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Which is the optimal fiscal rule in a monetary union? Targeting the structural, the global budgetary deficit, or the public debt?

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Abstract: The aim of our paper is to contribute to the debate on optimal fiscal rules in a monetary union: in terms of global budgetary deficit, of structural budgetary deficit, or of public debt. Indeed, these rules seem to be mixed in the framework of the European Economic and Monetary Union, with the new Fiscal Compact. With the help of a simple macroeconomic model, we show that a goal in terms of public debt is the most appropriate in order to decrease the indebtedness levels, but that it could increase the recessionary risks for the most indebted European countries. Goals in terms of global budgetary deficit or public debt are the most appropriate to limit the budgetary activism and to stabilize fiscal variables in case of demand or supply shocks. However, a goal in terms of structural budgetary deficit is the most appropriate in order to stabilize economic activity levels in case of asymmetric demand or supply shocks.

Keywords: monetary union, EMU, fiscal rules, public debt, structural budgetary deficit, global budgetary deficit

JEL classifications: E62, F02, F42

'Optimal fiscal rule: Structural, global budgetary deficit, or public debt?'

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S. Menguy teaches in France at the University Paris Descartes, in the department of social sciences. Her research activity mainly concerns the coordination of monetary and budgetary policies in the framework of the European Economic and Monetary Union, as well as the definition of optimal fiscal rules in a monetary union in order to provide a fiscal discipline which would not be harmful to economic efficiency.

PUBLIC INTEREST STATEMENT

The Fiscal Compact constrains the European countries to reduce their public debt levels at a sufficiently pace towards the reference value of 60% of GDP: their distance with respect to this reference value should decrease at a rate of the order of one-twentieth per year (on average over the previous three years). Indeed, the paper shows that a goal in terms of public debt is the most appropriate in order to decrease debt levels in a monetary union, though this could increase recessionary risks for the most indebted countries. Goals in terms of global budgetary deficit or public debt are also found most appropriate to limit budgetary activism and to stabilize fiscal variables in case of demand or supply shocks. However, the Fiscal Compact has combined the debt rule with a "Golden rule" in terms of structural budgetary deficit: the latter must remain below .5% of GDP. Indeed, such a goal seems the most opportune in order to stabilize prices and economic activity levels at times of asymmetric demand or supply shocks.

1. Introduction

In the framework of a monetary union, the necessity to introduce fiscal rules has widely been studied in the economic literature. In particular, in the European Economic and Monetary Union (EMU), the European Central Bank (ECB) conducts a common monetary policy for all member countries of the Euro-zone. Therefore, these countries have lost their independence to conduct independent monetary and exchange rates policies. Monetary unification removes from national authorities very useful instruments of economic policy, notably interest rates, and exchange rates variations. In these conditions, autonomous and active budgetary policies seem necessary to stabilize asymmetric shocks, and they shouldn't be heavily constrained. However, the width of fiscal externalities in a monetary union can justify the thresholds imposed on national budgetary policies, in order to avoid negative fiscal externalities on partner countries.

In a monetary union, the fiscal laxity of a country can increase interest rates and endanger financial stability for all partner countries. It is precisely what is happening since 2010 in Europe with the sovereign debt crisis. As mentioned by Buti and Franco (2005), monetary union membership can give rise to moral hazard problems: the fiscal discipline can be weakened if a fiscally lax country expects that the central bank or another country will bail it out. Besides, it can also imply a free riding behavior, which relies on the knowledge that a higher budgetary deficit would imply a smaller increase in average interest rates at the level of the whole monetary union. Indeed, financial markets are not always sufficiently efficient in order to discipline budgetary policies and to perfectly evaluate the risk premiums related to the public debts of the various member countries of a monetary union. Moreover, fiscal discipline is often considered as the necessary condition to allow an independent common central bank to pursue its fundamental goal of price stability in a monetary union. Indeed, too high indebtedness levels imply of risk of monetization of these public debts, and thus a risk of rising inflation in the whole monetary union.

To sum up, in a monetary union, the last autonomous instrument in the hands of national governments after monetary unification, i.e. budgetary policies, must keep some independence in order to stabilize asymmetric cyclical variations. However, fiscal rules are necessary and budgetary policies must also be constrained. As mentioned by the ECB (2012), the European sovereign debt crisis underlined that a stronger coordination of macroeconomic policies was necessary in the framework of a monetary union, as well as promoting convergence and competitiveness, and that imbalances were dangerous for the stability of the whole Euro-zone. This crisis has also reinforced the necessity and the urgency, for the European authorities, to strengthen the European institutional framework and governance, in particular by the way of more efficient fiscal rules. A fiscal rule is a permanent constraint on fiscal policy through simple numerical limits on budgetary aggregates. But what are the best fiscal rules, in a monetary union? Should the rule be related to the level of the budgetary deficit, and then, should it concern the global or the structural budgetary deficit? Or should the rule be related to the level of the public debt? More precisely, in Europe, the Stability and Growth Pact (SGP) was mainly constraining global budgetary deficits. On the contrary, the new Fiscal Compact reinforces the constraint on structural budgetary deficits, the latter being limited to .5% of GDP. Besides, it also insists on the reduction of the indebtedness levels of the European countries. Which of these rules are the most restrictive, and what are their respective advantages and drawbacks for the stabilization of various shocks?

The aim of the current paper is to contribute to this debate on optimal fiscal rules in a monetary union. The second section presents the fiscal rules in force in the EMU and gives a selected review of literature on optimal fiscal rules in a monetary union. The third section presents a simple model able to compare the respective advantages of distinct fiscal rules in order to stabilize various shocks. The fourth section studies the implications of a goal in terms of budgetary deficit or of public debt regarding economic stabilization. The fifth section analyzes the implications of various fiscal rules for the stabilization of demand or supply shocks. Finally, the sixth section concludes the paper.

2. European Framework and Review of Literature

2.1. From the SGP to the Fiscal Compact

In 1992, article 104c of the Maastricht Treaty and the annexed Protocol 5 included thresholds and constraints on the global budgetary deficit (which had to be below 3% of GDP) and on the public debt (which had to be below 60% of GDP). Afterwards, in 1996, the SGP added that national budgetary policies should "create room for maneuver in adapting to exceptional and cyclical disturbances," while avoiding excessive deficits. Hence, EMU member countries should have a medium-term budgetary objective "close to balance or in surplus," they should maintain a balanced budget over the economic cycle. This structural budgetary equilibrium should let enough room of maneuver for automatic stabilizers to play and for economic stabilization in the short run in case of macroeconomic shocks. However, we can mention that with a primary or global budgetary deficit nil in normal times, the public debt could tend towards zero, while public authorities need budgetary financing for projects which have long-run returns.

Buti and Giudice (2002), for example, underlined the problems of asymmetric working of the SGP, and of insufficient definition of medium term targets. Thus, after the reform of the SGP in 2005, there was more flexibility in the medium-term deficit target but also a more restrictive definition of the fiscal objective: it was clearly stated that the medium-term target should apply to the cyclically adjusted budget balance. Indeed, the primary budgetary deficit has two components: a structural part and a cyclical or conjunctural part. The conjunctural deficit is a decreasing function of the output gap, of the excess output in comparison with its potential value. On the opposite, the structural deficit is the public deficit which has been cleared of cyclical effects and one-off measures, as well as of temporary measures. In this framework, the main advantage of a shift to a structural deficit rule would be the increased flexibility in dealing with cyclical stabilization. On the other hand, the main difficulty relies naturally on the possibility to evaluate a structural deficit. Indeed, the operational simplicity of such an indicator is widely put into question given the reference to controversial magnitudes of the variables used in calculation, such as the output gap and the NAIRU.

On 29 September 2010, the European Commission proposed a revision of the SGP, a set of six legislative proposals aiming at strengthening the European economic governance (see e.g. European Commission [2011][1] or ECB [2012] for a complete description, or Tamborini [2011]). The 3% of GDP limit for budgetary deficits and the medium-term objective of budgetary positions in balance are maintained. But the so-called "Six Pack" has introduced a new set of rules for economic and fiscal surveillance in Europe.

- Countries will face sanctions if public spending increases more rapidly than GDP, unless this is compensated by a rise in taxation or if they run a budgetary surplus.
- Countries in situation of excessive deficit will face sanctions if they do not cut their structural deficit by .5% per year.
- Countries running a higher than 60% of GDP public debt ratio will be under an Excessive Deficit Procedure (EDP) if this debt ratio does not fall by 1/20th per year of the gap between the effective debt and the 60% reference value (on average over three years). Nevertheless, given that most countries are today in EDP in the European Union and have to comply with agreed fiscal consolidation paths, they are granted a three-year period following the correction of the excessive deficit for meeting the debt rule.

Therefore, the corrective part of the SGP has been strongly reinforced. Countries should submit each year a Stability and Convergence Program, and reduce their budgetary deficits according to a schedule proposed by the Commission. Besides, the indebtedness levels are now much more controlled and scrutinized in Europe. This legislative package, the so-called "Six Pack," is the fiscal part of a new "Treaty on Stability, Coordination and Governance," called the "Fiscal Compact," which entered into force on 1 January 2013, after ratification by 12 Euro Area countries.

Therefore, in conclusion, Verhelst (2012) mentions that in the framework of the Fiscal Compact, the "new Golden rule" (a structural deficit below .5% of GDP) is the most restrictive for a country whose growth rate is sufficiently high. However, according to Creel, Hubert, and Saraceno (2012b), the debt rule will remain the most binding as long as indebtedness levels will be excessively high in the European monetary union.

2.2. Economic Literature

The new rules mentioned in the Fiscal Compact have revived the debate, in the economic literature, about the most appropriate fiscal rules in a monetary union. Should they be related to the budgetary deficit or to the indebtedness level?

De Haan, Berger, and Jansen (2004) considered that SGP enforcement mechanisms were too weak, as they could be classified as "soft law" (possibility of deviation in specific circumstances, lack of impartial enforcement mechanism). They argued that stricter rules were necessary, which should not rely on cyclically adjusted deficit estimates, because it is impossible to evaluate precisely the output gap. Nevertheless, Schaechter, Kinda, Budina, and Weber (2012) mention that a budget balance rule has the advantage to target a variable which is largely under the control of governments, and which primarily influences the public debt. Debt rules are the most appropriate to the goal of long-run budgetary sustainability; however, debt levels take time to be impacted by budgetary measures and thus, they don't provide clear short-term guidance for policy-makers. Moreover, debt developments can be affected by parameters beyond the control of governments, as interest rates or exchange rates variations, or exceptional financial support or guarantees.

To protect the sustainability of public finances is the main goal, for the budgetary authorities. Indeed, with a two-country microfounded New-Keynesian model of a monetary union, Machado and Ribeiro (2012) find that a high public debt level hampers business cycle stabilization, by creating incentives to bias monetary policy from an inflation-stabilization assignment to a debt-stabilization assignment. Therefore, many authors insist on the fact that these are the indebtedness levels which must be scrutinized in a monetary union. Pisani-Ferry (2002) already proposed to put a threshold on the public debt levels. Indeed, he proposed the option, for EMU member countries, to subscribe voluntarily to a "Debt sustainability pact," where countries would engage to maintain their public debt below 50% of GDP for example, and would in exchange be exempted from any EDP.

Furthermore, regarding fiscal rules, Marattin and Marzo (2008) analyze the optimal stabilization policies in the framework of a highly distorted economy. They find that for the governments, a tax rule responding to public debt is welfare improving provided monetary policy's response to output is not zero; on the contrary, fiscal policy should better react to the budgetary deficit if monetary policy gives up on the output stabilization task. A target in terms of public debt would then be optimal provided the monetary authority can take care of short-run economic stabilization. So, Hughes Hallett and Jensen (2011) argue that fiscal targets should be long-term objectives, while the central bank should be involved with short-term stabilization. Indeed, fiscal policies aim at long-run objectives: public services, social support, education, research and development, or infrastructure. In this framework, the authors suggest the adoption of soft debt targets (a "fiscal space": a target, going with the definition of band and a ceiling), in order to provide a forward looking path for sustainable budgetary policies. In the same way, Checherita-Westphal, Hughes Hallett, and Rother (2012) underline the benefits of a debt ceiling combined with a fiscal adjustment rule. Indeed, such a rule allows short-run flexibility to adapt to cyclical fluctuations; it has the advantages of a structural rule without having to calculate accurately structural deficits. According to the authors, growth maximizing public debt ratios would then be around 50% of GDP for EMU member countries.

Finally, Creel, Hubert, and Saraceno (2012a) simulate the macroeconomic effects of the adoption of various fiscal rules in a reduced form small scale New-Keynesian model. They compare the so-called "Fiscal Compact": a balanced budget rule (a structural deficit at .5% of GDP) and the debt reduction rule, the Maastricht 3% deficit limit (status quo), and an "investment" rule (imposing budget balance over the cycle only for current spending, while allowing public investment to be financed through debt). They find that all rules guarantee long-run sustainability of public finances. However, the investment rule emerges robustly as the one guaranteeing the lowest output loss, followed by the status quo, whereas the "Fiscal Compact" rules appear to be more recessionary. Blanchard and Giavazzi (2004) also mention the advantages of a fiscal rule which would distinguish public investment from consumption expenditure in the EMU.

In the framework of this voluminous literature on optimal fiscal rules in a monetary union, the contribution of our paper is the following. With the help of a simple macroeconomic model, we analyze the detailed implications and respective advantages of a rule in terms of global or structural budgetary deficit or of public debt for the stabilization of various shocks. Therefore, the optimal rules that our model defines should be considered as normative principles for the design of the best EMU rules.

3. A Model Intended for Comparing Various Fiscal Rules

In this section, we consider a dynamic open macroeconomic New-Keynesian model, in order to compare the respective advantages of various fiscal rules: an objective in terms of structural budgetary deficit, of global budgetary deficit or of public debt, in order to stabilize demand or supply shocks in a monetary union. All variables are expressed in deviations from their long-run equilibrium values. For simplicity, the monetary union is supposed to be closed vis-à-vis the rest of the world. All potential fiscal rules are supposed to have the same goal in the long-term: the inter-temporal sustainability of fiscal policies. However, thanks to our model, we will try to define the most efficient fiscal rules for the short-term economic stabilization.

3.1. The Supply and Demand Functions

We use a stylized dynamic New-Keynesian model, which is broadly consistent with this literature, even if we don't detail here its underlying microeconomic structure (see e.g. Lieb [2012] or Galí and Monacelli [2008] for the microfoundations of the model). The linearized demand equation for the country (i) in period (t) is:

$$y_{i,t}^{d} = y_{i,t+1}^{e} - \sigma\left[i_t - \pi_{i,t+1}^{e} - r_{i,t}^{*}\right] + \eta(\pi_{j,t} - \pi_{i,t}) - (1-\sigma)\left(g_{i,t+1}^{e} - g_{i,t}\right) + \varepsilon_{i,t}^{d} \quad (1)$$

$0 < \sigma < 1$: Share of private consumption in GDP, in the efficient steady-state, in comparison with the share of public consumption: $(1-\sigma)$.

$0 < \eta < 1$: Openness to trade of the member countries of the monetary union.

With, in period (t) in the country (i): ($y_{i,t}$): output; ($g_{i,t}$): public expenditure; ($\pi_{i,t}$): inflation rate; ($r_{i,t}^{*}$): natural or equilibrium real interest rate, time discount rate which regulates the private sector's borrowing and lending; ($\varepsilon_{i,t}^{d}$): demand shock; ($\text{def}_{i,t}$): primary budgetary deficit net of cyclical component; and in the monetary union: (i_t): nominal interest rate; (^{e}) denotes the operator of anticipation, whereas (*) denotes a potential level when economic variables are at their equilibrium value.

Traditionally, in New-Keynesian models, aggregate demand is driven by the optimizing behavior of households, which maximize an intertemporal utility function. Thus, output depends on expected future output, because rational agents can smooth their consumption intertemporally. Variation in demand is also an increasing function of the variation in public expenditure in a given country in comparison with what is expected for the following period ($g_{i,t} - g_{i,t+1}^{e}$). It is a decreasing function of the excess of the anticipated real interest rate in the country (i) in comparison with its equilibrium value. Besides, net exports of the country (i) are an increasing function of its price competitiveness. Finally, variation in demand is also an increasing function of a national positive demand shock, which is a white noise: $E_t\left(\varepsilon_{i,t}^{d}\right) = 0$.

The primary budgetary deficit in the country (*i*) is the differential between its public expenditures and resources:

$$\text{def}_{i,t} = g_{i,t} - \tau_t y_{i,t} \tag{2}$$

with τ_t is the taxation rate; *k* is the GDP elasticity of the deficit, or automatic stabilizers.

The structural primary budgetary deficit depends on structural public expenditure net of structural tax revenues that are realized when the economy is in equilibrium at potential output:

$$\text{def}_{si,t} = g^*_{i,t} - \tau^*_t y^*_{i,t} \tag{3}$$

So, if we split the primary budgetary expenditures and resources of the country (*i*) between a structural and a conjunctural part, we obtain:

$$g_{i,t} = g^*_{i,t} - k(y_{i,t} - y^*_{i,t}) \tag{4}$$

$$\tau^*_t y_{i,t} = \tau^*_t y^*_{i,t} + \tau^*_t (y_{i,t} - y^*_{i,t}) \tag{5}$$

Therefore, by combining Equations 3 and 4, we have:

$$g_{i,t} = \text{def}_{si,t} + (\tau^*_t + k) y^*_{i,t} - k y_{i,t} \tag{6}$$

The linearized supply function for the country (*i*) in period (*t*) is represented by a forward looking New-Keynesian Phillips curve:

$$\pi_{i,t} = b\pi^e_{i,t+1} + v(y^s_{i,t} - y^*_{i,t}) - \varepsilon^s_{i,t} \tag{7}$$

with, in period (*t*) in the country (*i*): $(\varepsilon^s_{i,t})$**:** positive supply or productivity shock, or negative cost-push shock, which is a white noise. $0 < b < 1$: time discount factor; $0 < v < 1$.

Indeed, in New-Keynesian models, aggregate supply results from the behavior of firms that set prices for their products so as to maximize profits in a monopolistic competition setting. Inflation then depends on expectations about future prices, because of learning effects. Besides, the output gap expresses the demand-pull factor and tensions on the utilization of productive capacities. In this framework, (*v*) is a parameter representing the degree of price flexibility in the monetary union. It is an increasing function of the labor market flexibility, of the sensitivity of prices to marginal costs, of the sensitivity of real wages to the number of workers and to unemployment. Finally, $(\varepsilon^s_{i,t})$ captures a deflationary shock unrelated to excess demand (markup, etc.).

Let's suppose that the goal of the common central bank is to preserve price stability in all the monetary union ($\pi_t = 0$), according to the status of the ECB. In this context, the monetary authority should fix the common nominal interest rate at the following level (see Appendix A):

$$i_t = r^*_t - \frac{(1 - \tau^*_t + \sigma\tau^*_t)}{\sigma} y^*_t + \frac{(1-\sigma)}{\sigma} \text{def}_{s,t} \tag{8}$$

Therefore, in the framework of our model, the game between the governments and the common central bank can be summarized by conflicting economic policies in the following way. The higher the average structural budgetary deficit in all the monetary union, the higher the common interest rate and the more restrictive the common monetary policy. In the same way, the higher the common interest rate, the more expansionary are the budgetary policies in the member countries of the monetary union.

In this context, in the short run, according to Equations 1 and 6–8, demand and supply shocks affect prices and economic activity levels in the following way (see Appendix A for the calculations):

$$\begin{aligned}2\sigma(1+k-k\sigma)(1+k-k\sigma+2v\eta)\pi_{i,t} &= v\sigma(1-\sigma)(1+k-k\sigma)(\text{def}_{si,t}-\text{def}_{sj,t})\\ &+2\sigma v(1+k-k\sigma+2v\eta)\varepsilon_t^d+2\sigma v(1+k-k\sigma)\overline{\varepsilon_t^d}\\ &-2\sigma(1+k-k\sigma)^2\overline{\varepsilon_t^s}-2\sigma(1+k-k\sigma)(1+k-k\sigma+2v\eta)\varepsilon_t^s\\ &+f(y_{i,t}^*,y_{j,t}^*,r_{i,t}^*,r_{j,t}^*)\end{aligned} \tag{9}$$

$$\begin{aligned}2\sigma(1+k-k\sigma)(1+k-k\sigma+2v\eta)y_{i,t} &= \sigma(1-\sigma)(1+k-k\sigma)(\text{def}_{si,t}-\text{def}_{sj,t})\\ &+2\sigma(1+k-k\sigma+2v\eta)\varepsilon_t^d+2\sigma(1+k-k\sigma)\overline{\varepsilon_t^d}\\ &+4\sigma\eta(1+k-k\sigma)\overline{\varepsilon_t^s}+f(y_{i,t}^*,y_{j,t}^*,r_{i,t}^*,r_{j,t}^*)\end{aligned} \tag{10}$$

Therefore, in a given country (*i*), the inflation rate and the economic activity level are an increasing function of symmetric positive demand shocks. An asymmetric demand shock or an increase in the structural budgetary deficit higher than in the partner country increases inflation and economic growth in the positively affected country, whereas they imply a recession in the negatively affected country. Besides, symmetric positive supply shocks decrease prices in all the monetary union without affecting economic activity levels. On the contrary, an asymmetric supply shock implies expansionary and deflationary (recessionary and inflationary) tensions in the country of the monetary union positively (negatively) affected by the shock.

3.2. Determination of Optimal Budgetary Policies

We suppose that budgetary policies are endogenous, and are the result of the minimization of a complex loss function. Indeed, we consider that each budgetary authority has multiple goals, to which various weights are granted, in order to cover the larger range of potential behaviors. Therefore, the preferences of the government (*i*) are given by the following quadratic loss function:

$$L_{i,t}=\frac{1}{2}\left[\alpha_\pi\pi_{i,t}^2+\alpha_y(y_{i,t}-y_{i,t}^*)^2+\alpha_{g,s}(\text{def}_{si,t}-\text{def}_s^T)^2+\alpha_{g,g}(\text{def}_{i,t}-\text{def}^T)^2+\alpha_{g,d}(d_{i,t}-d^T)^2\right] \tag{11}$$

where the parameters (α_π), (α_y), and (α_g) express the respective weights of inflation, economic activity, and variation in the budgetary instrument for the government (*i*), and where (*T*) denotes a targeted value for each fiscal variable. The targeted variation in inflation is zero, in conformity with the goal of price stability, whereas the economic activity target is its potential level. Besides, this loss function allows studying the consequences of various budgetary goals pursued by the governments in a monetary union: in terms of structural budgetary deficit (def_s) with a weight ($\alpha_{g,s}$) or in terms of global budgetary deficit (def) with a weight ($\alpha_{g,g}$) inclusive of interest payments on the former public debt level, or in terms of public debt (*d*) with a weight ($\alpha_{g,d}$).[2] These weights are not necessarily equal; they depend on the political costs of potential sanctions that can be associated with the deviation of one budgetary instrument from its targeted value. Let's mention that for simplicity, this paper considers the global public debt level, without distinguishing between the indebtedness due to public investment expenditure which is supposed to generate future growth and future fiscal resources, justified from a productive point of view, and the indebtedness due to consumption expenditure, which should be forbidden. Indeed, this interrogation about justification and advantages of a "Golden rule" would be the subject of another complete paper.

Then, thanks to variations in the weights given to distinct budgetary instruments, we can define the best fiscal rules in order to stabilize economic variables (activity and prices) at the smallest cost for the variation in budgetary instruments. According to Equations 2, 6, and 10, the primary budgetary deficit of the country (*i*) in period (*t*) in percentage of GDP ($\text{def}_{i,t}$) is as follows:

$$
\begin{aligned}
&2\sigma(1+k-k\sigma)(1+k-k\sigma+2v\eta)\mathrm{def}_{i,t}\\
&\quad=\sigma(1+k-k\sigma)[2+(k-\tau_t)(1-\sigma)+4v\eta]\mathrm{def}_{si,t}-2\sigma(\tau_t+k)(1+k-k\sigma)\overline{\varepsilon_t^d}\\
&\qquad+\sigma(1-\sigma)(1+k-k\sigma)(\tau_t+k)\mathrm{def}_{sj,t}-2\sigma(1+k-k\sigma+2v\eta)(\tau_t+k)\varepsilon_t^d\\
&\qquad-4\sigma\eta(\tau_t+k)(1+k-k\sigma)\overline{\varepsilon_t^s}+f(y^*_{i,t},y^*_{j,t},r^*_{i,t},r^*_{j,t})
\end{aligned}
\tag{12}
$$

If we suppose that the public debt increases with the primary budgetary deficit and with the interest rate charges on this public debt [$d_{i,t}=(1+i_t)d_{i,t-1}+\mathrm{def}_{i,t}$], using Equations 8 and 12, the public debt of the country (i) in percentage of GDP ($d_{i,t}$) is:

$$
\begin{aligned}
&2\sigma(1+k-k\sigma)(1+k-k\sigma+2v\eta)(d_{i,t}-d_{i,t-1})\\
&\quad=[2\sigma(1+2v)+(k-\tau_t)\sigma(1-\sigma)+(1+k-k\sigma+2v\eta)(1-\sigma)d_{i,t-1}](1+k-k\sigma)\mathrm{def}_{si,t}\\
&\qquad+[(1+k-k\sigma+2v\eta)d_{i,t-1}+\sigma(\tau_t+k)](1-\sigma)(1+k-k\sigma)\mathrm{def}_{sj,t}\\
&\qquad-2\sigma(1+k-k\sigma+2v\eta)(\tau_t+k)\varepsilon_t^d-2\sigma(\tau_t+k)(1+k-k\sigma)\overline{\varepsilon_t^d}\\
&\qquad-4\sigma\eta(\tau_t+k)(1+k-k\sigma)\overline{\varepsilon_t^s}+f(y^*_{i,t},y^*_{j,t},r^*_{i,t},r^*_{j,t})
\end{aligned}
\tag{13}
$$

We suppose that the instrument of the government (i) (variable under control) is its structural primary budgetary deficit ($\mathrm{def}_{si,t}$), as the global budgetary deficit also depends on the economic conjuncture and is thus less easily controllable. The choice of an optimal structural primary budgetary deficit consists in fixing either the public expenditure for a given taxation rate, or the taxation rate for a given level of public expenditure. Therefore, using Equations 9, 10, 12, and 13, the optimal structural primary budgetary deficit of the country (i), which minimizes Equation 11, is the following:

$$
\begin{aligned}
m_{0,i}\mathrm{def}_{si,t}=\;&m_{3,i}(1-\sigma)\mathrm{def}_{sj,t}-2\sigma m_{1,i}\tfrac{(1+k-k\sigma+2v\eta)}{(1+k-k\sigma)}\varepsilon_t^d-2\sigma m_{1,i}\overline{\varepsilon_t^d}+2\sigma m_{2,i}\overline{\varepsilon_t^s}\\
&+2\sigma^2\alpha_\pi v(1+k-k\sigma+2v\eta)(1-\sigma)\varepsilon_t^s+4\sigma^2(1+k-k\sigma+2v\eta)^2\alpha_{g,s}\mathrm{def}_s^T\\
&+2\sigma^2(1+k-k\sigma+2v\eta)[2+(k-\tau_t)(1-\sigma)+4v\eta][\alpha_{g,g}\mathrm{def}^T+\alpha_{g,d}(d^T-d_{i,t-1})]\\
&+2\sigma\alpha_{g,d}(1-\sigma)d_{i,t-1}(1+k-k\sigma+2v\eta)^2(d^T-d_{i,t-1})+f(y^*_{i,t},y^*_{j,t},r^*_{i,t},r^*_{j,t})
\end{aligned}
\tag{14}
$$

Afterwards, we can find the optimal structural primary budgetary deficit ($\mathrm{def}_{si,t}$), and replace this equation in Equations 9, 10, 12, and 13 in order to obtain the optimal levels of all economic variables (see Appendix B for calculation and detailed values of m_i). What are then the implications of our model, regarding the advantages of the various potential fiscal goals of the budgetary authorities for economic stabilization?

4. The Various Goals of the Budgetary Authorities

4.1. Calibration of the Parameters of the Model

In order to obtain theoretical estimations of our results, we must first calibrate our model. In the rest of the paper, we will use the following basic calibration:

- Share of private consumption in GDP (σ): Galí and Monacelli (2008) and Lieb (2012) consider a high value for this parameter: σ=.75. Vogel, Roeger, and Herz (2006) suppose: σ=.73. Therefore, we will take the following value: σ=.75.
- Openness to trade: (η): Galí and Monacelli (2008) or Lieb (2012) consider a moderate openness: η=.3. On the contrary, Vogel et al. (2006) suppose a higher openness: η=.73. As the situation of the European countries is very heterogeneous regarding this criterion, we will take: η=.5.
- Price flexibility (v): Kienzler and Schmid (2010): v=.13 and Vogel et al. (2006): v=.26 consider that it is moderate in Europe. Thus, we will take: v=.2.

Table 1. Budgetary Deficits and Public Debts in EMU Member Countries, in 2012

	$\mathbf{def}_{si,t}$	$\mathbf{def}_{i,t}$	$d_{i,t}$		$\mathbf{def}_{si,t}$	$\mathbf{def}_{i,t}$	$d_{i,t}$
Austria	−.2	−.1	74.0	Belgium	−.2	.6	99.8
Cyprus	2.9	3.2	85.8	Estonia	1.0	.1	9.8
Finland	.0	.8	53.6	France	1.1	2.3	90.2
Germany	−2.5	−2.5	81.0	Greece	−1.6	4.0	156.9
Ireland	4.0	4.5	117.4	Italy	−4.2	−2.5	127.0
Latvia	−.6	.0	40.6	Luxembourg	−1.5	.1	21.7
Malta	−.2	.2	71.1	Netherlands	.8	2.2	71.3
Portugal	.4	2.1	124.1	Slovakia	1.9	2.7	52.4
Slovenia	.4	1.7	54.4	Spain	5.1	7.6	86.0

Source: AMECO database.

Notes: $\text{def}_{si,t}$: Primary structural budgetary deficit: net lending (−) or net borrowing (+) excluding interest—general government, percent of GDP at market prices—adjusted for the cyclical component, based on potential GDP.

$\text{def}_{i,t}$: Primary budgetary deficit: net lending (−) or net borrowing (+) excluding interest—general government, percent of GDP at market prices.

$d_{i,t}$: General government consolidated gross debt, percent of GDP at market prices.

- Automatic stabilizers (k): in conformity with the value usually reported in the literature (Creel and Saraceno [2010], Creel et al. [2012b]), we will take: $k=.5$.
- Taxation rates: we will suppose $\tau=.4$, in conformity with average taxation rates in the European Union.
- Indebtedness levels (d_{t-1}): we will take $d_{t-1}=.8$ (see Table 1).
- Finally, regarding the preferences of the governments, we will assume: $\alpha_{\pi}=.2$, $\alpha_{y}=1$, and $\alpha_{g}=.1$, as they mainly aim at stabilizing economic activity levels.

4.2. A Goal in Terms of Budgetary Deficit

According to Table 1, a goal in terms of budgetary deficit is harder to enforce in EMU member countries such as Spain, Ireland, Cyprus, or Greece. However, what are the theoretical teachings of our model regarding its implications? If the member countries of a monetary union have a goal in terms of budgetary deficit ($\alpha_{g,s}>0$ or $\alpha_{g,g}>0$), the latter increases in proportion to the targeted deficit in both countries. According to Equations 15 and 16, the effective (global or structural) budgetary deficit increases proportionally to its targeted level in both countries. Indeed, according to Equations B1, B2, and B5 in Appendix B, we obtain:

$$0<\frac{\partial \text{def}_{si,t}}{\partial \text{def}_{s}^{T}}=\frac{\sigma}{\left[\sigma+(1-\sigma)d_{i,t-1}\right]}\frac{\partial\left(d_{i,t}-d_{i,t-1}\right)}{\partial \text{def}_{s}^{T}}\underset{\alpha_{g,g}\to 0}{\longrightarrow}1 \tag{15}$$

$$0<\frac{\partial \text{def}_{i,t}}{\partial \text{def}^{T}}=\frac{\sigma}{\left[\sigma+(1-\sigma)d_{i,t-1}\right]}\frac{\partial\left(d_{i,t}-d_{i,t-1}\right)}{\partial \text{def}^{T}}\underset{\alpha_{g,s}\to 0}{\longrightarrow}1 \tag{16}$$

Therefore, with a higher targeted budgetary deficit, the public debt increases in the member countries of the monetary union, according to Equations 15 and 16. Besides, in a given country, the public debt increases more than proportionally to the targeted budgetary deficit, and all the more as a country was previously indebted in the former $(t-1)$ period. Indeed, a higher structural budgetary deficit increases the common interest rate in the monetary union according to Equation 8, and therefore the charges on the public debts of the member countries. For example, with the basic calibration of our model, for a country whose public debt represents 80% of its GDP, the

public debt would increase 26.67% more than the targeted structural or global budgetary deficit.

Nevertheless, Equations B1 and B2 in Appendix B show that combining a goal in terms of budgetary deficit with a goal in terms of public debt, as it is explicitly the case with the European "Fiscal Compact," can strongly reduce variations in the budgetary deficit in comparison with its targeted level. With such a combination of fiscal rules, the public debt can even decrease if the targeted budgetary deficit increases (then the derivatives in Equations 15 and 16 are smaller than one).

Finally, according to Equations 9 and 10, variations in inflation and in economic activity only depend on the differential between the domestic and foreign structural budgetary deficits. So, if both vary in proportion, targeted budgetary deficits don't influence inflation and economic activity levels according to Equations B3 and B4; we obtain:

$$\frac{\partial y_{i,t}}{\partial \text{def}_s^T} = \frac{\partial \pi_{i,t}}{\partial \text{def}_s^T} = \frac{\partial y_{i,t}}{\partial \text{def}^T} = \frac{\partial \pi_{i,t}}{\partial \text{def}^T} \underset{\alpha_{g,d}\to 0}{\longrightarrow} 0 \tag{17}$$

4.3. A Goal in Terms of Public Debt

If the governments have a goal in terms of public debt ($\alpha_{g,d}>0$), a former public debt level higher than the targeted level ($d_{i,t-1}>d^T$) in a given country necessitates to decrease its structural and global budgetary deficit, in order to reduce its indebtedness level, according to Equations B1 and B2 in Appendix B. For example, with the basic calibration of our parameters, if the public debt of a country represents 80% of its GDP (so, the targeted level: $d^T=60\%$ in the Maastricht Treaty is 25% smaller), to reduce optimally the public debt until 64.90% of GDP, we have: $\partial \text{def}_{si,t}=-18.45\%$; $\partial \text{def}_{i,t}=-16.70\%$; $\partial y_{i,t}=-1.94\%$; $\partial \pi_{i,t}=-.39\%$.

Therefore, if the member countries of a monetary union have a goal in terms of public debt, as soon as this level was formerly higher than the target, the budgetary policy of a given country must be contractionary in order to make its public debt converge towards this target. So, according to Equations B3 and B4 in Appendix B, this implies a limited recession and deflation in this country.

To conclude, a public debt goal can be appropriate in order to reduce the indebtedness level and to tend towards a specific and defined public debt level. Indeed, targeting a positive structural or global budgetary deficit risks increasing the public debt more than proportionally to the target. On the contrary, a goal in terms of public debt allows a decrease in the public debt towards a targeted level, and thus the long-term sustainability of public finances. In the framework of the sovereign debt crisis in Europe, with excessive debt levels in many European countries, a public debt goal is thus more relevant. Nevertheless, our model underlines the dangers of a goal in terms of public debt for economic growth. The Fiscal Compact could then imply higher recessionary risks for the very indebted countries than the former rules in terms of budgetary deficit. In the framework of a monetary union, a public debt goal has asymmetric consequences on the member countries according to their indebtedness levels. Thus, our model can contribute to explain the difficulties encountered today by the excessively indebted European countries (Greece, Portugal, Ireland or Italy according to Table 1), which have to undertake contractionary measures with dangerous recessionary consequences in order to reduce their public debts.

5. Stabilization of Demand or Supply Shocks

This section studies the stabilization of demand or supply shocks in case of various fiscal rules. Which rules are the most appropriate to stabilize various shocks?

5.1. Optimal Budgetary Policies in Case of Demand Shocks

A symmetric (ε_t^d) or asymmetric ($\overline{\varepsilon_t^d}$) positive demand shock increases the budgetary surplus above its equilibrium value (contractionary budgetary policies); there is a counter-cyclical reaction to

shocks. The public debt can then usually be reduced in the positively affected countries, whereas it increases in the negatively affected countries. Indeed, according to Equation B1 in Appendix B, we obtain:

$$\frac{\partial \text{def}_{si,t}}{\partial \varepsilon_t^d} < 0 \tag{18}$$

$$\frac{\partial \text{def}_{si,t}}{\partial \overline{\varepsilon_t^d}} < 0 \tag{19}$$

More precisely, in case of a positive symmetric demand shock (ε_t^d), economic policies are conflicting and the interest rate then usually decreases (expansionary monetary policy) in order to compensate for the contractionary budgetary policies. For example, with the basic calibration of our model, with a goal in terms of structural budgetary deficit ($\alpha_{g,s}$=.1), we obtain: i_t=−.28ε_t^d; $\text{def}_{si,t}$=−.85ε_t^d; $\text{def}_{i,t}$=−1.65ε_t^d. Moreover, a goal in terms of global budgetary deficit seems more efficient in order to limit the budgetary activism. Indeed, it takes into account the positive effect of the shock to improve fiscal resources, which allows showing smaller structural primary budgetary surpluses. Thus, with our basic calibration, if ($\alpha_{g,g}$=.1), we obtain: i_t=−.04ε_t^d; $\text{def}_{si,t}$=−.12ε_t^d; $\text{def}_{i,t}$=−.92ε_t^d. Finally, a goal in terms of public debt seems the most efficient in order to reduce the budgetary activism. Indeed, it takes into account the monetary reaction to fiscal shocks, and the necessity of weak structural budgetary surpluses in order to reduce variations in interest rates (both are then negligible) and in the public debt. Therefore, with our basic calibration, if ($\alpha_{g,d}$=.1), we obtain: $\text{def}_{i,t}$=−.80ε_t^d. Nevertheless, as long as stabilizing economic activity remains more important than the fiscal goal, the high primary budgetary surpluses allow a reduction in the public debt in case of a symmetric positive demand shock.

Besides, monetary policy can't stabilize asymmetric demand shocks ($\overline{\varepsilon_t^d}$); variations in interest rates are negligible. Budgetary surpluses in the positively affected countries are then compensated by similar budgetary deficits in the countries negatively affected by the shock. For example, with the basic calibration of our model, in case of a goal in terms of structural budgetary deficit ($\alpha_{g,s}$=.1), we have: $\text{def}_{si,t}$=−.61$\overline{\varepsilon_t^d}$; $\text{def}_{si,t}$=−1.81$\overline{\varepsilon_t^d}$. As previously mentioned, a goal in terms of global budgetary deficit ($\alpha_{g,g}$=.1) is then more efficient in order to limit the budgetary activism. So, with our basic calibration, we obtain: $\text{def}_{si,t}$=−.10$\overline{\varepsilon_t^d}$; $\text{def}_{si,t}$=−.76$\overline{\varepsilon_t^d}$. And a goal in terms of public debt ($\alpha_{g,d}$=.1) still reduces more the budgetary activism, in order to minimize variations in the public debt; so, we obtain: $\text{def}_{si,t}$=−.68$\overline{\varepsilon_t^d}$.

Furthermore, in case of demand shocks, the counter-cyclical budgetary reaction is all the more accentuated as stabilizing economic activity and inflation are fundamental goals for the governments (α_y and α_π are higher than α_g), but this result is not new. However, our paper also shows that if the member countries of the monetary union have a goal in terms of structural budgetary deficit ($\alpha_{g,s}$) and if the weight given to this goal is high, the structural budgetary surplus in the positively affected countries is fixed to a minimal value in order to stabilize this targeted variable, decreasing the global budgetary surplus, whereas the public debt is more weakly reduced. With a goal in terms of public debt ($\alpha_{g,d}$) whose weight is high, budgetary policies can even become expansionary. Indeed, there is usually a structural budgetary deficit in the countries positively affected by the shock, compensating the conjunctural increase in fiscal resources, and limiting the increase in the primary global budgetary surplus. Finally, with a goal in terms of global budgetary deficit ($\alpha_{g,g}$), budgetary policies can be still more expansionary. Indeed, structural budgetary deficits are then high in the countries positively affected by the shock, in order to reduce the targeted primary global budgetary deficit to a minimal value. In this context, in case of symmetric positive demand shocks, public debt levels can increase with the interest rates in all member countries of the monetary union.

To conclude, in case of demand shocks, the budgetary policies are all the more active and counter-cyclical as stabilizing economic activity and inflation are the most important goals for the governments. On the contrary, budgetary policies can become pro-cyclical if the governments have a large preference for public expenditure smoothing ($\alpha_{g,g}$ or $\alpha_{g,d}$ is higher than α_y and α_π). Indeed, the governments increase their structural budgetary deficit if a positive demand shock increases fiscal resources, whereas they decrease their structural deficit if a negative demand shock reduces fiscal resources, increases public expenditure, and the conjunctural deficit.

Moreover, in case of demand shocks, whatever the goal of the budgetary authorities, Equations B1 and B2 in Appendix B show that the budgetary policy is all the more counter-cyclical as the parameters (σ), (η), and (v) are small. Indeed, if the share of public consumption in GDP ($1-\sigma$) is high, budgetary policies are more efficient in order to stabilize demand shocks, and they are then more active. On the contrary, openness to trade (η) improves the automatic stabilization of demand shocks. Indeed, if a country is very open, a positive demand shock increases its imports and damages its price competitiveness, which contributes to reduce its nets exports and the expansionary consequences of the shock. High price flexibility (v) also increases the inflationary (deflationary) and stabilizing consequences of a positive (negative) demand shock, which allows a less active budgetary policy.

Regarding automatic stabilizers (k), if they are weak, budgetary policies must be more counter-cyclical to compensate. Therefore, countries positively (negatively) affected by demand shocks have higher structural budgetary surpluses (deficits). On the contrary, if automatic stabilizers (k) are strong, budgetary policies can be less active. Countries positively affected by demand shocks can even have structural budgetary deficits in case of fiscal rules targeting the global budgetary deficit or the public debt ($\alpha_{g,g}>0$ or $\alpha_{g,d}>0$). Finally, if the budgetary authorities target the global budgetary deficit or the public debt ($\alpha_{g,g}>0$ or $\alpha_{g,d}>0$), in case of positive demand shocks, higher taxation rates (τ_t) allow structural budgetary deficits in order to get the same budgetary balance, as the latter are compensated by more conjunctural fiscal resources. Anyway, in case of positive (negative) demand shocks, higher taxation rates increase (decrease) fiscal resources and increase the global primary budgetary surpluses (deficits) in a framework of economic growth (economic recession).

5.2. Stabilization of Economic Variables in Case of Demand Shocks

What are then the consequences of the previous budgetary policies on the stabilization of demand shocks? Equation B3 in Appendix B implies[3]:

$$\frac{\partial y_{i,t}}{\partial \varepsilon_t^d}\ \underset{(d_{i,t-1}\sim d_{j,t-1})}{\longrightarrow}\ \frac{1}{(1+k-k\sigma)}>0 \tag{20}$$

$$\frac{\partial y_{i,t}}{\partial \overline{\varepsilon_t^d}}<0 \tag{21}$$

Therefore, economic activity and prices increase in case of positive demand shocks. Indeed, fiscal rules prevent a perfect stabilization of these shocks by the budgetary authorities. For example, in case of symmetric positive demand shocks (ε_t^d), with the basic calibration of our model, we obtain: $y_{i,t}=.89\varepsilon_t^d$; $\pi_{i,t}=.18\varepsilon_t^d$.

Besides, monetary policy can perfectly stabilize the part of symmetric demand shocks which is not due to structural heterogeneities (in the previous public indebtedness levels) between the member countries of the monetary union, whatever the preferences of the budgetary authorities. However, in case of asymmetric demand shocks, economic growth is more moderate if the budgetary policy is more active and counter-cyclical, and thus, if inflation and economic activity are the main goals for the governments (α_y and α_π are higher than α_g). Therefore, according to the previous section, economic activity and inflation are better stabilized with a goal in terms of structural budgetary deficit.

For example, with the basic calibration of our model, if ($\alpha_{g,s}$=.1), we have $y_{i,t}$=.64$\overline{\varepsilon_t^d}$ and $\pi_{i,t}$=.13$\overline{\varepsilon_t^d}$, whereas if ($\alpha_{g,g}$=.1), we have: $y_{i,t}$=.74$\overline{\varepsilon_t^d}$ and $\pi_{i,t}$=.15$\overline{\varepsilon_t^d}$, and if ($\alpha_{g,d}$=.1), we obtain: $y_{i,t}$=.75$\overline{\varepsilon_t^d}$ and $\pi_{i,t}$=.15$\overline{\varepsilon_t^d}$.

Furthermore, in case of demand shocks, economic activity and inflation are better stabilized if the share of public consumption in GDP ($1-\sigma$) is high. Indeed, budgetary policies are then more efficient in order to stabilize economic variables. In case of asymmetric demand shocks, economic activity and inflation are also better stabilized if openness to trade (η) or price flexibility (v) is high in the monetary union. Indeed, if a country has a high openness to trade, an asymmetric positive demand shock contributes to increase its imports, to damage its relative price competiveness, and thus to reduce its net exports and the expansionary tensions in the national country. High price flexibility also accentuates the inflationary tensions due to a positive demand shock, and it reduces the price competitiveness and net exports of the national country.

Besides, in case of symmetric demand shocks, or of asymmetric demand shocks with a goal in terms of structural budgetary deficit ($\alpha_{g,s}>0$), economic variables are better stabilized if automatic stabilizers (k) are high and thus more efficient. On the contrary, in case of asymmetric demand shocks, and with fiscal rules in terms of global budgetary deficit or public debt ($\alpha_{g,g}>0$ or $\alpha_{g,d}>0$), economic variables are better stabilized if budgetary policies are more active to compensate for their weak efficiency regarding conjunctural stabilization, that is to say, if taxation rates (τ_t) or automatic stabilizers (k) are weak. Indeed, according to Equations 9 and 10, the differential in structural budgetary deficits between the member countries of the monetary union can then contribute to stabilize their differentials regarding economic activity or inflation. Finally, we can also mention that the public indebtedness level ($d_{i,t-1}$) of a given country usually damages its economic stabilization capacity in case of demand shocks, if the governments have a goal in terms of public debt ($\alpha_{g,d}>0$).

So, in conclusion, in case of demand shocks, the relative efficiency of the potential goals of the budgetary authorities in order to stabilize economic variables is not clear-cut. If the governments favor economic activity and prices stabilization, a goal in terms of structural budgetary deficit is the most appropriate in case of asymmetric demand shocks; however, fiscal variables are then less well stabilized. On the contrary, if governments favor the respect for fiscal rules, a goal in terms of global budgetary deficit or public debt is the most appropriate. However, economic activity and inflation are then less well stabilized in case of asymmetric demand shocks.

5.3. Optimal Budgetary Policies in Case of Supply Shocks

Afterwards, what are the implications of our model regarding optimal budgetary policies in order to stabilize supply shocks? Anything then depends on the relative preferences of the budgetary authorities. Indeed, in case of symmetric supply shocks, Equations B1 and B2 in Appendix B imply:

$$\frac{1}{\alpha_\pi v}\left(\frac{\partial \mathrm{def}_{si,t}}{\partial \varepsilon_t^s}\right)>0 \tag{22}$$

$$\frac{1}{\alpha_\pi v}\left(\frac{\partial \mathrm{def}_{i,t}}{\partial \varepsilon_t^s}\right)>0 \tag{23}$$

A positive (negative) symmetric supply shock (ε_t^s) has deflationary (inflationary) consequences in all member countries of the monetary union without much affecting their economic activity levels (see Equations 9 and 10). Therefore, the budgetary deficits (surpluses) of the member countries increase all the more as stabilizing prices (α_π) is important for the governments in comparison with stabilizing economic activity levels (α_y). The budgetary activism is also a decreasing function of the preference of the governments for the respect for fiscal rules (α_g).

Nevertheless, the budgetary activism remains always quite limited in case of symmetric supply shocks. Indeed, without any variation in economic activity levels, the expansionary (contractionary) budgetary policies only aim at compensating the deflationary (inflationary) consequences of the shock. Therefore, with the basic calibration of our model, if ($\alpha_{g,s}$=.1) or ($\alpha_{g,g}$=.1), we only have: (i_t=.03ε_t^s), $def_{si,t}$=$def_{i,t}$=.08ε_t^s). Moreover, budgetary policies are still less active with a goal in terms of public debt, in order to reduce fluctuations in the public debt level: if ($\alpha_{g,d}$=.1), we obtain: (i_t=.02ε_t^s), $def_{si,t}$=$def_{i,t}$=.06ε_t^s).

Besides, the budgetary activism of the member countries is an increasing function of price flexibility (v) in the monetary union. Indeed, the latter accentuates the inflationary consequences of fluctuations in economic activity levels. On the contrary, the budgetary activism is a decreasing function of the share of private consumption in GDP (σ), of openness to trade (η) and of the efficiency of automatic stabilizers (k) in the member countries of the monetary union. Indeed, automatic stabilization mechanisms then reduce the necessity of fiscal measures in order to stabilize prices in the member countries of the monetary union. If the budgetary authorities have fiscal rules in terms of global budgetary deficit ($\alpha_{g,g}$) or of public debt ($\alpha_{g,d}$), the budgetary activism is also an increasing function of taxation rates (τ_t), improving the efficiency of budgetary policies in economic stabilization.

Quite differently, an asymmetric supply shock ($\overline{\varepsilon_t^s}$) has deflationary and expansionary (inflationary and recessionary) consequences in the positively (negatively) affected member countries of the monetary union (see Equations 9 and 10). Monetary policy is then inefficient, whereas these shocks have ambiguous consequences on the budgetary balances of the countries, which face a conflict of goals between stabilizing economic activity and inflation. More precisely, according to Equations B1 and B2 in Appendix B, the budgetary surplus of a country affected by a positive supply shock increases all the more as avoiding the expansionary consequences of the shock (α_y) is a more important goal whereas stabilizing the deflationary tensions (α_π) is a more negligible goal for the governments.

Besides, the budgetary activism decreases all the more as stabilizing a fiscal variable is important for the governments. So, with the basic calibration of our model, if ($\alpha_{g,s}$=.1), we have ($def_{si,t}$=−.58$\overline{\varepsilon_t^s}$) and ($def_{si,t}$=−1.16$\overline{\varepsilon_t^s}$). But a high preference for stabilizing the structural deficit ($\alpha_{g,s}$) can create a moderate structural primary budgetary surplus and strongly reduce the global budgetary surplus of a country affected by a positive supply shock. Moreover, the budgetary policies are still less active in case of a goal in terms of global budgetary deficit. Indeed, if ($\alpha_{g,g}$=.1), we have ($def_{si,t}$=−.06$\overline{\varepsilon_t^s}$) and ($def_{i,t}$=−.73$\overline{\varepsilon_t^s}$). But if stabilizing the global deficit has a stronger weight in the preferences of the governments, the country affected by a positive supply shock should have a structural budgetary deficit in order to compensate for the consequences of the conjunctural surpluses on its global budgetary balance. The budgetary policy is still more expansionary with a goal in terms of public debt. Indeed, with our basic calibration, if ($\alpha_{g,d}$=.1), we have ($def_{si,t}$=.03$\overline{\varepsilon_t^s}$) and ($def_{si,t}$=−.65$\overline{\varepsilon_t^s}$). But the primary structural budgetary deficit of a country affected by a positive supply shock can strongly increase with the weight given to the achievement of a given public debt target.

Furthermore, Equations B1 and B2 in Appendix B show that the structural primary budgetary surplus (deficit) of a country affected by a positive (negative) supply shock increases all the more as the parameters (σ) and (v) are small. Indeed, if the share of public consumption in GDP ($1-\sigma$) is high, budgetary policies are more efficient in order to stabilize the consequences of asymmetric supply shocks. In the same way, weaker price flexibility (v) increases the expansionary (recessionary) consequences of positive (negative) supply shocks, which necessitates a more active budgetary policy. Budgetary policies are also more active if openness to trade (η) of the member countries of the monetary union is high. Indeed, this increases the expansionary consequences of the shock in the country positively affected whose price competitiveness improves, and on the contrary the recessionary consequences of the shock in the country negatively affected whose price competitiveness

is damaged. Global budgetary balances are also more accentuated if taxation rates (τ_t) are high, and if budgetary policies are then more efficient in economic stabilization. Finally, global budgetary balances are also accentuated to compensate for weak automatic stabilizers (k), but only in the framework of fiscal rules in terms of global budgetary deficit or of public debt (if $\alpha_{g,g}>0$ or $\alpha_{g,d}>0$).

5.4. Stabilization of Economic Variables in Case of Supply Shocks

What are the results of the former budgetary policies for the stabilization of economic activity and prices in case of supply shocks? The situation is then different whether the shock is symmetric or asymmetric.

In case of symmetric positive (negative) supply shocks, the deflationary (inflationary) tensions implied by the shocks are limited by the expansionary (contractionary) budgetary policies in the member countries of the monetary union. However, we have seen in the previous section that the budgetary activism then remains very limited, especially if stabilizing economic activity is the most important goal for the governments. Therefore, according to Equations B3 and B4 in Appendix B, we have:

$$\frac{1}{\alpha_\pi \alpha_{g,d} v}\left(\frac{\partial y_{i,t}}{\partial \varepsilon_t^s}\right)>0 \tag{24}$$

$$\frac{\partial \pi_{i,t}}{\partial \varepsilon_t^s}=-1+v\left(\frac{\partial y_{i,t}}{\partial \varepsilon_t^s}\right) \tag{25}$$

So, with a goal in terms of budgetary deficit ($\alpha_{g,s}>0$ or $\alpha_{g,g}>0$), economic activity levels are obviously perfectly stabilized ($y_{i,t}\to 0$). Indeed, symmetric supply shocks or symmetric structural budgetary balances between the member countries of the monetary union don't affect economic activity levels. In these conditions, prices strongly vary proportionally to symmetric supply shocks ($\pi_{i,t}\to -1$). Positive (negative) symmetric supply shocks imply a strong deflation (inflation) in the member countries of the monetary union. Furthermore, with a goal in terms of public debt ($\alpha_{g,d}>0$), according to the values mentioned in Appendix B, a higher debt level in a member country (i) of the monetary union ($d_{i,t-1}>d_{j,t-1}$) decreases its structural and global budgetary surpluses. Prices and economic activity levels are then marginally smaller in the country (i) and higher in the country (j) affected by a symmetric supply shock.

An asymmetric supply shock implies deflationary and expansionary tensions in the country of the monetary union positively affected, and inflationary and recessionary tensions in the country negatively affected by the shock. Indeed, according to Equations B3 and B4 in Appendix B, we have:

$$\frac{\partial y_{i,t}}{\partial \overline{\varepsilon_t^s}}<0 \tag{26}$$

$$\frac{\partial \pi_{i,t}}{\partial \overline{\varepsilon_t^s}}<0 \tag{27}$$

Therefore, in case of asymmetric supply shocks, there is a conflict of goals for the budgetary authorities between stabilizing economic activity and prices. In this context, whatever the fiscal rule chosen by the governments, differentials in economic activity levels are reduced but at the cost of higher prices differentials between the member countries of the monetary union if stabilizing activity is the most important goal (α_y) whereas stabilizing prices (α_π) or achieving a fiscal target (α_g) are minor goals for the governments. Besides, a goal in terms of structural budgetary deficit seems the most appropriate in order to stabilize economic activity levels, even if prices differentials are then slightly increased. Indeed, budgetary policies are then more counter-cyclical. So, with the basic calibration of our model, if ($\alpha_{g,s}=.1$), we have ($y_{i,t}=.65\ \overline{\varepsilon_t^s}$) and ($\pi_{i,t}=-.87\overline{\varepsilon_t^s}$). However, if ($\alpha_{g,g}=.1$), we have ($y_{i,t}=.74\overline{\varepsilon_t^s}$) and ($\pi_{i,t}=-.85\overline{\varepsilon_t^s}$) and if ($\alpha_{g,d}=.1$), we obtain: ($y_{i,t}=.76\overline{\varepsilon_t^s}$) and ($\pi_{i,t}=-.85\overline{\varepsilon_t^s}$).

In case of asymmetric supply shocks, we can also mention that economic stabilization is improved regarding both economic activity levels and prices if price flexibility (v) is high in the monetary union. Indeed, prices fluctuations have then weaker consequences on economic activity levels. Economic activity levels are also better stabilized but at the cost of higher prices differentials if the share of the public sector in GDP ($1-\sigma$) is high, if openness to trade (η) is weak or if taxation rates (τ_t) are weak in the member countries of the monetary union. Indeed, smaller price competitiveness in the country negatively affected by an asymmetric supply shock should decrease net exports and accentuate the recessionary tensions in this country, whereas on the contrary, stronger price competitiveness should increase net exports and accentuate the expansionary tensions in the country positively affected by the shock. Prices differentials are then automatically reduced, whereas economic activity differentials are accentuated. However, these automatic stabilization mechanisms are limited if the countries are weakly open or if the share of the private sector is weak.

6. Conclusion

The aim of our paper was to contribute to the debate on appropriate fiscal rules in a monetary union. The Fiscal Compact has introduced two kinds of rules for the member countries of the EMU: a debt rule (to reduce each year the public debt ratio by one twentieth of the difference with the 60% reference level), and a "Golden rule" (a structural budgetary deficit inferior to .5% of GDP). However, what are the respective advantages and drawbacks of these distinct fiscal rules regarding economic stabilization? A simple macroeconomic model shows that a public debt goal is the most appropriate in order to reduce the indebtedness levels of the member countries towards a specific target. Indeed, a goal in terms of global or structural budgetary deficit increases the budgetary deficit in proportion to its targeted level. The public debt increases then more than proportionally to the target and cannot be well stabilized. On the contrary, a goal in terms of public debt allows the long-term sustainability of public finances. In the framework of excessive indebtedness levels in many European countries, a public debt goal seems thus more relevant. However, for the more indebted European countries, such a target implies asymmetric contractionary measures which can have more dangerous recessionary consequences.

Furthermore, in case of demand shocks, budgetary policies are all the more active and counter-cyclical as stabilizing economic activity and prices is more important than achieving a fiscal goal for the governments. However, the relative efficiency of the potential goals of the budgetary authorities in order to stabilize national economic variables is not clear-cut. If the governments favor economic activity and prices stabilization, a goal in terms of structural budgetary deficit is the most appropriate in order to stabilize these economic variables in case of asymmetric demand shocks. However, fiscal variables are then less well stabilized. On the contrary, a goal in terms of global budgetary deficit or of public debt is the most appropriate if the governments favor the respect for fiscal rules; indeed, budgetary policies are less counter-cyclical. However, economic activity and prices are then less well stabilized in case of asymmetric demand shocks.

Besides, in case of supply shocks, the situation is still more complex. Positive (negative) symmetric supply shocks imply deflationary (inflationary) tensions in the member countries of the monetary union, without really affecting economic activity levels. Budgetary policies are then less active if stabilizing economic activity or a fiscal variable are the most important goals for the governments. Goals in terms of global budgetary deficit or of public debt are then marginally more efficient, as they slightly limit the budgetary activism without being harmful to economic stabilization. On the contrary, an asymmetric positive (negative) supply shock has expansionary and deflationary (recessionary and inflationary) consequences in the affected country. Therefore, it necessitates a budgetary surplus (deficit) in this country, all the more important as stabilizing economic activity is more important than stabilizing prices or achieving a fiscal goal. However, budgetary policies are more active with a goal in terms of structural budgetary deficit, and differentials in economic activity levels are then reduced, even if it is at the cost of higher prices differentials. Therefore, a structural budgetary deficit seems the most appropriate as soon as stabilizing economic activity is the main goal for the governments, which is empirically very likely.

In conclusion, in case of demand shocks or asymmetric supply shocks, a fiscal rule in terms of structural budgetary deficit seems to improve economic stabilization. However, obviously, the European authorities then have to face a dilemma between optimality and operational efficiency. Indeed, even if a target in terms of structural budgetary deficit is theoretically efficient, the problem of such a goal relies on the ambiguity for the measure of this indicator. Therefore, the Fiscal Compact has combined a rule in terms of structural budgetary deficit with a rule in terms of public debt, which is the most efficient in order to limit the budgetary cost of economic stabilization and to decrease the indebtedness levels of the European countries.

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Notes

1. See European Commission (2011, pp. 61–100), for a discussion of the European framework and of the reform of the SGP with new rules giving more importance to the debt criterion.
2. Goals in terms of structural budgetary deficit ($def_{si,t}$) or global budgetary deficit ($def_{i,t}$) inclusive of interest rate payments on the former public debt level ($+i_t d_{i,t-1}$), as mentioned in the Fiscal Compact, give exactly the same results as goals in terms of primary budgetary deficits. Indeed, the interest rate on the public debt (i_t) depends on potential economic activity and real interest rate, and on the structural budgetary deficit, which is already a potential variable.
3. According to Equation B4 in Appendix B, equations are similar for prices: $\frac{\partial(\pi_{i,t})}{\partial(\varepsilon_t^d)} = v\frac{\partial(y_{i,t})}{\partial(\varepsilon_t^d)}$; $\frac{\partial(\pi_{i,t})}{\partial(\overline{\varepsilon_t^d})} = v\frac{\partial(y_{i,t})}{\partial(\overline{\varepsilon_t^d})}$.
4. The values of the parameters ($m_{0,i}$) to (m_{10}) can be obtained by request from the author.

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Appendix A: Economic Activity and Inflation Rate Equations

By combining Equations 1 and 6, we obtain:

$$\begin{aligned}(1+k-k\sigma)\,y_{i,t}^{d} &= (1+k-k\sigma)\,y_{i,t+1}^{e}+(1-\sigma)\left(\tau_{t}^{*}+k\right)y_{i,t}^{*}-(1-\sigma)\left(\tau_{t+1}^{*e}+k\right)y_{i,t+1}^{*e}\\ &\quad+\eta(\pi_{j,t}-\pi_{i,t})-\sigma\left(i_{t}-\pi_{i,t+1}^{e}-r_{i,t}^{*}\right)+(1-\sigma)\left(\mathrm{def}_{si,t}-\mathrm{def}_{si,t+1}^{e}\right)+\varepsilon_{i,t}^{d}\end{aligned} \tag{A1}$$

Solving Equations A1 and 7 forwards, and under the assumption that: $\lim_{T\to\infty} y_{i,t+T}=\lim_{T\to\infty} y_{i,t+T}^{*}=\lim_{T\to\infty}\mathrm{def}_{si,t+T}=\lim_{T\to\infty}\pi_{i,t+T}=0$, we have:

$$\begin{aligned}(1+k-k\sigma)\,y_{i,t}^{d} &= (1-\sigma)\left(\tau_{t}^{*}+k\right)y_{i,t}^{*}-\sigma\left(i_{t}-r_{i,t}^{*}\right)-\sigma\sum_{T=0}^{\infty}\left(i_{t+T+1}^{e}-\pi_{i,t+T+2}^{e}-r_{i,t+T+1}^{*e}\right)\\ &\quad+\eta(\pi_{j,t}-\pi_{i,t})+\eta\sum_{T=0}^{\infty}\left(\pi_{j,t+T+1}^{e}-\pi_{i,t+T+1}^{e}\right)+\sigma\pi_{i,t+1}^{e}+(1-\sigma)\mathrm{def}_{si,t}+\varepsilon_{i,t}^{d}\end{aligned} \tag{A2}$$

$$\pi_{i,t}=v(y_{i,t}-y_{i,t}^{*})+v\sum_{T=0}^{\infty}b^{T+1}\left(y_{i,t+T+1}^{e}-y_{i,t+T+1}^{*e}\right)-\varepsilon_{i,t}^{s} \tag{A3}$$

Then, by combining Equations A2 and A3, and assuming that $\sum_{T=0}^{\infty}b^{T+1}(y_{i,t+T+1}^{e}-y_{i,t+T+1}^{*e})=\sum_{T=0}^{\infty}(i_{t+T+1}^{e}-r_{i,t+T+1}^{*e})=\sum_{T=0}\pi_{i,t+T+1}^{e}=0$, we obtain:

$$\begin{aligned}&(1+k-k\sigma)\left(1+k-k\sigma+2v\eta\right)\pi_{i,t}\\ &\quad=-v\left(1-\tau_{t}^{*}+\sigma\tau_{t}^{*}\right)\left(1+k-k\sigma+v\eta\right)y_{i,t}^{*}\\ &\qquad-v^{2}\eta\left(1-\tau_{t}^{*}+\sigma\tau_{t}^{*}\right)y_{j,t}^{*}-v\sigma(1+k-\sigma k+2v\eta)i_{t}+v\sigma(1+k-k\sigma+v\eta)r_{i,t}^{*}\\ &\qquad+v^{2}\sigma\eta r_{j,t}^{*}+v(1+k-k\sigma+v\eta)\left[(1-\sigma)\mathrm{def}_{si,t}+\varepsilon_{i,t}^{d}\right]-v\eta(1+k-k\sigma)\varepsilon_{j,t}^{s}\\ &\qquad+v^{2}\eta\left[(1-\sigma)\mathrm{def}_{sj,t}+\varepsilon_{j,t}^{d}\right]-(1+k-k\sigma)(1+k-k\sigma+v\eta)\varepsilon_{i,t}^{s}\end{aligned} \tag{A4}$$

On average values for all the monetary union, we have:

$$(1+k-k\sigma)\pi_{t}=-v\left(1-\tau_{t}^{*}+\sigma\tau_{t}^{*}\right)y_{t}^{*}+v(1-\sigma)\mathrm{def}_{s,t}-v\sigma\left(i_{t}-r_{t}^{*}\right)+v\varepsilon_{t}^{d}-(1+k-k\sigma)\varepsilon_{t}^{s} \tag{A5}$$

So, if expected demand and supply shocks are zero, and if the monetary authority aims at stabilizing average prices in the monetary union ($\pi_{t}=0$), the interest rate is fixed at the following level:

$$\sigma i_{t}=\sigma r_{t}^{*}-\left(1-\tau_{t}^{*}+\sigma\tau_{t}^{*}\right)y_{t}^{*}+(1-\sigma)\mathrm{def}_{s,t} \tag{A6}$$

We can then replace this value of (i_{t}) in Equation A4 order to obtain the inflation rate in the country (i). Afterwards, we can replace this value of ($\pi_{i,t}$) in Equation A2 in order to obtain the economic activity level ($y_{i,t}$).

Appendix B: Main Economic Variables

We suppose that all demand and supply shocks are fully independent. In these conditions, if we replace ($\mathrm{def}_{sj,t}$) in Equation 14 by its value, we obtain[4]:

$$\begin{aligned}&(1+k-k\sigma)\left[(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}\right]\mathrm{def}_{si,t}\\ &\quad=-\sigma[m_{4,i}m_{1,i}-\alpha_{g,d}(d_{j,t-1}-d_{i,t-1})(1-\sigma)^{2}(\tau_{t}+k)(1+k-k\sigma+2v\eta)m_{3,i}]\varepsilon_{t}^{d}\\ &\qquad-\sigma(1+k-k\sigma)[m_{5,i}m_{1,i}+\alpha_{g,d}(d_{j,t-1}-d_{i,t-1})\eta(1-\sigma)^{2}(\tau_{t}+k)m_{3,i}]\overline{\varepsilon_{t}^{d}}\\ &\qquad+2\alpha_{\pi}v\sigma^{2}(1+k-k\sigma)(1-\sigma)m_{4,i}\varepsilon_{t}^{s}+\sigma(1+k-k\sigma)[m_{5,i}m_{2,i}\\ &\qquad-2\alpha_{g,d}(d_{j,t-1}-d_{i,t-1})\eta(1-\sigma)^{2}(\tau_{t}+k)m_{3,i}]\overline{\varepsilon_{t}^{s}}+2\alpha_{g,s}\sigma^{2}(1+k-k\sigma)\\ &\qquad\times(1+k-k\sigma+2v\eta)m_{4,i}\mathrm{def}_{s}^{T}+\alpha_{g,g}\sigma^{2}(1+k-k\sigma)[2+(k-\tau_{t})(1-\sigma)\\ &\qquad+4v\eta]m_{4,i}\mathrm{def}^{T}-\alpha_{g,d}\sigma(1+k-k\sigma)[(1+\sigma)(1+2v\eta)+(1-\sigma)(k-\tau_{t}\sigma)][d_{i,t-1}m_{0,j}(d_{i,t-1}-d^{T})\\ &\qquad+d_{j,t-1}(1-\sigma)m_{3,i}(d_{j,t-1}-d^{T})]+f(y_{i,t}^{*},y_{j,t}^{*},r_{i,t}^{*},r_{j,t}^{*})\end{aligned} \tag{B1}$$

If we replace these values of ($def_{si,t}$) and ($def_{sj,t}$) in Equation 12, we obtain:

$$\begin{aligned}
&\left(1+k-k\sigma\right)\left[(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}\right]\mathrm{def}_{i,t}\\
&=-\left\{\left(\tau_t+k\right)\left[(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}\right]\right.\\
&\quad\left.+\sigma m_{4,i}m_{1,i}+\alpha_{g,d}\sigma(1-\sigma)^2(\tau_t+k)[m_{7,i}m_{1,i}+(1+k-k\sigma+2v\eta)(d_{i,t-1}-d_{j,t-1})m_{8,i}]\right\}\varepsilon_t^d\\
&\quad-(1+k-k\sigma)[\sigma m_{5,i}m_{1,i}+\alpha_{g,d}\sigma(d_{j,t-1}-d_{i,t-1})(1-\sigma)^2(\tau_t+k)m_{3,i}+2(\tau_t+k)m_{6,i}m_{6,j}\\
&\quad-\alpha_{g,d}(\tau_t+k)(1-\sigma)m_9]\overline{\varepsilon_t^d}+2\alpha_\pi v\sigma^2[m_{4,i}-\alpha_{g,d}(1-\sigma)^2m_{7,j}(\tau_t+k)](1+k-k\sigma)(1-\sigma)\varepsilon_t^s\\
&\quad-(1+k-k\sigma)\{4\eta(\tau_t+k)m_{6,i}m_{6,j}-\sigma m_{5,i}m_{2,j}2\sigma\alpha_{g,d}d_{j,t-1}-d_{i,t-1}\eta(1-\sigma)^2(\tau_t+k)m_{3,i}\\
&\quad+(\tau_t+k)(1-\sigma)m_{10}\}\overline{\varepsilon_t^s}+2\alpha_{g,s}\sigma^2(1+k-k\sigma)(1+k-k\sigma+2v\eta)[m_{4,i}-\alpha_{g,d}(1-\sigma)^2\\
&\quad\times(\tau_t+k)m_{7,j}]\mathrm{def}_s^T+\alpha_{g,g}\sigma^2(1+k-k\sigma)[2+(k-\tau_t)(1-\sigma)+4v\eta][m_{4,i}-\alpha_{g,d}(1-\sigma)^2\\
&\quad\times(\tau_t+k)m_{7,j}]\mathrm{def}^T-\alpha_{g,d}\sigma\left[(1+\sigma)\left(1+2v\eta\right)+(1-\sigma)\left(k-\tau_t\sigma\right)\right]\left\{\left[m_{0,j}-(1-\sigma)\left(\tau_t+k\right)m_{6,j}\right]\right.\\
&\quad\left.d_{i,t-1}\left(d_{i,t-1}-d^T\right)+[m_{3,i}+(\tau_t+k)m_{6,i}](1-\sigma)d_{j,t-1}(d_{j,t-1}-d^T)\right\}(1+k-k\sigma)+f(y_{i,t}^*,y_{j,t}^*,r_{i,t}^*,r_{j,t}^*)
\end{aligned}\tag{B2}$$

If we put the optimal primary structural budgetary deficit in Equation B1 in the economic activity Equation 9 and in the inflation rate Equation 10, we obtain:

$$\begin{aligned}
&\left(1+k-k\sigma\right)\left[(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}\right]y_{i,t}\\
&=\{(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}+\alpha_{g,d}\sigma\left(1-\sigma\right)^2m_{7,i}m_{1,i}\\
&\quad+\alpha_{g,d}\sigma\left(1-\sigma\right)^2\left(d_{i,t-1}-d_{j,t-1}\right)\left(\tau_t+k\right)\left(1+k-k\sigma+2v\eta\right)m_{6,i}\}\varepsilon_t^d\\
&\quad+(1+k-k\sigma)[2m_{6,i}m_{6,j}-(1-\sigma)\alpha_{g,d}m_9]\overline{\varepsilon_t^d}+2\alpha_\pi\alpha_{g,d}v\sigma^2(1+k-k\sigma)\\
&\quad\times(1-\sigma)^3m_{7,j}\varepsilon_t^s+(1+k-k\sigma)(1-\sigma)[m_{10}+4\eta m_{6,i}m_{6,j}]\overline{\varepsilon_t^s}\\
&\quad+2\alpha_{g,s}\alpha_{g,d}(1+k-k\sigma+2v\eta)(1-\sigma)^2\sigma^2(1+k-k\sigma)m_{7,j}\mathrm{def}_s^T\\
&\quad+\alpha_{g,g}\alpha_{g,d}\sigma^2(1-\sigma)^2(1+k-k\sigma)[2+(k-\tau_t)(1-\sigma)+4v\eta]m_{7,j}\mathrm{def}^T\\
&\quad+\alpha_{g,d}\sigma(1+k-k\sigma)(1-\sigma)[(1+\sigma)(1+2v\eta)+(1-\sigma)(k-\tau_t\sigma)]\\
&\quad\times[d_{j,t-1}m_{6,i}(d_{j,t-1}-d^T)-d_{i,t-1}m_{6,j}(d_{i,t-1}-d^T)]+f(y_{i,t}^*,y_{j,t}^*,r_{i,t}^*,r_{j,t}^*)
\end{aligned}\tag{B3}$$

$$\begin{aligned}
&\left(1+k-k\sigma\right)\left[(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}\right]\pi_{i,t}\\
&=v[(1-\sigma)\left(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}\right)+2\left(1+k-k\sigma+2v\eta\right)m_{6,i}m_{6,j}\\
&\quad+\alpha_{g,d}\sigma\left(1-\sigma\right)^2m_{7,i}m_{1,i}+\alpha_{g,d}\sigma(1-\sigma)^2(d_{i,t-1}-d_{j,t-1})(\tau_t+k)(1+k-k\sigma+2v\eta)m_{6,i}]\varepsilon_t^d\\
&\quad+v(1+k-k\sigma)[2m_{6,i}m_{6,j}-(1-\sigma)\alpha_{g,d}m_9]\overline{\varepsilon_t^d}-(1+k-k\sigma)[(1-\sigma)(m_{3,i}m_{6,j}+m_{3,j}m_{6,i})\\
&\quad+2(1+k-k\sigma+2v\eta)m_{6,i}m_{6,j}-2\alpha_\pi\alpha_{g,d}v^2\sigma^2(1-\sigma)^3m_{7,j}]\varepsilon_t^s\\
&\quad-(1+k-k\sigma)[(1-\sigma)(m_{3,i}m_{6,j}+m_{3,j}m_{6,i}-vm_{10})+2(1+k-k\sigma)m_{6,i}m_{6,j}]\overline{\varepsilon_t^s}\\
&\quad+2\alpha_{g,s}\alpha_{g,d}v(1+k-k\sigma+2v\eta)(1-\sigma)^2\sigma^2(1+k-k\sigma)m_{7,j}\mathrm{def}_s^T\\
&\quad+\alpha_{g,g}\alpha_{g,d}v\sigma^2(1-\sigma)^2(1+k-k\sigma)[2+(k-\tau_t)(1-\sigma)+4v\eta]m_{7,j}\mathrm{def}^T\\
&\quad+\alpha_{g,d}\sigma v(1+k-k\sigma)(1-\sigma)[(1+\sigma)(1+2v\eta)+(1-\sigma)(k-\tau_t\sigma)]\\
&\quad\times[d_{j,t-1}m_{6,i}(d_{j,t-1}-d^T)-d_{i,t-1}m_{6,j}(d_{i,t-1}-d^T)]+f(y_{i,t}^*,y_{j,t}^*,r_{i,t}^*,r_{j,t}^*)
\end{aligned}\tag{B4}$$

Using Equation 8 and the variation in the public debt, we also have:

$$(d_{i,t}-d_{i,t-1})=\mathrm{def}_{i,t}+\frac{(1-\sigma)d_{i,t-1}}{2\sigma}(\mathrm{def}_{si,t}+\mathrm{def}_{sj,t})+f(y_{i,t}^*,y_{j,t}^*,r_{i,t}^*,r_{j,t}^*)\tag{B5}$$

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