Proceedings of FEDSM2007
5th Joint ASME/JSME Fluids Engineering Conference
July 30-August 2, 2007 San Diego, California USA

FEDSM2007-37647

A ONE DIMENSIONAL MATHEMATICAL MODEL FOR URODYNAMICS

Ismail B Celik
West Virginia University

Asaf Varol
Firat University, Elazig, Turkey

Coskun Bayrak
University of Arkansas at Little Rock

Jagannath R Nanduri
West Virginia University

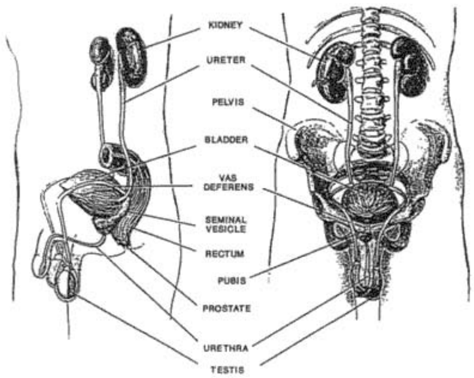
ABSTRACT

Millions of people in the world suffer from urinary incontinence and overactive bladder with the major causes for the symptoms being stress, urge, overflow and functional incontinence. For a more effective treatment of these ailments, a detailed understanding of the urinary flow dynamics is required. This challenging task is not easy to achieve due to the complexity of the problem and the lack of tools to study the underlying mechanisms of the urination process. Theoretical models can help find a better solution for the various disorders of the lower urinary tract, including urinary incontinence, through simulating the interaction between various components involved in the continence mechanism. Using a lumped parameter analysis, a one-dimensional, transient mathematical model was built to simulate a complete cycle of filling and voiding of the bladder. Both the voluntary and involuntary contraction of the bladder walls is modeled along with the transient response of both the internal and external sphincters which dynamically control the urination process. The model also includes the effects signals from the bladder outlet (urethral sphincter, pelvic floor muscles and fascia), the muscles involved in evacuation of the urinary bladder (detrusor muscle) as well as the abdominal wall musculature. The necessary geometrical parameters of the urodynamics model were obtained from the 3D visualization data based on the visible human project. Preliminary results show good agreement with the experimental results found in the literature. The current model could be used as a diagnostic tool for detecting incontinence and simulating possible scenarios for the circumstances leading to incontinence.

INTRODUCTION

Urinary incontinence has been reported to affect 35% of American women over 50 years of age an almost 15% who have leakage on a daily basis [1]. The common types of incontinence are (1) Stress incontinence (2) Urge incontinence (3) Overflow incontinence and (4) Functional incontinence. Approximately 60% of women with incontinence will have stress incontinence [2] where the urethral sphincter is not able to hold urine due to weakened pelvic muscles that support the bladder, or malfunction of the urethral sphincter [3]. Urge incontinence is also a storage problem in which the bladder muscle contracts regardless of the amount of urine in the bladder. Urge incontinence may occur without a recognizable prior disease or may result from neurological injuries, neurological diseases, infection, bladder cancer, bladder stones, bladder inflammation, or bladder outlet obstruction [4]. Overflow incontinence happens when there is an impediment to the normal flow of urine out of the bladder and the bladder cannot empty completely. Patients with functional incontinence have mental or physical disabilities that impair urination, although the urinary system itself is normal [5]. In order to understand and simulate the various incontinence mechanisms we intend to build a mathematical model to describe the hydrodynamic processes in the urinary tract.

We first present the anatomy of the human urinary tract. The lower part of the urinary tract consists of a sack like muscular storage organ called the bladder, that is found in the pelvis behind the pelvic bone (pubic symphysis) and a drainage tube, called the urethra, that exits to the outside of the body. The bladder is an organ where the urine filtered by kidneys is stored. The kidneys filter approximately 160 liters of blood a day in order to maintain the necessary fluid balance. Water makes up approximately 95 percent of the total volume of urine, with the remaining 5 percent consisting of dissolved solutes or wastes (i.e. urea, creatinine, uric acid and several electrolytes). Urine is steadily excreted from the kidneys then pumped down to the ureters to the bladder by means of muscle contractions and the force of gravity (Figure 1). Once in the bladder, the urine is temporarily stored until it is voided from the body through the urethra [6].



.

Figure 1 - Normal Male Genitourinary Tract [7]

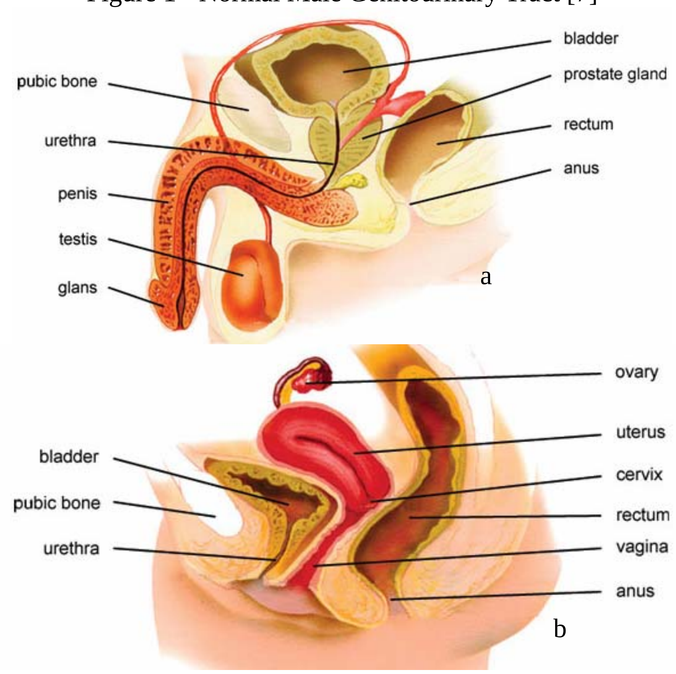


Figure 2 – Bladder of men (a) and women (b) [8]

The detrusor is a thick layer of smooth muscle which expands to store urine and contracts to expel urine. The urethra is a small tube which leads from the neck of the urinary bladder to the outside of the body. In men, the urethra is approximately 20 cm long. When it leaves the bladder, it passes downward through the prostate gland, the pelvic muscle and finally through the length of the penis until it ends at the urethral orifice or opening at the tip of the glans penis. In women, the urethra is approximately 4 cm long runs in front of the vagina. The urethral orifice or meatus is the outside opening of the urethra and is located between the clitoris and the vaginal opening [6]. Storage and emptying of urine in the bladder are regulated by the internal and external urethral sphincters in response to neural signals under normal circumstances. Sphincters are made up of ring-like band of muscle fibers that contract or expand to regulate urine discharge. Sphincters normally remain closed and need stimulation to open [6].

Figure 3 shows the bladder and its nerve systems. During filling of the bladder with urine, the bladder expands to accommodate urine flow from the kidneys. After the filling of the reservoir, signals are sent to the brain to warn that the reservoir is full under normal circumstances voluntary voiding occurs by sending signals from brain to the bladder to contract to the external urethral sphincter to open [9]. Detailed mathematical models for simulating the hydrodynamic processes in the urinary tract are scarce in the literature. In what follows we briefly review the relevant literature that we were able to find after an extensive search.

A mathematical model of the male urinary tract was reported by Kren et al. [10] to model the interaction between the urethra, bladder and urine flow. In this research, the flow is considered to be non-stationary, isothermal and turbulent. The elastic properties of the bladder and urethra have been modeled as a dynamic motion theory and has been created using D'Alambert principle. Kren et al. used a finite element mesh for their numerical solution, and they used an iterative method to solve the problem of bladder deformation. In this work, the flow is considered to be turbulent. This phenomenon is questionable because the urinary flows are not always turbulent (a typical Reynolds number $Re=(\rho u D)/\mu$ range is 300-4000). Both kinds of flows (i.e. laminar and turbulent) should be considered for creating the model. Kren et al. focus mostly on the mathematics and numerical methods rather than on the actual physics of urodynamics. The authors demonstrate that the predicted urine discharge rate would be different if the urethra walls are treated as compliant rather than as rigid walls. This conclusion seems to us a trivial outcome. In our opinion, this work is not complete and more research is required to relate the dynamics of compliant walls to urodynamics in general.

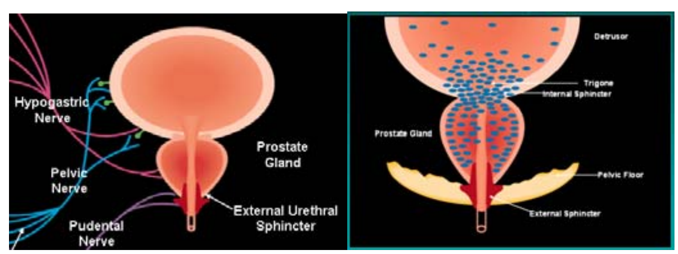


Figure 3 - The relationships between a bladder and nerve system [9]

According to the reference [7], the normal uroflow changes as shown in Figure 4. From the figure, it is seen that the maximum volumetric flow rate doesn't go up more than 240 ml/min, but this of course can change from person to person, and depending on the pressure inside the urethra. However, the Peak and the average flow is expressed in ml/sec. Generally normal peak flow in females is over 20 ml/sec and in males >15 ml /sec, normal flow can be up to 45-50 ml /sec but this is unusual. Voiding time is defined as the total time from the beginning to the end of the micturition. The flow time is defined as the total time when urine is actually flowing (Figure 5). When the uroflow is intermittent or abnormal, the voiding time and flow time can not be defined precisely [7]. But the International Society of Continence (ISC) suggested the flow time be calculated disregarding the time intervals between flow episodes.

One problem that we found in reviewing of such results in the literature is that the figures are only given as sketches and they rarely have time or length scales. This makes the interpretation of the results very difficult. For example, Figure 6 shows a conceptual pressure distribution as a function of time. According to this figure, after beginning of the contraction, the pressure goes up at first and reaches a maximum value and later decreases to a level of the pre-micturition pressure although the contraction continues. The figure does not indicate how long the filling process continues. Filling period is an important parameter that should be measured for accurate analysis of the system. It appears that during the contraction period, the flow rate of the urine has been taken as constant. This is not realistic because depending on the contraction rate, the velocity of the urine can change which may then result in an increase of the flow rate. The urine filtered from the kidneys should also be a function of time of the day as well as eating and drinking habits of a person.

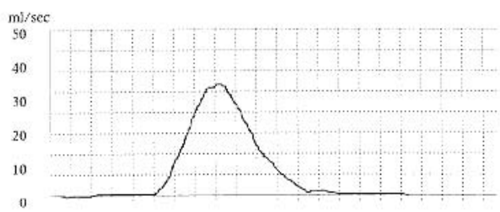


Figure 4 – The diagram of normal uroflow [7]

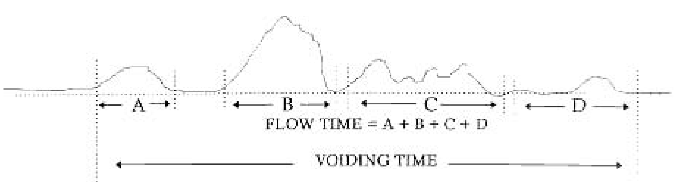


Figure 5 – Comparing the flow and voiding times [7]

Wang et al. [11] investigated the effect of the hydrostatic pressure on ion transport in the bladder uroepithelium. They showed that increasing hydrostatic pressure across the mucosal surface of the uroepithelium is accompanied by increases in ion transport. If this is true, measured electrical activity can be used as an indicator of the detrusor pressure.

Figure 7 shows [12] the relationships between the tension and pressure distribution based on the bladder volume. It is seen that the increasing of the volumetric flow rate does not mean an increase in the pressure. This phenomenon can be explained by using Laplace's equation which relates the wall tension, T, to the pressure, P, inside the bladder. For a sphere this equation yields

$$P = 2T / R \qquad (1)$$

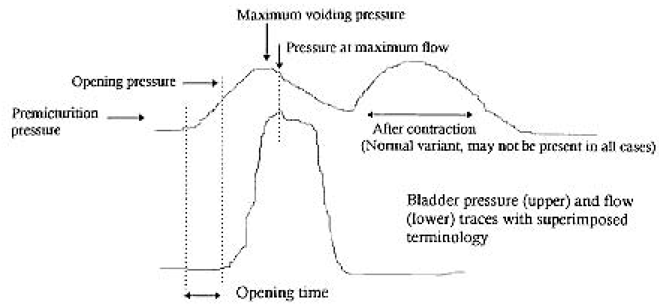


Figure 6 – Flow and Pressure distribution in the bladder

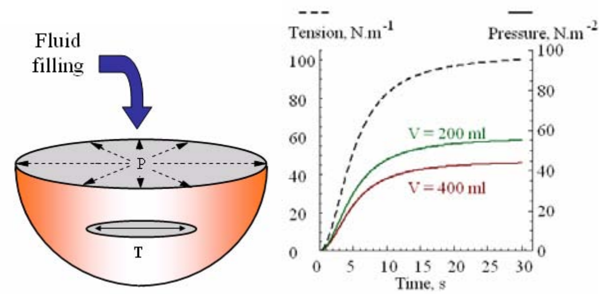


Figure 7 – Tension and Pressure Distribution in the bladder

Equation (1) indicates that as the volume decreases (i.e. the radius R decreases) the pressure increases for a constant wall tension. It also reveals a linear relationship between the detrusor force and the bladder circumference as suggested by Rikken et al. [13]. However the tension in the detrusor muscles is usually not constant. As revealed by the work of Damaser [14] the bladder has non-linear elastic, viscous and plastic mechanical properties. Hence, assuming a constant tension in Eq. (1) may lead to pressure-volume variation that is contrary to that observed in clinical trials [15], [16].

Observations [15], [16] indicate that during the filling period, the bladder will be expanded; at the beginning, this action will result in a relatively rapid increase in the pressure of the bladder. As the bladder is filled there is a mild increase in pressure followed by a rapid increase as the bladder reaches its capacity. After the voiding begins the pressure decreases to its normal level and the cycle repeats itself under normal circumstances.

Griffiths et al. [12] obtained pressure data from 32 men with lower urinary tract symptoms and from 7 asymptomatic volunteers. It is concluded that non-invasive voiding studies using the cuff inflation technique can provide useful information on obstruction [12]. On the other hand Pel and Mastright [17] argue that noninvasive pressure monitoring is, at present, not sensitive enough for clinical use. The mathematical models we propose to develop can help interpret the results from non-invasive monitoring better by expressing pressure, volume and discharge in a functional form.

We believe that fluid dynamic models when used in conjunction with structural dynamics models have great potential for understanding of many complex processes that occur in the urinary tract. Such models can also be used as diagnostic tools for detecting and finding the cause of symptoms or patient complaints that are difficult to understand by simple tests such as cystometrogram (CMG). The lack of rigorous mathematical models and detailed numerical simulations in the literature lead us to propose new computational models for the analysis of urine flow in the lower urinary tract.

METHODOLOGY

Before any diagnosis can be made of malfunctioning, the urodynamics of a normal person must be understood well, and the proposed model must be able to predict the observed behavior of a normal bladder say during a normal working day. We summarize a conceptual model to this regard.

We postulate that the bladder walls are compliant, that is they expand and contract to accommodate the volume of liquid accumulated inside the bladder in a natural manner. This assumption is based on the fact that the detrusor layer is made of so called 'smooth' cells that are less sensitive to impulse. There is evidence [18] of incomplete activation of the detrusor muscle during normal voiding. As depicted in Figure 3 the lower urinary track is innervated by three sets of peripheral nerves involving the pelvic parasympathetic, sympathetic, and somatic nervous systems. The neural signals supplied by this nervous system forms the bases of a dynamic control mechanism for the urination process. Hence we allow for the possibility that the bladder can contract in response to stimulation of the pelvic (parasympathetic) nerve. Moreover, pressure inside the bladder can be influenced by abdominal pressure which may increase during coughing or sneezing etc. Further, the tension in the detrusor produces a stress normal to the bladder walls which we denote as the detrusor pressure, P_d; this is usually taken as the difference of bladder pressure and the abdominal pressure. The tensile stress in the detrusor layer and P_d are related through geometrical parameters via Laplace law (see Eq. 1). For the preliminary model we assume that the change in the bladder pressure P_b is determined by the compliance equation

$$\frac{dP}{dV} = c_\tau + cV^\gamma \qquad (2)$$

Where c_τ and c are model parameters related to the compliance (1/elasticity) of the tissues that make up the bladder wall structure, γ is also a material property which expresses the non-linear behavior of volume as a response to the detrusor pressure. The model parameters c_τ and c should also be functions of the volume (e.g. stretching) itself and implicitly a function of the detrusor tension in a truly non-linear model. In order to account for voluntary or involuntary contraction of the bladder walls we model the tension as a sum of volumetric dependence and a voluntary contraction of detrusor as signaled by parasympathetic nerves. The resulting equation for the bladder/intravesical pressure is:

$$P_b = P_{ref}\left(\theta^n \tau\right) + P_a \qquad (3a)$$

$$\tau = \theta^m + f_{imp} \qquad (3b)$$

Here P_a is the normal abdominal pressure, θ is the dimensionless volume, τ is the dimensionless detrusor tension, and f_{imp} is an appropriate impulse function. We denote the urine influx from the kidneys by $Q_{in}(t)$ which should be determined empirically from collected data. This would change with age, eating and drinking habits; it also changes depending on the time of the day, and of course, on the condition of the kidneys. The bladder should be functioning continuously as long as the kidneys function normally, hence the volume, V of the bladder will be determined by

$$\frac{dV}{dt} = Q_{in}(t) - Q_{out}(t) \qquad (4)$$

where t is the time, and $Q_{out}(t)$ is the urine discharge rate which is the primary unknown to be determined as a function of time.

When the bladder volume reaches a certain critical value, say V_{cr} , which is less than the maximum capacity of the bladder, V_{max} a signal should be sent to the brain indicating urge to urinate. This signal may result from the detrusor tension reaching a critical value. During the voluntary waiting period the filling will continue but the pressure of the sphincter should increase. Voluntary voiding occurs by detrusor contraction and simultaneous relaxation of the urethral sphincters. The bladder neck and urethra internal sphincter will open and the urethra will fill with urine. For the time being we shall assume that this later process takes place smoothly after the critical volume is reached. Hence the cross-sectional area of the internal sphincter is given by

$$A_{i-sph} = A_{0\text{-}sph} H\left(V_{cr} - V_{min}\right) \qquad (5)$$

where $A_{0\text{-}sph}$ is the area of the internal sphincter at normal opening, and H is a unit square wave function such that when $V_{min} < V < V_{cr}; H = 0 \text{ } otherwise \text{ } H = 1$ or a sinusoidal oscillatory function which can represent intermittent voiding.

Equation 5 indicates that the internal sphincter relaxes naturally after receiving a signal from the spine's parasympathetic nerves that there is a need for urination and contracts when the emptying process is completed. After opening of the internal sphincter the urethral pressure, Pu, should attain the same value as the bladder pressure plus some negligible pressure rise caused by gravity, hence, we write:

Before opening of the internal sphincter

$$P_u = P_{u,\,normal} \qquad (6)$$

After opening of the internal sphincter

$$P_u = P_b + \rho g \Delta h \quad (7)$$

where ρ the urine density is, g is the gravitation; Δh is the vertical height from the external sphincter to the top of the liquid layer inside the bladder. After the internal sphincter opens, the control is transferred to the external sphincter, by which the person has the option to prevent micturition (peeing) until an appropriate time. The increase in the tension of the contracted sphincter ring should cause an increase in the bladder pressure through the balance of forces as related by Laplace's equation. A similar pressure change is possible by regulating the abdominal pressure (see Eq. 3). This pressure increase should not cause any significant change to the bladder volume as the volume of the liquid stored in it is incompressible. The bladder volume will continue to increase as a result of the influx of urine filtered through the kidneys.

During this waiting period the pressure, $P_{ext,sph}$, exerted by the sphincter should increase with time. We assume a function of the form

$$P_{ext,sph}(t) = P_b + \Delta P_u S\left(\frac{t - t_{fill}}{\Delta t_{wait}}\right) \quad (8)$$

Where ΔP_u is the maximum pressure that can be exerted by the external sphincter muscles in addition to the normal urethral pressure, Δt_{wait} is the waiting period permissible till the urine discharge begins, and S is a function to be determined.

After a reasonable period of waiting the voiding process should start and continue until the bladder is completely or partially emptied. The continuation or stoppage of urination is, of course, under the control of the person under normal circumstances. Hence, the closing and opening of the external sphincter ring should be a control function in the analysis given by

$$A_{ext,sph}(t) = f\left(t, P_a, P_b, P_{ext,sph}, V\right) \quad (9)$$

After the external sphincter opens, the fluid dynamics of the urine flow inside the urethral tube can be analyzed by a one dimensional transient model that allows compliant tube walls. The equations to be solved are

Conservation of mass:

$$\frac{dA}{dt} = -\frac{dQ}{dx} \quad (10)$$

Conservation of Momentum:

$$\frac{dQ}{dt} = \frac{d}{dx}(uQ) - \frac{A}{\rho}\frac{dP}{dx} - \frac{\xi \tau_w}{\rho} \quad (11)$$

Here x is the distance along the center of the urethral tube, A is the cross-sectional area of the urethral tube, ξ is the perimeter of the tube-line, and τ_w is the shear stress exerted by the fluid on the surface of the urethra. This shear stress can be determined using empirical information for friction coefficient that is a function of wall roughness and the Reynolds number, $\mathrm{Re} = \rho u D_h / \mu$, here D_h is the hydraulic diameter of the tube, and μ is the viscosity of the urine. The necessary geometrical parameters for the urodynamics model can be obtained from the 3D visualization of the environment that is based on visible human data sets. Such data can also be obtained from cystograms. Data from the literature obtained by way of the CT (Computed tomography), PET (Positron emission tomography), and MRI (Magnetic Resonance Imaging) can also be used [20].

In Eq. 9, the tubular area starting from the neck of the bladder and extending to the external sphincter can change under pressure or as a result of obstruction. To account for the elasticity of the urethra a constitutive equation must be formulated such as

$$A = f\left(P, \delta, E\right) \quad (12)$$

where P is the pressure inside the tube, δ is the tube-wall thickness, E is the modulus of elasticity. In a future study, this will be done following the model proposed in [25].

If the area of the urethra does not change with time, Equation 10 simply states that Q does not change with distance. By introducing similar simplifications Equation 11, can be reduced to a balance between the pressure differential, $\Delta P = \left(P_u - P_{atm}\right)$, P_{atm} being the atmospheric pressure, and the urine flow rate, Q given by

$$Q = K A_{ext,sph}(t)\left[\frac{2\left(\Delta P - \Delta P_{min}\right)}{\rho f_{cor}}\right]^{1/2} \quad (13)$$

where ΔP_{min} is the minimum pressure differential required for initiating flow, K is a discharge coefficient, f_{cor} is a correction factor that is a function of the Reynolds number, and the geometrical properties of the urethra and the external sphincter orifice.

QUANTITY	VALUE	COMMENT
Max. bladder volume (ml)	800	
Min. bladder volume (ml)	16	
Min. Pressure (cm of H_2O)	100	
Ref. Urethral Diameter (mm)	2.5	
Urethra Length (cm)	12 – 20	
Discharge Coefficient	0.10	
Avg. Urine inflow rate (ml/s)	0.03 ~ 35 ml/kg/day	Variable, defined by a function
Urine Viscosity (kg/m-s)	$1x10^{-3}$	
Exponent γ	- 4/3	
Exponent m	+ 2/3	

Table 1 – Parameter list for urodynamics model

RESULTS AND DISCUSSION

In what follows we present some preliminary results from a simple model that is constructed along the lines of the lumped parameter analysis described above. The physical and geometrical parameters specified for these simulations are listed in Table 1.0. The correction factor f_{cor} was calculated using laminar pipe flow assumption. The compliance function is prescribed from the non-linear relation,

$$c_\tau = \left(\theta^n \frac{\partial \tau}{\partial \theta} \right) \tag{14}$$

$$c = -n\tau \tag{15}$$

Where n = -1/3 (i.e. $R \sim V^{1/3}$)

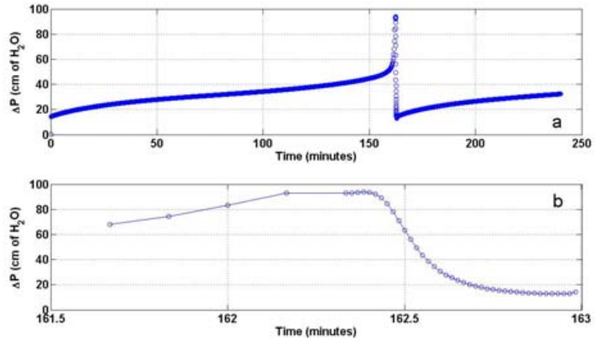


Figure 8 – Variation of bladder pressure with time (a), enlarged (b) near voiding

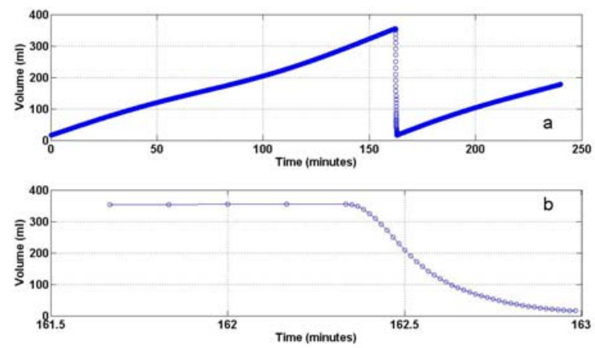


Figure 9 – Variation of bladder volume with time (a), enlarged (b) near voiding

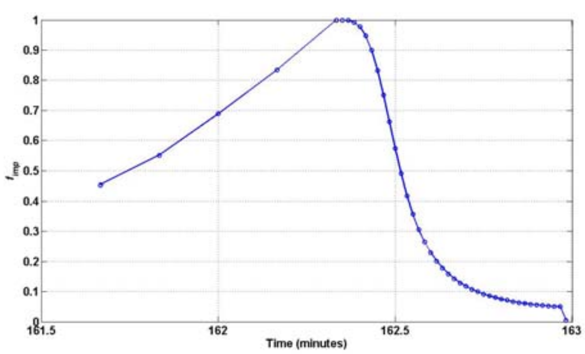


Figure 10 – Typical detrusor impulse function

A complete cycle of filling and voiding process is depicted in Figures 8 & 9. The pressure variation seen in the initial part of Figure 11a resembles closely the clinically observed trend [21] [15]. This pressure trend was obtained by selecting an appropriate impulse function (see Eq. 3b) as depicted in Figure 10 by trial and error. According to the diagrams shown in [21] the pressure reaches a plateau just before the maximum flow rate is attained, and then falls back to its normal level. The simulation results (Figs. 8b & 12) are consistent with such observations. Figure 9 depicts the change in bladder volume as a function of time. The increase in bladder volume is not linear resulting from a non-uniform kidney output as prescribed by a sinusoidal function. The voiding is completed in about half a minute.

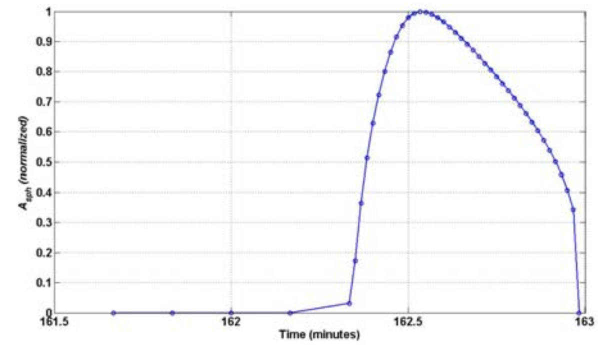


Figure 11– Variation of relative sphincter area with time

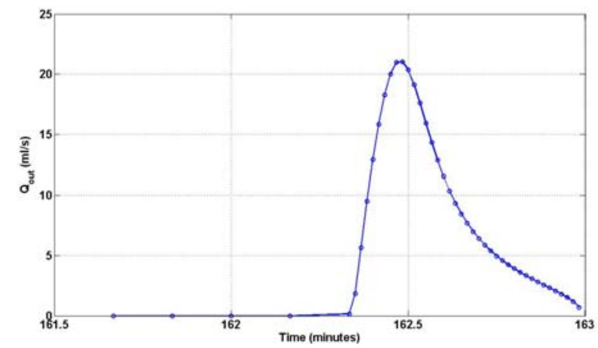


Figure 12 – Variation of urine outflow with time

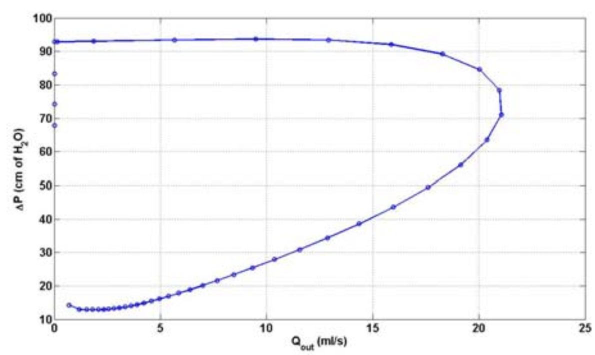


Figure 13 – Bladder pressure versus outflow rate

The variation of the sphincter area with time is shown in Figure 11 and the corresponding flow rate through the urethra is shown in Figure 12. The flow rate curve and the maximum flow rate of c.a. 30 ml/sec are in agreement with clinical observations [21] [16]. Moreover the flow-rate versus pressure diagram depicted in Figure 11 also agrees with the average curve as presented in [16]. We have run another case with urethra length of 20 cm (not shown here) which reduced the maximum discharge rate to about 20 ml/sec as expected due to frictional loss. This yielded a 50 second voiding time.

CONCLUSION

We have shown that a mathematical model can be used to simulate various functions and inter-coupling of different components of the human urinary tract. The purpose is to demonstrate how mathematical models of biological systems such as the bladder works. The key to success for realistic predictions is the calibration of the critical physiological parameters that are mentioned above

REFERENCES

1. National Association For Continence, <http://www.nafc.org>
2. Kuzmarov IW. "Urodynamic assessment and chain cystogram in women with stress urinary incontinence: clinical significance of detrusor instability". Urology1984;24:236-238
3. "The Pelvic Floor and Stress Incontinence", The Continence Foundation, Nov 2004, pp 19-22
4. Nissenkorn I, Shalev M, Radziszewski P, Dobronski P, Borkowski A, De Jong PR., "Patient adjusted intermittent electrostimulation for treating stress and urge urinary incontinence", BJU Int. 2004 Jul;94(1):105-9.
5. Urinary Incontinence, http://www.ucdmc.ucdavis.edu/ucdhs/health/az/50UrinaryIncontinence/doc50.html
6. How the Bladder works?, http://www.ucl.ac.uk/uro-neph/ppt/hb081102.ppt
7. Basic Urodynamics, http://www.life-tech.com/uro/urolib/index.html
8. http://homepages.cae.wisc.edu/~bme301/bladder/
9. A.G. Ramage D.Sc., 'Alpha1-adrenoceptors and BPH", Department of Pharmacology, University College London, Royal Free Campus, Slides notes, 2004
10. Kren, J.; Horak, M.; Zatura, F.; Rosenberg, M. "Methametical Model of the Male Urinary Tract", Biomed, Papers 145(2), 91-96 (2001)
11. Wang, Edward C. Y.; Lee, Jey-Myung Lee, John P. Johnson; Thomas R. Kleyman, Robert Bridges, and Gerard Apodace, "Hydrostatic Pressure-Regulated Ion Transport In Baldder Uroepithelium", Am J Physiol FRenal Physiol 285: F651-F663, 2003, First published May 27, 2003; 10.1152/ajprenal.2002
12. Griffiths, C. J., D. Rix, A. Macdonald, W. Robson, L. Kelly, M. Reddy, M J Drinnan, R S Picard, P D Ramsden, "Can non-invasive Bladder Measurements identify men with Bladder outflow Obstruction, Departments of Medical Physics and Urology, Freeman Hospital, Newcastle upon Tyne, UK, ICS 2000 Tampere.
13. Rickken, Berend ; Pel, Johan J. M. ; Mastrigt, Ron, Repeat Noninvasive Bladder Pressure Measurements with an External Catheter, Journal of Urology, 162(2); 474-479, August 1999
14. Damaser, M.S. "Whole bladder mechanics during filling," Scand J. Ural Nephrol Suppl. 201:51-58, 1999.
15. Guyton, A.C. Textbook of Medical Physiology, 3rd edition, W.B. Saunders Company, London, 1976.
16. Schmidt, F; Shin, Peter; Jorgensen, T.M.; Djurhuus, J.C.; and Constantinou, C.E.; "Urodynamic patterns of normal male micturition: Influence of water consumption on urine production and detrusor function" The Journal of Urology, American Urological Association Vol. 168, 1458-1463, Oct. 2002.
17. Pel, J. J. M. and van Mastrigt, R.: "Non-invasive Diagnosis of Intravesical Obstruction in Male Patients", Neurourol. Urodynam., 17: 394, 1998.
18. http://www.sghurol.demon.co.uk/urod/uth.htm
19. Medical Imaging, Seeing Inside, Scientific American, P. 92, August 2004, Workingknowledge@sciam.com
20. Taylor, C. A.; Draney, M. T.: Experimental and Computational Methods in Cardiovascular Fluid Mechanics, Annual Review of Fluid Mech. 2004. 36: 197-231.
21. Walsh, Patrick. C.; Gittes, Ruben, F; Perlmutter, Alan, D. Stamey, Thomas, A. : "Campbell's Urology", W.B. Sauders Company, Philadelphia, 1986