A Hybrid Sensitivity Analysis for Use in Early Design

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Sensitivity analyses are frequently used during the design of engineering systems to qualify and quantify the effect of parametric variation in the performance of a system. Two primary types of sensitivity analyses are generally used: local and global. Local analyses, generally involving derivative-based measures, have a significantly lower computational burden than global analyses but only provide measures of sensitivity around a nominal point. Global analyses, generally performed with a Monte Carlo sampling approach, and variation-based measures provide a complete description of sensitivity but incur a large computational burden and require information regarding the distributions of the design parameters in a concept. Local analyses are generally suited to the early stages of design when parametric information is limited, and a large number of concepts must be evaluated (necessitating a light computational burden). Global analyses are more suited to the later stages of design when more information about parametric distributions is available and fewer concepts are under consideration. Current derivative-based local approaches provide a different and incompatible set of measures than a global variation-based analysis. This makes a direct comparison of local to global measures ill posed. To reconcile local and global sensitivity analyses, a hybrid local variation-based sensitivity (HyVar) approach is presented. This approach has a similar computational burden to a local approach but produces measures or percentage contributions. The HyVar approach is directly comparable to global variation-based approaches. In this paper, the HyVar sensitivity analysis method is developed in the context of a functional based behavioral modeling framework. An example application of the method is presented along with a summary of results produced from a more comprehensive example. [DOI: 10.1115/1.4001408]

1 Introduction

Investigating the sensitivity of concept performance with respect to variation is an important task during the design of systems. High sensitivity to parameters that are noisy can lead to poor or unexpected performance. However, parameters with high sensitivities can also be used as tuning variables as changing them result in significant performance changes. As a result, knowledge of the relative sensitivity of design parameters can be as important as performance predictions when evaluating and selecting concepts during the early stages of the design process. Having sensitivity information during the conceptual design stage enables the designers to make better concept identification and selection decisions.

To quantify and mitigate (for noise) or utilize (for tuning) sensitivity during the design process, resources should be allocated to accurately identify and model the impact of sensitivity. Identifying sensitivity effects as early as conceptual design allows better resource allocation throughout the entire process. However, during the conceptual design of a system, little is known about the potential physical forms of the solution. Without comprehensive form and parameter information, it is difficult to define performance models and the probability distributions for the relevant design parameters.

Another challenge encountered in performing sensitivity analyses on models of complex systems is the computational burden. In general, local sensitivity analyses require at least two model evaluations per design variable in the model. In contrast, global analyses using Monte Carlo methods typically require large numbers of evaluations ($\geqslant 1000$) per design variable.

In this work, it is proposed that the task of performing a sensitivity analysis during the design of a system be decomposed into two phases: a local sensitivity analysis for screening a large number of concepts during conceptual design and a global sensitivity analysis performed during the later stages of design. The approach is recommended due to the inherent differences in the information required to make decisions in early design versus the later stages of design. To facilitate the transition from a local analysis to a global analysis, it is recommended that a sensitivity analysis method that produces comparable results be used in each design stage.

Typically, a local sensitivity analysis involves the use of derivative-based methods. For global analyses, variation-based methods are generally suggested [1–3]. In this work, a simplified local method similar to a variation-based approach is developed that allows a transition from a local analysis of several concepts to a global analysis of a few. Since the problem of efficiently performing global variation-based sensitivity analyses has been well researched, most of the work presented here focuses on using a variation like local method during the analysis of multiple conceptual solutions of a design problem.

A key contribution of this work includes the formal development of a sensitivity analysis method that integrates with a function-based design and modeling approach. Thus, the method can be used during conceptual design and other initial design tasks. The method developed here allows designers to explore system sensitivity as it relates to specific design parameters as well as system subfunctions via new quantitative sensitivity measures. Additionally, the method developed combines the positive features of global and local sensitivity analysis methods.

Results are presented in six sections. Section 2 outlines modern theories on the design process of systems and the role of behavioral modeling in these design processes. Section 3 details current methods for performing sensitivity analyses in design. Section 4 proposes a new hybrid local sensitivity analysis derived from global variation-based methods. In Sec. 5, an example is presented that demonstrates how to apply the hybrid method to a functionally decomposed behavioral model. Section 6 briefly summarizes a more complete example of the proposed sensitivity analysis as

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performed on a hybrid racecar powertrain. Finally, conclusions and extensions of the work are presented in Sec. 7.

2 The Design of Systems

Two broad schools of formal design practice for systems exist: the "European," or traditional school of design, and the "systems engineering" approach. The European, or traditional approach to design is well summarized by the works of Pahl et al. [4]. They defined the design process for a system in four basic phases: (1) product planning, (2) conceptual design, (3) embodiment design, and (4) detailed design. Other researches into design methodologies within this traditional design approach involve more or less the same four steps [5–7]. Some of these methodologies move the boundaries of the steps around and regroup the basic activities differently but in total represent the same basic sequence of events. (1) The first phase of design is to gather information about the system's user and their needs, and to map this information to the highest level functionality of the system. (2) The required functionality is explored in detail, and potential solutions for this functionality are created and evaluated. (3) After selecting a concept for production, the physical requirements for the concept are defined along with solutions for the product's auxiliary functionality. The overall performance of the concept relative to the user's needs is then assessed. (4) A complete description of the system is made to allow the product to be manufactured.

A full account of the systems engineering method of design can be found in Ref. [8]. Systems engineering focuses more on management and information control during the design process for large systems. It highly emphasizes the understanding and development of requirements. The classifications used in systems engineering, i.e., formulation, analysis, and interpretation, can be mapped to the first two phases in the traditional method of engineering design [8]. Formulation, which includes problem definition, value system design, and system synthesis, straddles the product planning and conceptual design phases of the traditional design method. The problem definition and value system design activities in systems engineering map to activities in the product planning and system synthesis phases of the conceptual design. The analysis classification includes system analysis and modeling along with refinement of alternatives. These activities are followed by an interpretation that includes decision making and planning for action. The activities that occur during the analysis and interpretation classification stages in systems engineering map to activities that occur during conceptual design in the traditional engineering design method.

In summary, both schools of engineering design promote the same basic order of operations during the initial stages of system design: determine what needs to be done at a functional level, find solutions that can potentially accomplish this functionality, compare the solutions through the use of models, and make a decision about which solutions to investigate further.

2.1 Functional Modeling During Design. The functional decomposition of a complex design problem is promoted in both traditional engineering design [4] and systems engineering [8]. A functional model is a graphical depiction of a detailed product functionality. Functional models include functions, generally represented as verbs, which describe the desired transformations of flows, which are generally described using nouns. The process for creating a functional model generally involves the following basic steps:

1. Create a black-box model that includes the overall functionality of the product along with external flows.
2. For each input flow in the black-box model, identify the sequence of functional transformations that are required to produce one or more of the output flows.
3. Aggregate these function sequences into a complete functional model for the product.
4. Assess the model's coverage of customer needs and system requirements, add functions/flows or decompose as required.

A range of reasons for creating a functional model during product design are detailed by Otto and Wood [9]. In general, the primary reason is to create a solution-neutral method of representing what a product needs to do without assuming how it is going to do it. This mapping of what to how represents the remainder of the conceptual design process. Several methods exist for performing this decomposition such as formal methods [5] and the function-flow block diagram (FFBD) approach as used in systems engineering [8,10].

The functional analysis approach used in this work evolved within the traditional engineering design approach beginning in the early 1960s [11]. The functional analysis approach employed here focuses on the use of verb-noun pairs along with an explicit breakdown of flows into energies, materials, and signals [12]. This flow-based approach has been extensively and significantly formalized in the works of Hirtz et al. [13]. The benefits of this approach include the use of a standard modeling language (the functional basis [13]) and an emphasis on the energy-, material-, and flow-based identifications of functions. A complete survey of this functional analysis approach appears in the works of Nagel et al. [14]. For the remainder of this work, a functional model will refer to a functional decomposition of a system using the flow-based methodology and the functional basis lexicon approach [15].

The functional decomposition of a system allows a complex design problem to be broken down into smaller elements based on those elements' intended functionality. These elements can then be analyzed in detail individually. In most formal system design methods, the functional decomposition is also recommended as the starting point for identifying potential solutions to the design problem. The functional decomposition decouples the task of representing what a system needs to do from how it is going to do it [3,4,6,16]. Thus, the desired functionality of a system can be described before actual solutions are identified. From morphological charts [5] to current knowledge driven concept generation algorithms [17], significant research has been conducted into expanding the ability to explore the solution space for a problem by using a function-based solution identification method.

Functional modeling provides the basic framework for the sensitivity analysis method developed in this article. The use of functional modeling has several positive contributions. One, functional modeling is used early in design. Thus, using a functional model as a framework for sensitivity analysis allows a straightforward transition from early conceptual design to concept selection and refinement. Additionally, using functional modeling as a framework allows a sensitivity exploration to be performed on a function-by-function basis. This is important as designers explore potential sources of noise or design control. In general, functional modeling has proven as a useful framework for approaching various design challenges including the creation of formal methods applicable during conceptual design [18–21]

2.2 Behavioral Modeling During Design. A behavioral model is a quantitative representation of a system, or a specific aspect of that system. Such models can be used to prescribe performance targets early in the design process and can predict the performance of systems relative to these targets later in the design process. Two general approaches for creating such models are the abstraction and component-based approaches.

In the abstraction approach, a behavior of interest is identified, and a model is created based on an abstraction of the system that exhibits the behavior of interest [22–24]. For example, if the performance of an internal combustion engine needs to be modeled, an abstracted model of the engine may be created by using an approximation of the combustion processes or results from dynamometer testing. Such a model represents a parameterized abstraction of the behavior of the system. The component-based ap-

Table 1 Characterization of sensitivity analysis methods

Characteristic	Method LD	ND	MCR	VB	SMF
Computationally efficient	X	X			X
Works directly on model	X	X	X	X	
Does not require additional information	X				
Dimensionless		X	X	X	X
Contributive measure			X	X	X
Local/global	L	L	G	G	G

proach for modeling the same system would be to model the constitutive elements of the engine separately and then combine them (generally through an automated or semi-automated process) to produce a complete system model.

Both approaches have strengths and weaknesses. Abstract models are generally more focused on the behavior of interest while component-based models are generally more closely associated with the system itself. As a result, an abstract model may provide a good description of the behavior of interest but may be of little use if the system changes or if the assumptions made about the system during the abstraction process are faulty. Conversely, a component-based model can provide a variety of information about a system but may not able to predict a specific aspect of its behavior as well as a model abstracted solely for that purpose. Essentially, these models answer two separate questions. Abstract models answer the question of *what* a system does, and component-based models answer the questions of *how* a system functions through the action of its constitutive elements.

The abstraction method of modeling systems is generally the method taught in engineering educations. Traditional engineering classes (physics, thermodynamics, etc.) focus on modeling systems by creating a set of equations or relationships that represent an abstracted behavior of a complete system. In general, the result of this type of modeling process is a set of algebraic or differential equations, which are then used to investigate some aspects of the system's performance. This method of modeling provides good insight into what a system does by providing elegant analytical equations but is generally limited to small systems that exhibit rather simple behavior (when compared with larger, more complex systems).

In contrast, generic component-based system modeling platforms such as SIMULINK [25], DYMOLA [26], and bond graph based applications [27] have been developed as a means to model complex systems across multiple domains. The DYMOLA or MODELICA approaches have roots in the object-oriented modeling approach developed by Elmqvist [16], while bond graphs originated from Paynter [28]. In these approaches, the behavior of a component of the system is modeled independently from the other components of system, and then these model elements are automatically assembled to produce a complete system model. Such an approach is useful for large complex systems and does a good job in modeling how system behavior results from the behavior of its constitutive elements.

3 Sensitivity Analyses in Engineering Design

Sensitivity analysis is the study of how system input variation creates system output variation. Frequently, sensitivity analysis is focused only on the model of a real system [2]. Such analyses can be qualitative or quantitative. A qualitative analysis identifies the relative importance of various design parameters to the overall sensitivity of the model. A quantitative analysis provides numerical measures of how sensitive a model is to variation in the design parameters.

Sensitivity analyses are used in a variety of fields in addition to engineering including the economic [29], environmental [30], and scientific [2] industries. As applied in engineering design, sensitivity analysis is often used to determine configuration robustness with respect to noise, determine parameters that provide design control, or perform system or component level tolerance analysis, allocation, and design.

System sensitivity is important for various reasons during conceptual design, particularly concept selection and performance prediction. In cases in which concept robustness (with respect to noise) is an important criterion, sensitivity analysis is needed to compare the potential concepts. In addition, sensitivity analysis is used to determine elements within the system that provide for significant design control. When evaluating elements of a system, sensitivity analysis is used to determine the relative contribution of each variable to the system's sensitivity, or, in the case of a functional decomposition of the system, the contribution of each function to overall system sensitivity.

There are multiple methods available for sensitivity analysis. A list of methods currently used follows along with a characterization of these methods in Table 1.

Local derivative (LD) is a sensitivity analysis approach characterized by the use of the local partial derivative of an output variable in response to an input variable of choice. The resulting measure carries units of the output variable divided by the input variable and does not require knowledge of the input distribution or estimation of the output distribution. Such approaches typically use two evaluations of the model for each parameter (at perturbed high/low values) to produce a second-order estimate of the local derivative [1].

Normalized derivative (ND) is a nondimensional normalization of a local derivative measure to the standard deviation of the input and output variables (other normalizations are sometimes used). This normalization requires knowledge of the input variable's distribution and an estimation of the output variable deviation [1].

Monte Carlo regression (MCR) is a linear model fit to the results of a Monte Carlo simulation of the system. This approach requires comprehensive knowledge of the input variable distributions. MCR analysis is more computationally complex than a local derivative approach as it requires numerous evaluations of the model ($\geqslant 1000$) for each parameterization. The resulting measures provide a global breakdown of the sensitivity contribution [1].

Variance-based (VB) is a sensitivity analysis performed using estimates of model variance and parametric contributions to variance. VB sensitivity analysis is analogous to MCR under certain conditions and has similar characteristics [1].

Simplified model fit (SMF) is a computationally efficient model (such as a Kriging model [31]) is fit to the system performance model. A sensitivity analysis is then performed on the resulting model. This approach shares characteristics of the variation-based analysis approach used but with a decrease in the computational burden at the expense of an increase in the model uncertainty (from the model fitting process). In certain cases, analytical solutions are possible [3].

During the design of engineering systems, several considerations must be made when selecting a sensitivity analysis, as follows:

- local information versus global information [29,32,33]

- computational burden [31,32,34]
- knowledge of design parameter distribution and distribution parameters
- usefulness of resulting knowledge
- modeling requirements

During conceptual design, limited information is available about potential concepts. Information limitations may prevent detailed knowledge of parametric distributions. As a result, it may not be possible to perform a full variation-based sensitivity analysis for concepts. Additionally, such analyses are computationally expensive (even with the more efficient approaches discussed in Refs. [3,32]). Local measures of sensitivity (derivative-based approaches) provide quantitative measures of each variable's effect on the system's performance at a single nominal operating point but cannot be used in a contributive manner or provide an overall sensitivity magnitude such as a variation-based approach. However, local measures do not require specific information about parametric distributions. Local measures are generally much faster to evaluate computationally (typically, two performance evaluations are required for each variable considered in the sensitivity analysis). Since the results of a typical local analysis are derivatives (normalized or not), and the result of a variation-based analysis is a set of effect contribution percentages and an overall variance, the results of the two analyses are not comparable.

To reconcile the various approaches to performing a sensitivity analysis in conceptual design, a new hybrid local variation-based (HyVar) method is developed here. The HyVar approach uses similar mechanics and provides the same output information as a traditional sample-based variation-method combined with the evaluation cost of a local derivative method. The HyVar approach augments the derivative-based local approach with a variation-based measure. In the HyVar approach, derivative information, variance like sensitivity magnitude, and main effect contributions are calculated for a given parameterization of each concept. The resulting measures provide the same results of a local derivative approach along with measures similar to that of a full variation-based analysis. The analysis is still a local analysis and does not replace full variation-based global sensitivity analysis, but requires significantly less knowledge about parameter distributions. Thus, it is suitable for screening a large number of parameterized concepts during the early stages of design.

The benefit of using the HyVar approach over local derivative methods are the compatibility of the results of the HyVar analysis with a full variation-based method while retaining a computational burden similar to a local method. Due to this reduced computational burden, it is feasible to perform the analysis on a full model without using a fitting process. Additionally, since the output parameters from the hybrid analysis have the same format as the output parameters of a full variation-based approach, the result of the two analyses can be directly compared. An example of where such a comparison would be useful is between a HyVar analysis performed during conceptual design versus a full variation-based approach performed on the same model during detailed design once the parametric distributions are known.

The sensitivity measures provided by the proposed HyVar approach allow grouping of sensitivity contributions by direct addition of percentages. This is a primary reason for implementing them over a traditional derivative-based approach and allows the approach to be used in a functionality decomposed system behavior model. A derivation of these measures with an illustration of how the measures are combined to represent the sensitivity contribution of functions is presented in Sec. 4.

4 Hybrid Variation-Based Local Sensitivity Measures

In this section, the equations used to perform a HyVar sensitivity analysis are developed. These equations are consistent with those used for existing sensitivity analysis methods. However, because the method is based on a formal functional decomposition of the system, the equations are developed to index over both the input parameters and functions. Additionally, new measures are developed to determine the system sensitivity to each function and compensate for the total number of functions in the system. These new formalisms allow the designer to explore sensitivity on a function-by-function basis.

By applying the same type of calculations used in a traditional variation-based global sensitivity analysis to a local analysis, it is possible to get contribution measures of a system with the same computational burden as a traditional derivative-based measure. The HyVar approach provides contribution measures that are not found with a derivative-based analysis. The format of the HyVar measures is compatible with those of a full variation-based approach (the output of both approaches is a set of sensitivity percentages that correspond to the contribution to total variation for each parameter).

The variance in a finite population can be calculated using the formula shown in Eq. (1). In the context of the results of a system performance model, Y_i would be the model output for a particular parameterization of the model, and $\bar{Y}$ would be the mean value of the outputs of N parameterizations

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (Y_i - \bar{Y})^2 \tag{1}$$

For a complex system behavioral model decomposed based on functionality, the input parameters will be grouped according to function. In the case with multiple performance outputs, the output is a vector rather than a scalar, and the inputs to the model are a series of vectors (one for each function in the model). A mathematical representation of such a model appears in Eq. (2), where Y_i is a specific row in the performance vector and $f_i(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_i,\ldots,\boldsymbol{x}_F)$ represents an evaluation of that performance by the set of input vectors $\boldsymbol{x}_i$ associated with each function up to a number of F total functions:

$$Y_i = f_i(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_i,\ldots,\boldsymbol{x}_F) \tag{2}$$

In the context of a design problem, a concept can be thought of as a unique parameterization of a model. For each parameter in the parameter vectors for each function in a concept, a slightly higher or lower value of the parameter can be substituted for the nominal value (as is done in partial differencing in a derivative-based local sensitivity analysis). For each high and low case, the output can be compared with the nominal value of performance. This comparison can be made using the same basic calculation as the variance of a population, where the population mean is replaced with the nominally predicted performance of the concept. This comparison can be performed for each parameter in each function for each performance output. The mathematical representation of this comparison appears in Eq. (3)

$$V_{i,j,k} = \frac{(Y_{i,x_{j,k+}} - Y_i)^2 + (Y_{i,x_{j,k-}} - Y_i)^2}{2} \tag{3}$$

In this equation, the index i represents the performance output, the index j represents the function, and the index k represents the local parameter in the function j. Since two comparisons are made for each of the high and low values, the squared deviation from the nominal performance is divided by 2. The result V_{ijk} is a measure of the average squared deviation from nominal performance found from perturbing the parameter of interest. For each function, these deviations can be summed as shown in Eq. (4). In this equation, $\mathbf{P}$ is a vector that contains the number of local parameters for each function

$$V_{i,j} = \sum_{k=1}^{P_j} V_{i,j,k} \tag{4}$$

The result is a measure of the deviation in the performance produced by a single functional element of the model. These functional deviations can then be summed as well (Eq. (5)) to produce a total measure of deviation in the model, where F is the number of product functions in the model

$$V_i = \sum_{j=1}^{F} V_{i,j} \tag{5}$$

As in a variation-based sensitivity analysis, the deviations of each parameter and each function can be divided by the total model variation to produce a percentage measure of contribution to the total deviation (Eqs. (6) and (7)). The resulting measures S_{ij} and S_{ijk} represent the relative contribution to the total deviation of the performance (i) for a specific function (j) and a parameter within a function (k) represented as a percentage. The measure S_{ijk} corresponds to the main effect sensitivity contribution as calculated in a full-variance based approach. The S_{ij} measure is a newly proposed measure that represents the impact of variations contained within a single function to system variation. The approach can be applied to a system without a functional decomposition by assuming a single functional element ($F=1$) with all model parameters in that functional element

$$S_{i,j} = \frac{V_{i,j}}{V_i} \times 100 \tag{6}$$

$$S_{i,j,k} = \frac{V_{i,j,k}}{V_i} \times 100 \tag{7}$$

If all functions in a system have a relatively equal contribution to the overall variation in the system, the S measure (taken as a ratio, not a percentage) for each function should be approximately $1/F$, where F is the number of functions in the system. As a result, a direct comparison between the sensitivity contribution of a function that appears in both large and small systems (large and small values of F, respectively) is ill posed. To normalize the sensitivity contribution measures to allow such comparisons, the S measure (here, taken as a ratio, not as a percentage) should be multiplied by F, which is the number of functions in the system, to produce a normalized, dimensionless ratio of sensitivity (Eq. (8)). A value of SR_{ij} equal to 1 indicates a sensitivity contribution of $1/F$ for function j with respect to performance variable i. This indicates that the function has a sensitivity contribution equal to its functional contribution. Values greater than 1 indicate a relatively higher contribution to sensitivity than the contribution to functionality. The opposite is true for values less than 1. This measure allows a particular function's tendency to be over- or undersensitive to be characterized outside of the context of the particular model or concept being studied. For systems with more than one performance variable, the sensitivity ratio can be averaged per Eq. (9), where Z is the number of performance variables considered in the analysis:

$$SR_{i,j} = S_{i,j} \cdot F \tag{8}$$

$$SR_j = \frac{1}{Z}\sum_{i=1}^{Z} SR_{i,j} \tag{9}$$

The use of these measures on a functionally decomposed behavioral model along with a discussion of the results appears in the example presented in Sec. 5.

5 Example

To illustrate the application of the hybrid variation-based local (HyVar) sensitivity analysis, an example is presented based on a simple three-function system. The system represents a simple combination of an electrical power source, an electrical-to-rotational energy conversion, and a rotational load. This system is represented with a formal functional model in Fig. 1. In this model, the three functional elements of the system are *provision electrical energy*, *convert electrical energy to rotational energy*, and *export rotational energy*. The provision function includes a charge *status signal* output, and the convert function includes a throttle *control signal*.

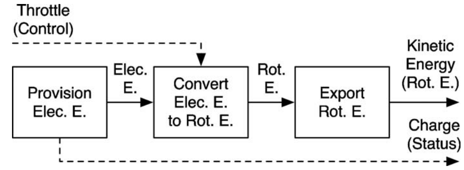

Fig. 1 Example functional model

The behavioral models for the first two functions are based on hybrid powertrain models appearing in Ref. [35]. The final function (export rotational energy) was created specifically for this example with a compatible interface to the energy conversion function. A description of the flow variables used in this example appears in Table 2 along with the mathematical relationships used in the behavioral model elements for each function in Table 3.

For each function in the model, a form solution was selected and used to create a behavioral model for that function. The provision electrical energy function was provided by a simple chemical battery. The energy conversion function was provided by an ac induction motor. The export function was provided by a simple rotational load with inertia and friction. The motor and battery parameterizations and modeling are based on the model elements used in the analysis of a hybrid Formula SAE racecar. The load model was derived and parameterized to interact with these components.

A function-based behavioral model assembly and solution process demonstrated in Ref. [35] was then used to create a well-posed model of the system that was capable of predicting the time required for the rotational load to reach a prescribed target speed from a standing start. Essentially, the model represented a battery connected to a motor, which is connected to a flywheel with a linear velocity-dependent friction force. Although this is a relatively simple system, it provides a useful illustration of how to apply the HyVar analysis. Each functional element in the model has a set of parameters associated with it. These parameters are tied to the mathematical relationship selected to represent the function, and are used to establish and distinguish various conceptual solutions to the system. A nominal set of parameters is used to represent a single concept and is shown in Table 4.

Based on this parameterization, a nominal performance vector can be calculated from the assembled system behavioral model. In this example, one performance variable is considered (time for the flywheel to accelerate to 200 rad/s).

Using this model, a variety of sensitivity analyses can be performed. In this example, a derivative-based local analysis is performed along with the HyVar method presented in Sec. 3. Addi-

Table 2 Model flow variables

Flow	Variable	Units	Description
Electrical energy	Voltage	V	Effort
	Current	A	Flow
Rotational energy	Moment	N m	Effort
	Ang. vel.	rad/s	Flow
Control signal	Analog	Varies	A single analog control signal
Status signal	Analog	Varies	A single analog status signal

Table 3 Model relationships

Function	Model description	Relationship
Provision electrical energy	Linear battery model with internal resistance	$V_{B,\text{nom}} - V_B - I_B \cdot R_B = 0$
		$\dot{C}_A = \dfrac{-I_B}{Q_{B,\max}}$
Convert electrical energy to rotational energy	Constant torque to constant power transitional electrical machine model with linear proportional control	$\omega_T = \dfrac{P_{\max} \cdot V_S}{M_{\max} \cdot V_{\text{rated}}}$
		if $(\omega_M > \omega_T)$: $M_M - M_{\max} \cdot C_M = 0$ else:
		$M_M - \dfrac{P_{\max} \cdot V_M \cdot C_M}{\omega_M \cdot V_{\text{rated}}} = 0$
		$I_S - I_{\text{rated}} \cdot C_M = 0$
Export rotational energy	Simple inertial and frictional load	$M_L - J_L \cdot \dot{\omega}_L - B_L \cdot \omega_L = 0$

tionally, a full variation-based Monte Carlo sensitivity analysis is performed as well with an assumed set of parametric distributions for the variables in each function.

The results of these analyses appear in Tables 5–7. Table 5 contains the results of three different applications of the HyVar method at three different variable perturbations (0.10%, 1.0%, and 10%). Using a constant perturbation percentage for each variable represents a constant coefficient of variation in a full variation-based approach. The HyVar sensitivity analysis uses three system simulations per parameter as compared with tens of thousands or more for a full Monte Carlo sensitivity. Thus, a variety of perturbation steps can be used. Smaller steps capture local effects better than larger steps but may not capture behavior that occurs further from the nominal performance. Performing three evaluations with three steps sizes varying by an order of magnitude provides a large range of coverage around each design variable. If significant variation in the results occurs between the three step sizes, it is recommended to perform a full Monte Carlo sensitivity analysis. At each percentage, the sensitivity contribution, derivatives, and normalized derivatives (normalized to the magnitude of the input variable perturbation) are tabulated along with the time required to perform the analysis and the nominal performance. As shown in this table, the most significant contribution to overall sensitivity in performance of the system is the nominal voltage of the battery. This is followed by the inertia of the load then the rated power of the motor, rated torque of the motor, and finally the friction of the load. The results of this analysis are charted in Fig. 2.

Table 6 shows the tabulated results of the HyVar sensitivity analysis at a 1% perturbation using the nomenclature and grouping established in Sec. 3. As shown in this table, the provision electrical energy function contributes 44.4% to the overall sensitivity of the system, followed by the export rotational energy function at 30.3%, and then the convert electrical energy to rotational energy function at 25.3%. Once normalized to the number of functions in the system, the sensitivity ratios for these functions become 1.33, 0.76, and 0.91, respectively. Thus, the provision

Table 4 Model parameters

Function	Symbol	Value	Units	Description
Provision electrical energy	$V_{B,\text{nom}}$	72.0	V	Nominal voltage
	R_B	0.1	W	Internal resistance
	$Q_{B,\max}$	60000.0	A s	Battery capacity
Convert electrical energy to rotational energy	V_{rated}	48.0	V	Rated voltage
	$P_{\max}$	13.41	kW	Maximum power
	I_{rated}	350.0	A	Rated current
Export rotational energy	B_{load}	2.0	N m s/rad	Load friction constant
	J_{load}	6.0	kg m^2	Load inertia

Table 5 HyVar results. The three columns under the different offset percentages represent the sensitivity contribution, the local derivative, and the normalized derivative response, respectively.

	Parameter offset percentages								
Variables		0.10%			1%			10%	
Nominal voltage	44.3%	−0.0239	−0.0034	44.36%	−0.0239	−0.0344	49.77%	−0.026	−0.379
Rated power	16.9%	-3.11×10^{-5}	−0.0021	16.93%	-3.11×10^{-5}	−0.0213	16.15%	-3.20×10^{-5}	−0.219
Rated torque	8.42%	−0.0053	−0.0015	8.41%	−0.0053	−0.0150	7.92%	−0.005	−0.154
Load friction	3.31%	0.235	0.0009	3.31%	0.2352	0.0094	2.88%	0.236	0.095
Load inertia	27.0%	0.224	0.0027	26.99%	0.2240	0.0269	23.27%	0.224	0.269
Analysis time (s)		0.0327			0.0338			0.0329	
Nominal performance (s)		1.3338			1.3338			1.3338	

Table 6 Sensitivity parameters

Contributor	Type	Label	Value
Nominal voltage	Parameter	$S_{1,1,1}$	44.36%
Rated power	Parameter	$S_{1,2,1}$	16.93%
Rated torque	Parameter	$S_{1,2,2}$	8.41%
Load friction	Parameter	$S_{1,3,1}$	3.31%
Load inertia	Parameter	$S_{1,3,2}$	26.99%
Provision electrical energy	Function	$S_{1,1}$	44.36%
Convert EE to RE	Function	$S_{1,2}$	25.34%
Export rotational energy	Function	$S_{1,3}$	30.30%
Provision electical energy	Function ratio	$SR_{1,1}$	1.33
Convert EE to RE	Function ratio	$SR_{1,2}$	0.76
Export rotational energy	Function ratio	$SR_{1,3}$	0.91

electrical energy function provides a larger contribution to the overall sensitivity than it does to functionality. The opposite is true with respect to the convert electrical energy to rotational energy and export rotational energy functions. A sensitivity contribution greater than 1 indicates that the parameters in the model of the functional element affect the performance of the system to a greater degree than the parameters in other functional elements in the system. Depending on the design problem, this information may be used for a variety of purposes. If the overall contribution of each functional element is desired to be equal, then the sensitivity ratios of each function should be driven to values around 1.0. If certain functions are desired to contribute more to the overall performance of a system, the sensitivity ratio of these elements should be driven to values greater than 1.0. If the opposite is true, and certain elements are desired to contribute less to the overall performance, the sensitivity ratios of these elements should be driven to values less than 1.0. If a specific sensitivity ratio profile is desired, a formal optimization problem can be used to create one. This allows not only the optimization of performance and overall sensitivity but also the sensitivity profile of the functional elements.

For reference, a full variation-based Monte Carlo sensitivity analysis was performed at a variety of sample sizes. The simulated annealing (SA) method presented in Ref. [2] was used to complete this analysis. In order to perform the analysis, parametric distributions for the variables in the model were required. As mentioned before, these distributions are not commonly available or applicable in early designs but are necessary to apply a full global variation-based approach. As seen in Table 7, for small numbers of samples ($\ll 1000$) the results are unreliable (variations should not have negative values). At a sample size of 1000, the results begin to stabilize. After 10,000 samples, the precision of the analysis improves. However, this increased precision requires a significant computational burden as seen by the increase in the analysis time. The resulting sensitivity contribution measures are charted in Fig. 3. The relative contribution of each parameter differs from that of the HyVar analysis but that is to be expected as the two analyses are working on separate sources of input (the full analysis considers the actual distribution of each parameter versus the HyVar analysis that considers a perturbation of each parameter). The primary result of this analysis is the relative magnitude of the analysis time required for each method. The HyVar method required 0.0338 s versus 184.2 s for a reliable full Monte Carlo analysis. It should be noted that the HyVar method is not intended as a replacement for a global analysis but rather a complement for the early stages of design where such a large computational burden (even with efficiency improvements demonstrated by Ullman [7] and Chen et al. [3]) and additionally required information are not practical. As mentioned before, the HyVar analysis provides the same type of information as a global analysis (relative contribution) but uses a method that is feasible for use in early design when considering multiple concepts.

Table 7 Global variation-based results

	Monte Carlo samples			
Variables	10	100	1000	10000
Nominal voltage (%)	87.18	61.69	62.02	63.63
Rated power (%)	−6.58	−3.73	1.04	0.58
Rated torque (%)	33.76	4.41	7.69	8.20
Load friction (%)	−7.83	0.40	8.63	7.02
Load inertia (%)	−28.66	17.46	25.34	22.48
Analysis time (s)	0.1886	1.8039	18.20	184.2
Mean performance (s)	1.3498	1.3482	1.3467	1.3457

6 Hybrid Racecar Example

A more comprehensive example application of the HyVar sensitivity analysis is shown in Ref. [35]. In this work, a complete early design process and behavioral analysis was performed for a hybrid racecar powertrain. The results of this work are briefly summarized here.

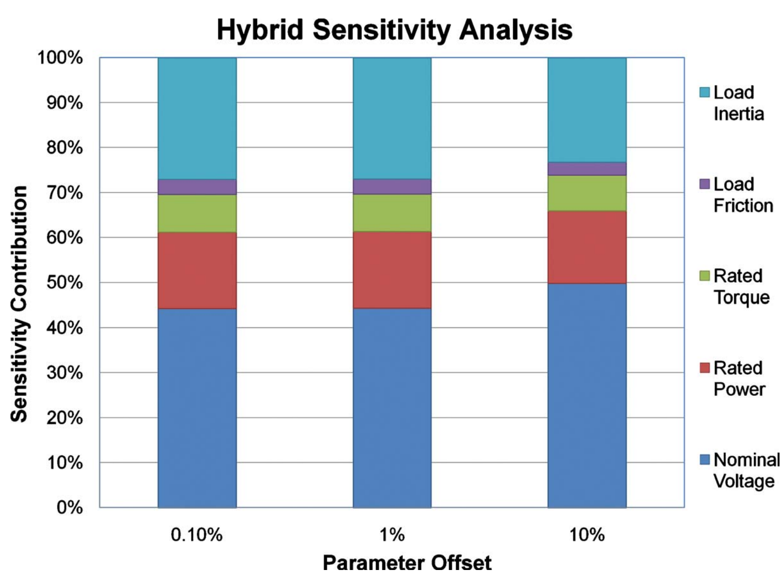

Fig. 2 HyVar results charted

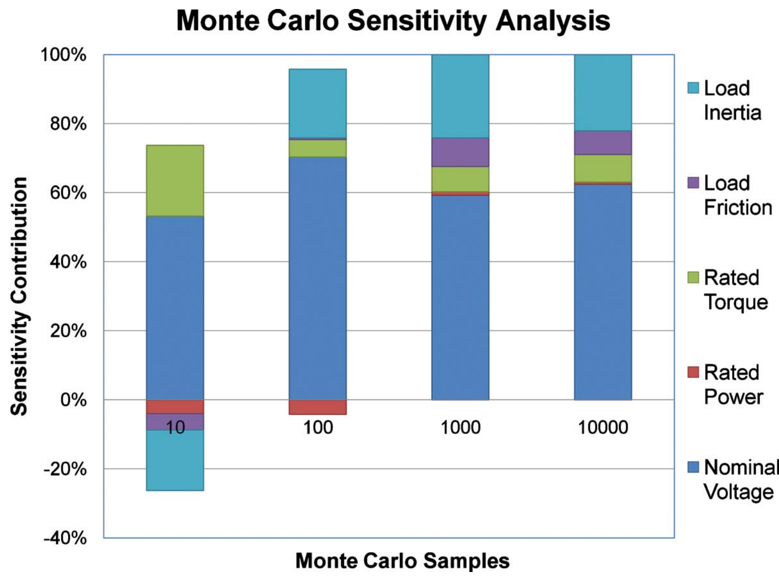

Fig. 3 Global variation-based results charted

For the hybrid powertrain, a functional decomposition was performed just as with the three-function system shown in Sec. 4. The resulting functional model appears in Fig. 4. In this model, the provision electrical energy function represents the storage and supply of electrical energy from the racecar's electrical power accumulator (mandated by the rules of the competition being entered). The convert electrical energy to rotational energy function represents the electrical machine that converts stored electrical energy to propulsive energy (and vice-versa for regenerative braking). The *distribute rotational energy* function represents the distribution of propulsive energy through the system. The *convert chemical energy to rotational energy* function represents the conversion of stored fuel energy to propulsive forces. The conversion processes are controlled by the *process control* function. The *distribute mechanical energy* function represents the distribution of energies through the chassis of the racecar. Finally, the two *transfer mechanical energy* functions represent the transmission of energies through the suspension and wheels of the racecar. A behavioral model was created based on this functional decomposition. The complete details of this model are shown in Ref. [35] and are omitted from this work for brevity.

The behavioral model created for this system was used to investigate the performance of a large number of powertrain concepts using both ac and dc electrical machines. For each concept, a HyVar sensitivity analysis was performed to assist the concept selection process. Example results for both ac and dc concepts are shown in Fig. 5. The ac concepts are labeled as 1 and 2, while the dc concepts are labeled as 3 and 4.

By using a functional grouping of the parametric sensitivities, it is possible to directly compare the impact of each powertrain function on the overall performance of the system among various configurations. As shown in Fig. 5, the significance of each function can vary greatly between different physical solutions. The sensitivity with respect to the convert electrical energy to rotational energy function was roughly equal for the ac and dc concepts as was the sensitivity with respect to the convert chemical energy to rotational energy function. However, the DC concepts proved to be much more sensitive to the variation in the functions performed by the rear suspension/wheels/tire (transfer mechanical energy-*R*). This result at first seems counterintuitive, but originates from the sensitivity of the system to the overall mechanical advantage between the motor and tire/ground interface. Ac motors, which operate primarily in a constant power regime, are much less sensitive than the dc motors used, which operate in a constant torque/current limited regime as implemented (with fixed ratio gearing). As a result, the dc motor is rarely at its peak power level. Performing a HyVar analysis during this design process allowed this behavior to be identified and considered for the remainder of the design process. For a more detailed discussion of this example, see Ref. [35].

7 Conclusions

The HyVar sensitivity analysis approach presented in this work is a local sensitivity analysis method that uses an approach similar to a full variation-based analysis. The method provides the same qualitative measures as a full approach with the computational burden of a derivative-based local approach. This approach provides a bridge between the local analysis of many concepts during early design and a global analysis of a smaller set of concepts later

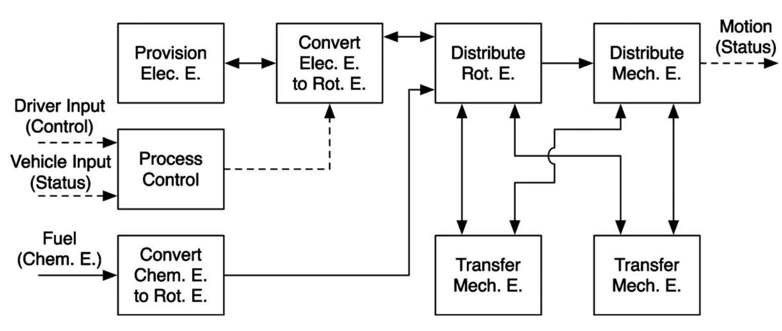

Fig. 4 Hybrid racecar powertrain functional model

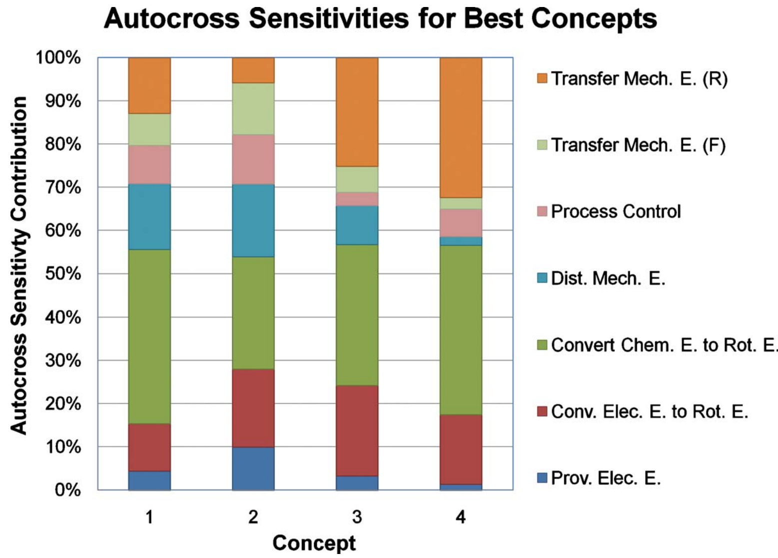

Fig. 5 Autocross sensitivities for best concepts

in the design process. The approach does not require detailed information regarding the distributions of the various parameters used in the model of a concept, and thus is appropriate for early concept analysis when this information is not be available. The HyVar method has a computational burden similar to a derivative-based approach and can be completed at the same time as such an approach with little overhead. As a result, it is recommended to augment current local derivative-based sensitivity approaches with the HyVar analysis in order to obtain the contribution measures along with the information typically provided by the derivative-based approach. The resulting contribution measures can then be used in the same manner as the contribution measures produced by a full variation-based analysis.

If a full variation-based analysis is required, it should be performed once a relatively small set of feasible concepts have been developed and there is sufficient knowledge to establish parametric distributions. In this case, the measures from the full analysis can be directly superseded by the measures generated by the HyVar analysis.

The HyVar approach also allows the direct addition of single parameter measures that provide the means of assessing the sensitivity contribution of a function in a functionally decomposed system behavioral model. The normalization of the measures to the functional size of system allows the measure to be used outside of the context of a particular system design and is conducive to design repository storage (see Ref. [36] for a discussion of a function-based design repository).

The method as developed here is posed to perform sensitivity analysis at the system level. As such, it is generally applicable to problems characterized as complex in the sense that the systems consist of multiple elements with energy flows in multiple domains and are conveniently modeled using a functional approach. The sensitivity example summarized in Sec. 5 represents a system model with over 45 design parameters, two energy domains, and eight functions. Formal methods were not used to compare the reduction in the computational time required between the HyVar method and the full Monte Carlo approach. A casual comparison showed the HyVar computation time on the order of a few minutes with full Monte Carlo simulations on the order of days or greater (a full Monte Carlo simulation was not run as it was not feasible in the time constraints of the analysis).[1]

Nevertheless, there are notions of system complexity for which we have not explored the applicability of the approach. For example, the systems modeled here are not chaotic. Neither the HyVar method nor the function-based behavior modeling approach on which it is based contain explicit compensation for complexity that is reflected in chaotic dynamic behavior. Also, we have not explored the systems that exhibit self-adaptation—another type of system complexity of the design interest.

The framework used here is based on a functional representation, a representation that is commonly used to simplify and abstract complex engineered systems. Nevertheless, based on the work presented here, we cannot claim that the approach scales linearly, or accurately, to systems of specific or arbitrary complexity or size. Useful design models of such systems will still likely require abstraction and modeling insight from the modeler. Developing and testing the framework used here to model the different types of complex systems and systems of an arbitrary number of elements remain as a future work.

[1]Simulations were run on an Apple Mac Pro with two 3 GHz dual-core Xeon processors and 8 Gbyte of 667 MHz RAM.

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