STAT321, AN ONLINE STATISTICS TEXTBOOK BASED ON INTUITIVE INFERENTIAL REASONING

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Research in human cognition has shown that people possess intuitive notions of statistics, but may fail to generate correct judgments in a number of situations. These characteristics of the human mind can be used to introduce statistical concepts and warn against misconceptions by showing parallels and divergences between proper statistical thinking and intuition. To date, this approach has been little investigated resulting in a lack of resources for the interested teacher. Here, I present Stat321, a project of an atypical online statistics textbook which uses the naive statistical thinking of the reader to explain the what, why and how of Statistics. Stat321 is designed to evolve with comments, ideas and the advances in statistics education research. Initial feedback from undergraduate students is discussed.

INTRODUCTION

In cognitive science, one of the leading theories for brain mechanism considers the human brain as a sort of computational probability machine which constantly performs predictions about the world and updates it with what its body senses (Friston, 2010). Such a mechanism would imply that human intuitive reasoning is statistical by nature and may therefore present some similarities with proper statistical thinking.

This hypothesis is supported by many studies showing that people, including children, intuitively perform on a regular basis all sorts of statistical computations (Ariely, 2001; Chong & Treisman, 2003; Cosmides & Tooby, 1996; Griffiths & Tenenbaum, 2006; Téglás et al., 2011).

It is therefore reasonable to try to identify parallels between statistical concepts or methods and our everyday thinking. Showing these similarities, but also the eventual divergences could greatly help teachers introducing statistical concepts and revealing common misconceptions.

OWN EXPERIENCE AND EMERGENCE OF THE PROJECT

Revealing Student Statistical Intuition

To give a concrete example, on my very first undergraduate statistics class at INSA Lyon, I gave my students (who did not have had any statistics class before) a little challenge. Here is more or less what I told them :

> *This morning, when I arrived at work, the first thing I did was to take a piece of paper and to let a pen fall on it* (cf. Figure 1). *The result I got on the page looked like the first image on the whiteboard* (cf. Figure 2-a). *Do you think I am lying ? I then did it a second time and got something close to the second image on the whiteboard* (cf. Figure 2-b). *Do you think I am lying for this one?*
>
> *Now here is my challenge. You need to think carefully at both questions and after discussion, to all agree for an answer to each question. Your possible responses are "I am lying", "I am saying the truth" or "We don't know".*

To spice up the challenge, I also told them that for each answer "I am lying" or "I am saying the truth", they would all win one point on their course grade if they were right and they would all lose two points if they were wrong, the answer "We don't know" having no effect on their grade.

The students started to discuss. Quickly, one student spoke up and said that she just had tried with her pen to do the same thing I did and she affirmed that the first pattern (cf. figure 2-a) was not possible. Another student added that they could all try the little experiment and see if someone could get a result similar to the first pattern. Not all tried it, but some did. This shows that some students intuitively get this very statistics idea of simulating the distribution of the possible results under the hypothesis that I was not lying.

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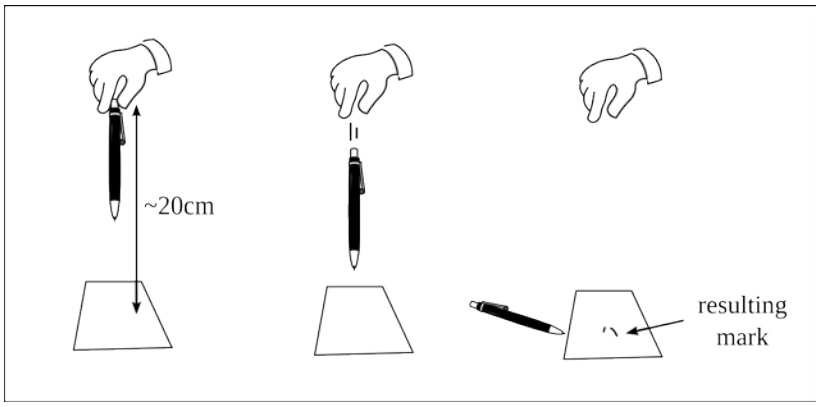

Figure 1

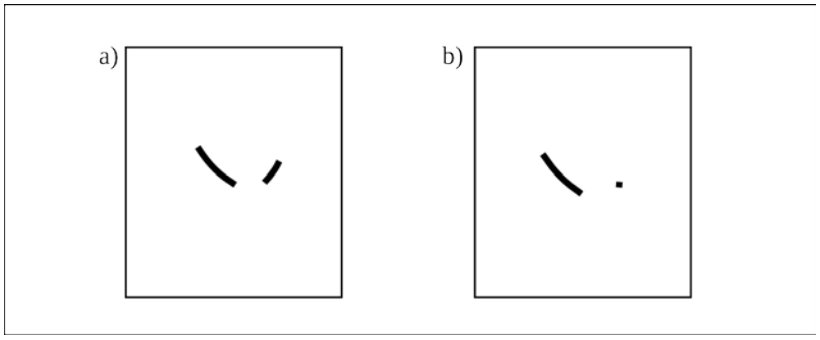

Figure 2

Some students asked me if the piece of paper was horizontal, what pen I used, from what height I let the pen fall or if there were already some marks on the paper. I answered their questions and showed them the pen. Some of them tried again the experiment with this pen. This shows that they are intuitively concerned with the way the data was collected and they try to make sure they are getting the right null distribution from their simulations (or from their thinking for those who do not try it themselves).

When they were trying to agree on an answer, one student went to the whiteboard and wrote the different combinations of answers, with the possible consequences in term of points. One student said it was best to answer "I don't know" to be sure not to lose any points. Another one said they should take the risk. This shows that students also get naturally the idea of considering the different risks when making their choices.

And it seems that for the first question, they considered the risk of losing being low enough so that they chose to answer "I'm lying". For the second question, one student said that it was probable that I was telling the truth, but I could as well have said a lie and they finally answered "We don't know".

Overall, their reasoning was very similar - although less rigorous - to the statistical thinking of a statistician when performing a statistical test of hypothesis. In fact, I did lie for both questions and they all won one point on their final grade.

The Importance Given to Statistical Intuition in the Research on Statistics Education

In cognitive science, it is a rather common idea to relate people's natural intuition to Statistics (Tenenbaum, Kemp, Griffiths, & Goodman, 2011). It does not seem to be the case however in the statistics education community, where it is surprising to see so many articles related to student misconceptions (Cooper & Shore, 2008; Sotos, Vanhoof, Noorthate, & Onghena, 2009; Watier, Lamontagne, & Chartier, 2011) and so little talking about students' powerful statistical intuition. This asymmetry may find its origin in the widely spread work of Kahneman and Tversky on people's judgment under uncertainty for which Kahneman received the Nobel Prize in

Economics in 2002. Indeed, many articles dealing with statistics education cite their famous 1974 science article " Judgment under Uncertainty: Heuristics and Biases " (Tversky & Kahneman, 1974) when talking about student misconceptions, but I haven't seen any citation of the articles cited in the introduction. Ironically, Kahneman himself agrees that humans compute naturally and efficiently several statistical properties in various everyday situations (Kahneman, 2003).

Despite the fact that part of the Statistics education community may not be aware of some striking human cognition results, it seems that recently there has been an increasing interest on the potentialities of students' intuitive thinking, called Informal Inferential Reasoning (IIR), for conveying certain statistical concepts (see for example the special issue on IIR in the volume 7, number 2 of the Journal of Statistics Education). However, this interest seems for the moment limited to basic notions and there is still a lack of resources for the teacher interested in using the approach.

That's why the project of an online textbook based on people's everyday thinking called Stat321 was launched.

PHILOSOPHY, ORGANISATION OF STAT321 TEXTBOOK

The Content

The hypothesis behind this project is that any statistical concept can somehow be related to the way we normally think in our everyday life. Also the idea is not to make another practical statistics textbook full of examples and exercises for learning how and when to use one statistical method or another, but to expose all the statistics theory in a manner that would be intuitive to the reader. The curriculum is therefore similar to those of statistical theory textbook, but with barely any math and a different organization.

The Curriculum Organization

The course organization is not intended to be fixed. However, the current idea is to avoid starting from probability or descriptive statistics as it is usually the case in most statistics textbooks. Instead, after a small introduction, the course quickly goes to the ideas behind hypothesis testing. The rationale is that probability or descriptive statistics may not appear very useful for students at the beginning, which is a problem in a course intended to be intuitive. Hypothesis testing on the contrary is a powerful and engaging concept that can be of direct utility for students.

The Approach

Stat321 always tries to relate the different statistical concepts to people's everyday intuition. Also, instead of using the traditional "Here is the theory / Here is what you have to do", the approach is more about a "Here is what you intuitively do / Here is what Statisticians do" and then discuss the differences. This way, it is expected that the reader will not only learn the different statistical concepts and techniques, but may also be tempted to apply them on real life while being aware that they may lead to erroneous conclusions on certain situations.

The Tools

One big advantage online textbooks have over classical paper ones is the possibility to integrate multimedia or interactive content. This can be of great help to make parallels between everyday intuition and proper statistical thinking.

To give an example, consider the scene in the French movie "La chèvre" where the detective Campana (played by Gérard Depardieu) set up a little experiment to check if his colleague François Perrin (Pierre Richard) would not be unusually unlucky. While Perrin is in the bathroom, Campana places in front of Perrin's plate two salt cellars, one closed and one slightly opened so that all the salt would directly fall on the plate if Perrin had to use it. When Perrin comes back, he takes the good salt cellar, but has to give it to a man asking for salt and he is finally stuck with the bad one.

When introducing students to experimental designs, the scene represents well the kind of "bad" experiment a normal person would naturally set up. We can then compare it with a somewhat similar but much more carefully designed experiment designed by statistician Ronald Fisher: the

famous "lady tasting tea" experiment where Fisher tried to test if a lady could really tell milk or tea had been poured first in a cup (Fisher, 1935).

The Look and Feel

Stat321 aims to participate in the enhancement of statistical literacy among the general population. As such, Stat321 needs to be attractive and there is an important effort deployed to make it modern, pleasant, good looking and to enhance curiosity through the use of hot topics, movies extracts or computer games.

The Future

Another big advantage of an online textbook is that we can easily make it evolve. This is something essential for a textbook based on people's intuitive thinking. Even if most of the current content has been positively tested on students, it still needs to adapt to people's reactions and should try to integrate new ideas taken from the statistical education community. For all these reasons, Stat321 is designed to easily evolve. It allows the modification, addition or removal of content and will integrate a system of comments and rating to facilitate the collect of people's reactions.

CONCLUSION

Stat321 is a new kind of statistics textbook. It aims to explain statistics theory through intuition rather than with mathematical formulas. In order to achieve this goal Stat321 presents several specificities that differ from the usual statistics textbooks: It starts from hypothesis testing rather than probability or descriptive statistics, it contains very few mathematics, it uses multimedia contents, tries to interact with its reader and is meant to evolve with people's reaction and our knowledge of what works and what does not.

In order to help in the latter, please do not hesitate to express your ideas, doubts, suggestions, opinions, comments, advices, interrogations, propositions...

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