DIGITAL LONGITUDINAL FEEDBACK SYSTEMS IN SYNCHROTRONS

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Abstract

The stability of a beam in synchrotrons with a digital longitudinal feedback system is treated. A longitudinal feedback system is required in synchrotrons to stabilize the high intensity beams against longitudinal instabilities and to damp the phase injection errors of a bunch. Damping rates of the digital longitudinal feedback system in dependence of its gain and delay are analysed.

INTRODUCTION

Longitudinal feedback systems are necessary in synchrotrons to reach the accuracy and stability required for reproducible beam performance [1, 2]. A digital bunch-by-bunch feedback [3] individually steers each bunch by applying electromagnetic kicks every time the bunch passes through the kicker (DK). The kick value is in proportion to the bunch deviation from the synchronous phase at the beam position monitor (BPM) location. The combiner (see Fig. 1) generates the wideband horizontal, vertical or sum signal, which is then demodulated to base-band by the detector. A stable beam rejection module removes useless

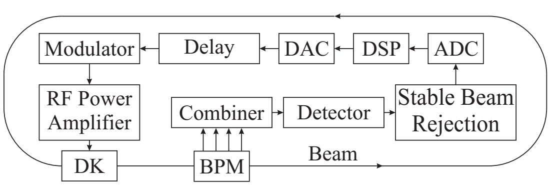

Figure 1: Block diagram of a digital feedback system

stable beam components from the signal, which is eventually digitized (ADC), processed (DSP), and re-converted (DAC) to analog by the digital processor. While in transverse feedback systems amplifier and kicker operate in base-band, longitudinal feedbacks require a modulator that translates the correction signal to the kicker operation frequency. The delay line adjusts the timing of the signal to match the bunch arrival time. Conditions for damping of the coherent synchrotron oscillations are discussed below.

BASIC NOTIONS

The RF system accelerates particles of a bunch in synchrotrons by producing a time-varying electric field in an accelerating station. The particle's energy growth per turn is given by [4, 5, 6]

$$E[n+1]-E[n]=q\widehat{V}_{\text{RF}}\sin\phi[n]\,, \qquad (1)$$

where q is the charge of the particle, $q\widehat{V}_{\text{RF}}$ is the maximal energy gain per turn and $\phi[n]$ is the phase of the RF when the particle crosses the middle of the accelerating station at the point s_{RF} of the orbit in the moment $t[n]$ at the n-th turn (the origin of time for the RF phase is taken at zero crossing of the RF voltage with positive slope; s is the coordinate along the orbit). It is also assumed in (1) that the electric field is the even function of $(s-s_{\text{RF}})$. Details of mode structure, geometry, transit time factor, etceteras are ignored in (1) by including these features in $\widehat{V}_{\text{RF}}$.

It will be assumed further that the particle with the energy $E[n]=mc^2\gamma[n]$ passes the n-th turn with the speed $c\beta[n]$ along the orbit with the circumference $C[n]$ so that the angular frequency $\omega[n]$ and the revolution period are $T[n]=2\pi/\omega[n]=C[n]/(c\beta[n])$; here c is the speed of light, m is the particle mass, and $\gamma=1/\sqrt{1-\beta^2}$. In accordance with these definitions the RF phase growth per turn can be written for the particle as follows:

$$\phi[n+1]-\phi[n]=\int_{t[n]}^{t[n+1]}\widehat{\omega}_{\text{RF}}(t)dt=\omega_{\text{RF}}[n+1]\,T[n+1], \quad (2)$$

where $\omega_{\text{RF}}[n+1]$ is the average value of the angular RF frequency $\widehat{\omega}_{\text{RF}}$ during the turn $T[n+1]\equiv t[n+1]-t[n]$.

The phase $\phi[n]$ can be kept unchanged (modulo 2π) at the value ϕ_{s} when the particle returns to the same accelerating section after one revolution period. The phase ϕ_{s} is also called the synchronous phase. In what follows, the subscript "s" is used for synchronous quantities. The synchronous phase growth per turn in accordance with (2) is

$$\phi_{\text{s}}[n+1]-\phi_{\text{s}}[n]=\omega_{\text{RF}}[n+1]\,T_{\text{s}}[n+1]=2\pi h_{\text{RF}}\,, \quad (3)$$

where $h_{\text{RF}}=(\omega_{\text{RF}}/\omega_{\text{s}})$ is the RF harmonic number. Let $\rho_{\text{s}}B_{\text{s}}$ be the momentum rigidity of the synchronous particle moving along the orbit with the circumference $2\pi R_{\text{s}}$. Then the synchronous particle's energy growth pur turn is

$$E_{\text{s}}[n+1]-E_{\text{s}}[n]=qV_{\text{RF}}\sin\phi_{\text{s}}[n]=q\,2\pi R_{\text{s}}\rho_{\text{s}}\dot{B}_{\text{s}}\,. \quad (4)$$

Eq. (3) and Eq. (4) are the system of difference equations for definition of $\phi_{\text{s}}[n]$ and $E_{\text{s}}[n]$. Let us assume that these solutions have been found. Therefore the motion of an arbitrary particle relative to a hypothetical synchronous particle can be examined.

Acceleration in presence of perturbations

Let $\delta E[n]$ and $\delta\phi[n]$ be small deviations of energy and phase of the particle from corresponding synchronous values at the n-th turn:

$$\delta E[n]\equiv E[n]-E_{\text{s}}[n], \qquad \delta\phi[n]\equiv\phi[n]-\phi_{\text{s}}[n]\,.$$

Let ΔV_{RF}, $\Delta\omega_{\text{RF}}$ and ΔB_{s} be small deviations of the accelerating voltage, the angular RF frequency and the magnetic field from designed values. Therefore the particle's energy deviation from the synchronous one gained per turn in accordance with (1) and (4) is

$$\delta E[n+1] - \delta E[n] = q(V_{\text{RF}} + \Delta V_{\text{RF}}[n]) \sin(\delta\phi[n] + \phi_{\text{s}}) - qV_{\text{RF}} \sin\phi_{\text{s}} . \quad (5)$$

In what follows, δ represents a difference taken with respect to the synchronous quantity at a given time (consequently at the given turn) and Δ represents an increment during acceleration. The phase deviation gained per turn in accordance with (2) and (3) is given by

$$\delta\phi[n+1] - \delta\phi[n] = \int\limits_{t[n]}^{t[n+1]} (\omega_{\text{RF}}(t) + \Delta\omega_{\text{RF}}(t))dt - 2\pi h_{\text{RF}} = \omega_{\text{RF}}[n+1]\, T[n+1] - 2\pi h_{\text{RF}} + \Delta\phi_{\text{RF}}[n+1] , \quad (6)$$

where the phase shift due to a small modulation of the RF frequency is

$$\Delta\phi_{\text{RF}}[n+1] = \int\limits_{t[n]}^{t[n+1]} \Delta\omega_{\text{RF}}(t)dt . \quad (7)$$

The deviation of the revolution period $T[n]$ from $T_{\text{s}}[n]$ is determined by [4, 7] the energy deviation $\delta E[n]$ and the magnetic field error $\Delta B_{\text{s}}[n]$:

$$T[n] \approx \left(1 + \frac{\eta_{\text{s}}\, \delta E[n]}{\beta_{\text{s}}^2[n]\, E_{\text{s}}[n]} - \frac{\alpha_{\text{s}}\, \Delta B_{\text{s}}[n]}{B_{\text{s}}[n]}\right) T_{\text{s}}[n] .$$

Here $\eta_{\text{s}} = \alpha_{\text{s}} - (1/\gamma_{\text{s}}^2)$ is the phase slip factor and α_{s} is the momentum compaction factor [7]. Therefore the phase deviation gained per turn is

$$\delta\phi[n+1] = \delta\phi[n] - \frac{4\pi^2\nu_{\text{s}}^2}{qV_{\text{RF}}\cos\phi_{\text{s}}}\, \delta E[n+1] + \Delta\phi_{\text{RF}}[n+1] - 2\pi\alpha_{\text{s}} h_{\text{RF}}\, \frac{\Delta B_{\text{s}}[n+1]}{B_{\text{s}}[n+1]} , \quad (8)$$

where the synchrotron oscillation tune ν_{s} is given by [7]:

$$\nu_{\text{s}} = \sqrt{-\frac{h_{\text{RF}}\eta_{\text{s}}}{2\pi\beta_{\text{s}}^2 E_{\text{s}}} qV_{\text{RF}}\cos\phi_{\text{s}}} .$$

For an adiabatic process of acceleration it is possible to neglect dependences of the coefficient before δE in Eq. (8) on time and n.

Eq. (8) and Eq. (5) are the system of difference equations for definition of $\delta\phi[n]$ and $\delta E[n]$. It corresponds to differential equations in [4] for synchrotron oscillations in presence of pertirbations and coincides with difference equations in [6, 7] in the case of $\Delta V_{\text{RF}} = \Delta\phi_{\text{RF}} = \Delta B_{\text{s}} = 0$. The system of Eq. (8) and Eq. (5) can be solved, for example, using the Z-transform [8] in a such way that was done for the transverse feedback system in [9].

Synchrotron oscillations

If $\Delta V_{\text{RF}} = \Delta\phi_{\text{RF}} = \Delta B_{\text{s}} = 0$ then for small $\delta\phi$ linearization of Eq. (5) in combination with Eq. (8) gives the synchrotron oscillation equation as difference equations in the matrix form:

$$\begin{pmatrix} \delta\phi[n+1] \\ \delta E[n+1] \end{pmatrix} = \widehat{M} \times \begin{pmatrix} \delta\phi[n] \\ \delta E[n] \end{pmatrix} , \quad (9)$$

where

$$\widehat{M} \equiv \begin{pmatrix} 1 - 4\pi^2\nu_{\text{s}}^2 & -\dfrac{4\pi^2\nu_{\text{s}}^2}{q\widehat{V}_{\text{RF}}\cos\phi_{\text{s}}} \\ q\widehat{V}_{\text{RF}}\cos\phi_{\text{s}} & 1 \end{pmatrix} .$$

Consequently the particle dynamics is determined by roots z_k of the characteristic equation:

$$\det\left(z_k\widehat{I} - \widehat{M}\right) = z_k^2 - (2 - 4\pi^2\nu_{\text{s}}^2)z_k + 1 = 0 , \quad (10)$$

where $\widehat{I}$ is the identity matrix. Eigenvalues of Eq. (10) are

$$z_{\pm} = \exp(\pm j2\pi\nu) , \qquad \sin\pi\nu = \pi\nu_{\text{s}}$$

that is well known result [5].

Dipolar motion approximation

Let us consider the bunch as a statistical collection of many particles [2], each particle oscillating around the synchronous phase. Let $f(\phi, E)\, d\phi\, dE$ be the probability that the RF phase is between ϕ and $\phi + d\phi$ and the particles energy is between E and $E + dE$ when the particle crosses the middle of the RF cavity at the n-th turn with phase $\phi[n]$ and energy $E[n]$. The first order moment (centre of charge) of $\phi[n]$ at the n-th turn is

$$\langle\phi[n]\rangle \equiv \iint \phi\, f(\phi, E)\, d\phi\, dE , \quad \iint f(\phi, E)\, d\phi\, dE = 1,$$

where the integration covers the entire bunch. Consequently with respect to synchronous phase and energy one can write:

$$\langle\delta\phi[n]\rangle = \iint (\phi - \phi_{\text{s}}[n])\, f(\phi, E)\, d\phi\, dE ; \quad (11a)$$

$$\langle\delta E[n]\rangle = \iint (E - E_{\text{s}}[n])\, f(\phi, E)\, d\phi\, dE . \quad (11b)$$

Let us now assume a stationary distribution so that $f(\phi, E)$ does not depend on t and n. Applying (11) to (8) and (5) for small $\delta\phi$ one can obtain:

$$\langle\delta\phi[n+1]\rangle = \langle\delta\phi[n]\rangle - \frac{4\pi^2\nu_{\text{s}}^2}{qV_{\text{RF}}\cos\phi_{\text{s}}}\langle\delta E[n+1]\rangle + \langle\Delta\phi_{\text{RF}}[n+1]\rangle - 2\pi h_{\text{RF}} \left\langle \frac{\Delta B_{\text{s}}[n+1]}{B_{\text{s}}[n+1]} \right\rangle ; \quad (12a)$$

$$\langle\delta E[n+1]\rangle = \langle\delta E[n]\rangle + qV_{\text{RF}}\langle\delta\phi[n]\rangle\cos\phi_{\text{s}} + q\,\langle\Delta V_{\text{RF}}[n]\rangle\sin\phi_{\text{s}} + q\,\langle\Delta V_{\text{RF}}[n]\,\delta\phi[n]\rangle\cos\phi_{\text{s}} . \quad (12b)$$

The above equations are identical to Eq. (8) and Eq. (5) describing the motion of each particle that cannot be individually observed by the instrumentation. On the other hand the motion of the centre of charge of the bunch, also called *dipolar motion*, can easily be monitored.

LONGITUDINAL FEEDBACK SYSTEMS

Let a pick-up (see Fig. 1) measure $\langle\delta\phi[n]\rangle$ for the bunch in place close to the accelerating section. The signal from the BPM can be used for the correction of the RF parameters via the modulator in the feedback loop when the same bunch crosses the accelerating section after one turn.

RF voltage modulation. Feedback is achieved passing the beam phase information through the loop to modify the amplitude at the RF cavity. Let the kick $\Delta V_{\text{RF}}[n]$ be in proportion to $\langle\delta\phi[n-1]\rangle$ but $\Delta\phi_{\text{RF}} = \Delta B_{\text{s}} = 0$:

$$\Delta V_{\text{RF}}[n] = -\frac{g_\phi\, qV_{\text{RF}}}{4\pi^2\nu_{\text{s}}^2}\, u[n-1]\,\langle\delta\phi[n-1]\rangle\,. \qquad (13)$$

Here $u[n]$ is the Heaviside step function and g_ϕ is the gain of the feedback loop. Consequently Eqs. (12) can be written in the matrix form that coinsides with (9) but

$$\widehat{M} \equiv \begin{pmatrix} 1-4\pi^2\nu_{\text{s}}^2+\hat{g} & -\dfrac{4\pi^2\nu_{\text{s}}^2(1-\hat{g})}{qV_{\text{RF}}\cos\phi_{\text{s}}} \\ \left(1-\dfrac{\hat{g}}{4\pi^2\nu_{\text{s}}^2}\right) qV_{\text{RF}}\cos\phi_{\text{s}} & 1-\hat{g} \end{pmatrix},$$

where $\hat{g} \equiv g_\phi \tan\phi_{\text{s}}$. The particle dynamics is determined by roots z_k of the characteristic equation:

$$z_k^2 - (2-4\pi^2\nu_{\text{s}}^2)z_k + 1 - g_\phi\tan\phi_{\text{s}} = 0\,. \qquad (14)$$

Eigenvalues of Eq. (14) for a small g_ϕ are

$$z_\pm = \exp\left(-\frac{g_\phi}{2}\tan\phi_{\text{s}}\right)\exp(\pm j2\pi\nu)\,,$$

$$\cos 2\pi\nu = (1-2\pi^2\nu_{\text{s}}^2)\exp\left(\frac{g_\phi}{2}\tan\phi_{\text{s}}\right)\,.$$

The damping regime corresponds to $g_\phi\tan\phi_{\text{s}} > 0$. It can be obtained during the B_{s} ramp only ($\tan\phi_{\text{s}} \neq 0$). It should be noted that the bunch returns to the synchronous phase as quickly as possible without coherent oscillations ($\nu = 0 \;\Rightarrow$ *critically damped* oscillator) for

$$(g_\phi)_{\text{opt}} \equiv g_\phi^* = 4\pi^2\nu_{\text{s}}^2(1-\pi^2\nu_{\text{s}}^2)/\tan\phi_{\text{s}}\,.$$

RF phase modulation. Let the phase shift due to a small modulation of the RF frequency in accordance with (7) is chosen in proportion to $\langle\delta\phi[n]\rangle$ but deviations $\Delta V_{\text{RF}} = \Delta B_{\text{s}} = 0$:

$$\langle\Delta\phi_{\text{RF}}[n+1]\rangle = -g_\phi\langle\delta\phi[n]\rangle\,, \qquad (15)$$

where g_ϕ is the gain of the feedback loop. Consequently Eqs. (12) can be written in the matrix form that coincides with (9) but

$$\widehat{M} \equiv \begin{pmatrix} 1-4\pi^2\nu_{\text{s}}^2-g_\phi & -\dfrac{4\pi^2\nu_{\text{s}}^2}{q\widehat{V}_{\text{RF}}\cos\phi_{\text{s}}} \\ q\widehat{V}_{\text{RF}}\cos\phi_{\text{s}} & 1 \end{pmatrix}.$$

The particle dynamics is determined by roots z_k of the characteristic equation:

$$z_k^2 - (2-g_\phi-4\pi^2\nu_{\text{s}}^2)z_k + 1 - g_\phi = 0\,. \qquad (16)$$

Eigenvalues of Eq. (16) for a small g_ϕ are

$$z_\pm = \exp\left(-\frac{g_\phi}{2}\right)\exp(\pm j2\pi\nu),\; \sin\pi\nu = \pi\nu_{\text{s}}\exp\left(\frac{g_\phi}{4}\right).$$

Therefore the damping regime corresponds to $g_\phi > 0$ and

$$(g_\phi)_{\text{opt}} \equiv g_\phi^* = 4\pi\nu_{\text{s}} - 4\pi^2\nu_{\text{s}}^2\,.$$

Feedback loop with delay. In practice there is a delay between the measurement of $\langle\delta\phi[n]\rangle$ and the RF kick. For example, one can write for (15):

$$\langle\Delta\phi_{\text{RF}}[n+1]\rangle = -g_\phi u[n-\hat{q}]\,\langle\delta\phi[n-\hat{q}]\rangle\,, \qquad (17)$$

where $\hat{q}$ is the number of turns in the feedback loop delay. Applying the bilateral Z-transform [8] in Eqs. (12) and Eq. (17) it is not difficult to obtain the characteristic equation for calculation of eigenvalues:

$$z_k^2 - (2-g_\phi z_k^{-\hat{q}} - 4\pi^2\nu_{\text{s}}^2)z_k + 1 - g_\phi z_k^{-\hat{q}} = 0\,. \qquad (18)$$

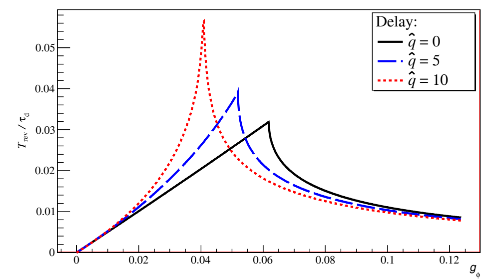

Figure 2: Dependences of maximal damping rates on g_ϕ

Dependences of maximal damping rates $T_{\text{rev}}/\tau_{\text{d}}$ = MAX $|(\ln|z_k|)|$ on feedback gains g_ϕ are shown in Fig. 2 for $\nu_{\text{s}} = 0.005$. It should be emphasised that g_ϕ^* values decrease with the growth of $\hat{q}$ values.

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