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# Synchronization in a Class of Complex Dynamical Networks with Nonlinear Coupling

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**Abstract:** This paper deals with a synchronization problem for a class of complex delayed dynamical networks with nonlinear coupling, including known and uncertain networks. Based on the Lyapunov stability theory, we propose controllers for both known and uncertain networks to realize synchronization. Numerical example is also given to illustrate the effectiveness of the proposed synchronization criteria.

**Keywords:** Complex networks; nonlinear coupling; time-delays; synchronization

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## 1 Introduction

Synchronization of complex networks has been an important issue within science and technology communities since its application in many different fields, such as the synchronous transfer of digital or analog signals in the communication networks, and the synchronous information exchange in the Internet and WWW [1–3]. A large number of studies have been reported on synchronization [4–22].

In work [7,8,9,12,13], the authors investigated the synchronization of linearly coupled dynamical networks. The synchronization of nonlinearly coupled networks was studied in [14,15]. In Ref. [16,17], the synchronization of an array of linearly coupled networks with constant single time delay was explored. Actually, the time delays presented in many synchronization schemes are in form of multicoupling time delays [18,19]. Adaptive method is employed to realize the synchronization of complex network and the efficient controller is designed in [21,22]. However, they require the coupling function to satisfy the Lipschitz condition. We notice that in most exist works there is a basic assumption: the nodes of the complex networks are coupled linearly by their state variables. However, in many circumstances this simplification does not match the peculiarities of real networks. On the other hand, in practice, it is often difficult to get the exact estimation of the coupling coefficients; sometimes the state equation of nodes is also uncertain.

Motivated by the above discussions, in this paper, we aim to solve the synchronization about a class of delayed dynamical complex networks with nonlinear coupling, including known and uncertain networks. By employing the Lyapunov stability theory, some criteria for the synchronization are derived.

The left paper is organized as follows. In Section 2, a model of delayed dynamical complex networks with nonlinear coupling is presented, and then a criterion for this sort of network to reach synchronization is deduced. Section 3 first introduces a complex dynamical network which nodes's state equation includes uncertain parameters and with unknown coupling coefficients, then a sufficient condition for the synchronization is deduced. In Section 4, two numerical examples are given to demonstrate the effectiveness of the proposed controller design methods in section 2 and section 3. Finally, conclusions are given in section 5.

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## 2 A generally complex network with nonlinear coupling

### 2.1 Model and assumption

Consider a heterogeneous time-delayed dynamical network consisting of  $N$  nonlinearly and diffusively coupled identical nodes, where each node is an  $n$ -dimensional dynamical system. The state equations of the entire network are given below

$$\dot{x}_i(t) = f(x_i(t)) + \sigma_0 \sum_{j=1}^N g_{(0)ij} H_0(x_j(t)) + \sigma_1 \sum_{j=1}^N g_{(1)ij} H_1(\overline{x_j(t-\tau)}) + u_i, i = 1, 2, \dots, N, \quad (1)$$

where  $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in R^n$ ,  $u_i \in R^n, i = 1, 2, \dots, N$  are the state variables and the input variables of node  $i$ , respectively.  $\overline{x_j(t-\tau)} = [x_{j1}(t-\tau_1), x_{j2}(t-\tau_2), \dots, x_{jn}(t-\tau_n)]^T$ , constant  $\tau_i > 0 (i = 1, 2, \dots, n)$  are the time delays.  $f : R^n \rightarrow R^n$  is a smooth nonlinear vector function; the constant  $\sigma_0, \sigma_1 > 0$  denote the coupling strength,  $H_l : R^n \rightarrow R^n (l = 0, 1)$  is sufficiently smooth nonlinear vector function,  $G_0 = (g_{(0)ij})_{N \times N}$  is the coupling configuration matrix representing topology of the network, If there is a connection between node  $i$  and node  $j$ , then  $g_{(0)ij} = g_{(0)ji} > 0$ ; otherwise  $g_{(0)ij} = 0$ , and the diagonal elements of  $G_0$  are definite by

$$g_{(0)ii} = - \sum_{j=1, j \neq i}^N g_{(0)ij}. \quad (2)$$

$G_1 = (g_{(1)ij})_{N \times N}$  is the delayed matrix; similar to the definition of  $G_0$ , satisfying  $g_{(1)ij} = g_{(1)ji} \geq 0$ , and

$$g_{(1)ii} = - \sum_{j=1, j \neq i}^N g_{(1)ij}. \quad (3)$$

The dynamical network (1) is said to achieve (asymptotical) synchronization if

$$x_1(t) \rightarrow x_2(t) \rightarrow \dots \rightarrow x_N(t) \rightarrow s(t), \text{ as } t \rightarrow \infty. \quad (4)$$

Because of the coupling configuration, the synchronous state  $s(t) \in R^n$  is a solution of an individual node, satisfying

$$\dot{s}(t) = f(s(t)), \quad (5)$$

Here,  $s(t)$  can be an equilibrium point, a periodic orbit, or even a chaotic orbit. The objective of control here is to find some controllers such that the solutions of systems (1) is synchronize with the solution of (5), in the sense that

$$\lim_{t \rightarrow \infty} \|x_i(t) - s(t)\| = 0, \quad i = 1, 2, \dots, N. \quad (6)$$

where the norm  $\|\cdot\|$  of a vector  $x$  is defined as  $\|x\| = (x^T x)^{1/2}$ .

Subtracting (5) from (1) gives the error dynamical system

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) + \sigma_0 \sum_{j=1}^N g_{(0)ij} h_0(e_j(t)) + \sigma_1 \sum_{j=1}^N g_{(1)ij} h_1(e_j(t-\tau)) + u_i, \quad (7)$$

where

$$\begin{aligned} e_i(t) &= x_i(t) - s(t), \\ h_0(e_j(t)) &= H_0(x_j(t)) - H_0(s(t)), \\ h_1(e_j(t-\tau)) &= H_1(\overline{x_j(t-\tau)}) - H_1(\overline{s(t-\tau)}), \\ \overline{s(t-\tau)} &= (s_1(t-\tau_1), s_2(t-\tau_2), \dots, s_n(t-\tau_n))^T. \end{aligned} \quad (8)$$

The following condition is needed for the solutions of (7) to achieve the objective (6).

**Assumption 1 (A1)** Suppose that there exists a nonnegative constant  $\alpha$  such that

$$\|f(x_i) - f(s)\| \leq \alpha \|x_i - s\|, \quad i = 1, 2, \dots, N. \quad (9)$$

## 2.2 Controller design

**Theorem 1** Suppose that (A1) hold. Then the complex dynamical network (1) is synchronized under the controllers

$$u_i = -(\eta - \sigma_0 g_{(0)ii} - \sigma_1 g_{(1)ii})e_i + \sigma_0 g_{(0)ii}v_i + \sigma_1 g_{(1)ii}w_i, \quad (10)$$

$$v_i = \begin{cases} e_i \frac{h_0^T(e_i)h_0(e_i)}{e_i^T e_i}, & e_i^T e_i \neq 0 \\ 0, & e_i^T e_i = 0, \end{cases} \quad (11)$$

$$w_i = \begin{cases} e_i \frac{h_1^T(e_i)h_1(e_i)}{e_i^T e_i}, & e_i^T e_i \neq 0 \\ 0, & e_i^T e_i = 0, \end{cases} \quad (12)$$

where  $\eta$  is a positive constant satisfying  $\gamma = \eta - \alpha > 0$ ,  $g_{(0)ii} < 0$ ,  $g_{(1)ii} < 0$ ,  $i = 1, 2, \dots, N$  are defined in equality (2) and (3).

**Proof.** Select the following Lyapunov-krasovskii functional candidate

$$V(t) = \sum_{i=1}^N e_i^T(t)e_i(t) - 2 \sum_{i=1}^N \sigma_1 g_{(1)ii} \int_{t-\tau}^t h_1^T(e_i(\xi))h_1(e_i(\xi))d\xi, \quad (13)$$

The time derivative of  $V(t)$  along the solution of the error system (7) is

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t)[f(x_i(t)) - f(s(t)) + \sigma_0 \sum_{j=1}^N g_{(0)ij}h_0(e_j(t)) + \sigma_1 \sum_{j=1}^N g_{(1)ij}h_1(e_j(t-\tau)) + u_i] \\ & - 2\sigma_1 \sum_{i=1}^N g_{(1)ii}[h_1^T(e_i(t))h_1(e_i(t)) - h_1^T(e_i(t-\tau))h_1(e_i(t-\tau))], \end{aligned} \quad (14)$$

With property (2) and (3), we have

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t)\{f(x_i(t)) - f(s(t)) + \sigma_0 \sum_{j=1, j \neq i}^N g_{(0)ij}[h_0(e_j(t)) - h_0(e_i(t))] \\ & + \sigma_1 \sum_{j=1, j \neq i}^N g_{(1)ij}[h_1(e_j(t-\tau)) - h_1(e_i(t-\tau))] + u_i\} \\ & - 2\sigma_1 \sum_{i=1}^N g_{(1)ii}h_1^T(e_i(t))h_1(e_i(t)) - 2\sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}h_1^T(e_i(t-\tau))h_1(e_i(t-\tau)), \end{aligned} \quad (15)$$

Substituting the controllers (10)–(12) into (15) and considering Assumption 1, we have

$$\begin{aligned} \dot{V}(t) \leq & 2 \sum_{i=1}^N (\alpha - \eta)e_i^T(t)e_i(t) + 2\sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij}e_i^T(t)h_0(e_j(t)) - 2\sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij}e_i^T(t)h_0(e_i(t)) \\ & + 2\sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}e_i^T(t)h_1(e_j(t-\tau)) - 2\sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}e_i^T(t)h_1(e_i(t-\tau)) \\ & - 2\sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij}e_i^T(t)e_i(t) - 2\sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}e_i^T(t)e_i(t) \\ & - 2\sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij}h_0^T(e_i(t))h_0(e_i(t)) - 2\sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}h_1^T(e_i(t-\tau))h_1(e_i(t-\tau)). \end{aligned} \quad (16)$$

Obviously, we have

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij}h_0^T(e_i(t))h_0(e_i(t)) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij}h_0^T(e_j(t))h_0(e_j(t)), \quad (17)$$

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}h_1^T(e_i(t-\tau))h_1(e_i(t-\tau)) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij}h_1^T(e_j(t-\tau))h_1(e_j(t-\tau)). \quad (18)$$

Therefore, it follows that

$$\begin{aligned} \dot{V}(t) \leq & -2 \sum_{i=1}^N \gamma e_i^T(t) e_i(t) - \sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij} (e_i(t) + h_0(e_i(t)))^T (e_i(t) + h_0(e_i(t))) \\ & - \sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij} (e_i(t) - h_0(e_j(t)))^T (e_i(t) - h_0(e_j(t))) \\ & - \sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij} (e_i(t) + h_1(e_i(t - \tau)))^T (e_i(t) + h_1(e_i(t - \tau))) \\ & - \sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij} (e_i(t) - h_1(e_j(t - \tau)))^T (e_i(t) - h_1(e_j(t - \tau))). \end{aligned}$$

Note that  $\gamma > 0$ ,  $g_{(0)ij} > 0$ ,  $g_{(1)ij} > 0 (i, j = 1, 2, \dots, N, i \neq j)$ ,  $\sigma_0 > 0$ ,  $\sigma_1 > 0$ , so we have  $\dot{V}(t) < 0$ . By Lyapunov stability theory, the error dynamical system (7) is asymptotically stabilized under the controllers (10)–(12), i.e., the error  $e_i(t)$  converges to zero as  $t \rightarrow \infty$ , or (6) is obtained, so the complex dynamical network (1) is synchronized under controllers (10)–(12). This completes the proof. ■

### 3 A complex network with uncertain parameters and unknown coupling coefficients

#### 3.1 Model and assumption

In reality, it is often difficult to get exact information of coupling coefficients and the state equation of each node may include uncertain parameter. Therefore, we consider a class of complex network that is described by

$$\dot{x}_i(t) = F(x_i(t)) + G(x_i(t))\theta + \sigma_0 \sum_{j=1}^N g_{(0)ij} H_0(x_j(t)) + \sigma_1 \sum_{j=1}^N g_{(1)ij} H_1(\overline{x_j(t - \tau)}) + u_i, \quad i = 1, 2, \dots, N, \quad (19)$$

where  $F : R^n \rightarrow R^n, G : R^n \rightarrow R^{n \times m}$  are smooth nonlinear vector functions;  $\theta \in R^m$  is the unknown or uncertain parameter. The explanations of rest notations are the same as the explanations in system (1), where  $g_{(0)ij}$  and  $g_{(1)ij}$  are the unknown coupling coefficients. Similarly, the synchronous state  $s(t) \in R^n$  is a solution of an individual node, satisfying

$$\dot{s}(t) = F(s(t)) + G(s(t))\theta, \quad (20)$$

Subtracting (20) from (19) gives the error dynamical system

$$\dot{e}_i(t) = F(x_i(t)) - F(s(t)) + [G(x_i(t)) - G(s(t))]\theta + \sigma_0 \sum_{j=1}^N g_{(0)ij} h_0(e_j(t)) + \sigma_1 \sum_{j=1}^N g_{(1)ij} h_1(e_j(t - \tau)) + u_i, \quad (21)$$

where

$$\begin{aligned} h_0(e_j(t)) &= H_0(x_j(t)) - H_0(s(t)), \\ h_1(e_j(t - \tau)) &= H_1(\overline{x_j(t - \tau)}) - H_1(\overline{s(t - \tau)}). \end{aligned}$$

In order to obtain Theorem 2, we need the following Assumption 2.

**Assumption 2 (A2)** Suppose that there exists a nonnegative constant  $\alpha$  such that

$$\|F(x_i) - F(s)\| \leq \alpha \|x_i - s\|, \quad i = 1, 2, \dots, N.$$

#### 3.2 Controller design

**Theorem 2** Suppose that (A2) hold. Then the complex network (19) is synchronized under the controllers

$$u_i = -(G(x_i) - G(s(t)))\hat{\theta} - (\eta - \sigma_0 \hat{g}_{(0)ii} - \sigma_1 \hat{g}_{(1)ii})e_i + \sigma_0 \hat{g}_{(0)ii} v_i + \sigma_1 \hat{g}_{(1)ii} w_i, \quad (22)$$

$$v_i = \begin{cases} e_i \frac{h_0^T(e_i) h_0(e_i)}{e_i^T e_i}, & e_i^T e_i \neq 0 \\ 0, & e_i^T e_i = 0, \end{cases} \quad (23)$$

$$w_i = \begin{cases} e_i \frac{h_1^T(e_i) h_1(e_i)}{e_i^T e_i}, & e_i^T e_i \neq 0 \\ 0, & e_i^T e_i = 0, \end{cases} \quad (24)$$

and the updating laws

$$\dot{\hat{g}}_{(0)ii} = -e_i^T e_i - h_0^T(e_i)h_0(e_i), \quad (25)$$

$$\dot{\hat{g}}_{(1)ii} = -e_i^T e_i - h_1^T(e_i)h_1(e_i), \quad (26)$$

$$\dot{\hat{\theta}} = (G(x_i) - G(s(t)))^T e_i, \quad (27)$$

where  $\eta$  is a positive constant satisfying  $\gamma = \eta - \alpha > 0$ ,  $g_{(0)ii} < 0$ ,  $g_{(1)ii} < 0$ ,  $i = 1, 2, \dots, N$  are defined in equality (2) and (3).  $\hat{g}_{(0)ii}$ ,  $\hat{g}_{(1)ii}$  are the estimates of the unknown coupling coefficients  $g_{(0)ii}$ ,  $g_{(1)ii}$  respectively and  $\hat{\theta}$  is the estimates of the unknown parameter  $\theta$ .

**Proof.** Select the following Lyapunov-krasovskii functional candidate

$$V(t) = \sum_{i=1}^N e_i^T(t)e_i(t) + \sigma_0 \sum_{i=1}^N \tilde{g}_{(0)ii}^2 + \sigma_1 \sum_{i=1}^N \tilde{g}_{(1)ii}^2 + \tilde{\theta}^T \tilde{\theta} - 2 \sum_{i=1}^N \sigma_1 g_{(1)ii} \int_{t-\tau}^t h_1^T(e_i(\xi))h_1(e_i(\xi))d\xi, \quad (28)$$

where  $\tilde{\theta} = \theta - \hat{\theta}$ ,  $\tilde{g}_{(0)ii} = g_{(0)ii} - \hat{g}_{(0)ii}$ ,  $\tilde{g}_{(1)ii} = g_{(1)ii} - \hat{g}_{(1)ii}$ .

The time derivative of  $V(t)$  along the solution of the error system (21) is

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) \{ F(x_i(t)) - F(s(t)) + [G(x_i(t)) - G(s(t))] \theta + \sigma_0 \sum_{j=1}^N g_{(0)ij} h_0(e_j(t)) \\ & + \sigma_1 \sum_{j=1}^N g_{(1)ij} h_1(e_j(t-\tau)) + u_i \} + 2\sigma_0 \sum_{i=1}^N \tilde{g}_{(0)ii} (-\dot{\hat{g}}_{(0)ii}) + 2\sigma_1 \sum_{i=1}^N \tilde{g}_{(1)ii} (-\dot{\hat{g}}_{(1)ii}) \\ & + 2\tilde{\theta}^T (-\dot{\hat{\theta}}) - 2\sigma_1 \sum_{i=1}^N g_{(1)ii} [h_1^T(e_i(t))h_1(e_i(t)) - h_1^T(e_i(t-\tau))h_1(e_i(t-\tau))], \end{aligned} \quad (29)$$

Substituting the controllers (22)–(24) and the updating laws (25)–(27) into (29) and considering Assumption 2, we have

$$\begin{aligned} \dot{V}(t) \leq & 2 \sum_{i=1}^N (\alpha - \eta) e_i^T(t) e_i(t) + 2\sigma_0 \sum_{i=1}^N \sum_{j=1}^N g_{(0)ij} e_i^T(t) h_0(e_j(t)) + 2\sigma_1 \sum_{i=1}^N \sum_{j=1}^N g_{(1)ij} e_i^T(t) h_1(e_j(t-\tau)) \\ & + 2\sigma_0 \sum_{i=1}^N g_{(0)ii} e_i^T(t) e_i(t) + 2\sigma_0 \sum_{i=1}^N g_{(0)ii} h_0^T(e_i) h_0(e_i) \\ & + 2\sigma_1 \sum_{i=1}^N g_{(1)ii} e_i^T(t) e_i(t) + 2\sigma_1 \sum_{i=1}^N g_{(1)ii} h_1^T(e_i(t-\tau)) h_1(e_i(t-\tau)), \end{aligned}$$

With property (2), (3) and (17), (18), we have

$$\begin{aligned} \dot{V}(t) \leq & -2 \sum_{i=1}^N \gamma e_i^T(t) e_i(t) - \sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij} (e_i(t) + h_0(e_i(t)))^T (e_i(t) + h_0(e_i(t))) \\ & - \sigma_0 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(0)ij} (e_i(t) - h_0(e_j(t)))^T (e_i(t) - h_0(e_j(t))) \\ & - \sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij} (e_i(t) + h_1(e_i(t-\tau)))^T (e_i(t) + h_1(e_i(t-\tau))) \\ & - \sigma_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N g_{(1)ij} (e_i(t) - h_1(e_j(t-\tau)))^T (e_i(t) - h_1(e_j(t-\tau))) \\ & < 0. \end{aligned}$$

The rest of the proof is similar to that of Theorem 1 and omitted here. This completes the proof. ■

## 4 Example and simulation

We use the unified new chaotic system [23] to describe the dynamics of single oscillator, which can be described as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -x_2 x_3 \\ x_1 x_3 \\ 1/3 x_1 x_2 \end{pmatrix} + \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \triangleq f(x) = F(x) + G(x)\theta. \quad (30)$$

It is chaotic when  $a = 5.0, b = -10.0, c = -3.8$ , with initial condition  $(0.5, -1.0, 1.5)^T$ . The entire networked system is given as

$$\dot{x}_i = f(x_i) + \sigma_0 \sum_{j=1}^{50} g_{(0)ij} H_0(x_j) + \sigma_1 \sum_{j=1}^{50} g_{(1)ij} H_1(\overline{x_j(t-\tau)}), \quad i = 1, 2, \dots, 50, \quad (31)$$

where  $g_{(0)ii} = -2, i = 1, \dots, 50; g_{(0)i,i-1} = g_{(0)i-1,i} = 1, g_{(0)i,i+1} = g_{(0)i+1,i} = 1, i = 2, \dots, 49, g_{(0)1,50} = g_{(0)50,1} = 1$ , and the others  $g_{(0)ij} = g_{(0)ji} = 0; g_{(1)11} = -49, g_{(1)ii} = -1, g_{(1)1i} = g_{(1)i1} = 1, i = 2, \dots, 50$ , and the others  $g_{(1)ij} = g_{(1)ji} = 0; \sigma_0 = \sigma_1 = 1; \tau_i = 0.1 (i = 1, 2, 3)$ . The coupling functions are

$$H_0(x_j(t)) = \begin{pmatrix} x_{j1}(t) \\ x_{j2}(t) \\ x_{j3}(t) - x_{j1}(t)x_{j2}(t) \end{pmatrix}, \quad H_1(\overline{x_j(t-\tau)}) = \begin{pmatrix} \sin x_{j1}(t-\tau_1) \\ \cos x_{j2}(t-\tau_2) \\ \sin x_{j3}(t-\tau_3) \end{pmatrix}, \quad j = 1, 2, \dots, 50. \quad (32)$$

### 4.1 A certain network with nonlinear coupling

Let  $m = 2, \theta = (a, b, c)^T = (5.0, -10.0, -3.8)^T$ , using the controller in Theorem 1, the initial values are given as follows:  $\eta = -100, s(0) = (0.5, -1, 1.5)^T, x_i(0) = (0.5 + i \times 0.05, -1 + i \times 0.05, 1.5 + i \times 0.05)^T, i = 1, \dots, 25, x_i(0) = (0.5 + (i - 160) \times 0.05, -1 + (i - 160) \times 0.05, 1.5 + (i - 160) \times 0.05)^T, i = 25, \dots, 50$ . Since the attractor is confined to a bounded region, there exists a constant  $M > 0$ , satisfying  $\|x_i\| \leq M, \|s\| \leq M$ , therefore

$$\|f(x_i) - f(s)\| = \sqrt{(ae_{i1} - x_{i3}e_{i2} - e_{i3}s_2)^2 + (be_{i2} + x_{i1}e_{i3} + e_{i1}s_3)^2 + (ce_{i3} + 1/3x_{i1}e_{i2} + 1/3e_{i1}s_2)^2} \leq \sqrt{2(a^2 + b^2 + c^2 + 4M^2)} \|e_i\|, \quad i = 1, 2, \dots, 50.$$

Function  $f(x)$  satisfies Assumption 1, and then we can realize the synchronization of this complex system by employing the controller (10)–(12), the error curves are shown in Figs. 1. It can be seen that the designed controller can quickly stabilize the error system and realize the synchronization.

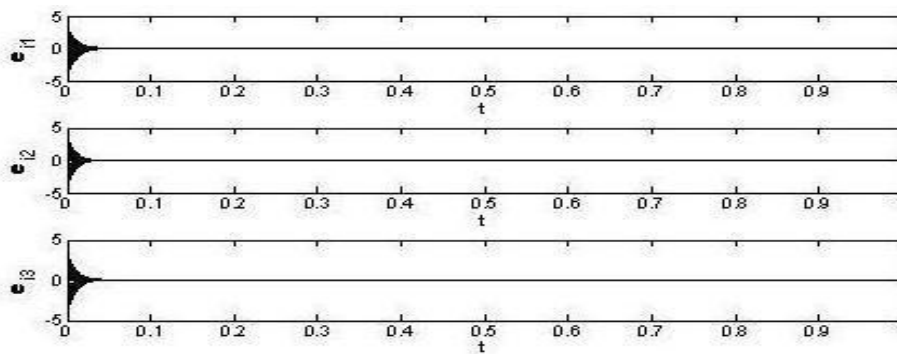


Figure 1: The synchronization errors  $e_{i1}, e_{i2}, e_{i3}$  of certain network.

### 4.2 A complex network with uncertain parameters and unknown coupling coefficients.

We still use network (31),  $\theta = (a, b, c)^T$  is uncertain parameters, it is easy to know function  $F(x)$  satisfies Assumption 2, using the controller in Theorem 2, the initial values are given as follows:  $\eta = -100, s(0) = (0.5, -1.0, 1.5)^T, x_i(0) = (0.5 + i \times 0.05, -1 + i \times 0.05, 1.5 + i \times 0.05)^T, i = 1, \dots, 25, x_i(0) = (0.5 + (i - 160) \times 0.05, -1 + (i - 160) \times 0.05, 1.5 + (i - 160) \times 0.05)^T, i = 26, \dots, 50, \theta(0) = (3.6, -11.4, -5.1)^T, \hat{g}_{(0)ii}(0) = -20; \hat{g}_{(1)ii}(0) = -1$ .

The synchronization errors  $e_{i1}, e_{i2}, e_{i3} (i = 1, \dots, 50)$  and the estimations  $\hat{a}, \hat{b}, \hat{c}, \hat{g}_{(0)ii}, \hat{g}_{(1)ii}$  of  $a, b, c, g_{(0)ii}, g_{(1)ii} (i = 1, \dots, 50)$  are shown respectively in Fig. 2 and Fig. 3. The numerical results show that controller is effective in Theorem 2.

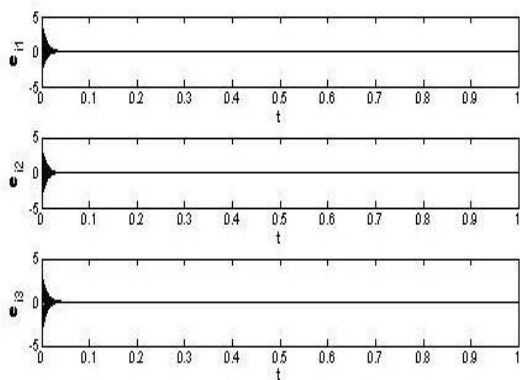


Figure 2: The synchronization errors  $e_{i1}$ ,  $e_{i2}$ ,  $e_{i3}$  of uncertain network.

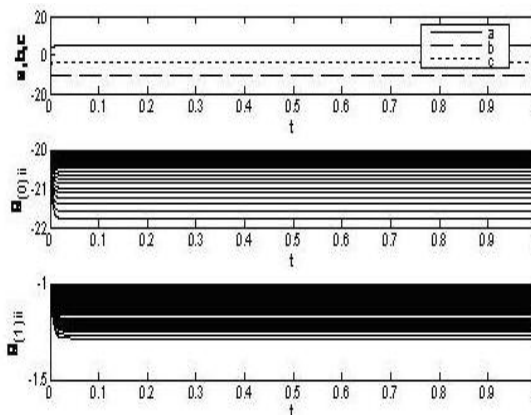


Figure 3: The estimations  $\hat{a}$ ,  $\hat{b}$ ,  $\hat{c}$ ,  $\hat{g}_{(0)ii}$ ,  $\hat{g}_{(1)ii}$  of  $a$ ,  $b$ ,  $c$ ,  $g_{(0)ii}$ ,  $g_{(1)ii}$ .

## 5 Conclusion

In this paper, we have investigated synchronization dynamics of a class of complex delayed networks with nonlinear coupling. Both known and uncertain networks are considered, some synchronization criteria have been deduced based on Lyapunov function method. The complex network which node is a new chaotic system is used to verify the effectiveness of the proposed method. This work extends the study of complex networks synchronization with nonlinear coupling.

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