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SCIENCE DIRECT
Journal of Magnetism and Magnetic Materials 258–259 (2003) 369–371
Journal of magnetism and magnetic materials
www.elsevier.com/locate/jmmm

Stochastic resonance in a superparamagnetic particle

Yu.L. Raikher[a,*], V.I. Stepanov[a], P.C. Fannin[b]

[a] Institute of Continuous Media Mechanics, Ural Division of RAS, Russian Academy of Science, 1, Korolyov st., Perm 614013, Russia
[b] Department of Electronic & Electrical Engineering, Trinity College, Dublin 2, Ireland

Abstract

The stochastic resonance (SR) effect in a single-domain particle is investigated for the case where the exciting field is imposed not parallel to the anisotropy axis but at an arbitrary angle. We show that despite the fact that for the transverse case there is no SR at all, the intermediate cases yield signal-to-noise ratios much higher than the well-investigated longitudinal case. The frequency range over which the effect is observable is estimated.

Keywords: Superparamagnetic dynamics; Stochastic resonance; Signal-to-noise ratio

The phenomenon of stochastic resonance (SR) is inherent to noise-driven multistable systems. As it always happens with the effects related to Brownian motion, it has a very wide range of applicability [1]. Manifestation of SR is simple. Let a bistable system containing a source of fluctuations (e.g. being in contact with a heat bath) be subjected to an oscillating field of a frequency Ω favouring the transitions between the equilibrium states. It turns out then that under enhancement of the noise intensity the signal-to-noise ratio (SNR) determined from the spectral density function $Q(\omega)$ at $\omega = \Omega$, passes through a distinctive maximum.

In magnetism, the SR effect turns up in several situations. In particular, in a single-domain ferromagnetic particle with uniaxial anisotropy. In the absence of interaction with the neighbours the particle orientation-dependent energy is

$$U = -\mu H(\boldsymbol{eh}) - KV(\boldsymbol{en})^2, \tag{1}$$

where $\boldsymbol{e}$, $\boldsymbol{n}$ and $\boldsymbol{h}$ are the unit vectors of the particle magnetic moment, anisotropy axis and the external field, respectively; K is the effective anisotropy constant (for uniaxial anisotropy it is essentially positive), $\mu = I_s V$ is the magnetic moment of a single-domain particle, I_s its magnetization and V its volume. As Eq. (1) shows, without external fields the component of the magnetic moment $\mu(\boldsymbol{en})$ along the anisotropy axis has two, energetically equivalent orientations, viz., $\boldsymbol{e}\|\boldsymbol{n}$ and $\boldsymbol{e}\| -\boldsymbol{n}$, thus making a one-dimensional bistable system. The rate of transition between those potential wells is controlled by the parameter $\sigma = KV/k_B T$. Assuming that the barrier height KV is fixed, one may regard $1/\sigma$ as the dimensionless temperature, i.e., the noise level. As soon as one adds to this situation a linearly polarized AC magnetic field parallel to the particle easy axis, all the necessary conditions for SR of the longitudinal component $\mu(\boldsymbol{en})$ are satisfied. Exactly this situation was analysed in detail in a number of works, see Refs. [2–9]. Reducing the problem to a one-dimensional equation (as is typical for the basic SR theory), the authors find that in the limit $\omega \to 0$ and in the linear response theory approximation, magnetic stochastic resonance is described by some universal curve, $\mathrm{SNR}_0(1/\sigma)$. This curve corresponds to the cross-section $\beta = 0$ of the surface in Fig. 1 and also to the dashed line in Fig. 2. Returning to the basic framework of the problem, and, in particular, looking at expression (1), one realizes that for a magnetic particle the one-dimensional formulation is valid only as long as the probing AC field is perfectly parallel to the particle easy axis. Indeed, it is just for $\boldsymbol{h}\|\boldsymbol{n}$ that it suffices to have one angle to characterize a position of vector $\boldsymbol{e}$ with respect to the other two. As soon as the parallelism is broken, two angles (and,

*Corresponding author. Tel.: +7-342-239-1324; fax: +7-342-233-6957.
E-mail address: raikher@icmm.ru (Y.L. Raikher).

0304-8853/03/$ - see front matter
PII: S0304-8853(02)01112-5

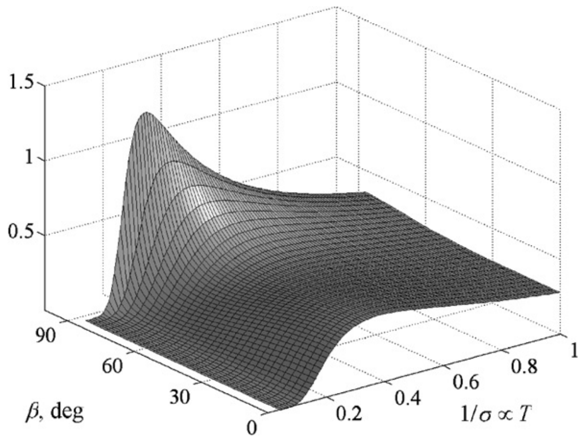

Fig. 1. Signal-to-noise ratio (arb. units) as a function of the dimensionless temperature and the angle between the field and the particle anisotropy axis; the plotted part of the surface ends at $\beta = 87°$, further behaviour follows from the cross-sections given in Fig. 2.

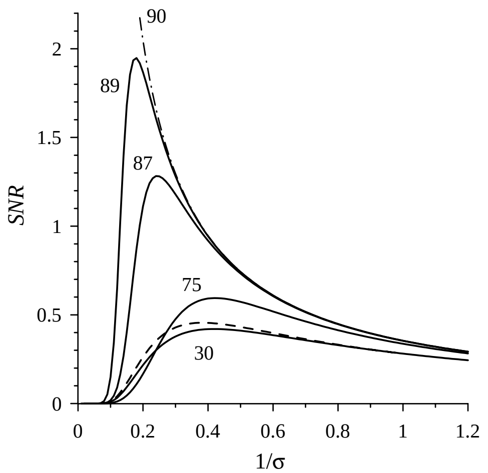

Fig. 2. Signal-to-noise ratio vs. dimensionless temperature for varying value of the tilt angle; the transverse case (90°) is shown by dash-dots, the longitudinal case (0°) by dashes, marks at the solid lines show the respective values of β.

accordingly, two equations of motion) are necessary to describe the particle magnetic moment $\boldsymbol{e}$. Concerning magnetic SR, this simple observation means that all the existing theory was focused only on just one limiting case of all the possible situations. Denoting the angle between vectors $\boldsymbol{h}$ and $\boldsymbol{n}$ as β, one may say that nothing is known for $\beta \neq 0$. Meanwhile, finite angles are of particular interest especially for particle assemblies where a statistical spread of the grain axes is always present.

Recognizing the fact that in general the magnetic moment of a single-domain particle has two orientational degrees of freedom, we arrive at the necessity to solve the problem of SR anew. It becomes rather complicated for strong probing fields, but simplifies considerably in the linear response approximation. There we have to expand the AC magnetic field in two components, longitudinal and transverse with respect to $\boldsymbol{n}$, find independently the responses to each of them, sum up the results and make the projection on the direction of $\boldsymbol{h}$. These data are used then to evaluate the sought for SNR. Fortunately, a lot of technical details are spared by the relationship

$$\mathrm{SNR} = (\pi H_1^2/4kT)[\omega|\chi(\omega)|^2/\chi''(\omega)] \tag{2}$$

derived in Ref. [10]. It renders the SNR for a system in terms of its dynamic susceptibility; here the formula is specified for the magnetic case with H_1 being the amplitude of the exciting field.

Calculation of the dynamic susceptibilities of a single-domain particle is based on a numeric solution of the micromagnetic Fokker-Planck equation for the orientational distribution function of the particle magnetic moment. Nowadays it is a standard technique, see Refs. [7,11], for example. In the oblique probing field the particle susceptibility is obtained in the form

$$\chi = \chi_{\parallel}(\sigma, \omega)\cos^2\beta + \chi_{\perp}(\sigma, \omega)\sin^2\beta, \tag{3}$$

where $\chi_{\parallel}$ and $\chi_{\perp}$ are the dynamic susceptibilities along and across the particle easy axis. These functions cannot be written down analytically, but their general behaviour is well known and the numeric procedures for them are well established [11]. In Eq. (3) the longitudinal term reflects the effect of spontaneous and induced flips of the magnetic moment over the potential barrier of internal anisotropy. These motions are called *interwell* transitions and they are the direct cause of the superparamagnetic behaviour of fine particles. The transverse susceptibility refers to a magnetic moment confined in one and the same potential well, i.e., describes the *intrawell* motion.

The enhancement of SR in the oblique field may be found (and we have done that) directly from a consistent realization of the above-outlined numeric procedures. The results are given in Fig. 1, where SNR is plotted as the function of $1/\sigma \propto T$ and β. However, to understand the origin of the effect it suffices to consider a simple scheme. Let us denote

$$\chi = \chi_0[f_{\parallel}(1 + \mathrm{i}\omega\tau)^{-1} + f_{\perp}(1 + \mathrm{i}\omega\tau_0)^{-1}],$$

$$f_{\parallel} = \tfrac{1}{3}(1 + 2S_2)\cos^2\beta, \quad f_{\perp} = \tfrac{1}{3}(1 - S_2)\sin^2\beta, \tag{4}$$

where $\tau = \tau_0 \exp(\sigma)$ is the Néel superparamagnetic relaxation time, $\tau_0 \sim 10^{-10} - 10^{-9}$ s is the reference intrawell time, and χ_0 is the static susceptibility for isotropic particles. At $\sigma > 1$ all the other response times are much shorter than τ, and thus the transverse susceptibility may be taken in its equilibrium form. The function S_2 is the internal order parameter of the particle, which is zero at low σ and tends to unity at $\sigma \gg 1$. Substituting expressions (3) in Eq. (2), one finds

$$\mathrm{SNR} \propto \sigma^2 \frac{(f_\parallel + f_\perp)^2 + \omega^2\tau_0^2 f_\perp^2 e^{2\sigma}}{\tau_0 f_\parallel e^\sigma + \tau_0 f_\perp (1 + \omega^2\tau_0^2 e^{2\sigma})}. \tag{5}$$

Note that we had to keep the imaginary part of the transverse term in χ'' since it is divided by ω in Eq. (2).

Let us compare formula (5) with the pure longitudinal (conventional) case

$$\mathrm{SNR}_0 = \sigma^2 e^{-\sigma} f/\tau_0$$

that follows from the same equation at $f_\perp = 0$. One then sees that at $\beta \neq 0$ the SR behaviour (SNR being a non-monotonic function of σ) requires the condition $\sigma < -\ln(\omega\tau_0)$ which ensures that the last terms in the numerator and denominator do not dominate. Otherwise—with the leading terms $e^{2\sigma}$—SNR (5) becomes proportional to σ that is a monotonic function inherent to the purely transverse case. If, however, the frequency range is appropriate ($\omega < 1/\tau_0 e^\sigma$, i.e., σ is not too large) from Eq. (5) one gets

$$\mathrm{SNR} \propto \sigma^2 \frac{(f_\parallel + f_\perp)^2}{\tau_0 (f_\parallel e^\sigma + f_\perp)}. \tag{6}$$

Let β be close to 90°. Moving in formula (6) from the high-temperature end, i.e., increasing σ, we see that at first SNR closely resembles that of the transverse case since the term $f_\parallel e^\sigma$ contains a small coefficient $\cos^2\beta$ and is insignificant. In Fig. 2 this behaviour is well visible as a coincidence of the right-hand wings of the curves corresponding to $\beta = 87{-}90°$. But as the temperature diminishes (σ grows), the exponent in the denominator overtakes causing a drastic reduction in SNR and, by this, the SR peak. Consequently, at $\beta \to 90°$ the stochastic resonance is considerably enhanced in comparison to the conventional one, see Fig. 2.

Quantification of SNR, as already mentioned, is possible only with numeric calculations. The surface presented in Fig. 1 is obtained in this way and it gives an overview of the behaviour of the SNR function with respect to its two main governing parameters. However, in a 3D plot some important details are difficult to discern. For them, in Fig. 2 we present cross-sections β=const of the surface of Fig. 1. These projections reveal that the angular dependence of SNR in a single-domain particle happens to be non-monotonic. Namely, as β increases from zero, SNR first goes down making SR even less sharp than it is in the longitudinal case. This suppression lasts until $\beta \approx 60°$ and only then the enhancement effect of the transverse modes begins to work.

The evidence rendered by the qualitative considerations and confirmed by the numerical analysis can be summarized as follows:

- magnetic SR in single-domain particles is the phenomenon, which in a general case results from the joint contribution of interwell and intrawell relaxation processes;
- magnetic SR in the longitudinal configuration is well known, in the transverse case it does not exist at all whilst in the intermediate situation, quite surprisingly, SR displays considerable enhancement with respect to the longitudinal case;
- this enhancement is the greater the closer the situation is to the transverse one, but at any angle the crucial condition for the existence of SR is $\omega\tau \ll 1$; this means that the longitudinal projection of the magnetic moment oscillates in a quasi-equilibrium regime;
- the SNR maximum under an oblique field has as its limiting value the SNR level inherent to the same particle at the same temperature in the purely transverse geometry.

We acknowledge partial financial support from the International Association for the Promotion of Co-operation with Scientists from the New Independent States of the Former Soviet Union (INTAS) under Grant 01-2341 and from the Award PE-009-0 of the US Civilian Research & Development Foundation for the Independent States of the Former Soviet Union (CRDF).

P.C.F. and Yu.L.R. gratefully acknowledge support from Enterprise Ireland under their International Collaboration Programme, 2001.

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