close to replicating the marked positive autocorrelation in U.S. output growth over short horizons. Our model thus provides an answer to Cogley and Nason (1995) who show that the RBC benchmark but also extensions with adjustment costs or time-to-build lags in investment fail to perform satisfactorily along these two key dimensions. At the same time, the model as it stands currently generates counterfactual investment dynamics – an issue that needs to be investigated in future work.

Both the costly capital allocation and the credit market friction are central for the strong internal propagation of exogenous shocks in our model. On the one hand, the credit market friction implies that capital separations vary inversely with the business cycle. This is consistent with the aforementioned firm-level evidence but also with the countercyclical behavior of both default rates on corporate bonds and liabilities of business failures. As a result, exogenous shocks directly affect the *productive capital stock* rather than just *investment in capital*. For example, a temporary positive technology shock increases firm profits and decreases the separation of capital from production. The subsequent increase in the capital stock is therefore much stronger than in the RBC benchmark and more than outweighs the return of technology back to trend after the shock. This leads to an amplified and humpshaped response in both hours worked and output. On the other hand, the credit market friction alone is not sufficient on its own to generate these results. Without costly capital allocation, the separated capital could be immediately reaffected to new firms and thus, costly state verification alone would only affect investment.

The significance of the credit market friction *in combination with* costly capital allocation suggest that credit market frictions are not only important to rationalize firm behavior – as documented by a rich microeconomic literature – but also matter crucially for the business cycle.¹ This contrasts with recent studies on the topic. In particular, models with asymmetric information between lenders and firms such as Bernanke and Gertler (1989) or Kiyotaki and Moore (1997) imply that the firm’s ability to finance investment varies inversely with the value of its collateral and thus with the business cycle. This financial accelerator mechanism has the potential to generate amplified and persistent output effects in response to small shocks. Yet, simulations in a DGE context by Chari, Kehoe and McGrattan (2006), Dib and Christensen (2005) or Petrosky-Nadeau (2005) suggest that credit market frictions of this type typically imply procyclical default rates and fail to generate *quantitatively* important business cycle fluctuations.² Similar to models with adjustment costs or time-to-build lags in investment, this lack of internal propagation can be traced back to the assumption of costless capital allocation, which implies that credit market frictions only affect the dynamics of aggregate investment. But investment is small relative to the productive capital stock and thus, the impact on output remains insignificant. Furthermore, indirect effects through labor supply shifts induced by changes in consumption patterns are limited by offsetting movements in interest rates.

Our strategy to formalize costly capital allocation is inspired by the now widely employed search-and-matching approach to model labor market frictions, as pioneered by Diamond (1981) and Mortensen and Pissarides (1994). Recently, Dell’Ariccia and Garibaldi (2000), Den Haan,

¹Empirically, panel data studies find that small firms with more difficult access to credit pay fewer dividends, take on more debt, and have investment rates that are more sensitive to cash flows even after controlling for future profitability. See Hubbard (1998) and Stein (2000) for surveys. Theoretically, numerous papers show how optimizing models of the firm with incomplete contract enforcement and asymmetric information in the lending process can rationalize the observed correlation of firm size and age with mean growth (negative) and survival rates (positive). See Cooley and Quadrini (2001) or Clementi and Hopenhayn (2002) for examples.

²For example, Petrosky-Nadeau (2005) simulates the financial accelerator model of Bernanke, Gertler and Gilchrist (1998) in a New Keynesian context. He finds that the financial accelerator contributes only about 0.05% to the response of output to shocks and fails to generate persistence in output growth. This latter result contrasts with Carlstrom and Fuerst (1997) who restrict the agency problem to capital-producing entrepreneurs. While their model does not imply more amplification, it generates persistence in output growth. But this persistence comes at the price of a counterfactual response in household consumption and a procyclical default rate.

Ramey, and Watson (2003) and Wasmer and Weil (2004) have motivated the application of this matching process to capital as the result of financing frictions between firms and lenders. By contrast to these studies, we incorporate our model in a modern DGE framework with endogenous labor supply and intertemporal consumption/savings decisions. The *quantitative* implications of our model can therefore be readily compared to the RBC benchmark that our model nests as a special case, but also to the aforementioned financial accelerator models.³ Furthermore, our interpretation of capital matching extends beyond financing frictions, which may be highly relevant for new entrepreneurs and small firms but seem less obvious for large firms that account for the bulk of aggregate capital accumulation. Instead, search and matching in our model applies directly to the *allocation of physical capital*, which – similar to the allocation of workers to jobs in the labor market – has its origins in the limited yet state-dependent availability of investment opportunities, capital suppliers and financiers. Such a description of investment is at least in principle consistent with the well-documented fact that at any given time, the distribution of investment rates across individual firms is wide, with large mass of both firms with zero investment and firms with investment spikes.

To our knowledge, only few papers have so far examined the business cycle implications of costly capital entry/exit together with credit market frictions. One of them is Cooley, Marimon and Quadrini (2003) who derive credit market frictions from limited contract enforceability and allow for heterogeneity in firm size. This heterogeneity makes aggregation and the computation of the equilibrium a non-trivial issue. By contrast, our modeling approach bypasses the issue of firm size by assuming constant returns to scale production and the equilibrium is solved for a loglinear approximation around the balanced growth path. This greatly facilitates computation and allows for straightforward comparison with well-known business cycle models.

The remainder of the paper is organized as follows. Section 2 reviews the empirical evidence on investment flows and capital stocks. Section 3 presents the model. Section 4 discusses functional specifications and calibration. Sections 5 and 6 report quantitative results and assess their robustness. Section 7 concludes.

³Moran (2005) and Pierrard (2005) also incorporate credit matching frictions into a business cycle context. However, they do not model endogenous capital separation and reallocation. In line with our results, their models fail to generate endogenous amplification and persistence.

2 Empirical evidence

To motivate our extension of the business cycle model, we first review the computation of investment flows and capital stocks in the U.S. national accounts. Second, we document firm-level evidence on capital allocation and separation.

2.1 NIPA investment flows and capital stocks

For the National Income and Production Accounts (NIPA), the Bureau of Economic Analysis (BEA) computes investment flows and aggregate capital stocks (called fixed reproducible tangible wealth) using a supply-side top-down approach.⁴ Investment flows by asset type are measured as the real value of shipments from capital goods producing industries after subtracting inventory changes, net exports abroad as well as private and government consumption of these assets. Capital stocks for each asset are then inferred from the respective investment flows using the perpetual inventory method

$$K_{a,t} = \sum_{j=0}^{\infty} \omega_{ajt} I_{a,t-j},$$

where $K_{a,t}$ is the capital stock of asset a in period t ; $I_{a,t-j}$ is the real investment flow into asset a at $t-j$; and ω_{ajt} is the weight given to vintage j of asset a . This weight embodies the depreciation (decline in value) of the asset due to wear and tear, obsolescence, accidental damage, and aging. For most asset types, depreciation schedules are assumed to decline geometrically over time.⁵

For the sample 1950-1995, the annual investment flow for private non-residential assets averages 9.4% of its capital stock. The capital stock series is very smooth. Its Hodrick-Prescott filtered standard deviation for 1950-1995 is a mere 0.08. The low variability is one of the main reasons why many business cycle researchers abstract from capital when computing Solow residuals (see for example King and Rebelo, 2000).

Aside from the many problems associated with measuring and appropriately deflating shipments of capital goods, there are three reasons why NIPA's investment and capital stock series

⁴Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) provide a more detailed description of these computations and discuss the associated problems.

⁵See Katz and Herman (1997) for a description.

are problematic quantities for business cycle researchers. First, shipments of capital goods only provides information about investment flows of new assets and the BEA adjusts these series only for net transfers of used capital from consumers, government and foreign countries. Inter- and intra-industry transfers are completely missed. Second, the BEA's depreciation schedules in ω_{ajt} are supposed to reflect the service life of an asset, which implicitly assumes that capital from sales and exiting businesses is transferred costlessly to other productive units. To the extent that capital separation (i.e. exit of firms and sales) is an important phenomenon, ω_{ajt} therefore underestimates the loss in capital value due to specificity and other reallocation frictions. Both of these points imply that total annual investment in new and used capital goods may be substantially larger than 9.4%. Third, if capital separation and reallocation vary over the cycle, the NIPA capital stock measure may be much too smooth.

2.2 Firm-level evidence on gross capital flows

To quantify the importance of gross capital flows, we need to adopt a bottom-up approach and look at firm-level data on investment expenditures and disinvestment. Conceptually, capital separation from a productive process and subsequent reallocation occurs either because a continuing firm sells property, plant and equipment (PP&E); because a firm is liquidated; or because of mergers/acquisitions. We include this latter case in our investigation since mergers/acquisitions not only represent a change of ownership but often involve important modifications to the composition and use of existing capital.⁶ Here, we review results from different U.S. firm- and establishment-level surveys on these quantities. None of the surveys is entirely representative and each one of them suffers from its own shortcomings.⁷ They nevertheless provide valuable evidence on the importance of gross capital flows.

One of the first studies with firm-level data is Ramey and Shapiro (1998) who use the Compustat survey to document gross flows of capital. For their full sample (1959-1995), they find that roughly 70% of gross investment flows come from expenditures in new PP&E by existing firms, about 25% come from purchases of used capital, while entry of new firms contributes only 5%. These addition rates exhibit large fluctuations and comove with the cycle. For capital

⁶See Ramey and Shapiro (1998) or Eisfeldt and Rampini (2005a) and references therein.

⁷See Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) for a detailed discussion about the shortcomings of some of these surveys.

separations, in turn, retirements (71%) – which can be interpreted as the physical result of depreciation – are the most important component, followed by sales (21%) and exits due to mergers and bankruptcies (9%). By contrast to additions, these rates vary countercyclically around trend, resulting in a correlation coefficient with unemployment of 0.52.

Overall, Ramey and Shapiro’s gross flows of capital additions and subtractions average 9.7% and 7.3% of undepreciated capital stocks, respectively, which is comparable to the job creation and destruction rates reported in Davis, Haltiwanger and Schuh (1996). These flows may not be entirely comparable to the extent that depreciation and replacement of capital is a more important phenomenon than retirement of old / entry of young workers in the labor market. At the same time, Ramey and Shapiro’s gross flows of capital should be considered as a lower bound because they do not take into account acquisitions and because their measure abstracts from depreciation of capital in use.⁸ When taking depreciation into account, the gross flow of additions jumps up to 17.3% of total capital stocks, with investment in new capital representing 12.3%. Part of these depreciation rates probably represent accounting standards rather than actual decreases in the value-of-use. Nevertheless, it remains true that reallocation of used capital accounts for an important part of investment that is entirely missed in the NIPA tables.

The findings of Ramey and Shapiro are broadly confirmed by another study with Compustat data by Eisfeldt and Rampini (2005a). Based on a sample from 1971 to 2000, they also report that reallocation of used capital makes up 25% of gross investment, where reallocation is measured by sales of PP&E *plus* acquisitions and gross investment is defined as the sum of capital expenditures and acquisitions.⁹ Sales of PP&E represent about one third of these reallocation flows, with the remaining two thirds coming from acquisitions. Both components fluctuate over

⁸This may explain why Ramey and Shapiro’s ratio of new investment to total capital averages only 6.9%, which is substantially less than reported in the NIPA tables.

⁹There is some disagreement between Ramey and Shapiro (1998) and Eisfeldt and Rampini (2005a) about the treatment of acquisitions in the measure of reallocation. Ramey and Shapiro claim in their appendix that total capital expenditures (schedule v30 in Compustat) also includes the book value of the PP&E of acquired companies. Eisfeldt and Rampini, by contrast, claim that this variable excludes the net asset of businesses acquired and measure acquisitions with a separate variable (schedule v129) instead. A further difference between the two studies is that by contrast to Eisfeldt and Rampini, Ramey and Shapiro convert book values to current values, which necessitates strong assumptions about price deflators and the age of the different capital vintages in each firm.

the cycle, with a combined H-P filtered correlation coefficient with output of 0.64. Furthermore, Eisfeldt and Rampini's reallocation measure represents 1.5% of capital stocks if capital stocks are defined as total assets but 5.5% if capital stocks are defined by total PP&E. This implies that annual gross investment flows average between 6% and 22% of total capital stocks. Since total assets comprise a substantial part of non-tangible capital, the ratio of gross investment to physical capital that is relevant for traditional business cycle models such as the one presented here is likely to be substantially larger than the net investment ratio of 9.4% reported in the NIPA tables.¹⁰

One concern with Compustat is that it covers only corporations that file with the SEC. Other proprietorships and partnerships as well as establishments held by foreign firms not registered with the SEC are not part of their capital stock measure. Small and medium-size firms are thus underrepresented. Given that it is exactly these firms that are most likely to undergo major changes (merger/acquisition, bankruptcy, structural reorganisation) and invest in used capital, separation and reallocation flows are likely to be larger for the economy as whole. This conjecture receives some support from Eisfeldt and Rampini (2005b) who use data from the Annual Capital Expenditure Survey (ACES) to document differences in investment behavior between large and small firms. In existence since 1993, ACES is a nationally representative firm-level survey of capital expenditures in new and used structures and equipment. The survey does not provide information about reallocation that is due to the acquisitions of existing firms, however. For the 2004 survey, investment in used PP&E averages 8.7% of total capital expenditures over all firms. But as Eisfeldt and Rampini report, this fraction decreases considerably with firm size. For firms in the lowest asset decile, the fraction averages 28% while for firms in the highest asset decile (which make up the bulk of total investment and capital stocks), it represents only 10%. The low average compared to these larger fractions for the two extreme deciles indicate that aggregate capital flows are mostly driven by large firms. At the same time, the differences suggest that the investment flows from Compustat discussed above would be even higher if it also included small firms that are not listed with the SEC.

¹⁰Following the business cycle literature, we abstract from non-tangible capital here but acknowledge that it may play an important role for fluctuations and represents an interesting dimension to investigate in future research.

2.3 Distribution of investment flows across firms

Studies by Caballero, Engel and Haltiwanger (1995), Doms and Dunne (1998) or Cooper, Haltiwanger and Power (1999) show that investment on the microlevel is lumpy, with a wide distribution across firms. At any given point in time and even for narrowly defined sectors, there is a substantial mass of firms with zero investment that coexists with firms that exhibit very high investment rates, so called investment spikes.

Becker et al. (2005) reconfirm these findings in their summary using firm-level data from the Annual Survey of Manufacturers (ASM). Over the sample 1972 to 2001, they find that the fraction of plants with zero investment varies between 25 and 10% (with a slightly decreasing tendency). Furthermore, establishments are much more likely to have zero investment in structures (up to 62%) than equipment (as low as 5%). On the other end of the distribution, between 25 and 10% of all plants have investment rates in equipment and structures that exceed 20% of their capital stock (not counting acquisitions). In addition, the share of plants with such investment spikes is procyclical.

2.4 Sources of capital separation and reallocation costs

Separation of capital from productive processes occur, of course, for many different reasons. One of these reasons are credit constraints that require firms to sell assets, reorganize or even declare bankruptcy. While it is difficult to quantify how much of all separations are due to credit constraints, there is convincing evidence to suggest that its effects on separation are countercyclical. First, Covas and Den Haan (2005) document that in the U.S., default rates for corporate bonds peak at the end of recessions. The H-P filtered contemporaneous correlation of default rates with real GDP is -0.33 for the period 1971-2004, and -0.77 for the period 1986-2004. Second, we find that the series of current liabilities of business failures taken from DRI (mnemonic: fail) is countercyclical, with a H-P filtered correlation coefficient with real GDP of -0.36 for the sample 1948-2002.

Capital separations due to credit constraints and subsequent reallocation is likely to be associated with a substantial loss in value relative to its replacement cost at the original place of use. For example, Ramey and Shapiro (2000) argue that reselling capital is a time-consuming and costly process because of thinness in used-capital markets and sectoral specificity of capital.

Their argument is based on equipment level data about closures of aeronautical plants. They find that other aerospace companies are overrepresented among buyers, and that even after taking into account age-related depreciation, the average resale value of equipment is only 28% relative to replacement cost.¹¹

Becker et al. (2005) corroborate Ramey and Shapiro's (2000) findings with ACES data. Every year, ACES selects a new probability sample that can be used to compute the capital stock of firms that disappear, either because they cease to be active or because they continue to operate under a different firm. This series of capital separation due to exit/acquisition can then be compared with the following year's series of used capital expenditures and other additions and acquisitions. Over their 8-year sample, the thus defined absorption rate equals on average 64% of total separations. Since this measure also includes assets sold by continuing firms, the absorption of separated capital from firm death is likely to be lower.

3 The Model

As in the frictionless RBC benchmark, our model is populated by two agents: firms that produce using capital and labor; and households who decide on optimal consumption, leisure and investments in either riskless bonds or productive capital.

We add two frictions to this benchmark. First, the allocation of capital from households to firms involves a costly and time-consuming matching process. Second, the capital lending relationship between households and firms is subject to a credit market friction. Following the costly state verification literature initiated by Townsend (1979), this credit market friction takes the form of an idiosyncratic productivity shock that households can only observe at a cost. This asymmetric information assumption gives rise to an agency problem that results in a debt contract with endogenous separation of capital from firms whose productivity falls below a state-dependent threshold.

For the sake of simplicity, we abstract from a distinct sector for capital allocation. Instead, households act directly as capital lenders. Furthermore, we assume that the same matching friction applies to the allocation of both new and used (i.e. previously separated) capital. This

¹¹Even for machine tools, which typically have a better resale value than specialized aerospace equipment, the resale value is only about 40% relative to the replacement cost.

assumption simplifies our model because we do not need to keep track of different types of capital.

3.1 Search and matching in the capital market

Capital is either in a productive state or in a liquid state. We define by K_t the capital stock that enters the production function of a representative firm in period t . We do not distinguish among different firms because, as discussed below, our modeling assumptions imply that all firms are identical and that firm size is indeterminate.

Liquid capital L_t , in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households. Since we do not distinguish between the two types, a negative flow of new capital simply implies that households reffect separated capital to consumption.

To undertake investments, firms must post projects and search for liquid capital at cost κ per project. We denote by V_t the number of posted projects of a representative firm in period t . Actual investment (i.e. new capital allocations) in period t then is the result of a matching process $m(L_t, V_t)$ that is a positive function of the total amount of liquid capital L_t and the total number of project postings V_t . A firm's probability to find capital is therefore given by $p(\theta_t) = \frac{m(V_t, L_t)}{V_t}$ with $\partial p(\theta_t)/\partial \theta_t > 0$, where $\theta_t = \frac{L_t}{V_t}$ may be interpreted as a measure of relative capital market liquidity. Likewise, the probability of liquid capital being matched to a firm equals $q(\theta_t) = \frac{m(V_t, L_t)}{L_t}$ with $\partial q(\theta_t)/\partial \theta_t < 0$.¹² We will assume that $m(L_t, V_t)$ exhibits constant returns to scale and thus $p(\theta_t) = \theta_t q(\theta_t)$.

Capital matched to a firm in period $t - 1$ enters production in period t . This relationship between firm and capital continues to hold in $t + 1$ with probability $(1 - s_t)$ and so on for the periods thereafter. If the relationship is terminated, which happens with probability s_t , the capital is separated and returned to the household net of depreciation δ . Both the matching probability and the separation rate are taken as exogenous by the firm but depend on the state of the economy, as will be described below. Given these assumptions, the dynamics of the productive capital stock evolves according to the following law of motion

$$K_{t+1} = (1 - \delta)(1 - s_t)K_t + m(L_t, V_t). \quad (1)$$

¹²In addition, to ensure that $p(\theta_t)$ and $q(\theta_t)$ are between 0 and 1, we require that $m(L_t, V_t) \leq \min[L_t, V_t]$

3.2 Firms

At the beginning of each period, the firm observes exogenous aggregate technology X_t . Given its existing capital stock K_t , the firm then posts new projects V_t at unit cost κ and hires labor N_t at wage rate W_t to produce output Y_t with technology

$$Y_t = a_t f(X_t N_t, K_t), \quad (2)$$

with $f_N, f_K > 0$ and $f_{NN}, f_{KK} < 0$. The variable $a_t > 0$ denotes the realization of an idiosyncratic productivity shock that is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$. This shock is assumed to occur after the firm's optimal decisions have taken place and after the factor price equilibria are established.

Given these assumptions, the profit maximization problem of the firm is described by the following Bellman equation

$$\begin{aligned} J(K_t) &= \max_{N_t, V_t} \left\{ \int_{\bar{a}_t}^{\infty} [af(X_t N_t, K_t) - \rho_t K_t] dH(a) - W_t N_t - \kappa V_t + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J(K_{t+1}) \right\} \\ \text{s.t. } K_{t+1} &= (1 - \delta)(1 - s_t)K_t + p(\theta_t)V_t, \end{aligned}$$

where ρ_t is the rental rate of capital; and $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}$ is the discount factor of future cash flows. Several comments are in order about this expression. First, the discount factor is a function of the marginal utility of consumption Λ because the firm transfers all profits to the household. Second, the firm expects to pay the wage bill and the cost of posting vacancies in full but maximizes only over the portion of the revenue net of capital rental costs that is expected to yield positive profits, $\int_{\bar{a}_t}^{\infty} [af(X_t N_t, K_t) - \rho_t K_t] dH(a)$ where \bar{a}_t is defined as the break-even point associated with zero profits; i.e. \bar{a}_t such that $\bar{a}_t f(X_t N_t, K_t) = \rho_t K_t + W_t N_t + \kappa V_t$. As explained below, this is because under optimal contracting, any revenues associated with productivity shocks below \bar{a}_t are absorbed either by the capital lender (in case of continuation of the capital match) or by an insurance (in case of capital separation).¹³ Third, the firm takes both W_t and ρ_t as exogenous. The exogeneity of W_t is a direct consequence of our assumption of competitive labor markets. The exogeneity of ρ_t , in turn, implies that firms in our model do not internalize the effects of

¹³Alternatively, we could have assumed that the firm expects to pay the wage bill and vacancy costs only in case of positive profits. This would result, however, in substantial overhiring on part of the firm and thus a labor share that is too high.

their capital stock on the marginal productivity of capital and thus on the negotiation of ρ_t discussed below.

The first-order conditions of the optimization problem are

$$(N_t) : \int_{\bar{a}_t}^{\infty} a f_N(X_t N_t, K_t) dH(a) = w_t \quad (3)$$

$$(V_t) : \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}) = \frac{\kappa}{p(\theta_t)} \quad (4)$$

where $J_K(K)$ is the marginal value to the firm of an additional matched unit of capital. Differentiating the firm's value function with respect to capital, the definition of $J_K(K_t)$ is

$$J_K(K_t) = \int_{\bar{a}_t}^{\infty} [a f_K(X_t N_t, K_t) - \rho_t K_t] dH(a) + (1 - \delta)(1 - s_t) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} J_K(K_{t+1}). \quad (5)$$

This equation states that the value to the firm of an additional unit of capital is worth today's marginal product of capital net of the rental rate (in case the firm posts positive profits) plus its expected future value net of depreciation in case the project is continued.

3.3 Households

Before considering the household's optimal program, it is useful to define the rental contract that leads to optimal capital separation. Specifically, we assume that capital matches are discontinued either for exogenous reasons or for reasons associated with credit frictions in the capital lending relationship between households and firms. We define the total separation rate as

$$s_t = s^x + s_t^e,$$

where s^x denotes (constant) exogenous separation and s_t^e denotes endogenous separation.

To model the latter, we postulate that while firms perfectly observe the realization of the idiosyncratic shock a_t , households can only do so at an auditing cost. This asymmetric information assumption creates an agency problem because in the absence of auditing, the firm would always want to misreport a_t . The debt contract to deal with this problem is structured as follows.¹⁴ Households and firms negotiate the rental rate ρ_t per unit of matched capital *prior* to the realization of the idiosyncratic productivity shock a_t . Then, if $a_t \geq \bar{a}_t$ the firm pays $\rho_t K_t$, the household refrains from auditing and the capital match continues. If, on the other hand,

¹⁴We need to investigate whether this debt contract is actually optimal.

$a_t < \bar{a}_t$ the firm is unable to pay the negotiated capital rental because we assume that the wage bill $W_t N_t$ and the cost of posting vacancies κV_t need to be covered first. In this situation, the household pays the auditing cost to verify the firm's production and decides on the continuation of the capital match. If a_t is above some threshold \underline{a}_t that is associated with the household's choice of optimal separation, the household takes all of the firm's production and covers for the totality of $W_t N_t$ and κV_t so as to continue the capital match into the next period. Note that if a_t is sufficiently low, this may entail injecting additional funds. If instead a_t is below the optimal threshold \underline{a}_t , the household separates the match and takes back its capital stock without receiving nor paying anything. In this case, the firm bankrupts and the difference between production and the cost of $W_t N_t$ and κV_t is picked up by an insurance that is funded with the dividends from profit-making firms (see below).

Given these assumption, the endogenous part of separation is defined as

$$s_t^e = F(\underline{a}_t)$$

and the household's expected revenue from matched capital equals

$$R_t^K = \int_{\bar{a}_t}^{\infty} \rho_t K_t dH(a) + \int_{\underline{a}_t}^{\bar{a}_t} [(1 - \phi)af(X_t N_t, K_t) - W_t N_t - \kappa V_t] dH(a), \quad (6)$$

where we posit that the auditing cost to the household is a fixed proportion $\phi > 0$ of the firms revenue in case the match continues, $\int_{\underline{a}_t}^{\bar{a}_t} \phi af(X_t N_t, K_t) dH(a)$.¹⁵

With this information at hand, we formulate the household's optimal program. Specifically, households maximize the expected discounted flow of utility $u(C_t, 1 - N_t)$ over consumption C_t , leisure $1 - N_t$, risk-free bond holdings B_{t+1} , the amount of liquid capital L_t destined for matching with firms, and the optimal separation threshold \underline{a}_t . Time spent working yields revenue $W_t N_t$ while risk free bond holdings carry a net rate of return r_t in the following period. Matched capital, in turn, yields expected revenue R_t^K , while any capital unmatched is carried over into the next period with zero net return. Formally, this problem is described by the following

¹⁵There are of course many other possible definitions of this auditing cost. As we note in the quantitative part of the paper, the present definition has the counterfactual implication that auditing costs increase during a business cycle upturn. In future versions of the paper, we will change this.

Bellman equation

$$\begin{aligned}
V(U_t, K_t, B_t) &= \max_{C_t, N_t, L_t, B_{t+1}, \underline{a}_t} [u(C_t, 1 - N_t) + \beta E_t V(U_{t+1}, K_{t+1}, B_{t+1})] \\
&\quad + \Lambda_t [W_t N_t + R_t^K + (1 - \delta) s_t K_t + U_t + B_t + D_t - C_t - L_t - \frac{B_{t+1}}{(1 + r_t)}] \\
\text{s.t. } K_{t+1} &= (1 - \delta)(1 - s_t) K_t + q(\theta_t) L_t
\end{aligned}$$

where $U_t = (1 - q(\theta_{t-1}))L_{t-1}$ is the quantity of unmatched liquid capital in $t - 1$; D_t are firm profits transferred to households, and $(1 - \delta)s_t K_t$ is the value of capital separated from firms and returned into the budget constraint. Similar to the firm's optimization problem, we assume that the household considers the wage rate W_t and its matching probability $q(\theta_t)$ as exogenous.

The first-order conditions of this optimization problem are

$$(C_t) : u_C = \Lambda_t \quad (7)$$

$$(N_t) : u_N = \Lambda_t W_t \quad (8)$$

$$(B_{t+1}) : \beta E_t [\Lambda_{t+1}(1 + r_t)] = \Lambda_t \quad (9)$$

$$(L_t) : \beta E_t [V_U(U_{t+1}, K_{t+1}, B_{t+1})(1 - q(\theta_t)) + V_K(U_{t+1}, K_{t+1}, B_{t+1})q(\theta_t)] = \Lambda_t \quad (10)$$

$$(\underline{a}_t) : \Lambda_t \left[\frac{\partial R_t^K}{\partial \underline{a}_t} + (1 - \delta)f(\underline{a}_t)K_t \right] = (1 - \delta)f(\underline{a}_t)K_t \beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}) \quad (11)$$

The first three conditions are standard. The fourth condition for the household's choice of L_t states that the discounted expected utility of the marginal unit of liquid capital available for investment must equal the expected discounted return from investing in the riskless bond. With probability $(1 - q(\theta_t))$ a unit of liquid capital remains unmatched and is worth $V_U(U_{t+1}, K_{t+1}, B_{t+1})$ to the household, while with probability $q(\theta_t)$ it is matched with a project and turned into productive capital with marginal value $V_K(U_{t+1}, K_{t+1}, B_{t+1})$. From the above Bellman equation and the definition of R_t^K in (6), we can work out these marginal values as

$$V_U(U_t, K_t, B_t) = \Lambda_t \quad (12)$$

$$V_K(U_t, K_t, B_t) = \Lambda_t \{ \rho_t [1 - H(\bar{a}_t)] + (1 - \phi)(\underline{\mu}_t - \bar{\mu}_t) f_K(X_t N_t, K_t) + (1 - \delta) s_t \} \quad (13)$$

$$+ (1 - \delta)(1 - s_t) \beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}), \quad (14)$$

where $1 - H(\bar{a}_t) = \int_{\bar{a}_t}^{\infty} dH(a)$ and $\underline{\mu}_t = \int_{\underline{a}_t}^{\infty} a dH(a)$, $\bar{\mu}_t = \int_{\bar{a}_t}^{\infty} a dH(a)$ denote partial expectations.

Note that V_K is dynamic because with probability $1 - s_t$ the investment relationship between household and firm continues into the next period.

Finally, the fifth condition states that the optimal separation threshold \underline{a}_t is such that the marginal utility from capital revenue plus the last unit of capital separated equals the expected discounted value of the last unit of matched capital carried over into the next period. Applying the fundamental theorem of calculus on $\partial R_t^K / \partial \underline{a}_t$, we can write this condition more explicitly as

$$\begin{aligned} \Lambda_t(1 - \delta)h(\underline{a}_t)K_t &= \Lambda_t [(1 - \phi)\underline{a}_t f(X_t N_t, K_t) - W_t N_t - \kappa V_t] h(\underline{a}_t) \\ &\quad + (1 - \delta)h(\underline{a}_t)K_t \beta E_t V_K(U_{t+1}, K_{t+1}, B_{t+1}) \end{aligned}$$

This condition implicitly defines the optimal separation threshold \underline{a}_t . It can be shown that $\underline{a}_t f(X_t N_t, K_t) < W_t N_t + \kappa V_t$; i.e. the household is willing to inject additional funds into loss-making firms to cover $W_t N_t + \kappa V_t$ so as to continue the capital match. This is because separating capital and rematching it with a new firm is costly (there is a probability of no match at which the liquid capital unit yields zero return).

3.4 Rental rate of capital

To determine the rental rate of capital, we assume that once matched, households and firms split the surplus of their relationship according to a Nash bargaining process. As discussed above, this bargaining process takes place before the idiosyncratic shock a_t is realized. The surplus is the sum of marginal benefits to each party, $S_t = J_K(K_t) + \frac{V_k(U_t, K_t, B_t) - V_U(U_t, K_t, B_t)}{\Lambda_t}$. Define η as the household's relative bargaining power. The household then receives $\frac{V_k(U_t, K_t, B_t) - V_U(U_t, K_t, B_t)}{\Lambda_t} = \eta S_t$, while the firm's share is $J_K(K_t) = (1 - \eta)S_t$. After some algebraic manipulations that are detailed in the appendix, we obtain the following expression for the rental rate

$$\begin{aligned} \rho_t &= \eta \left[\bar{\mu}_t f_K(X_t N_t, K_t) + (1 - \delta)(1 - s_t) \frac{\kappa}{\theta_t} \right] + (1 - \eta)\delta \\ &\quad + \left[\rho_t H(\bar{a}_t) - (1 - \eta)(1 - \phi)(\underline{\mu}_t - \bar{\mu}_t) f_K(X_t N_t, K_t) \right]. \end{aligned} \tag{15}$$

The first term in brackets is the maximum amount the firm is willing to pay per unit of capital. It equals the marginal product of capital conditional on making a profit plus the average cost that is saved by entering the proposed capital match rather than continuing the search. The

second term in brackets is the household's opportunity cost to enter the proposed capital match, which equals the fraction not lost to depreciation when capital remains liquid δ . Finally, the third term in brackets represents the default risk-premium that arises because households do not receive the full contractual payment ρ_t when the firm's idiosyncratic shock drops below \bar{a}_t (zero profit).

3.5 Aggregation and equilibrium

The micro literature on firm dynamics usually assumes decreasing returns to scale production (see for example Cooley and Quadrini, 2001 or Esteban-Rossi and Wright, 2005). Here, for reasons of tractability, we follow the traditional macro literature and assume that the production function $f(\cdot)$ exhibits constant returns to scale. Under this assumption, it is straightforward to show that the capital labor ratio of all firms is the same. Hence, all optimality conditions are independent of firm size and the rental rate is identical for all firms.

The constant returns assumption justifies our derivation of the optimality conditions in a representative firm framework, but at the same time bypasses any issues that arise from firm size heterogeneity. These issues are admittedly important but taking them into account would render our model less tractable and complicate the quantitative analysis. In particular, we would no longer be able to draw direct comparisons with other representative agents models such as the frictionless RBC benchmark or the financial accelerator model of Bernanke, Gertler and Gilchrist (1998).

To compute the Rational Expectations equilibrium, we first need to define aggregate dividends D_t that firms return each period to the household. As mentioned above, we assume that there exists a state contingent insurance that covers for any shortfall in wage payments and costs of project postings left over by bankrupt firms (firms with productivity below \underline{a}_t for which the household refuses to inject funds to continue the lending relationship). This insurance is financed by the profits of firms with productivity above \bar{a}_t . Hence, aggregate dividends returned to the household are defined as

$$D_t = \int_{\bar{a}_t}^{\infty} [af(X_t N_t, K_t) - \rho_t K_t - W_t N_t - \kappa V_t] dH(a) + \int_0^{\underline{a}_t} [af(X_t N_t, K_t) - W_t N_t - \kappa V_t] dH(a).$$

Second, we combine different expressions to substitute out the marginal value function terms. The resulting system of equations together with assumptions about the exogenous shock process X_t defines the equilibrium dynamics of our model. Details of this system are provided in the appendix.

3.6 Comparison with the baseline RBC model

Before continuing to the quantitative evaluation, it is useful to compare our model with the baseline RBC model (see for example King and Rebelo, 2000) in which both costly capital allocation and credit market frictions are absent. In particular, the RBC model describes a world in which the cost of project postings κ is zero and thus, firms post an infinity of projects. Moreover, all capital is returned to the household (net of depreciation) at the end of each period and is reallocated at no cost at the beginning of following period.

In terms of our model, these assumptions translate into $s_t = 1$, $q(\theta_t) = 1$ and $U_t = 0$. Furthermore, it can easily be shown that $\rho_t = f_K(X_t N_t, K_t)$: the repayment on liquidity is equal to the marginal product of capital.¹⁶ Finally, from the law for productive capital one sees that to choose liquidity then amounts to choosing capital in the following period; i.e. $L_t = K_{t+1}$. This implies a value of matched liquidity $V_K(U_t, K_t, B_t) = \Lambda_t[\rho_t + (1 - \delta)]$, and the optimality condition for the choice of liquidity becomes a standard Euler equation:

$$\beta E_t \Lambda_{t+1} [\rho_{t+1} + (1 - \delta)] = \Lambda_t.$$

4 Shocks, functional forms and calibration

4.1 Shocks

Following much of the RBC literature we assume that our model economy is perturbed by an exogenous labor augmenting shock X_t that has both a deterministic trend part \bar{X}_t and a stochastic transitory part A_t . In particular $X_t \equiv A_t^{1/(1-\alpha)} \bar{X}_t$. The deterministic trend part evolves according to

$$\bar{X}_t = g \bar{X}_{t-1},$$

¹⁶The value of bargaining power η is irrelevant in the RBC setting as the competitive nature of the capital market rules out any positive surplus between matched firms and lenders.

and the stochastic transitory part evolves according to

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

with $\varepsilon_t^A \sim (0, \sigma_A^2)$.¹⁷

4.2 Functional forms

For household preferences, we follow King and Rebelo's (2000) baseline specification and define the family's period utility as $u(C, 1 - N) = \log C + \frac{\omega}{1-\xi}(1 - N)^{1-\xi}$. For production, we assume a Cobb-Douglas function with constant returns to scale of the form $af(XN, K) = aA(\bar{X}N)^{1-\alpha}K^\alpha$ with $0 < \alpha < 1$. The idiosyncratic shock a is assumed to follow a log-normal distribution with mean $E(a) = 1$; i.e. $\log(a) \sim N(-\frac{\sigma_{\log(a)}^2}{2}, \sigma_{\log(a)}^2)$. Finally, the matching technology takes the form similar to the one used in the labor literature, $m(V, L) = \chi V^\epsilon L^{1-\epsilon}$ with $0 < \epsilon < 1$.

4.3 Calibration

We calibrate our model to quarterly data. For the parameters that are common with the RBC benchmark, we use calibrations that are standard in the literature. We set $g = 1.004$ and $\beta = 0.992$, which implies an annual trend growth rate of 1.6% and an average annual real yield on a riskless 3-month treasury bill of 4.95%. For the labor supply, we fix the parameter ω such that the average fraction of hours worked equals $n = 0.214$. Together with $\xi = 4$, this results in a Frisch elasticity of labor supply of 1. Furthermore, we set the share of capital in the production function to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$, which corresponds to an annual decline of productive use of capital of 10%. Finally, to calibrate the exogenous driving process for the temporary technology shock, we extract a Solow residual from the data and then subtract a linear trend with average growth rate g . Estimation of the resulting AR(1) process yields $\rho_A = 0.98$ and $\sigma_A = 0.0072$ (as in King and Rebelo, 2000).

For the parameters that are proper to the capital matching model, we proceed as follows. We choose a quarterly steady state separation rate of $s = 0.01$ such as to match the long-run

¹⁷Alternatively, we could have specified a stochastic technology shock that is difference stationary and supplement it with an additional transitory shock (e.g. a labor supply or a government spending shock). We check in future versions of the paper that our results are robust to such a shock process specification.

averages from the firm-level data discussed in Section 2. Together with $\delta = 0.025$, this calibration implies that 71% of all separations are due to depreciation (i.e. retirement) and 29% are due to sales and firm exits / acquisitions, which is consistent with the proportions reported by Ramey and Shapiro (1998). Furthermore, the values of s , δ and g result in the following steady state *gross* investment rate (using the capital accumulation equation (1))

$$\frac{m(V, L)}{K} = [g - (1 - \delta)(1 - s)] = 0.03875,$$

which translates into a yearly investment rate of 15.5% – a rate that corresponds to the range of values computed by Ramey and Shapiro (1998) and Eisfeldt and Rampini (2005a).

For the remaining parameters where we have no direct microeconomic information (yet), we select a combination of values that implies steady state ratios of different macro aggregates that are broadly in line with the data and deliver interesting business cycle dynamics. We then assess the robustness of these results to alternative calibrations. Specifically, we assume that it takes on average 2 quarters before capital is (re-)allocated and becomes productive; i.e. $q = 0.5$. Furthermore, we assume that half of all separations are due to reasons other than our credit market friction; i.e. $s^x = 0.005$; set the household’s bargaining weight to $\eta = 0.5$; and select a fraction of output lost to auditing equal to $\phi = 0.1$. These calibrations are sufficient to endogenously determine the rest of the parameters (i.e. θ , κ , v , σ ,...) such that the system of steady state equations is satisfied (see the appendix for details). We obtain the following long-run averages: the cost of posting vacancies relative to output amounts to $v\kappa/y = 0.78\%$, the consumption-output ratio equals 72.6%, and the standard deviation of the idiosyncratic productivity shock is $\sigma_a = 0.30$ (with a standard deviation of the underlying normal $\sigma_{\log(a)} = 0.29$). Together with our calibrations of r , g , δ and s , these values result in an average annualized spread of the rental rate over the riskless rate (net of depreciation) of 3.13%.¹⁸ This number lies in-between the spread of the average Aaa corporate bond yield over

¹⁸We can derive an explicit expression for the steady state rental rate of capital by combining the equations for optimal project postings (4), optimal liquidity (??) and the rental rate (15)

$$\rho = (r + \delta) + \left(\frac{1-q}{q}\right) \left[r - (1-\delta)(1-s) \left(1 - \frac{\beta}{g}\right) \right] + \left[\rho H(\bar{a}) - (1-\phi)(\underline{\mu} - \bar{\mu})\alpha \frac{y}{k} \right].$$

The first term in brackets represents the steady state rental rate in the RBC benchmark without frictions. The second term compensates for imperfect capital allocation ($q < 1$), and the third term is a risk premium for the incomplete payment of ρ if idiosyncratic productivity falls below \bar{a} .

the 3-month Treasury bill of 1.87% and the average equity risk premium for the U.S. of 7.58% (for 1951-2000).

The final parameter we need to calibrate is the elasticity of the matching function ϵ . This parameter has no influence on the steady state values but only affects the dynamics. We set it to $\epsilon = 0.15$ and then explore the implications of other calibrations.

5 Simulation results

We analyze the empirical performance of our model in two stages. First, we consider impulse response functions (IRFs) of different aggregates with respect to the aggregate technology shock. The goal of this exercise is to graphically explore the workings of our model relative to the RBC benchmark. Second, we report a variety of unconditional second moments. To put the different results in perspective, we compare them to the RBC benchmark as well as a non-monetary version of Bernanke, Gertler and Gilchrist's (1998) financial accelerator model.

The assumption of a deterministic trend in labor productivity implies that we need to normalize all aggregates by \bar{X}_t so as to obtain a stationary system that we can simulate using log-linear solution techniques. Once normalized, we compute the rational expectations solution of the log-linear system of equations with the algorithm developed by King and Watson (1998).¹⁹

5.1 Impulse response functions

Figure 1 plots the IRFs of prominent macro aggregates to a persistent but temporary technology shock.

¹⁹We thank Bob King for providing us with the relevant Matlab code.

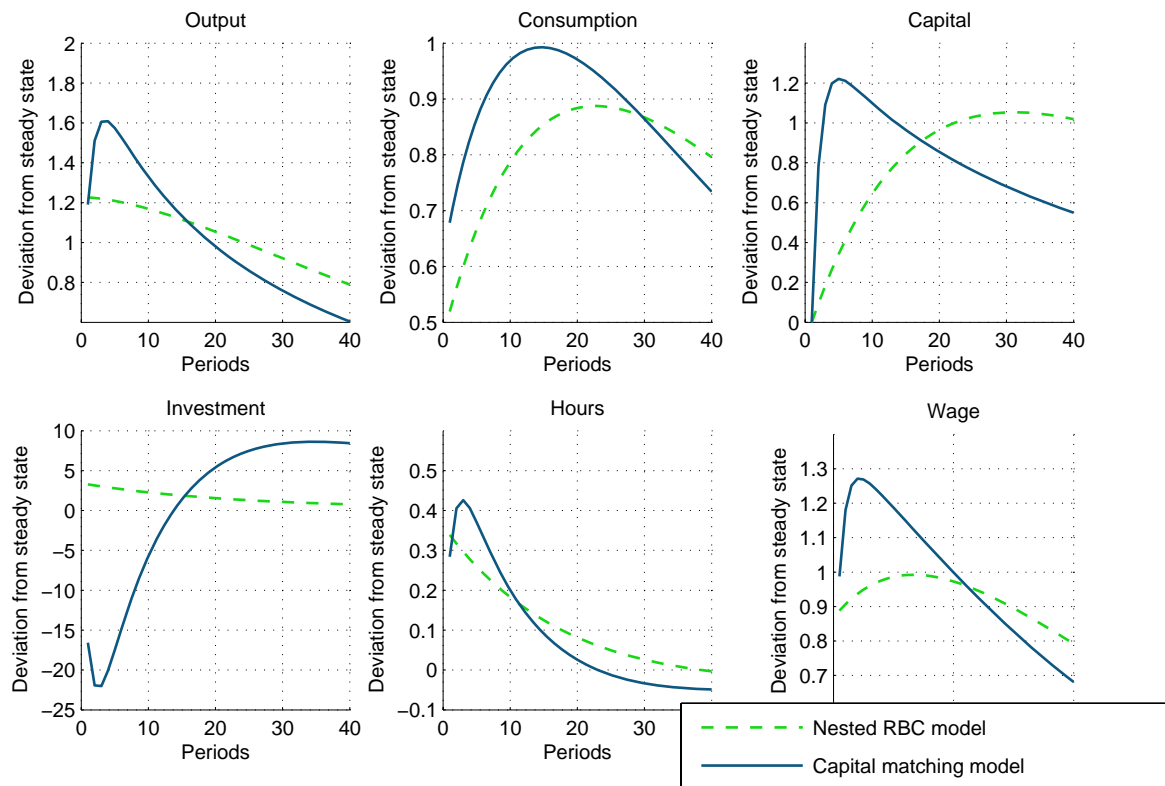


Figure 1: IRFs to a persistent technology shock

As is immediately apparent from the top-left panel, our capital matching model (solid lines) generates an amplified and markedly humpshaped response of output compared to the RBC benchmark (dotted lines). Whereas output in the RBC benchmark peaks upon impact and then gradually decreases in line with the technology shock, output in the capital matching model peaks only after 3 periods and the maximum response is roughly 33% larger.

Both humpshape and amplification have their origins in the state-dependent nature of the credit constraint and the capital allocation friction. To obtain this effect, the gradual decrease in the aggregate productivity variable A_t after impact needs to be more than compensated with higher labor and/or capital input. In our model, the higher aggregate productivity level decreases the fraction of firms with negative profits and thus, capital separation drops precipitously (see bottom-left panel of Figure 2 below). As a result, the increase in capital starting in the period after the shock is more rapid and amplified compared to the RBC benchmark, even though the

response of investment is strongly negative (we discuss this issue below). This directly affects output via the production function. In addition, the larger capital stock shifts up labor demand even more, with the associated substitution effect in the labor market leading to a hump-shaped response of hours and thus output

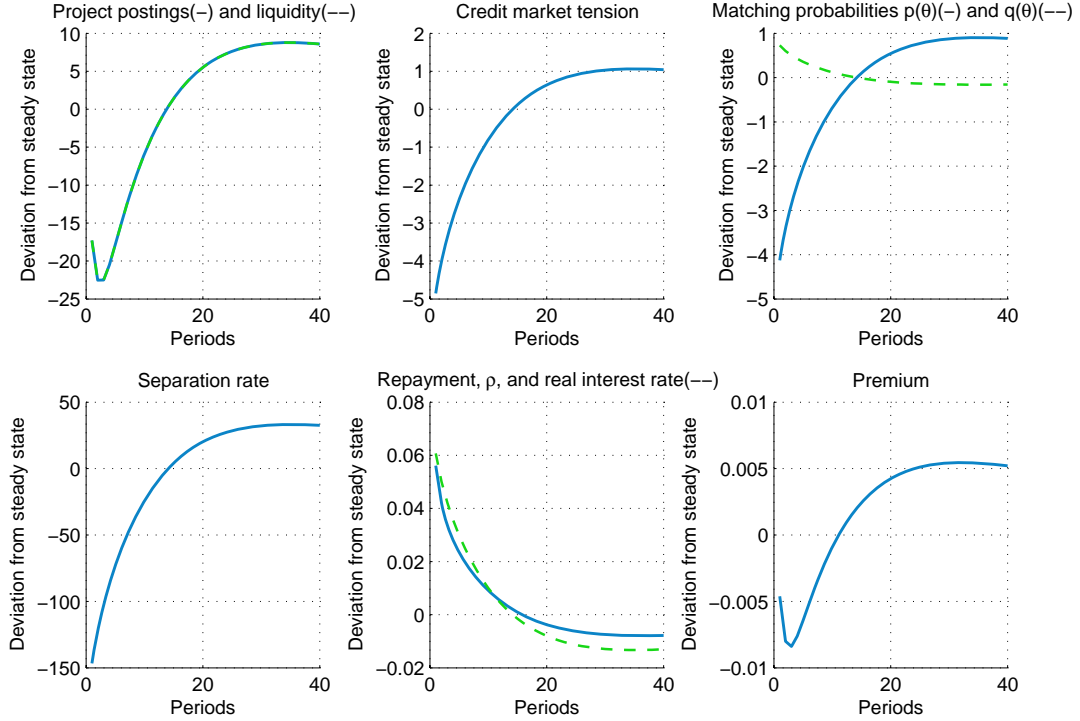


Figure 2: IRFs of capital matching variables to persistent technology shock

Both firms and households respond to the technology shock with an decrease in project postings V_t and liquid capital L_t , respectively. Since, L_t drops more than V_t , capital liquidity $\theta_t = L_t/V_t$ drops. This in turn implies that the probability of locating funds for a project $p(\theta_t)$ decreases while the probability of locating a project $q(\theta_t)$ increases. From the point of view of the household, this represents a smaller degree of capital reallocation friction as the likelihood of matching L_t with a firm and thus obtaining a positive return in the following periods increases.

Another interesting feature is the response of the risky rental rate of capital ρ_t compared to the risk-free rate $r_t + \delta$. Unsurprisingly, the risk free rate increases as households need a higher return to save for future consumption. The rental rate also increases, but somewhat less which implies that the premium of the risky rate over the riskless rate moves inversely. To shed more

light on this result, reconsider the definition of the rental rate from equation (15)

$$\begin{aligned} \rho_t = & \eta \left[\bar{\mu}_t f_K(X_t N_t, K_t) + (1 - \delta)(1 - s_t) \frac{\kappa}{\theta_t} \right] + (1 - \eta) \delta \\ & + \left[\rho_t H(\bar{a}_t) - (1 - \eta)(1 - \phi)(\underline{\mu}_t - \bar{\mu}_t) f_K(X_t N_t, K_t) \right]. \end{aligned}$$

Several forces are at work here. On the one hand, the firm's marginal productivity of capital and the partial expectation $\bar{\mu}_t$ increase, thus putting upward pressure on ρ_t . Likewise, the risk premium (expression on the second line) increases slightly on impact.²⁰ On the other hand, the drop in the separation rate lowers the required minimum return of the household, thus putting downward pressure on ρ_t . Furthermore, the decrease in θ_t and s_t have inverse effects on the expected average cost of obtaining future capital for the firm. A priori, it is not clear which of these effects prevail in general equilibrium. As it turns out, however, the drop in the premium is a relatively robust to alternative calibrations.

A problem with our model as it stands currently is the large drop in investment upon impact of the shock (see bottom left panel of Figure 1). This results is contradicted by the data where both investment in new capital and reallocation of used capital are strongly procyclical. To take a closer look at the sources of this response, we combine the household's budget constraint with the definition of aggregate profits to obtain

$$Y_t[1 - \phi(\underline{\mu}_t - \bar{\mu}_t)] = C_t + [L_t + \kappa V_t] - [\phi s_t(1 - \delta)K_t + U_t]. \quad (16)$$

The term $Y_t\phi(\underline{\mu}_t - \bar{\mu}_t)$ on the left-hand side represents the output lost to monitoring. The first term in brackets on the right-hand side represents the resources devoted to investment by households and firms. The second term in brackets represents returns from current capital stocks $s_t(1 - \delta)K_t$ plus resources not matched in the previous period U_t . Since s_t drops after the shock, less capital is separated from firms and returned to the household's budget constraint. Furthermore, U_t also varies countercyclically in the periods after the shock since both L_t and the probability of non-allocation $1 - q(\theta_t)$ drop. The two effects imply that less resources are

²⁰The increase in the risk premium upon impact is somewhat counterintuitive and has to do with the fact that we applied the monitoring cost only on the portion of the productivity shock for which no separation occurs. Since separations drop in response to the positive technology shock, $\underline{\mu}_t$ decreases, thus increasing the portion for which monitoring costs apply. We will change the definition of monitoring cost in future versions of the model so as to avoid this implication.

available in each period to finance consumption and liquid capital set aside for investment. As it turns out, this effect is not reversed by the increase in output and the lesser costs of project postings implied by the drop in V_t .²¹ The large drop in L_t and the resulting drop in investment $q(\theta_t)L_t$ is clearly an issue for our model. We need to assess the exact determinants behind this response and how it could be reversed.

5.2 Unconditional second moments

5.2.1 Autocorrelation of output growth

One of the great challenges in business cycle macroeconomics is the positive autocorrelation of output growth over several quarters in the data. As Cogley and Nason (1995) document, the RBC model completely fails to generate such positive autocorrelation and researchers have proposed different theories that could potentially explain this pattern. However, the results so far have been mixed at best.

Figure 3 displays the autocorrelation function for output growth for the data (green line), our model (blue line), the RBC model (dotted line) and a non-monetary version of Bernanke, Gertler and Gilchrist's (1998, BGG henceforth) financial accelerator model (solid lines with stars).²²

²¹Note that part of the increase in output is absorbed by the larger monitoring costs. As discussed in the previous footnote, however, this effect is counterintuitive and an artifact of our definition of monitoring costs.

²²See Petrosky-Nadeau (2005) for a description of the BGG model. The calibration of this model is similar to the one reported in BGG.

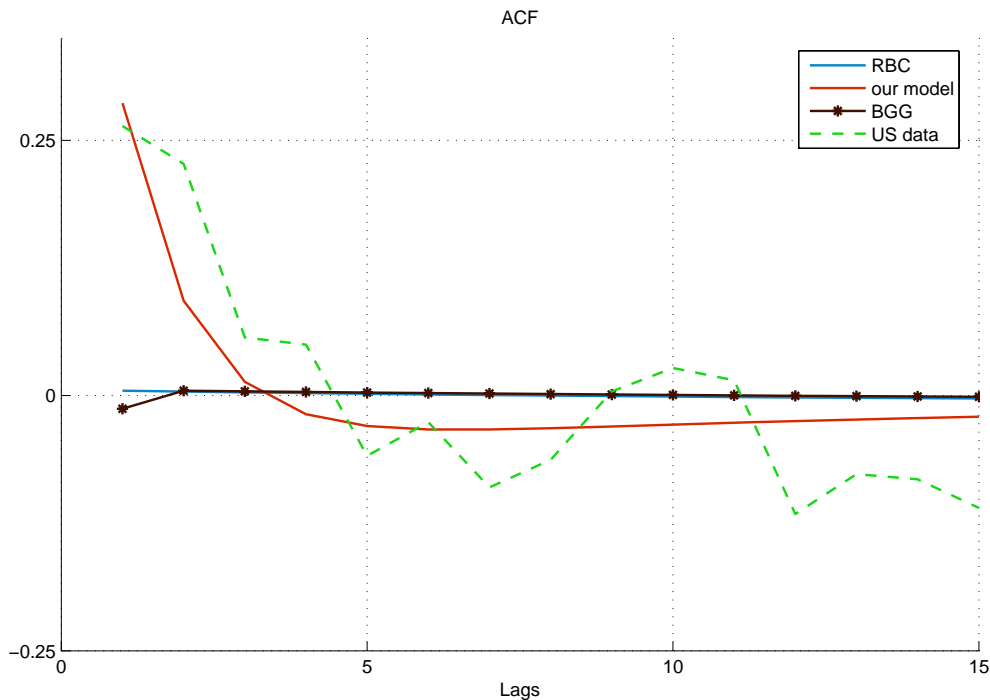


Figure 3: Autocorrelation function for output growth

As is immediately apparent, both the RBC benchmark and BGG’s financial accelerator model fail to generate any autocorrelation in output growth. By contrast, our capital matching model tracks the empirical autocorrelation of output in the data much better. The fit is admittedly imperfect since the autocorrelation drops one lag too early. What is remarkable, however, is the high value for the correlation at the first lag. To our knowledge, very few parsimonious models manage to generate such high values without creating substantial positive autocorrelation at lags four and beyond.²³ This goes to show that credit constraints together with costly capital allocation generates substantial internal propagation.

²³Chiang, Gomes and Schorfheide (2003), for example, propose a model with learning-by-doing to generate persistence in output growth. While their model implies sizable persistence for lags 1 and 2, it also generates substantial persistence at lags 3 and thereafter. Gilchrist and Williams (2000) is another example that generates some persistence in output growth.

5.2.2 Business cycle volatilities and cross-correlations

Table 1 presents unconditional second moments for the growth rates of different prominent macro aggregates for quarterly U.S. data, the RBC benchmark, our capital matching model, and BGG’s financial accelerator model.

	Table 1: Second moments for baseline calibration							
	U.S data (i)		RBC benchmark		Matching model		BGG model	
	a	b	a	b	a	b	a	b
c	0.58	0.69	0.45	0.96	0.46	0.96	0.62	0.97
n	0.95	0.87	0.27	0.98	0.29	0.97	0.2	0.94
w	0.44	0.14	0.73	0.99	0.76	0.99	0.81	0.99
k			0.27	0.1	0.73	0.81	0.29	0.13
i	2.89	0.87	2.6	0.99	15.9	-0.94	3.45	0.98
$premium$	0.54	-0.59	0.003	0.98	0.006	-0.91	0.08	-0.034
$\sigma(y)$	1.66		1.12		1.45		1.05	

a: Standard deviation relative to output.

b: Contemporaneous correlation with output.

All moments are Hodrick-Prescott filtered.

Data sources: (i) DRI Basic Economics.

There are several striking features. First, our model generates substantial internal amplification compared to the two other models. This effect is due to the somewhat more volatile dynamics of hours and the markedly more volatile and more correlated capital stock dynamics. In the RBC benchmark and BGG’s financial accelerator model, by contrast, capital stocks do not exhibit much volatility and are hardly correlated with output. Changes in the capital stock due to endogenous separation thus represent an important channel through which business cycle dynamics are affected. These findings could also have important consequences for the measurement of the Solow residual and the thus resulting technology shocks. In particular, technology shocks as computed in Section 4 have been criticized for their large volatility that imply a substantial probability of technological regress (see for example the discussion in King and Rebelo, 2000). A more volatile and procyclical capital stock as generated in our model has the potential

to reduce the size of the technology shock, thus addressing one of the main criticism of the RBC paradigm.

A second interesting feature of our model is that it generates a countercyclical premium of the capital rental rate over the riskfree rate, which is in line with the data (where we computed the premium as the spread between a Baa corporate bond yield and a risk-free 3-month T-bill yield). By contrast, the RBC benchmark implies a procyclical premium and BGG model generates a premium that is acyclical.

At the same time, and as discussed above, the positive results reported here come at the price of strongly counterfactual investment dynamics. Specifically, investment is much too volatile and negatively correlated with output.

6 Assessing the effects of the different frictions

The purpose of this section is to assess the quantitative importance of the different frictions by resimulating the model, first with the credit constraint turned off (i.e. $\sigma_a^2 = 0$ such that $s_t = s$) and second with perfect matching of liquid capital to projects (i.e. $q = 1$).

A fair question to ask, of course, is to what extent our results are robust to the calibration of the other parameters that are specific to our capital matching model. In particular, one may wonder how our results change for different values of η , s or ϵ . We will provide more information about these robustness checks in future versions of the paper.

6.1 The effect of removing credit market frictions

Consider first the case without idiosyncratic productivity shocks. This corresponds to a situation where credit market frictions do not apply because firms production never falls short of covering their factor costs. Hence, the separation rate is constant over the cycle, i.e. $s_t = s = s^x = 0.01$.

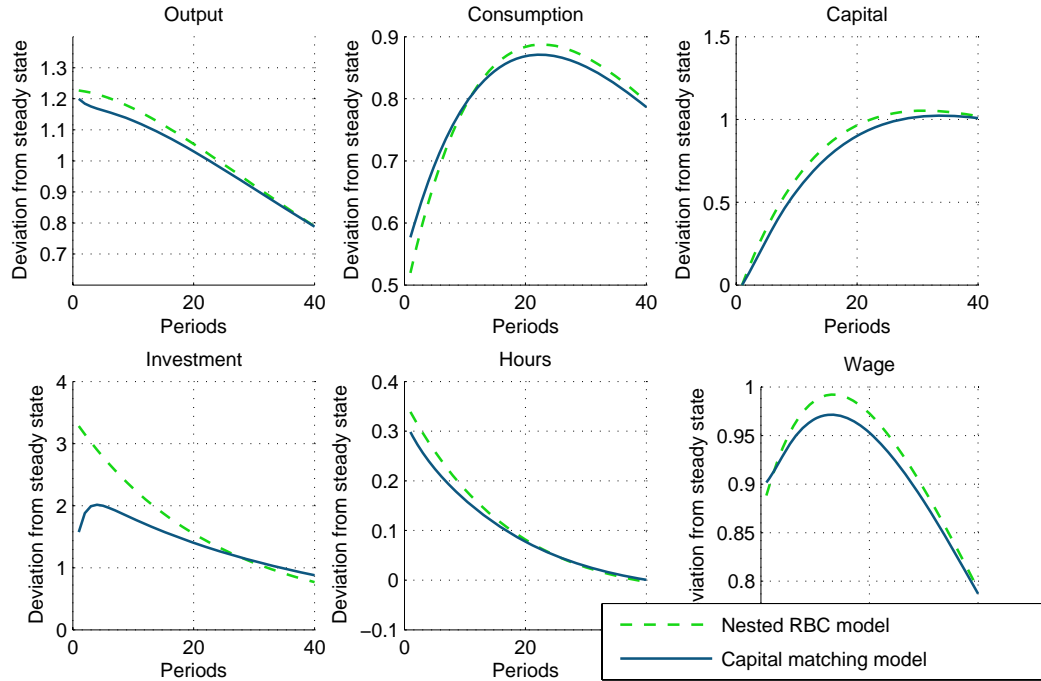


Figure 4: IRFs to a technology shock if no credit market frictions

As is evident from the top-left panel of Figure 4, the response of output to a technology shock is no longer amplified and humpshaped. In fact, the response even falls slightly below the RBC benchmark as the response of capital and thus hours react less to the technology shock. The reason for the more muted response of capital is two-fold. First, separations are constant by definition; and second, the response of investment is positive but less volatile than in the RBC benchmark. The investment response in our model is smaller because the cost of posting projects is costly.

Figure 5 provides more detail about the variables related to capital matching. By contrast to our baseline case, both postings and liquid capital increase. Furthermore, the response of the rental rate is now above the response of the riskless rate, thus implying a slight increase in the premium.

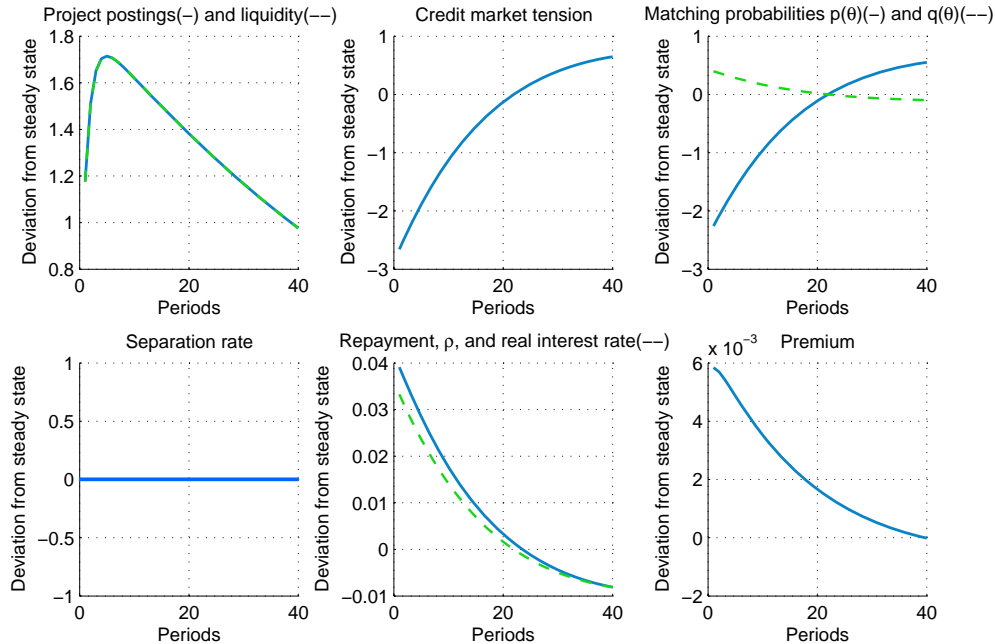


Figure 5: IRFs to a technology shock if no credit market frictions

These results suggest that the capital matching friction alone does not have sizable quantitative effects. To the contrary, the friction on investment acts to reduce the internal propagation of exogenous shocks.

6.2 The effect of removing costly capital allocation

The second experiment to consider is the removal of imperfect matching of capital with firms, i.e. $q(\theta_t) = 1$. This corresponds to a situation where firms post an infinity of new projects every period because costs are zero ($\kappa = 0$). Capital markets therefore become perfectly competitive and there is no longer any rent to share between capital providers (households) and firms. However, the rental rate of capital still involves a risk premium because firms may default on the rental payment, in which case the household audits the firm and may need to reinject funds to cover for the wage bill and project postings.²⁴ This risk premium affects the cost of capital and thus investment.

²⁴Despite constant returns to scale production, firms do not make zero profit. This is because their optimal decision with respect to capital applies only over the part of production with positive expected profits.

Specifically, we derive the following expression for the marginal product of capital is determined by (see appendix for details)

$$\beta E_t \left\{ \Lambda_{t+1} \left[f_K(X_{t+1}N_{t+1}, K_{t+1})[\bar{\mu}_{t+1} + (1 - \phi)(\underline{\mu}_{t+1} - \bar{\mu}_{t+1})] + 1 - \delta \right] \right\} = \Lambda_t,$$

with the optimal separation threshold determined by $\underline{a}_t f(X_t N_t, K_t) = W_t N_t$. In the RBC benchmark without credit market frictions, by contrast, the marginal product of capital equals $\beta E_t \{ \Lambda_{t+1} [f_K(X_{t+1}N_{t+1}, K_{t+1}) + 1 - \delta] \} = \Lambda_t$. The wedge $[\bar{\mu}_{t+1} + (1 - \phi)(\underline{\mu}_{t+1} - \bar{\mu}_{t+1})]$ internalizes the risk premium. It varies over the cycle, depending on expected default and separation probabilities.

Figure 6 shows the reactions of the different macro aggregates to the technology shock for this case.

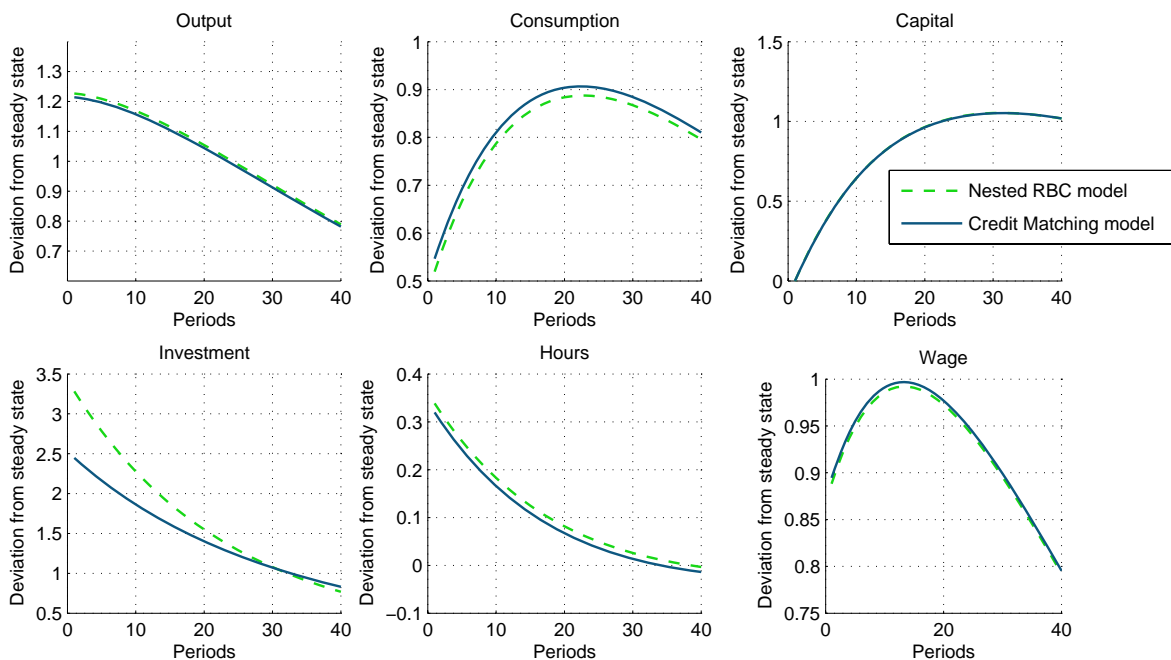


Figure 6: IRFs to a technology shock without costly capital allocation

As the top-left panel shows, output responds almost identically than in the RBC benchmark. Hence, the credit market friction on its own has almost no quantitative effect. This finding mirrors recent simulations by Chari, Kehoe and McGrattan (2006), Dib and Christensen (2005) or Petrosky-Nadeau (2005) who find that financial accelerator model of the type proposed in Bernanke, Gertler and Gilchrist (1998) on their own fail to generate *quantitatively* important

business cycle fluctuations.

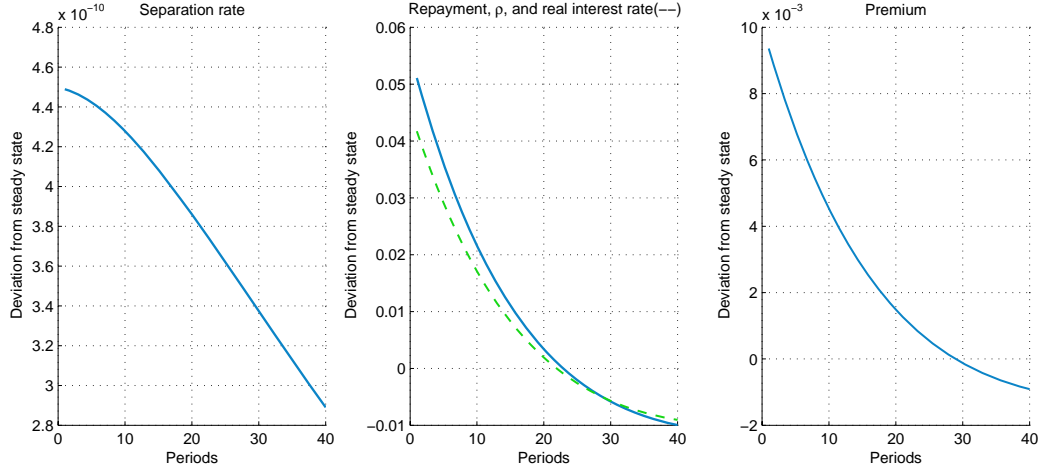


Figure 7: IRFs to a technology shock if no costly capital allocation

The reason for this lack of amplification is that the change in separations are very small (see Figure 7) and, more importantly, that separated capital can be costlessly reallocated for production in the following period. Interestingly, the separation rate increases rather than decreases, which coincides with the prediction of financial accelerator models but is contradicted in the data. Furthermore, the premium increases slightly and the response of investment falls slightly below the one in the RBC benchmark.

In sum, the robustness analysis in this section illustrates that it is the combination of costly capital allocation and credit market friction that generates the strong internal propagation results of our model. This conclusion is summarized in Table 2 that compares key second moments of our baseline model to the cases with each of the two frictions removed.

Table 2: Comparison of second moments when frictions are removed

	RBC benchmark	Baseline model with both frictions	No credit market friction	No capital matching
$\sigma(y)$	1.12	1.45	1.08	1.11
$corr(\Delta y, \Delta y_{-1})$	0.005	0.28	-0.005	0.005
$corr(\Delta y, \Delta y_{-2})$	0.004	0.93	-0.001	0.004
$corr(\Delta y, \Delta y_{-3})$	0.003	0.14	0.001	0.003

Notes: Standard deviation of output is H.-P. filtered.

Autocorrelations of growth rates are unfiltered

7 Conclusion

to be added

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