EFFECT OF GAS-PROPERTIES EVALUATION METHOD ON THE OPTIMUM POINT OF GAS TURBINE CYCLES

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ABSTRACT

Recent work has revealed that the assumption regarding the behavior of gases (perfect, ideal, real) and, consequently, the way their properties are evaluated may alter critically the picture obtained about the performance of gas turbine systems. This fact prompted an investigation of how the aforementioned assumption may affect the optimal design point of gas turbine systems. Three systems have been selected for study and three optimization problems have been formulated and solved for each system: two thermodynamic and one thermoeconomic. The results demonstrate that the method (assumption) used for the evaluation of properties of gases has a very significant effect on the optimal point of each system.

Keywords: Gas properties, Gas turbine cycles, Optimization, Thermoeconomics.

NOMENCLATURE

C_n	Capital cost of component n (installed)
c_f	Unit cost of fuel
c_p	Specific heat capacity at constant pressure
$\tilde{c}_p$	Molar heat capacity at constant pressure
c_v	Specific heat capacity at constant volume
$\tilde{c}_v$	Molar heat capacity at constant volume
FCR	fixed charge rate
f	fuel to air ratio: $f = \dot{m}_f / \dot{m}_a$
H_u	Lower heating value of the fuel
k	Specific heat ratio: $k = c_p / c_v$
$\dot{m}_a$	air mass flow rate
$\dot{m}_f$	fuel mass flow rate
$\dot{m}_g$	exhaust gas mass flow rate
P	Pressure
r	Pressure ratio
$\dot{Q}$	Useful heat rate of the cogeneration system (production of steam)
$\tilde{R}$	Universal gas constant
T	Absolute temperature $[K]$
T_1	Compressor inlet temperature
T_3	Turbine inlet temperature (simple cycle)
t	period of operation during a year
$\dot{W}$	Net power to the load
w	Specific work, as defined by Eq. (17)
x	set of independent variables for optimization
x_i	molar fraction of species i in a mixture
Z	Annualized cost rate of a system, in $ (including capital as well as operation and maintenance expenses)

Greek letters

γ	$\gamma = (k-1)/k$
η	Efficiency
η_B	Efficiency of the combustor
η_C	Isentropic efficiency of the compressor
η_J	Efficiency of the Joule cycle
η_m	Mechanical efficiency
η_T	Isentropic efficiency of the turbine
τ_i	Temperature ratio: $\tau_i = T_i / T_1$
ϕ	Maintenance factor

Subscripts

A	Air standard gas turbine cycle
a	Air
B	Combustor

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C	Compressor
f	fuel
g	exhaust gases
I	Intercooler
T	Turbine
R	Exhaust gas boiler
X	Air preheater
0	Standard conditions: 25°C, 1.01325 bar

Superscripts

*	Optimum value

1. INTRODUCTION

It is common knowledge that the efficiency of a simple gas turbine cycle increases monotonically with the maximum cycle temperature for constant pressure ratio [1]. In order to be more specific, the efficiency of the air standard cycle (assumption of perfect gas with no change of mass flow rate due to fuel addition and no pressure losses in the ducts and the combustion chamber) is given by the equation

$$\eta_A = \frac{\eta_C \eta_T \eta_J \tau_3 - \left(r^k - 1\right)}{\eta_C \left(\tau_3 - 1\right) - \left(r^k - 1\right)} \tag{1}$$

where

$$\eta_J = 1 - \frac{1}{r^k} \tag{2}$$

is the Joule cycle efficiency, i.e. the ideal cycle with isentropic compression and expansion and no losses. Starting with Eq. (1) it is easily proved that, if the turbine temperature is increased, keeping the pressure ratio constant, the thermal efficiency of the air standard cycle increases continuously and asymptotically it reaches the limit:

$$\lim_{\tau_3 \to \infty} \eta_A = \eta_T \eta_J \tag{3}$$

It was tacitly assumed that the general trend was the same even if a change of specific heat capacity or of the mass flow rate due to fuel addition were considered. Thus, it was a surprise to read in Ref. [2] that, if the assumption of a working substance of constant quality and quantity is relaxed, then the behavior changes drastically: for a constant pressure ratio, the efficiency initially increases with the turbine-inlet temperature, it reaches a maximum value and then it decreases. Detailed studies of these effects appear in Refs. [3]-[5]. This remark prompted the investigation reported here.

Many publications on optimization of gas turbine cycles, including Refs. [6]-[9], are based on the assumption of perfect gas with different values for the specific heats of air and exhaust gases, in order to decrease the inaccuracy. After the aforementioned, the question arises: "how is the optimum point affected if the properties of gases are evaluated with a higher accuracy?" An answer to this question is attempted in the following, using as examples three different system configurations.

2. EVALUATION OF GAS PROPERTIES

A clarification of terminology is useful at this point. The specific heat capacities of a *real gas* are functions of both temperature and pressure:

$$c_p = c_p(p, T), \qquad c_v = c_v(p, T) \tag{4}$$

For an *ideal gas*, they are functions of the temperature only:

$$c_p = c_p(T), \qquad c_v = c_v(T) \tag{5}$$

For a *perfect gas*, they are constant:

$$c_p = \text{const.}, \qquad c_v = \text{const.} \tag{6}$$

This 'textbook material' is repeated here because it is often written in related publications that 'real-gas' effects are studied, while in fact the gases are considered ideal. Thus, the reader should be careful.

In the present work, it is considered that air consisting of N_2, O_2, CO_2 and H_2O is compressed and then it reacts with a fuel having the general composition $C_\alpha H_\beta$ in a complete combustion to produce exhaust gases consisting of N_2, O_2, CO_2 and H_2O. For simplicity, minor constituents (such as CO, NO_x, etc) due to additional reactions, dissociation, impurities or other reasons are not considered here. The properties of each species are evaluated by the following equations obtained from Refs. [10] and [11]:

$$\frac{\tilde{c}_{p0}}{\tilde{R}} = a_1 \cdot T^{-2} + a_2 \cdot T^{-1} + a_3 + a_4 \cdot T + a_5 \cdot T^2 + a_6 \cdot T^3 + a_7 \cdot T^4 \tag{7}$$

$$\frac{\tilde{h}_0}{\tilde{R}} = -a_1 \cdot T^{-1} + a_2 \cdot \ln(T) + a_3 \cdot T + \frac{1}{2} a_4 \cdot T^2 + + \frac{1}{3} a_5 \cdot T^3 + \frac{1}{4} a_6 \cdot T^4 + \frac{1}{5} a_7 \cdot T^5 + b_1 \quad (8)$$

$$\frac{\tilde{s}_0}{\tilde{R}} = -\frac{1}{2} \cdot a_1 \cdot T^{-2} - a_2 \cdot T^{-1} + a_3 \cdot \ln(T) + + a_4 \cdot T + \frac{1}{2} \cdot a_5 \cdot T^2 + \frac{1}{3} \cdot a_6 \cdot T^3 + + \frac{1}{4} \cdot a_7 \cdot T^4 + b_2 - \ln(P) \quad (9)$$

The numerical values of the parameters a_i depend on the species and the temperature range, and they are given in Ref. [11].

The properties of air and exhaust gases are evaluated with the assumption that they are ideal mixtures; for example the molar heat capacity is calculated by the equation

$$\tilde{c}_p = \sum_i x_i \tilde{c}_{pi} \quad (10)$$

Thus, the perfect gas assumption of previous works has been replaced here with the ideal gas assumption. The effect of pressure is still considered negligible for the pressure ranges used in the systems that are studied here, as justified by values obtained for the compressibility factor.

3. SYSTEMS STUDIED

3.1 Description of the Systems

Three systems have been selected in order to study the effect of the method used for property evaluation on the optimal design point.

System I consists of a simple, open-cycle gas turbine (Figure 1). An approach for its thermodynamic and thermoeconomic optimization based on the perfect gas assumption has been presented in Refs. [6] and [7].

System II is a cogeneration plant consisting of a regenerative gas turbine with an exhaust gas boiler producing saturated steam (Figure 2) of a given quality and quantity; it is the system of the CGAM problem [8,9].

System III is an inter-cooled, regenerative gas turbine with a twin spool gas generator and a power turbine (Figure 3).

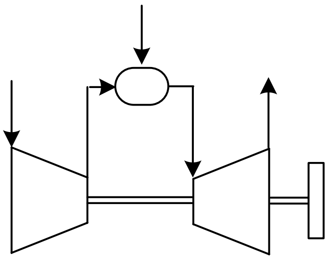

Figure 1: System I.

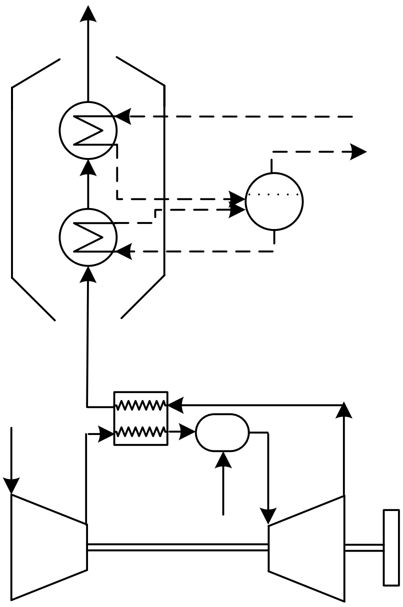

Figure 2: System II.

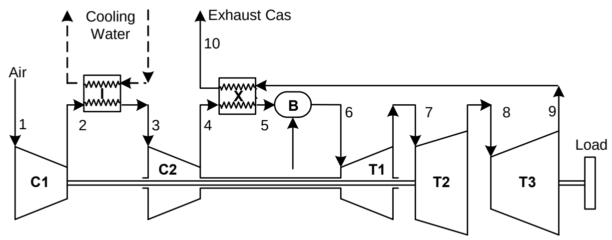

Figure 3: System III.

3.2 Mathematical Models of the Systems

Thermodynamic model of System I

The air temperature at the exit of the compressor and the exhaust gas temperature at the exit of the turbine are evaluated by the equations

$$T_2 = T_1 \cdot \left[1 + \left(r_C^{\frac{k_{12}-1}{k_{12}}} - 1 \right) \cdot \frac{1}{\eta_C} \right] \quad (11)$$

$$T_4 = T_3 \cdot \left[1 - \left(1 - r_T^{\frac{1-k_{34}}{k_{34}}} \right) \cdot \eta_T \right] \quad (12)$$

where

$$k_{ij} = \frac{\tilde{c}_{pij}}{\tilde{c}_{pij} - \tilde{R}} \quad (13)$$

$$\tilde{c}_{pij} = \frac{1}{T_j - T_i} \int_{T_i}^{T_j} \tilde{c}_p(T)\,dT \quad (14)$$

Since k_{12} and k_{34} depend on the temperatures T_2 and T_4, respectively, Eqs. (11) and (12) are used in an iterative procedure in order to obtain the temperatures T_2 and T_4.

An energy balance in the combustion chamber gives the equation

$$f H_u \eta_B = (1+f)(h_3 - h_0) - (h_2 - h_0) \quad (15)$$

which can be solved for f, if the temperature T_3 is given. The composition of the exhaust gases is determined by the reaction of combustion for any specified fuel. Since the temperature T_3 and the composition of exhaust gases are interrelated through the temperature-dependent properties of the constituents, an iterative procedure is applied also here.

The system efficiency is given by the equation

$$\eta_I = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_3 - h_4) - (h_2 - h_1)}{f H_u} \quad (16)$$

The specific work is also of interest:

$$w = \frac{\dot{W}}{\dot{m}_a} \quad (17)$$

Thermodynamic model of System II

Equations (11)-(15) and (17) are valid for System II, with a proper adjustment of certain numerical indexes. The effectiveness of the air pre-heater is given by the equation

$$\varepsilon_X = \frac{h_3 - h_2}{h_{a5} - h_2} \quad (18)$$

The subscript 'a5' is used in Eq. (18) in order to make it clear that h_{a5} is the enthalpy of air at temperature T_5. There is no ambiguity about h_2 and h_3. The following efficiencies are defined for this system.

Net shaft-power efficiency:

$$\eta_{II} = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_4 - h_5) - (h_2 - h_1)}{f H_u} \quad (19)$$

Efficiency of providing the useful heat:

$$\eta_{II,Q} = \frac{\dot{Q}}{\dot{m}_f H_u} = \frac{\dot{m}_s (h_9 - h_8)}{\dot{m}_f H_u} \quad (20)$$

Total efficiency:

$$\eta_{II,tot} = \eta_{II} + \eta_{II,Q} = \frac{\dot{W} + \dot{Q}}{\dot{m}_f H_u} \quad (21)$$

The model of System II consists of many more equations, which are given in Ref. [9], but they are not repeated here due to space limitations.

Thermodynamic model of System III

For the compression, combustion and expansion processes, equations similar to those of the System I are used. The effectiveness of the air pre-heater is given by an equation similar to Eq. (18), with proper adjustment of the numerical indexes. In addition, the following equalities are taken into consideration:

$$\dot{W}_{C1} = \dot{W}_{T2}, \qquad \dot{W}_{C2} = \dot{W}_{T1} \quad (22)$$

The division of the pressure ratio between the low-pressure and high-pressure spool is determined by an iterative procedure so that Eqs. (22) are satisfied.

The system efficiency is given by the equation

$$\eta_{III} = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_8 - h_9)}{f H_u} \quad (23)$$

The specific work is given by Eq. (17).

Thermoeconomic models of the systems

The cost functions for Systems I and II appear in Refs. [6]-[9]. For System III, equations available for Systems I and II have been properly modified and used. Space limitations do not allow giving the complete set of equations here.

4. PERFORMANCE OF SYSTEMS WITH ALTERNATIVE METHODS FOR EVALUATION OF PROPERTIES

For the performance evaluation and for the optimization of the systems, certain values have been considered for the various parameters involved, which are given in Table 1.

Table 1. Values of parameters.

System I	System II	System III
$r_B = 0.975$	$r_B = 0.975$	$r_{C1} = r_{C2}$
$\eta_B = 0.99$	$\eta_B = 0.99$	$r_I = 0.98$
$\eta_m = 0.99$	$r_{Xa} = 0.975$	$T_3 = T_1$
	$r_{Xg} = 0.965$	$r_B = 0.975$
Air	$\Delta T_{X\min} = 20\,K$	$\eta_B = 0.99$
N_2: 77.82%	$\eta_m = 0.99$	$r_{Xa} = 0.975$
O_2: 20.68%	$r_R = 0.95$	$r_{Xg} = 0.965$
CO_2: 0.03%	$\Delta T_{p\min} = 15\,K$	$\Delta T_{X\min} = 20\,K$
H_2O: 1.47%	$T_{7\min} = 373.15\,K$	$\eta_m = 0.99$
$T_{amb} = T_0$=25°C	$\dot{m}_{st} = 14\ kg/s$	$r_R = 0.95$
$P_{amb} = P_0$	$P_8 = 20\,bar$	
= 1.01325 bar	$T_8 = 298.15\,K$	
Fuel: CH_4	$T_{8P} = T_9 - 15\,K$	
$H_u = 50000 \frac{kJ}{kg}$	$P_9 = 20\,bar\,(sat.)$	

The first step in this investigation has been the study of the effect of properties evaluation on the simple cycle efficiency (System I). The results depicted in Figure 4 are revealing: the perfect gas assumption, as expected, gives an efficiency continuously increasing with the turbine inlet temperature. With the ideal gas assumption and properties evaluated by Eqs. (7)-(10), the efficiency exhibits a maximum at a temperature of about 1600 K (for the parameter values considered here). Thus, the related results of Ref. [5] are reproduced to a very close approximation (small differences are due to different values of parameters and to the different sources of equations for evaluation of gas properties).

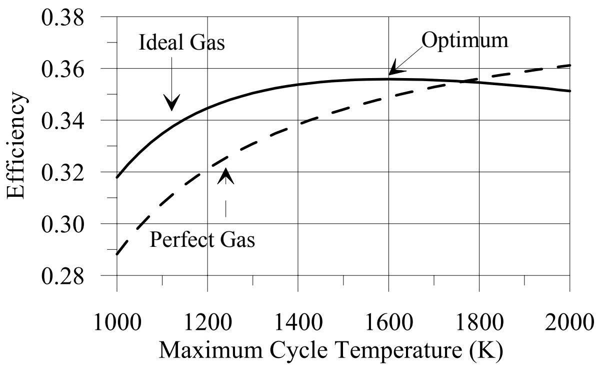

Figure 4: Efficiency of System I as a function of turbine inlet temperature with two gas models: perfect gas, ideal gas; η_C =0.90, η_T =0.92, r=10.

With the perfect gas model, the specific work increases continuously with turbine inlet temperature for a certain pressure ratio. This trend remains the same with the ideal gas model too.
In Figures 5 and 6, the effect of the gas model assumption on the system efficiency and specific work as functions of pressure ratio is shown. It is clarified that the graphs of Figures 5B and 6B correspond to the system of Figure 2 but without the exhaust gas boiler.

The coordinates of the optimum points in Figures 5 and 6 are given in Tables 2 and 3, respectively.

A change from perfect to ideal gas model changes the pressure ratio for maximum efficiency by +37.04%, –14.29% and –15.38% for the simple cycle, the regenerative cycle and the intercooled-regererenerative cycle, respectively. The optimum efficiency increases by 11.11%, 13.08% and 10.10%, respectively.

A change from perfect to ideal gas model increases the pressure ratio for maximum specific work by 16.67%, 28.57% and 50.00% for the simple cycle, the regenerative cycle and the intercooled-regererenerative cycle, respectively. The optimum specific work increases by 15.49%, 13.25% and 15.85%, respectively.

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$$\mathbf{x}_{I,\eta} = (r, \tau_3) \quad (31)$$

The solution of this problem gives extremely and unrealistically high optimum values r* and τ_3^* for the values of η_C and η_T considered here. This is why only the pressure ratio has been considered as independent variable, while τ_3 is a parameter.

The effectiveness ε_X of the air pre-heater could be an independent variable for problems (a) and (b) of System II also. In such a case, maximization of the specific work would result in elimination of the air-pre-heater, changing the structure of the system. In order to keep the structure the same as in the CGAM problem, it was decided to treat ε_X as a fixed parameter for these problems.

It is interesting to note that, as we go from the simple cycle to the intercooled regenerative cycle, the optimum value of the annualized cost rate decreases by 25.22% (from $9.914 \cdot 10^6$\$ to $7.417 \cdot 10^6$\$), in spite of the fact that the system becomes more complex. The most important reasons for this decrease are the significant decrease of the pressure ratio (which decreases the capital cost of certain components) and the significant increase of the system efficiency (which decreases the fuel cost).

CONCLUSION

A preliminary performance evaluation followed by the solution of three optimization problems for each one of three different gas turbine system configurations has demonstrated that a change from the perfect gas to ideal gas model for evaluation of properties has a very significant effect on the results, which cannot be ignored. With the computing capabilities of today, the necessary calculations are conveniently performed.

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