JOURNAL OF AIRCRAFT
Vol. 47, No. 4, July–August 2010

Engineering Notes

Induced-Drag Compressibility Correction for Three-Dimensional Vortex-Lattice Methods

T. Melin*
Linköping University, 58183 Linköping, Sweden
A. T. Isikveren†
University of Bristol, Bristol, England BS8 1TR, United Kingdom
and
M. I. Friswell‡
Swansea University, Swansea, Wales SA2 8PP, United Kingdom

DOI: 10.2514/1.C000197

Nomenclature

C_D	=	drag coefficient
C_{Di}	=	induced-drag coefficient
C_{D0}	=	zero-lift drag coefficient
C_L	=	lift coefficient
$C_{L\alpha,c}$	=	compressible lift slope, 1/deg
$C_{L\alpha,i}$	=	incompressible lift slope, 1/deg
C_P	=	pressure coefficient
C_{P0}	=	incompressible pressure coefficient
F	=	force, N
k	=	induced-drag factor
$\hat{L}$	=	vortex segment direction vector
M	=	Mach number
V_{eff}	=	effective velocity at vortex core, m/s
V_{rot}	=	velocity induced by rigid-body rotations, m/s
V_{Γ}	=	velocity induced by vorticity, m/s
$V_{\Gamma c}$	=	velocity induced by compressible vorticity, m/s
V_{∞}	=	freestream velocity, m/s
w	=	aerodynamic influence matrix
Γ	=	vortex strength, m^2/s
Λ	=	aspect ratio

Introduction

THIS Note covers the implementation of the Prandtl–Glauert (PG) rule for compressibility corrections at high subsonic Mach numbers in the subset of panel methods known as vortex-lattice methods (VLM). The PG compressibility correction is based on the Prandtl–Glauert rule, the similarity rule between flows in the incompressible and the compressible plane. The flowfield in the VLM is resolved in the incompressible plane and the results are transformed to the compressible plane. While the PG rule is an exact relation for inviscid flow, the term *correction* is more appropriate when comparing computed data with experimental results.

Panel methods used in conceptual design of aircraft calls for a correction to be applied to the basic incompressible results in order to be more accurate, and comparable to experimental results, at high subsonic Mach numbers. Several different corrections are available for this purpose. These corrections are applicable in the speed region before onset of transonic effects ($M = 0$ to ~ 0.5), such as wave drag created by local supersonic flow. The most common compressibility correction is the PG transformation [1] shown in Eqs. (1a) and (1b).

$$C_P = \frac{1}{\beta} C_{P0} \tag{1a}$$

$$\beta = \sqrt{1 - M^2} \tag{1b}$$

The PG correction is simple to implement but will underestimate the pressure coefficient at velocities just below the transonic region. High-end two-dimensional panel methods, such as XFOIL [2], use the more complex Kármán–Tsien relationship [3,4], which is more accurate but will slightly overestimate the pressure coefficient. The higher-order methods model the nonlinear aspect of the flow, needed for an accurate representation of the compressible flowfield.

Both of these methods and several of the other compressibility corrections available (such as the Küchemann–Weber, Wilby, or Laitone rules [5,6]) were developed for two-dimensional flows. Hence, they require special treatment to be applicable to three-dimensional flows. The induced drag, which is not present in the two-dimensional flow, requires special consideration.

The higher-order methods require direct access to the pressure coefficient C_P, which is not available in a thin-airfoil vortex-lattice method. VLMs typically produce a delta pressure coefficient, describing the difference in pressure on the top and bottom airfoil surfaces. This makes the higher-order corrections nonapplicable. Therefore, the best choice of compressibility correction for a VLM would be the PG rule. However, comparative investigations of the accuracy of different compressibility corrections, such as the one performed by Nordstrud [7], suggest the PG correction is the least accurate of the available methods.

The scope of the PG rule is to achieve an accurate description of the pressure distribution and lift produced. However, applying the PG correction factor directly to the incompressible pressure coefficient in a three-dimensional vortex lattice would result in a pressure distribution that would not accurately describe the induced drag; it would be severely underpredicted. For most aircraft configurations, the drag polars do not change in shape for Mach numbers in the range before the onset of transonic effects. The lift and drag at a specific angle of attack will increase, but not the drag as a function of lift, as shown in [8,9].

Lift and drag coefficients are proportional to the pressure coefficient, as shown in Eq. (2):

$$\begin{matrix} C_L \\ C_D \end{matrix} \propto C_P \tag{2}$$

As both lift and drag are computed as the surface integral of pressure projected onto the lift axis system, they keep their proportionality to the pressure coefficient if the PG correction is used on the pressure coefficient directly. However, in three-dimensional flows, the self-induced downwash from the lift will affect the pressure distribution, causing the proportionality at a given angle of

Received 27 November 2009; revision received 1 March 2010; accepted for publication 1 March 2010.

*Research Associate, Division of Fluid and Mechanical Engineering, Department of Management and Engineering; tomas.melin@liu.se.
†Senior Lecturer, Department of Aerospace Engineering, Queens Building, University Walk; askin.t.isikveren@bristol.ac.uk. Member AIAA.
‡Professor, Aerospace Structures School of Engineering, Singleton Park; m.i.friswell@swansea.ac.uk.

attack to break down. Lift and drag coefficients will still be proportional to the local pressure coefficient, but not to the incompressible pressure distribution. Indeed, such proportionality would violate the drag polar equation (3), where the induced drag is proportional to the lift squared:

$$C_{di} = kC_L^2 \tag{3}$$

The PG correction could therefore be used to correct lift directly, while the induced drag can be corrected using Eq. (3). However, this approach would leave the other stability and control derivatives untreated. The method proposed below allows for a more stringent treatment of the induced drag. Additionally, applying Eq. (3) to compute the induced drag, as recommended in [10], would be computationally inefficient. The proposed method computes the compressible force distribution directly, from which the coefficients and pressure distribution are evaluated. To transform all coefficients and the pressure distribution would require more computational overhead than transforming the vorticity distribution. By transforming the vorticity distribution, all of the secondary derivatives (such as yaw moment due to aileron deflection) are treated.

Proposed Method

In the compressible-speed region, the following approach may be used to correct results for the effects of compressibility. The VLM is initiated as usual, solving for the incompressible vortex strength distribution Γ_i. Once the incompressible vortex strengths are computed, the compressible vortex distribution can be computed through Eq. (4), where the incompressible vortex strength at each panel is multiplied with the PG correction factor:

$$\Gamma_c = \frac{1}{\beta}\Gamma_i \tag{4}$$

The induced velocity at the vortex core is then computed using the compressible vortex strength as

$$V_{\Gamma c} = w\Gamma_c \tag{5}$$

The compressible vortex induced velocity in three-dimensional space form is added to the velocity contributions from the freestream and from the aircraft solid-body rotations to give

$$\bar{V}_{\text{eff}} = \bar{V}_\infty + \bar{V}_{\text{rot}} + \bar{V}_{\Gamma c} \tag{6}$$

The compressible vorticity and effective velocity at the vortex core is then used to compute the force vector acting on each panel, i.e., the Kutta–Joukowski theorem in rewritten form, as

$$\bar{F} = \rho\Gamma_c(\hat{L} \times \bar{V}_{\text{eff}}) \tag{7}$$

The force distribution is integrated to global forces and moments or projected onto the surface normal to yield a pressure distribution. All of the aerodynamic coefficients in this method are linear functions of the global forces and moments.

As for the traditional two-dimensional correction, this three-dimensional variant is limited to thin airfoils at small angles of attack. This means that the velocity potential remains the same and all assumptions connected to the solution of Laplace's equation still hold.

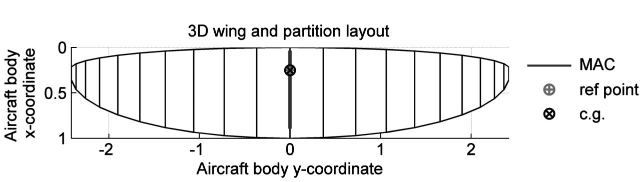

Fig. 1 Simulation geometry; elliptic plan form with an aspect ratio of 6. MAC is the mean aerodynamic chord.

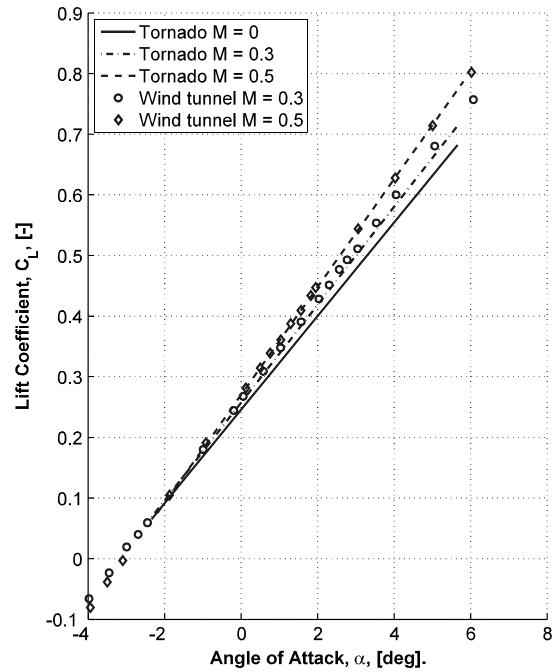

Fig. 2 Comparison of measured and computed lift as a function of angle of attack for different Mach numbers.

Validation

This method is implemented in the Tornado VLM software package, version 135 [11] (and later), and the validation results are presented below. Figure 1 shows the plan form of an elliptical wing with aspect ratio of 6, and an unswept quarter-chord line. The wing was modeled with four panels chordwise and 10 panels semi-spanwise. The appropriate number of panels was found with an induced-drag grid convergence study, with the drag change between iterations less than 0.5% as the convergence criterion. The panel distribution was linear chordwise and cosine spanwise. Figure 2 shows the lift as a function of angle of attack at different Mach numbers. The drag as a function of lift, both for incompressible and compressible flow, is shown in Fig. 3.

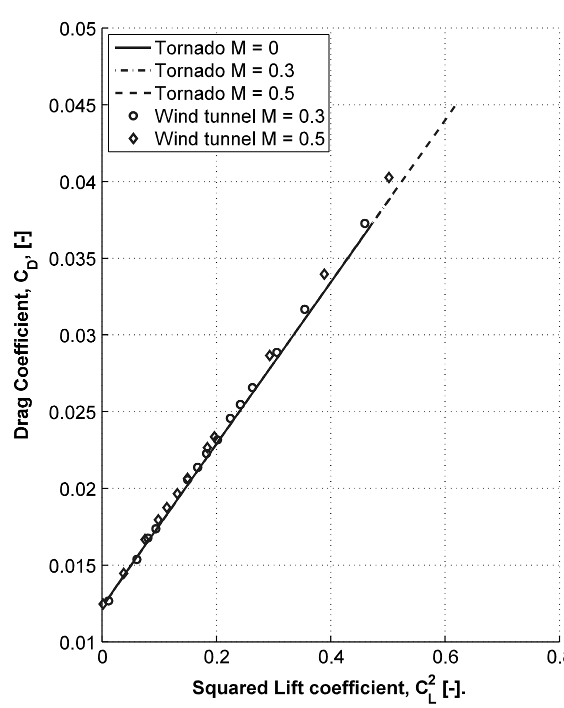

Fig. 3 Comparison of experimental and numerical drag as a function of lift squared. Tornado results are adjusted for zero-lift drag.

Table 1 Lift slopes for experimental and computed lift as at different Mach numbers

Lift slope $C_{L\alpha,c}$	$M = 0.0$	$M = 0.3$	$M = 0.4$	$M = 0.5$
Lifting line (LL)	0.0790	0.0816	0.0838	0.0869
Experiment (Exp)	——	0.0816	0.0839	0.0873
Corrected VLM	0.0772	0.0809	0.0841	0.0891
Residual, LL-VLM	0.0018	0.0007	−0.0003	−0.0022
Difference, LL-VLM	2.4%	0.9%	−0.4%	−2.4%
Residual, Exp-VLM	——	0.0007	0.0002	−0.0018
Difference, Exp-VLM	——	0.9%	0.2%	−2%

Table 2 Polar curve fit of experiments and corrected vortex-lattice results

Induced-drag factor k	$M = 0.0$	$M = 0.3$	$M = 0.4$	$M = 0.5$
Lifting line	0.053	0.053	0.053	0.053
Experiments	——	0.055	0.05585	0.055
Corrected VLM	0.053	0.053	0.05278	0.053
Residual	——	0.002	0.003	0.002
Difference	——	−4%	−5%	−4%

The computed data are compared with theoretical lifting-line results [Eq. (8)] taken from [10], and experimental results, taken from [9]. The validation case is an elliptical wing of aspect ratio of 6 is tested at Mach numbers 0.3 and 0.5 and Reynolds number 2.1×10^6 in the NASA Langley Research Center wind-tunnel facility:

$$C_{L\alpha} = \frac{2\pi\Lambda}{\sqrt{\beta^2\Lambda^2 + 4} + 2} \tag{8}$$

The incompressible vortex-lattice vorticity is corrected as described above and the results are presented below. The compressible lift slopes at different Mach numbers are shown in Table 1. The VLM data are adjusted to show the same zero-lift angle of attack as the wind-tunnel model.

The difference between computed and experimental data is within the error margin of a vortex-lattice code, which has about 5% uncertainty based on the internal assumptions and a 0.5% error from the grid convergence cutoff. Additionally, the residual in the curve fit of the experimental data is on the order of 1%. In this case, the comparison suffers most from the fact that the wind-tunnel model is fairly thick at 16%, which is not optimal for the VLM's thin-wing approximation. Figure 3 shows the drag as a function of lift in the form described by

$$C_D = C_{D0} + kC_L^2 \tag{9}$$

The experimental and numerical data are curve fit to Eq. (9), and the resulting induced-drag factor k is compared between the numerical and experimental methods in Table 2. For an elliptically loaded wing, the induced-drag factor k can also be computed with the lifting-line theory valid for all Mach numbers, according to

$$k = \frac{1}{\pi\Lambda} \tag{10}$$

Again, a difference of 5% is within the error margin of the VLM. The proposed correction appears to underpredict the polar curve-fit factor when compared with the experiments, which could be due to viscous effects not modeled in the VLM. The agreement with the lifting-line data is very good, which is expected as the two methods are similar.

Conclusions

The proposed method appears to model the compressible effects on the induced drag for a three-dimensional vortex-lattice model to within method assumptions accuracy. The good agreement between experimental and computational data is particularly noteworthy, as the thickness of the experimental wing, 16%, is too thick to be considered thin. A thin wing is one of the assumptions in the VLM, as is the small-angle-of-attack assumption. Both of these are a part of the small-disturbance hypothesis. The robustness of the accuracy of the results at the edges of the envelope of validity is one of the large benefits of the VLM.

In addition to lift and drag, all stability derivatives may also be treated by the same model. It has therefore been included in the vortex-lattice method implementation of Tornado, version 135 [11] and later. The reader is reminded that the proposed method cannot predict transonic effects such as drag creep, lift divergence, or shock formation.

References

[1] Glauert, H., "The Effects of Compressibility on the Lift of an Aerofoil," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 118, No. 779, March 1928, pp. 113–119. doi:10.1098/rspa.1928.0039

[2] Drela, M., and Youngren, H., "XFOIL 6.9 User Primer," Nov. 2001, http://web.mit.edu/drela/Public/web/xfoil/xfoil_doc.txt, [retrieved 28 Oct. 2009].

[3] von Kármán, T., "Compressibility Effects in Aerodynamics," *Journal of the Aeronautical Sciences*, Vol. 8, No. 9, July 1941, pp. 337–356.

[4] Tsien, H. S., "Two-Dimensional Subsonic Flow of Compressible Fluids," *Journal of the Aeronautical Sciences*, Vol. 6, No. 10, 1939, pp. 399–407.

[5] "Second-Order Method for Estimating the Subcritical Pressure Distribution on a Two-Dimensional Aerofoil in Compressible Inviscid Flow," IHS EDSU, Rept. ESDU 72025, London, 2010.

[6] Laitone, E. V., "New Compressibility Correction for Two-Dimensional Flow," *Journal of the Aeronautical Sciences*, Vol. 18, No. 5, May 1951, p. 350.

[7] Nordstrud, H., "A Correction for Compressible Subsonic Planar Flow," *Journal of Aircraft*, 0021-8669 Vol. 8, No. 2, 1971, pp. 123–125. doi:10.2514/3.44241

[8] Mineck, R. E., and Vijgen, P. M. H. W., "Wind-Tunnel Investigation of Aerodynamic Efficiency of Three Planar Elliptical Wings with Curvature of Quarter Chord Line," NASA TP-3359, Oct. 1993.

[9] Roskam, J., *Airplane Design Part VI: Preliminary Calculation of Aerodynamic, Thrust and Power Characteristics*, Roskam Aviation and Engineering Corp., Ottawa, KS, 1987.

[10] Schlichting, H., and Truckenbrodt, E., "Aerodynamik des Flugzeuges," *Zweiter Band*, Springer–Verlag, Berlin, 1969.

[11] Melin, T., "Tornado Version 135 Source Code Specification," Linköping Univ., Rept. LIU-IEI-R09/0081SE, Linköping, Sweden, Nov. 2009.