# Selective Precision Synthesis of the Four-Bar Motion Generator With Prescribed Input Timing

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*The recently developed Selective Precision Synthesis technique has been extended to the four-bar motion generating mechanism with prescribed input timing. A designer using this method can determine several mechanisms whose coupler triangle positions and orientations will be coordinated with the input crank rotations. The unique feature in the Selective Precision Synthesis formulation is that the path, orientations and rotations are specified along with allowable limits of accuracy creating an error envelope for each of these parameters. This modification removes the limiting conditions imposed by the precision point approach so that standard nonlinear programming techniques can be used to determine several mechanism solutions. It was found that the method yields fundamentally stable solutions rarely encountered in closed-form methods of mechanism synthesis. The problem of dyadic construction error in the original SPS technique is eliminated and the method developed here is well suited to batch or interactive computer-aided design. The computer program of this method is being made available to interested readers.*

## Introduction

One of the most widely used mechanisms in industry is the four-bar motion generator with prescribed input timing. This planar mechanism can guide a rigid body through a specific path with specific orientations while coordinating this motion with rotations (or speeds) of the input crank of the mechanism. In the precision point approach to the solution of the problem, the number of attainable precision conditions is only three due to the balance of unknowns and independent loop closure equations. Although this mechanism solution will provide exact agreement with input data at these precision points, the actual rigid body motion and timing may deviate considerably from the desired motion and timing between these points.

In practice, however, the structural error at any point need never be zero provided it does not exceed some prescribed limit of accuracy. In this way several mechanism solutions can often be found which generate a larger number of these accuracy (although not exact) conditions. The accuracy conditions are defined by discrete tracer point positions, coupler link rotations and input crank rotations and each of these quantities is associated with an allowable deviation from the nominally specified values.

## Mathematical Formulation

The four-bar motion generating mechanism with prescribed input timing is shown in Fig. 1 in its initial and $j$th position. The input crank, $\mathbf{Z}_1$, will rotate through prescribed values of $\theta_j$ and within limits of accuracy of plus or minus $h_j''$. The coupler triangle will rotate through prescribed values of $\phi_j$ and within limits of accuracy of plus or minus $h_j'$. The tracer point of the coupler triangle will intercept the accuracy neighborhoods defined by position vectors $\mathbf{R}_j$ and maximum allowable deviations $h_j$. The quantities specified by the designer are $\theta_j$, $h_j''$, $\phi_j$, $h_j'$, $\mathbf{R}_j$ and $h_j$. The subscript $j$ is a counter for each position and varies from one to $m$ planar positions. Although there is no mathematical limit of the value of $m$, designer judgement is necessary in deciding its value as well as reasonable values of the input parameters. Since $\theta_j$ and $\phi_j$ represent rotations from the initial position, $\theta_1$ and $\phi_1$ are each equal to zero. For simplicity the values of $h_1''$, $h_1'$ and $h_1$ are also equal to zero with no loss of accuracy to the mechanism solution.

By introducing two unknowns at each specified mechanism position, the rotations of the coupler triangle and input crank can be represented respectively by:

$$\left.\begin{matrix}\phi_j + \lambda_j h_j' \\ \theta_j + \mu_j h_j''\end{matrix}\right\} \quad \text{for} \quad j = 2, 3 \ldots m \tag{1}$$

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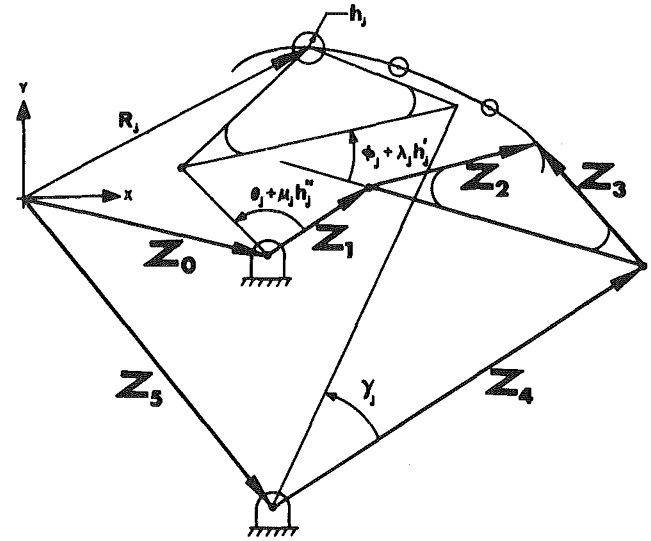


**Fig. 1 The four-bar motion generating mechanism with prescribed input timing in the initial and jth position.**

where $\lambda_j$ and $\mu_j$ are $2(m - 1)$ unknown scalar quantities.
Using this representation, the values of the coupler and input rotations will not exceed their error envelopes provided that:

$$\left.\begin{aligned} |\lambda_j| &\leq 1 \quad (2)\\ &\text{and} \\ |\mu_j| &\leq 1 \quad (3)\end{aligned}\right\} \quad \text{for} \quad j = 2, 3 \ldots m$$

The dyadic loop closure equations for the first position are:

$$\mathbf{Z}_0 + \mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{R}_1 \qquad (4)$$

$$\mathbf{Z}_5 + \mathbf{Z}_4 + \mathbf{Z}_3 = \mathbf{R}_1 \qquad (5)$$

In the $j$th position, the dyadic loop closure equations are:

$$\mathbf{Z}_0 + \mathbf{Z}_1 \exp\left[i(\theta_j + \mu_j h_j'')\right] + \mathbf{Z}_2 \exp\left[i(\phi_j + \lambda_j h_j')\right] = \mathbf{R}_j + \mathbf{d}_j \qquad (6)$$

$$\mathbf{Z}_5 + \mathbf{Z}_4 \exp\left[i\gamma_j\right] + \mathbf{Z}_3 \exp\left[i(\phi_j + \lambda_j h_j')\right] = \mathbf{R}_j + \mathbf{d}_j' \qquad (7)$$

and $\mathbf{d}_j$ and $\mathbf{d}_j'$ are vectors representing the dyadic deviations between the tracer point position and the precision point position for $j = 2, 3, \ldots m$. From the Selective Precision Synthesis technique [1–3] the tracer point of the mechanism will fall within the specified accuracy neighborhoods when $\mathbf{d}_j$ and $\mathbf{d}_j'$ are equal to each other and their magnitudes do not exceed the value of $h_j$ for $j = 2, 3, \ldots m$. By combining equations (4–7), the displacement equations can be obtained as follows:

$$\mathbf{Z}_1(\exp(i\theta_j)\exp(i\mu_j h_j'') - 1) + \mathbf{Z}_2(\exp(i\phi_j)\exp(i\lambda_j h_j') - 1) = \mathbf{R}_j - \mathbf{R}_1 + \mathbf{d}_j \qquad (8)$$

and

$$\mathbf{Z}_4(\exp(i\gamma_j) - 1) + \mathbf{Z}_3(\exp(i\phi_j)\exp(i\lambda_j h_j') - 1) = \mathbf{R}_j - \mathbf{R}_1 + \mathbf{d}_j' \qquad (9)$$

Since $\mu_j h_j''$ and $\lambda_j h_j'$ have very small radian values, the approximation:

$$\exp(ix) \simeq 1 + ix \qquad (10)$$

can be used to simplify the displacement equations. Solving for $\mathbf{d}_j$ and $\mathbf{d}_j'$ using this simplification and by replacing the exponentials by their trigonometric representations, one obtains:

$$\mathbf{d}_j = \mathbf{Z}_1\mathbf{A}_j + \mathbf{Z}_2\mathbf{B}_j + \mathbf{R}_1 - \mathbf{R}_j \qquad (11)$$

$$\mathbf{d}_j' = \mathbf{Z}_4(e^{i\gamma_j} - 1) + \mathbf{Z}_3\mathbf{B}_j + \mathbf{R}_1 - \mathbf{R}_j \qquad (12)$$

where

$$\mathbf{A}_j = \cos\theta_j - 1 + i\sin\theta_j + \mu_j h_j''(i\cos\theta_j - \sin\theta_j) \qquad (13)$$

and

$$\mathbf{B}_j = \cos\phi_j - 1 + i\sin\phi_j + \lambda_j h_j'(i\cos\phi_j - \sin\phi_j) \quad \text{for} \quad j = 2, 3, \ldots m \qquad (14)$$

Since $\mathbf{d}_j$ and $\mathbf{d}_j'$ must be equal, the value of $e^{i\gamma_j}$ can be found from equating equations (11) and (12):

$$e^{i\gamma_j} = 1 + \frac{\mathbf{Z}_1\mathbf{A}_j + (\mathbf{Z}_2 - \mathbf{Z}_3)\mathbf{B}_j}{\mathbf{Z}_4} \quad \text{for} \quad j = 2, 3, \ldots m \qquad (15)$$

For $\gamma_j$ to have a real value insuring closure of the mechanism, the following equality relations must hold:

$$1 + \left|\frac{\mathbf{Z}_1\mathbf{A}_j + (\mathbf{Z}_2 - \mathbf{Z}_4)\mathbf{B}_j}{\mathbf{Z}_4}\right| = 1 \quad \text{for} \quad j = 2, 3, \ldots m \qquad (16)$$

For the mechanism to intercept the accuracy neighborhoods, the magnitudes of $\mathbf{d}_j$ must not exceed the values of $h_j$:

$$|\mathbf{Z}_1\mathbf{A}_j + \mathbf{Z}_2\mathbf{B}_j + \mathbf{R}_1 - \mathbf{R}_j| \leq h_j \quad \text{for} \quad j = 2, 3, \ldots m \qquad (17)$$

For the Selective Precision Synthesis of the four-bar motion generating mechanism with prescribed input timing the inequality constraint relations that need to be satisfied are systems (2), (3) and (17). Equation (16) represents the equality constraint relations that must be satisfied . The numerical methods used in the solution of the problem become exceedingly more efficient by approximating the equality constraints with inequality constraints. This can be accomplished by specifying that the difference between the left and right-hand sides of equation (16) should not exceed "$\Delta$", a very small positive quantity. This approximation works quite well in that convergence is greatly enhanced with no noticeable loss of accuracy.

For $m$ discrete planar positions, the number of inequality constraint relations is $(4m - 4)$. The number of independent scalar unknowns is $(2m + 6)$. This includes $(m - 1)$ variables denoted by $\mu_j$, $(m - 1)$ variables denoted by $\lambda_j$, and 8 variables representing the horizontal and vertical components of link vectors $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_4$.

The method of solution of these inequality constraints employed here is the Fiacco-McCormick Technique and the Hooke and Jeeves Search method [1, 2, 8]. These methods are standard non-linear programming techniques that are well suited to the types of equations in this formulation.

In order to utilize these techniques, the independent variables must be denoted as:

$$X_i \quad \text{for} \quad i = 1, 2, \ldots (2m + 6) \qquad (18)$$

and the inequality constraints must be expressed in the form:

$$G_k(X_i) \leq 0 \quad \text{for} \quad k = 1, 2, \mathbf{H}. (4m - 4). \qquad (19)$$

The independent variables are represented by $X_i$ as follows:

$$\begin{aligned} \mathbf{Z}_1 &= X_1 + X_2 \\ \mathbf{Z}_2 &= X_3 + iX_4 \\ \mathbf{Z}_3 &= X_5 + iX_6 \\ \mathbf{Z}_4 &= X_7 + iX_8. \end{aligned} \qquad (20)$$

The next $(m - 1)$ values of $X_i$ represent the set of $\mu_j$ and the last $(m - 1)$ values of $X_i$ represent the set of $\lambda_j$.

To start the search method, an initial $X_i$ vector is assumed. The designer may choose any eight numbers representing $X_1$ through $X_8$ and the initial guesses for the remaining $(2m - 2)$ values of $X_i$ are equal to zero. The search method then attempts to find an optimum solution for $X_i$ such that all constraints are


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satisfied. Once this is accomplished, a refinement of the solution is attempted. Starting from the current solution and with the maximum allowable deviations and the value of "Δ" being halved, a new search is attempted. The answer to this problem, if found, will approximate the desired path more closely and still guarantee the values of the rotations $\theta_j$ and $\phi_j$ to be within limits. This process is repeated until convergence is no longer obtained.

Once a solution is found the locations of the fixed pivots can easily be determined as:

$$\mathbf{Z}_0 = \mathbf{R}_1 - \mathbf{Z}_1 - \mathbf{Z}_2$$
$$\mathbf{Z}_5 = \mathbf{R}_1 - \mathbf{Z}_3 - \mathbf{Z}_4 \quad (21)$$

The rotation of the output crank can be found from equation (15):

$$\gamma_j = \arg\left[1 + \frac{\mathbf{Z}_1\mathbf{A}_j + (\mathbf{Z}_2 - \mathbf{Z}_3)\mathbf{B}_j}{\mathbf{Z}_4}\right] \text{ for } j = 2, 3, \ldots m \quad (22)$$

Several initial $X_i$ vectors may be needed to find an acceptable mechanism since convergence is not assured. Designer judgement is necessary since the four-bar mechanism has obvious geometric limitations. Experience has shown that the numerical methods are extremely efficient and that the initial guesses need not be near the optimal value. Convergence is greatly enhanced, however, by educated guesses using standard kinematic synthesis techniques such as: Burmester theory, albegraic, matrix, graphical and vector methods.

## Numerical Example 1

A four-bar motion generating mechanism with prescribed input timing is desired which will guide a rigid body through six accuracy neighborhoods defined by position vectors, $\mathbf{R}_1$ through $\mathbf{R}_6$ and by maximum allowable deviations, $h_1$ through $h_6$. The rigid body rotations (from the initial position) of $\phi_2$ through $\phi_6$ plus or minus $h_2'$ through $h_6'$ respectively. This motion is to be coordinated (or timed) to input crank rotations (from the initial position) of $\theta_2$ through $\theta_6$ plus or minus $h_2''$ through $h_6''$. All of the above parameters are specified by the designer, are illustrated in Fig. 2 and are listed in Table 1. In this case, six planar positions will create a system of eighteen independent variables and twenty inequality constraints.

With these input parameters the computer program initiates a search with an initial guess for the eight scalar unknowns mentioned in equation (20). Of the ten initial guesses attempted, five converged to mechanisms. One of these mechanisms, however, had values for links $\mathbf{Z}_0$, $\mathbf{Z}_1$ and $\mathbf{Z}_2$ almost identical to values for links $\mathbf{Z}_5$, $\mathbf{Z}_4$ and $\mathbf{Z}_3$. In other words, two nondistinct dyads were found and a single degree-of-freedom mechanism could not be constructed. Of the remaining four answers, the one which approximates the desired motion most closely had the following initial guess:

$$X_{\text{initial}} = \{-1, 1, 2, -1, -1, -1, 1, 3\} \quad (23)$$

for the first eight and zero for the remaining ten variables.

The optimal value obtained was:

$$X_{\text{optimal}} = \{-0.80000 \quad 0.52000 \quad 2.71989 \quad -1.07994$$
$$-0.83999 \quad -1.51981 \quad 1.19989 \quad 4.03990$$
$$0.65999 \quad 0.47999 \quad 0.73999 \quad 0.21999 \quad -0.49999$$
$$-0.06000 \quad 0.20999 \quad 0.98249 \quad -0.06000 \quad 0.82249\} \quad (24)$$

The values of the link vectors can be found from equations (20) and (21). They are:

$$\left.\begin{aligned}
\mathbf{Z}_0 &= 0.21411 + 0.05994\,i \\
\mathbf{Z}_1 &= -0.80000 + 0.52000\,i \\
\mathbf{Z}_2 &= 2.71989 - 1.07994\,i \\
\mathbf{Z}_3 &= -0.83999 - 1.51981\,i \\
\mathbf{Z}_4 &= 1.19989 + 4.03990\,i \\
\mathbf{Z}_5 &= 1.77410 - 3.02008\,i
\end{aligned}\right\} \quad (25)$$

**Table 1 Mechanism requirements for example 1**

| Position Number, j | Input Crank Rotation $\theta_j \pm h_j''$ (degrees) | Coupler Triangle Rotation $\phi_j \pm h_j'$ (degrees) | Position Vector $\mathbf{R}_j$ | Maximum Allowable Deviation, $h_j$ |
|---|---|---|---|---|
| 1 | 0.0 ± 0.0 | 0.0 ± 0.0 | 2.134 - 0.500 i | 0.000 |
| 2 | 60.0 ± 0.5 | 16.0 ± 2.0 | 2.300 - 0.600 i | 0.050 |
| 3 | 120.0 ± 0.5 | 24.0 ± 1.0 | 3.100 - 0.700 i | 0.100 |
| 4 | 180.0 ± 0.5 | 0.0 ± 2.0 | 3.800 - 1.500 i | 0.100 |
| 5 | 240.0 ± 0.5 | -14.0 ± 2.0 | 3.500 - 1.200 i | 0.100 |
| m = 6 | 300.0 ± 0.5 | -10.0 ± 3.0 | 2.750 - 0.560 i | 0.100 |

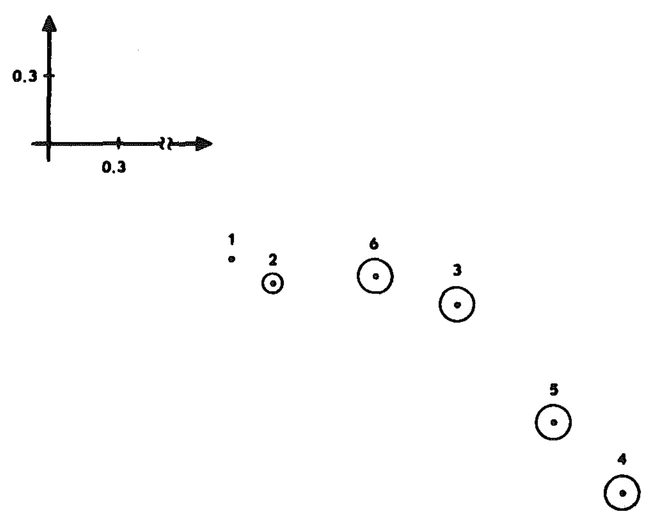


**Fig. 2 Graphical representation of mechanism requirements for example 1**

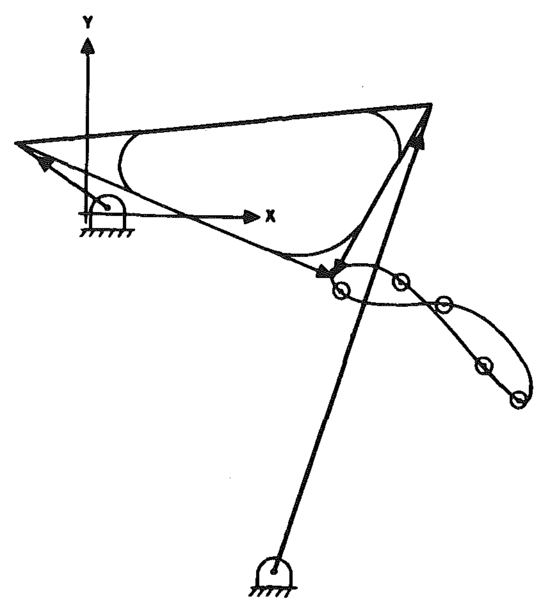


**Fig. 3 One of four optimum mechanisms determined for example 1**



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The values of the coupler link and input crank rotations can be found from equation (1). For $j$ = 2, 3, 4, 5 and 6 these values are:

$$\phi_j = \{17.31998 \quad 24.47998 \quad 1.47999 \quad -13.56001 \quad -11.49996\}$$
$$\theta_j = \{59.96999 \quad 120.10500 \quad 180.49124 \quad 239.96999 \quad 300.41113\}. \tag{26}$$

The values of the linear deviations from each precision point can be found from equation (11) or (12). For $j$ = 2, 3, 4, 5 and 6 these values are:

$$d_j = \{0.01868 \quad 0.05011 \quad 0.05011 \quad 0.05010 \quad 0.04465 \quad 0.05008\}. \tag{27}$$

These values compare favorably with the values of the maximum allowable devations listed in Table 1. In fact, the actual deviations are approximately half of the allowable deviations.

The values of the output crank rotations can be found from equation (22). For $j$ = 2, 3, 4, 5 and 6 these values are:

$$\gamma_j = \{4.80179 \quad -3.36047 \quad -25.48753 \quad -26.04752 \quad -12.22070\}. \tag{28}$$

The mechanism is illustrated in Fig. 3.

## Numerical Example II

A straight line path is to be approximated by nine uniformly-spaced accuracy neighborhoods having unequal maximum allowable deviations. The coupler link rotations are to be zero degrees within allowable limits. This motion is to be coordinated with equally spaced input crank rotations thereby approximating a constant speed rigid body translation corresponding to prescribed input timing conditions. Such a mechanism is used in packaging machines, transfer mechanisms and mechanical controller systems. Table 2 lists the values of the input data representing the above motion.

For nine accuracy neighborhoods, there were twenty-four (24) independent variables and thirty-two (32) inequality constraint relationships that need to be satisfied. Of the thirteen (13) initial guesses, eight (8) converged to optimal mechanisms. The best mechanism had the following link vectors:

$$\left.\begin{aligned} \mathbf{Z}_0 &= -3.14961 \quad -5.36008\,i \\ \mathbf{Z}_1 &= 0.26999 \quad -1.32957\,i \\ \mathbf{Z}_2 &= 2.87962 \quad 6.68965\,i \\ \mathbf{Z}_3 &= 1.92471 \quad 0.55996\,i \\ \mathbf{Z}_4 &= 4.27958 \quad -3.59962\,i \\ \mathbf{Z}_5 &= -6.20429 \quad 3.03966\,i \end{aligned}\right\} \tag{29}$$

The values of $\phi$, $\theta$, $\gamma$ and $d$ for $j$ = 2, 3, .... 9 are as follows:

$$\phi_j = \{1.99 \quad 3.19 \quad 3.60 \quad 3.48 \quad 2.70 \quad 1.29 \quad -0.79 \quad -3.99\} \tag{30}$$

$$\theta_j = \{17.22 \; 31.56 \; 44.65 \; 57.13 \; 72.12 \; 86.25 \; 102.72 \; 121.75\} \tag{31}$$

$$\gamma_j = \{2.45 \;\; 5.13 \;\; 8.09 \;\; 11.16 \;\; 14.96 \;\; 18.47 \;\; 22.13 \;\; 25.53\} \tag{32}$$

$$d_j = \{0.021 \; 0.028 \; 0.028 \; 0.032 \; 0.021 \; 0.032 \; 0.029 \; 0.024\}. \tag{33}$$

The above values were computed for mechanism number 3 listed in Table 3. An analysis of this mechanism showed that the velocities along the prescribed path were approximately equal in magnitude and direction. Comparison of the computed results with input data reveals that all rotations are within allowable limits and that the linear deviations at the nine precision points do not exceed the maximum allowable deviations. The mechanism is illustrated in Fig. 4.

**Table 2 Mechanism requirements for example 2**

| Position Number, $j$ | Input Crank Rotation $\theta_j \pm h_j''$ (degrees) | Coupler Triangle Rotation $\phi_j \pm h_j'$ (degrees) | Position Vector $R_j$ | Maximum Allowable Deviation, $h_j$ |
|---|---|---|---|---|
| 1 | 0.0 ± 0.0 | 0.0 ± 0.0 | 0.0 + 0.0 i | 0.000 |
| 2 | 15.0 ± 3.0 | 0.0 ± 4.0 | 0.150 + 0.260 i | 0.050 |
| 3 | 30.0 ± 3.3 | 0.0 ± 4.5 | 0.300 + 0.520 i | 0.050 |
| 4 | 45.0 ± 3.5 | 0.0 ± 4.5 | 0.450 + 0.779 i | 0.060 |
| 5 | 60.0 ± 3.8 | 0.0 ± 5.0 | 0.600 + 1.039 i | 0.060 |
| 6 | 75.0 ± 4.0 | 0.0 ± 5.0 | 0.750 + 1.299 i | 0.070 |
| 7 | 90.0 ± 3.8 | 0.0 ± 4.5 | 0.900 + 1.559 i | 0.060 |
| 8 | 105.0 ± 3.5 | 0.0 ± 4.5 | 1.050 + 1.819 i | 0.060 |
| m=9 | 120.0 ± 3.3 | 0.0 ± 4.0 | 1.200 + 2.078 i | 0.050 |

## Conclusions

This extension to the recently developed Selective Precision Synthesis techniques gives the mechanism designer the capabilities of determining several four-bar motion generating mechanisms with prescribed input conditions. Previously only three precision conditions were obtainable using the precision point approach. With the extension presented in this paper, "$m$" prescribed accuracy conditions are obtainable.

The numerical technique used in the formulation was the Hooke-and-Jeeves search method because of its speed and ac-

**Table 3**

| Number | Initial Guess | | | | | | | | Optimum Solution: $X_1$ through $X_8$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -2 | 1 | 2 | -1 | -3 | -1 | 2 | 0.2400 | -1.3241 | 3.1195 | 7.1659 | -0.2299 | -0.6449 | -6.0842 | 2.3945 |
| 2 | -1 | -1 | 2 | 3 | -3 | 2 | 1 | 3 | 0.2000 | -1.3586 | 4.0393 | 6.9594 | 2.0797 | -0.8401 | -12.1588 | 5.6394 |
| 3 | -1 | 1 | 2 | 2 | -1 | 2 | 1 | 2 | 0.2700 | -1.3296 | 2.8796 | 6.6897 | 1.9247 | 0.5600 | 4.2796 | -3.5996 |
| 4 | 0 | 1 | 2 | -2 | -2 | -2 | -2 | 2 | 0.5319 | -1.0797 | 0.0343 | -14.4909 | 1.0798 | 1.3198 | -9.7996 | 4.6797 |
| 5 | 0 | 1 | 2 | -2 | -2 | -2 | 0 | 2 | 0.5706 | -1.0048 | 1.0272 | -16.5244 | 3.3098 | 2.8473 | -10.8501 | 6.4522 |
| 6 | -1 | 2 | 1 | 3 | -1 | 2 | -1 | 3 | 0.4413 | -1.1998 | 1.8075 | 12.0249 | -3.2223 | 1.1199 | -7.4797 | 3.3998 |
| 7 | 1 | 0 | 1 | 2 | -1 | 2 | 1 | 1 | 0.4425 | -1.1995 | 2.1211 | 12.0074 | -3.3884 | 2.0798 | 9.0799 | -6.3596 |
| 8 | 1 | 1 | 2 | 2 | -2 | 2 | -1 | 1 | 0.3850 | -1.2397 | 2.0449 | 10.0049 | -3.6198 | 0.6750 | -5.7548 | 2.3248 |
| 9 | 1 | 0 | -1 | 2 | -2 | 2 | 0 | 2 | Did Not Converge (DNC) | | | | | | | |
| 10 | 1 | 0 | -1 | -2 | -2 | -2 | 0 | -1 | DNC | | | | | | | |
| 11 | -1 | 0 | 2 | 2 | -2 | 0 | -1 | 2 | DNC | | | | | | | |
| 12 | 0 | -1 | 3 | -1 | 2 | -2 | -3 | -2 | DNC | | | | | | | |
| 13 | 2 | 2 | -1 | -1 | 2 | -1 | 1 | 0 | DNC | | | | | | | |


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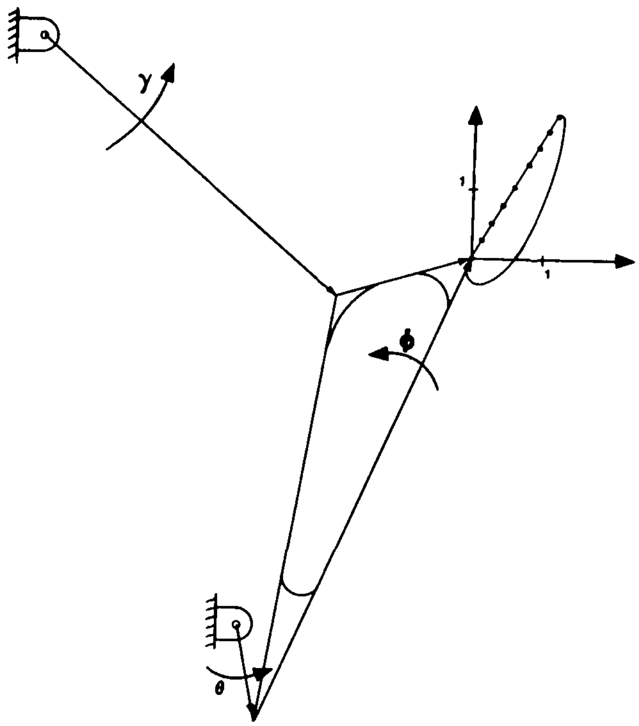


Fig. 4 One of eight optimum mechanisms determined for example 2

curacy for the types of functions encountered in most mechanism synthesis problems. Recent developments in the area of optimization techniques and their applications to mechanism synthesis problems involve slightly more efficient methods. Work is currently in progress by this author to incorporate a search technique which can directly handle equality constraints. In this way, equation (16) need not be modified and the search technique would deal with the constraint as well as the inequality constraints represented by equations (2), (3), and (17).

The optimization methods under consideration are the generalization reduced gradient method [8, 10], the projected method [9, 10] and the method of successive linear approximations [6, 7, 9]. Although the Hooke-and-Jeeves search method provided accurate results with search times of five to twenty seconds on an IBM (Model 75) computer, it is hoped this can be improved with one of the other methods. This is the subject of current research by this author.

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