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ANALYTICAL SOLUTIONS TO AXISYMMETRIC PLANE STRAIN POROUS MEDIA-TRANSPORT MODELS IN LARGE ARTERIES

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INTRODUCTION

Theoretical and numerical finite element models (FEMs) have been developed for analysis of coupled structural-fluid-species transport in soft tissues [1-3]. Here analytical solutions for coupled diffusive-convective transport of a single, neutral species in soft tissues are presented. Based on experimental observations [4], osmotic pressure and partial Onsager coupling of species transport can be neglected for large mobile species in rabbit carotid arteries. These analytical solutions provide a starting point for development of solutions to more complex problems and allow verification of the associated FEMs under development in our laboratory. The analytical solutions will allow comparison of elastic and poroelastic-species transport for axisymmetric, plane strain in thick-walled arteries including expressions for displacement, strain, stress, pore fluid pressure, and concentration fields,. The initial models considered here will be steady state (SS) solutions for compressible, linear, isotropic materials undergoing small strains.

THEORY

Fully coupled governing equations for porohyperelastic transport-swelling models for a single neutral species have been developed [1]. Based on the assumption of weak coupling between species and fluid transport and neglecting the contribution of osmosis [4], the partially coupled ABAQUS [5]-based PHEXPT formulation and FEMs can be developed. The initial linear, isotropic formulation is denoted as the PEXPT model here.

The infinitesimal engineering strains in a PEXPT plane strain model of a cylinder (artery) are

$$e_R = \frac{\partial u}{\partial R},\ e_\Theta = \frac{u}{R},\ e_z = 0 . \tag{1}$$

The equilibrium equations are

$$\frac{dT_R}{dR} + \frac{1}{R}(T_R - T_\Theta) = 0, \quad \frac{\partial T_Z}{\partial Z} = 0 \tag{2}$$

Now $T = \int_{R_i}^{R_e} T_\Theta dR = P_i R_i - P_e R_e$ is the circumferential wall force and the total engineering stresses are T_R, T_Θ, and T_Z. The internal and external pressures and radii are P_i, P_e and R_i, R_e. The effective stress principle is

$$T_I = T_I^{eff} - p^f \quad (I = R, \Theta, Z) \tag{3}$$

where the effective stress is $T_I^{eff} = \lambda_0 e_{II} + 2\mu_0 e_I$ and the pore fluid pressure is p^f. Now $\mathrm{v}^{fr} = n(\mathrm{v}^f - \mathrm{v})$ is the apparent relative fluid velocity (flux) that is related to p^f by conservation of mass and the Darcy law as

$$\frac{\partial \mathrm{v}^{fr}}{\partial R} = 0, \quad \frac{\partial p^f}{\partial R} + \frac{\mathrm{v}^{fr}}{k_0} - 0 . \tag{4}$$

The hydraulic permeability is k_0 and the current porosity is n. The molar concentration (in the pore fluid) is c. Conservation of mass and the diffusive/convective Fick law are

$$\frac{\partial j^{cr}}{\partial R} = 0, \quad j^{cr} = -d_0 \frac{\partial c}{\partial R} + b_0 c \mathrm{v}^{fr} \tag{5}$$

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The apparent relative species flux is $j^{cr} = c\,n(\mathrm{v}^c\text{-}\mathrm{v})$; diffusivity is d_0 ; and a convection coupling coefficient is b_0 . Note that the Peclet number is related to $k_0 b_0 / d_0$. The boundary conditions at R_i are $T_R = -p^f = -P_i$; $c = c_i$; at R_e :, $c = c_e$ and $e_z = 0$.

The steady state solution for the PEXPT model begins with of the above equation yielding the following total stresses given in Tong and Fung [7] and a linear p^f derived from

$$T_R = T_R(R) = -P_i\,[\frac{(R_e/R\ )^2-1}{(R_e/R_i)^2-1}] - P_e\,[\frac{1-(R_i\ /R\ )^2}{1-(R_i\ /R_e)^2}]$$

$$T_\Theta = T_\Theta(R) = +P_i\,[\frac{(R_e/R\ )^2+1}{(R_e/R_i)^2-1}] - P_e\,[\frac{1+(R_i\ /R\ )^2}{1-(R_i\ /R_e)^2}] \qquad (6)$$

$$p^f = \frac{R_e-R}{R_e-R_i}P_i + \frac{R\ -R_i}{R_e-R_i}P_e$$

The effective stress, strain, displacement, and concentration fields are determined as

$$T_R^{eff} = T_R + p^f,\ T_\Theta^{eff} = T_\Theta + p^f,\ T_Z^{eff} = \nu_0(T_R^{eff} + T_\Theta^{eff})$$

$$e_R = [T_R^{eff} - \nu_0(T_\Theta^{eff} + T_Z^{eff})]/E_0$$

$$e_\Theta = [T_\Theta^{eff} - \nu_0(T_Z^{eff} + T_R^{eff})]/E_0$$

$$u = R\,e_\Theta,\ c = c(R) = \frac{\mathrm{B}_0 + \mathrm{A}_0 c_i}{\mathrm{A}_0}\exp[\mathrm{A}_0(\mathrm{R}\ - R_i\ )] - \frac{\mathrm{B}_0}{\mathrm{A}_0} \qquad (7)$$

$$\mathrm{A}_0 = (\frac{k_0 b_0}{d_0})\frac{(P_i\ - P_e\ )}{(R_e - R_i\ )},\ B_0 = A_0\frac{c_i\exp[A_0(R_e - R_i\ )] - c_e}{1-\exp[A_0(R_e - R_i\ )]}$$

The porosity is $n = 1 - J^{-1}(1+n_0)$ where the initial porosity is n_0 and volume strain is $J = 1 + e_R + e_\Theta$. The values for the material properties were taken from the literature and data from our laboratory [4]. Note that the corresponding linear elastic (LE) solution is a special case where $T_I\ = T_I^{eff}$ and $p^f = c = 0$.

METHODS

The analytical LE and PEXPT solutions were evaluated with MATLAB. The LE and PEXPT response were compared for a rabbit carotid artery whose baseline internal and external radii are 0.54mm and 0.75mm, respectively. A remodeling sequence in response to elevated hypertensive internal pressure (200mmHg) was considered. The sequence demonstrated the effects of increased arterial wall thickness on stresses and pore fluid pressures and included the following three cases: (1) "normal vessel", applied pressure, 100 mm Hg and carotid arterial dimension, *Ri*=0.54 mm, *Re*=0.74 mm , and D/h= 6.4 (a relatively thick tube); (2) "hypertensive", all parameters as in (1) except applied pressure is increased to 200 mm Hg; (3) "remodeled", all parameters as in case (2) with *Ri*=0.50 mm, reduced to decrease wall stresses.

RESULTS

The analytical LE and PEXPT solutions were evaluated with MATLAB (Figure 1 and 2). The LE and PEXPT response were compared for a rabbit carotid artery. A remodeling sequence in response to elevated hypertensive internal pressure demonstrated the effects of increased arterial wall thickness on stresses and pore fluid pressures and in each of the three cases.

DISCUSSION

The results here demonstrate the differences between LE and PEXPT simulations of a typical thick walled artery. These solutions allow a clear view of the coupled deformation and fluid/species transport response in the arterial wall. Significant differences exist between LE and PEXPT, especially since the LE models are restricted to pure diffusion where as the PEXPT models couple convective and diffusive effects. The PEXPT model is a linear version of the more general nonlinear PHEXPT theory and ABAQUS-based FEMs. The solutions here provide a quantitative model for validation of the FEMs. Possible remodeling (thickening) in the arterial wall was studied with these initial linear SS models for a tissue response to hypertension (a known risk factor for arterial disease). These models are being extended to include pre-stress states associated with "opening angles" [7] arterial wall remodeling. This study will be the basis for future consideration of layered, anisotropic soft arterial wall tissues subjected to large strains and exhibiting highly nonlinear material responses.

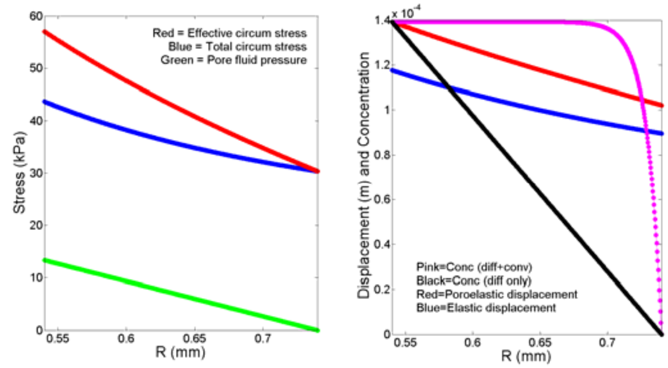

Figure 1. Circumferential stress (left), concentration (arbitrary units), and displacements (right) for poroelastic versus elastic rabbit carotid artery.

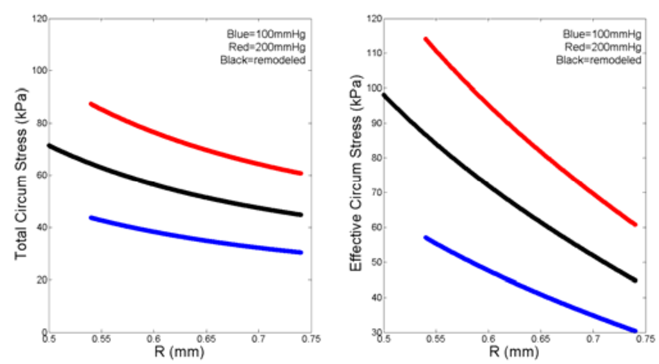

Figure 2. Increase in circumferential effective stress (left to right), and effect of remodeling (blue to red to black)

REFERENCES

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