

# A Control Method in dq Synchronous Frame for PWM Boost Rectifier under Generalized Unbalanced Operating Conditions

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**Abstract**— This paper proposes a new control scheme for regulating the instantaneous power for pwm boost type rectifiers under generalized unbalanced operating conditions. By nullifying the oscillating components of instantaneous power at the poles of the converter instead of the front-end through solving a set of nonlinear control equations in real time, the harmonics in the output dc voltage can be eliminated more effectively under generalized unbalanced operating conditions in the ac input side. The control scheme allows the pwm rectifier to generate a dc output without substantial even-order harmonics and to maintain nearly unity power factor under generalized unbalanced operating conditions, which makes it possible to reduce the size of the dc-link capacitor and ac inductors leading to reduced total cost. Simulation results along with experimental results for the open-loop control using a laboratory prototype converter confirm the feasibility of the new control method.

## I. INTRODUCTION

The pwm ac/dc converter has been increasingly employed in recent years owing to its advanced features including sinusoidal input current at unity power factor and high quality dc output voltage with a filter capacitor of small size. These features are not necessarily achieved under the operating conditions of unbalanced input supply and unbalanced input impedances. Such a generalized unbalanced operating condition is quite common in power systems, particularly in a weak ac system. Unevenly distributed single-phase loads or nonsymmetrical transformer windings as well as faults could lead to a generalized unbalanced network. It has been shown that unbalanced input voltages or impedances result in the appearance of even harmonics at the dc output and odd harmonics in the input currents [1-3]. Therefore, pwm ac/dc converters under generalized unbalanced operating conditions necessitate the use of input/output filters of large size and thereby completely offset several advantages of the pwm ac/dc converter [2].

Stankovic and Lipo [3] and Enjeti and Choudhury [2] have proposed methods to eliminate the input-output harmonics of the boost and buck type rectifiers, respectively. These methods suffer from two disadvantages: the power factor cannot be adjusted and the methods cannot operate well under the extreme unbalanced operating conditions such as unbalanced two-phase input or single-phase systems that often appear during the faults in power systems. Rioual *et al.* [4] and Song and Nam [5] derived control schemes regulating the instantaneous power at the input of the converter in dq synchronous frame for eliminating harmonics of the boost type rectifier under unbalanced input supply. These approaches cannot be

applied under the extreme unbalanced input supply. Stankovic and Lipo [6] addressed the cases of the generalized unbalanced operating conditions. However, the proposed control algorithm is a feed-forward method in phasor notation that does not regulate the instantaneous power explicitly so that it is not suitable for implementation with the feedback controlled space vector pwm in a dq synchronous frame.

This paper proposes a new control scheme for the pwm boost type rectifier under generalized unbalanced operating conditions. The positive and negative sequence dq components of the input voltages, pole voltages, and currents in the synchronous frame are employed to accurately describe the behavior of the ac/dc converter. The proposed technique nullifies the oscillating components of the instantaneous power at the poles of the converter so that harmonics in the output dc voltage can be eliminated under generalized unbalanced operating conditions in the ac input side. It is necessary to compute a set of nonlinear equations in real time for obtaining the references of the input currents in the synchronous frame in order to regulate the instantaneous power at the poles of the converter. The proposed control scheme allows the pwm rectifier to generate a dc output without substantial even-order harmonics and to maintain nearly unity power factor under generalized unbalanced operating conditions, which makes it possible to reduce the size of the dc-link capacitor and ac inductor. An unbalanced condition of a transformer sinusoidal secondary voltage with center tapped neutral and unbalanced impedances was chosen to be an example of the extreme unbalanced operating condition for the purpose of simulation and experiment. This particular operating condition is commonly found in residential areas where both 110Vac and 220Vac input voltages are available. Simulation and experimental results confirm the proposed control method.

## II. MODEL OF PWM BOOST RECTIFIER UNDER GENERALIZED UNBALANCED OPERATING CONDITIONS

It is possible to represent an unbalanced three-phase input voltage without a zero sequence as the orthogonal sum of positive and negative sequence.  $E_{dq}^p$  and  $E_{dq}^n$  are transformed space vectors in the positive and negative synchronous rotating frame respectively.  $E_a^p(t)$  is a positive sequential component of  $E_a(t)$  while  $E_a^n(t)$  is a negative sequential component of  $E_a(t)$ .  $E_a^p(t)$  and  $E_a^n(t)$  are summed to represent  $E_a(t)$ .

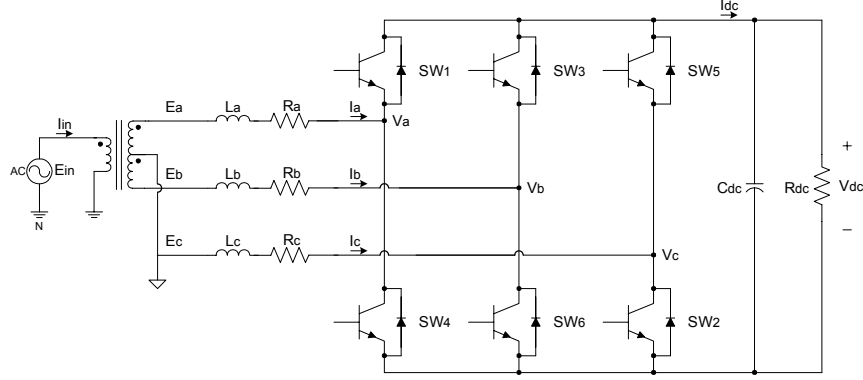


Fig. 1. Three-pole pwm ac/dc converter with input center tapped transformer.

$$E_{dqs} = e^{j\omega t} E_{dq}^p + e^{-j\omega t} E_{dq}^n = \frac{2}{3}(E_a + a E_b + a^2 E_c) \quad (1)$$

$$E_{dq}^p = \frac{2}{3}(E_a^p + a E_b^p + a^2 E_c^p) e^{j\omega t} = E_d^p + j E_q^p \quad (2)$$

$$E_{dq}^n = \frac{2}{3}(E_a^n + a E_b^n + a^2 E_c^n) e^{j\omega t} = E_d^n + j E_q^n \quad (3)$$

where  $a = e^{j(2\pi/3)}$ .

$I_{dq}^p, I_{dq}^n, V_{dq}^p$ , and  $V_{dq}^n$  can be defined in the same manner as in  $E_{dq}^p$  and  $E_{dq}^n$ . The conventional electrical equations on the ac side of the pwm ac/dc converter are shown in (4) and (5).

$$E_{dq}^p = V_{dq}^p + L \frac{d}{dt} I_{dq}^p + j\omega L I_{dq}^p + R I_{dq}^p \quad (4)$$

$$E_{dq}^n = V_{dq}^n + L \frac{d}{dt} I_{dq}^n - j\omega L I_{dq}^n + R I_{dq}^n \quad (5)$$

The input complex power of the converter is given in (6). At most six real and imaginary terms are found in  $S_{in}$  considering only the first harmonics of the input voltage and current.

$$S_{in} = E_{dqs} I_{dqs}^* = \frac{3}{2} (e^{j\omega t} E_{dq}^p + e^{-j\omega t} E_{dq}^n) (e^{j\omega t} I_{dq}^p + e^{-j\omega t} I_{dq}^n)^* \quad (6)$$

$$S_{in} = (P_o^{in} + P_{c2}^{in} \cos(2\omega t) + P_{s2}^{in} \sin(2\omega t)) + j(Q_o^{in} + Q_{c2}^{in} \cos(2\omega t) + Q_{s2}^{in} \sin(2\omega t)) \quad (7)$$

In order to cancel 120-Hz ripple in the dc output,  $P_{s2}^{in}$  and  $P_{c2}^{in}$  are set to zero [4-5]. The average reactive power exchanged between the utility source and the converter becomes zero by nullifying  $Q_o^{in}$  leading to unity power factor [4-5]. These conditions are incorporated into the following matrix equation (8). The positive and negative dq-components of the currents in the synchronous frame are regulated to the reference values obtained from (9).

$$\frac{2}{3} \begin{bmatrix} P_o^{in} \\ Q_o^{in} \\ P_{s2}^{in} \\ P_{c2}^{in} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} P_o^{in} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} E_d^p & E_q^p & E_d^n & E_q^n \\ E_q^p & -E_d^p & E_q^n & -E_d^n \\ E_q^n & -E_d^n & -E_q^p & E_d^p \\ E_d^n & E_q^n & E_d^p & E_q^p \end{bmatrix} \begin{bmatrix} I_d^p \\ I_q^p \\ I_d^n \\ I_q^n \end{bmatrix} \quad (8)$$

$$[I_{dq}] = [E_{dq}]^{-1} [S_{in}] \quad (9)$$

In a specific case of unbalanced operating conditions introduced in this paper as shown in Fig. 1, this approach cannot be applied because the matrix of  $[E_{dq}]$  is singular.

$$E_d^p = -E_d^n, E_q^p = -E_q^n \quad (10)$$

$$\text{Determinant of } [E_{dq}] = (E_d^p)^2 + (E_q^p)^2 - (E_d^n)^2 - (E_q^n)^2 = 0 \quad (11)$$

This singularity comes from the fact that the first and the fourth rows of the matrix  $[E_{dq}]$  are linearly dependent. This physically means that it is impossible to make the oscillating component of the instantaneous input power,  $P_{c2}^{in}$ , zero while having nonzero average input power,  $P_o^{in}$ .

### III. CONTROL STRATEGY

The complex power in the dq reference frame is conserved not only in the whole but also in each term by term as expected from the orthogonality among average, cosine, and sine terms in real and imaginary parts of the complex power:  $P_o^{in}, P_{c2}^{in}, P_{s2}^{in}, Q_o^{in}, Q_{c2}^{in}, Q_{s2}^{in}$ . Therefore, if the terms of  $P_{s2}^{in}$  and  $P_{c2}^{in}$  are compensated in the inductors then  $P_{s2}^{out}$  and  $P_{c2}^{out}$  calculated at the three poles of the converter vanish resulting in dc output without 120-Hz ripple. This can also be achieved by regulating  $P_{s2}^{out}$  and  $P_{c2}^{out}$  to zero directly. Even-order harmonic contents in a dc output voltage are directly caused by oscillatory active power components applied through three-pole ac/dc converters. The output complex power,  $S_{out}$ , is described as the following:

$$S_{out} = V_{dqs} I_{dqs}^*$$

$$= \frac{3}{2} (e^{j\omega t} V_{dq}^p + e^{j\omega t} V_{dq}^n) (e^{j\omega t} I_{dq}^p + e^{j\omega t} I_{dq}^n)^* \quad (12)$$

Therefore,

$$\text{Real}[S_{out}] = P_o^{out} + P_{c2}^{out} \cos(2\omega t) + P_{s2}^{out} \sin(2\omega t) \quad (13)$$

where:

$$\frac{2}{3} P_{c2}^{out} = V_d^n I_d^p + V_q^n I_q^p + V_d^p I_d^n + V_q^p I_q^n \quad (14)$$

$$\frac{2}{3} P_{s2}^{out} = V_q^n I_d^p - V_d^n I_q^p - V_q^p I_d^n + V_d^p I_q^n \quad (15)$$

Applying (4) and (5) into (14) and (15) after being decomposed into d-axis and q-axis under the assumptions of steady state condition and zero input resistances yield (16) and (17), respectively.

$$\begin{aligned} \frac{2}{3} P_{c2}^{out} &= E_d^n I_d^p - \omega L I_q^n I_d^p + E_q^n I_q^p + \omega L I_d^n I_q^p + E_d^p I_d^n \\ &+ \omega L I_q^p I_d^n + E_q^p I_q^n - \omega L I_d^p I_q^n \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{2}{3} P_{s2}^{out} &= E_q^n I_d^p + \omega L I_d^n I_d^p - E_d^n I_q^p + \omega L I_q^n I_q^p - E_q^p I_d^n \\ &+ \omega L I_d^p I_d^n + E_d^p I_q^n + \omega L I_q^p I_q^n \end{aligned} \quad (17)$$

In order to cancel 120-Hz ripple in a dc output voltage,  $P_{c2}^{out}$  in (16) and  $P_{s2}^{out}$  in (17) should be set to zero.

There is growing interest in definitions and interpretations of notions of reactive power in polyphase systems, particularly with nonsinusoidal waveforms and unbalanced networks [7-8]. The definition of average reactive power in (7) appears to have inconsistency with the classical notion of power factor especially in the case of extreme unbalanced operating conditions such as a center tapped split single-phase input supply.

The more relevant description of average reactive power in synchronous rotating frame is derived on the basis of the following classical definition of reactive power in sinusoidal steady state condition:

$$\begin{aligned} Q_o^{in} &= E_{arms} I_{arms} \sin\theta_{e-i}^a + E_{brms} I_{brms} \sin\theta_{e-i}^b \\ &+ E_{crms} I_{crms} \sin\theta_{e-i}^c \end{aligned} \quad (18)$$

where  $E_{arms}$  and  $I_{arms}$  are the rms values of  $E_a(t)$  and  $I_a(t)$ .  $\theta_{e-i}^a$  is the phase angle between  $E_a(t)$  and  $I_a(t)$ . After applying a few trigonometric identities with Park Transformation, the equation of (18) can be simplified to (19).

$$Q_o^{in} = E_q^p I_d^p - E_d^p I_q^p - E_q^n I_d^n + E_d^n I_q^n \quad (19)$$

The equations of (8) and (19) are linear while the equations of (16) and (17) are nonlinear with respect to the four unknown input current components;  $I_d^p$ ,  $I_q^p$ ,  $I_d^n$ , and  $I_q^n$ . Therefore, the following set of nonlinear equations needs to be solved in real time to obtain the references of input currents.

$$F(X) = \begin{bmatrix} f_1(I_d^p, I_q^p, I_d^n, I_q^n) \\ f_2(I_d^p, I_q^p, I_d^n, I_q^n) \\ f_3(I_d^p, I_q^p, I_d^n, I_q^n) \\ f_4(I_d^p, I_q^p, I_d^n, I_q^n) \end{bmatrix} = 0 \quad (20)$$

where  $f_1$ ,  $f_2$ ,  $f_3$ , and  $f_4$  represent (8), (19), (16), and (17) with the appropriate conditions on  $P_o^{in}$ ,  $Q_o^{in}$ ,  $P_{c2}^{out}$ , and  $P_{s2}^{out}$ .

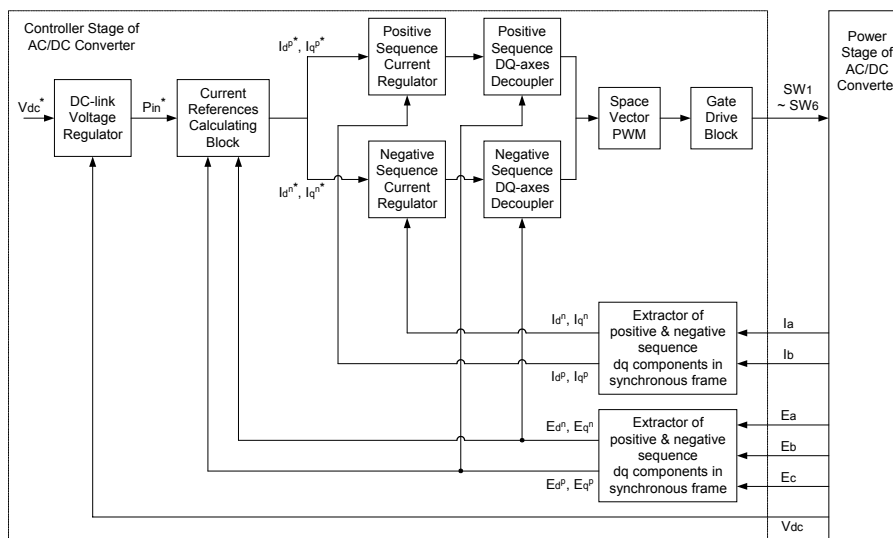


Fig. 2. System block diagram.



due to the intrinsic characteristic of the algorithm that needs at most  $2/3$  of input period ( $16.67 \text{ ms} \times 2/3 = 11.11 \text{ ms}$ ) to generate  $E_d^p, E_d^n, E_q^p, E_q^n$ .

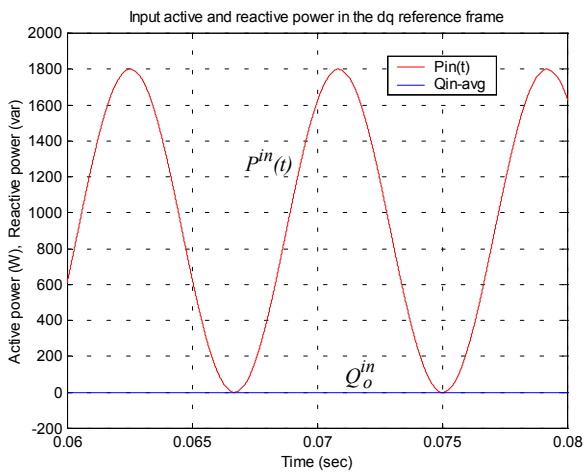


Fig. 5. Input active and reactive power in the dq synchronous reference frame of the simulation.

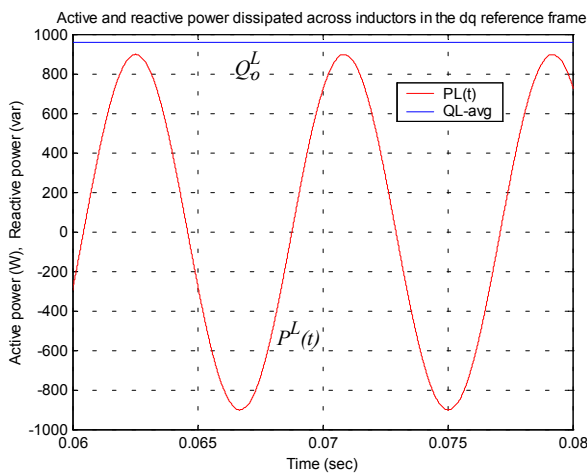


Fig. 6. Active and reactive power dissipated across the inductors in the dq synchronous reference frame of the simulation.

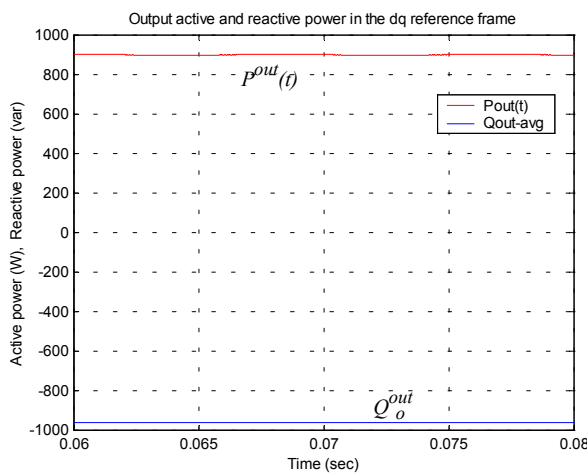


Fig. 7. Output active and reactive power in the dq synchronous reference frame of the simulation.

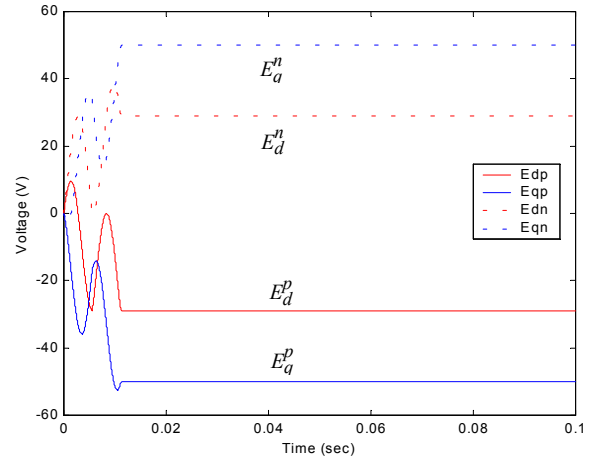


Fig. 8. Positive & negative sequence dq components of the input voltages derived from the Extractor Block in the simulation.

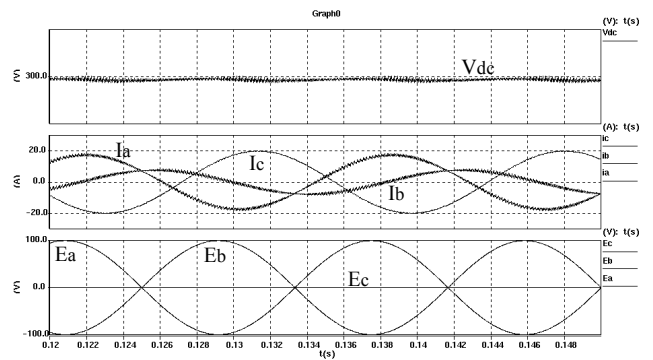


Fig. 9. Waveforms of the simulation.

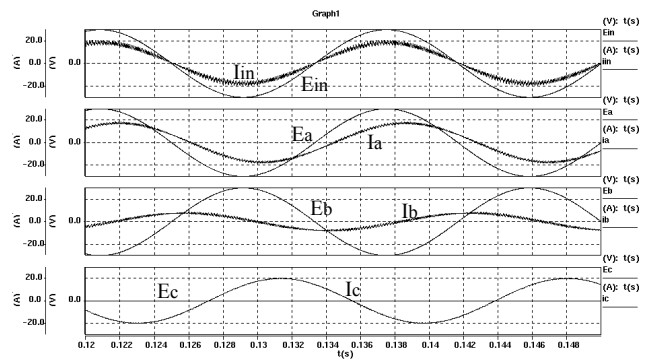


Fig. 10. Waveforms of the simulation.

Experimental results on a laboratory prototype pwm rectifier are shown in Fig. 11 verifying the simulation results. Smooth DC bus voltage can be observed despite the severe unbalance. The DSP processor TI TMS320F240<sup>®</sup> was employed in the control board. The waveforms in Fig. 11 were obtained under an open-loop control scheme. Algorithms for closed loop control are presently under investigation.

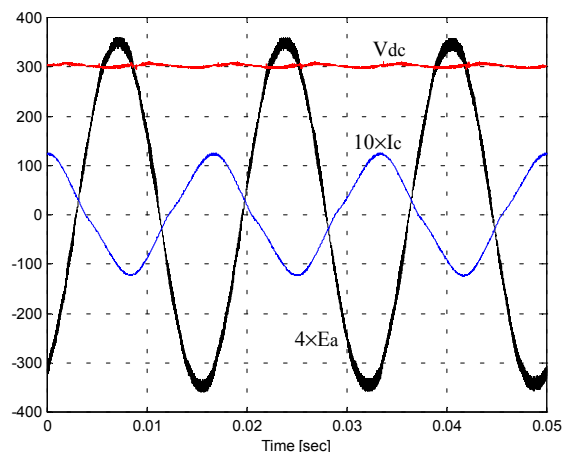


Fig. 11. Waveforms of the experimental prototype.

## V. CONCLUSION

This paper proposes a new control method for a line side connected pwm ac/dc converter operating under generalized unbalanced operating conditions. By nullifying the oscillating components of the instantaneous power at the poles of the converter instead of the front-end, the harmonics in an output dc voltage can be eliminated under generalized unbalanced operating conditions in ac input sides. A proposed control scheme needs to solve a set of nonlinear control equations with respect to the input currents in real time. A nonlinear equation solving algorithm called Broyden method was selected and successfully built into the current reference calculating block. An extreme unbalanced operating condition of a transformer sinusoidal secondary voltage with center tapped neutral and the unbalanced impedances was chosen in this paper. The dc output without substantial even-order harmonics and nearly unity power factor in ac side of the rectifier make it possible to reduce the size of the output dc-link capacitor and ac side inductors leading to reduced total system cost.

Simulation results along with experimental results for the open-loop control using a laboratory prototype converter confirm the proposed control method. Experimental verification for the complete closed-loop control is in progress.

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## REFERENCES

- [1] L. Moran, P.D. Ziogas, G. Joos, "Design Aspects of Synchronous PWM Rectifier-Inverter Systems under Unbalanced Input Voltage Conditions", IEEE Transactions on Industry Applications, Vol. 28, No. 6, November 1992, pp. 1286-1293.
- [2] Prasad N. Enjeti, Shamin A. Choudhury, "A New Control Strategy to Improve the Performance of a PWM AC to DC Converter under Unbalanced Operating Conditions", IEEE Transactions on Power Electronics, Vol. 8, No. 4, October 1993, pp. 493-500.
- [3] Ana V. Stankovic, T. A. Lipo, "A Novel Control Method for Input Output Harmonic Elimination of the PWM Boost Type Rectifier Under Unbalanced Operating Conditions", in Conf. Rec. APEC 2000, Vol. 1, pp. 413-419.
- [4] Pascal Rioual, Herve Pouliquen, Jean-Paul Louis, "Regulation of a PWM Rectifier in the Unbalanced Network State Using a Generalized Model", IEEE Transactions on Power Electronics, Vol. 11, No. 3, May 1996, pp. 495-502.
- [5] Hong-seok Song, Kwanghee Nam, "Dual Current Control Scheme for PWM Converter Under Unbalanced Input Voltage Conditions", IEEE Transactions on Industrial Electronics, Vol. 46, No. 5, October 1999, pp. 953-959.
- [6] Ana V. Stankovic, T. A. Lipo, "A Generalized Control Method for Input-Output Harmonic Elimination for the PWM Boost Rectifier Under Simultaneous Unbalanced Input Voltages and Input Impedances", in Conf. Rec. PESC 2001, Vol. 3, pp. 1309-1314.
- [7] A. M. Stankovic, H. Lev-Ari, "Frequency-Domain Observations on Definitions of Reactive Power", IEEE Power Engineering Review, June 2000, pp. 46-48.
- [8] IEEE PES Working Group on Nonsinusoidal Situations, "Practical Definitions for Powers in Systems with Nonsinusoidal Waveforms and Unbalanced Loads: A Discussion", IEEE Transactions on Power Delivery, Vol. 11, No. 1, 1996, pp. 79-101.