Trends in Mathematics, © 1999 Birkhäuser Verlag Basel/Switzerland

Active control of turbulence in boundary layer flows

P. J. Ioannou, B. F. Farrell

1 Introduction

Preventing the transition to turbulence of a laminar flow and suppressing the variance of a turbulent flow, perhaps with the ultimate goal of inducing relaminarization, or alternatively controlling eddy fluxes produced by a given level of fluctuations are in themselves problems of great practical importance. In addition, understanding the physical mechanism of turbulence and turbulent transition should lead either to methods of control or to an explanation of why such control is not possible. From this perspective the control problem is seen as a test of physical theory. From the viewpoint of practical engineering, a comprehensive theory of the transition process and of the maintenance of fully developed turbulence that both implied new control mechanisms and provided a means of testing proposed mechanisms would be of great utility even if the result were only to discourage the search for e.g. a passive compliant membrane that relaminarized the turbulent boundary layer or an acoustic intervention to control turbulence in a free jet. Extensive attempts to reduce drag in turbulent boundary layer flow by imposing a variety of active and passive control measures have shown that in the absence of applicable theory it is very unlikely that an optimal method can be identified. Similar problems are faced if the aim is to control pressure fluctuations in order to reduce acoustic radiation or if we aim at increasing mixing in order to enhance chemical reactions in combustion problems.

At the outset we must address the question whether control can be achieved in principle. It would seem that the answer is affirmative: if it were possible to observe the entire disturbance field an appropriate force field could be chosen to produce any desired level of variance. Equally if we were able to continuously disrupt the disturbance field at constant energy then inspection of the expression for the total fluctuation energy tendency shows decrease of fluctuation energy at the rate of trace($\mathbf{A}$), where $\mathbf{A}$ is the linearized operator associated with the stationary flow, which for viscous flows is typically negative. And indeed such a limitation on the coherence of motions produced by self-interaction of the fluctuations restricts the temporal correlation of fluctuations and provides a limit on the variance that could be maintained by a turbulent flow.

However control is a practical problem. The question is whether feasible observations linked to feasible control actions would result in a desired level of fluctuation variance. For example it is not obvious that both observation and control of a single variable at the surface would be sufficient to produce suppression of turbulence throughout the boundary layer.

Further, it is not obvious, that success in controlling a flow implies the possibility of control of all flows. In fact it is quite likely that some flows may be more easily controlled than others. Specifically optimism concerning the control of boundary layer turbulence appears more justified in view of recent advances in theoretical understanding than does control of free jets and free shear layers in which robust inflectional instabilities exist and in which there is no natural platform for observations as in the case of boundary layer turbulence.

It is useful to draw a distinction between controlling transition to turbulence and intervening to control the fluctuations in a fully turbulent flow. In the first problem the control strategy derives from theoretical understanding of the transition process, while the second derives from understanding of how the energy injection occurs in turbulent flows.

While essentially nonlinear theories for transition have been advanced there is at least a large class of transition scenarios, commonly referred to as bypass transition, in which small but finite perturbations induce the transition and for which linear mechanisms are clearly implicated. An example is the rapid transition at high Reynolds numbers in flows with naturally free stream fluctuations. For these flows it has been demonstrated that the perturbation growth can be traced to non-modal transient mechanisms due to the non-normality of the linear operator associated with the highly sheared boundary layer (Farrell, 1988; Gustavsson, 1991; Butler & Farrell, 1992; Reddy & Henningson, 1993; Trefethen *et al.*, 1993; Farrell & Ioannou, 1993a,1993b). Consequently designing an optimal control strategy for such flows proceeds from identification and suppression of those perturbations in the free stream turbulence with the potential for growth.

Moreover, this theory of by-pass transition has recently been extended to a theory for the maintenance of the turbulent state in which the spectral transfer arising from the non-linear interactions of the fluctuations is parameterized as stochastic noise and the linear interaction of the fluctuations with the background flow predominantly determines the energy injection properties. Consequently, we believe that for fully developed stationary boundary layer turbulence the process of injection of energy is essentially linear and the optimal control strategy adopted for the transitional flow is in principle a guide for the control of the fully turbulent flow. In what follows we describe the method and provide an example of how the proposed control strategy is applied (*cf* Farrell and Ioannou, 1996).

2 Formulation of the active control problem

Consider the evolution of small perturbations imposed on a steady channel flow with streamwise (x) background velocity $U(y)$ varying only in the cross-stream direction (y). Harmonic perturbations with streamwise wavenumber k and spanwise (z) wavenumber l obey the linear equation:

$$\frac{d\phi}{dt} = \mathbf{B}\,\phi \ , \tag{1}$$

where the state variable is $\phi = [\hat{v}\ , \hat{\eta}]^T$, in which $\hat{v}$ is the cross stream perturbation velocity, and $\hat{\eta} = il\ \hat{u} - ik\hat{w}$ is the cross stream perturbation vorticity ($\hat{u}, \hat{w}$ are the perturbation streamwise and spanwise velocities respectively). The operator is given by:

$$\mathbf{B} = \begin{bmatrix} \mathbf{L} & 0 \\ \mathbf{C} & \mathbf{S} \end{bmatrix}, \tag{2}$$

The components of the dynamical operator (2) are the Orr-Sommerfeld operator, **L**, the coupling operator between cross-stream velocity and vorticity, **C**, and the diffusion Squire operator, **S** (*cf* Butler & Farrell, 1992).

We choose to impose symmetric control at the channel walls $y = \pm 1$ in reaction to observations of a field variable at $Y_1^{ob} = -1 + Y_o$ and at $Y_2^{ob} = 1 - Y_o$. By cross-stream velocity control we mean that observations of the cross stream velocity at Y_1^{ob} , Y_2^{ob} are used to impose a cross stream velocity at $y = \pm 1$ according to:

$$\hat{v}(-1) \; = \; C\,\hat{v}(Y_1^{ob}) \quad , \quad \hat{v}(1) \; = \; C\,\hat{v}(Y_2^{ob})\,, \tag{4}$$

where C is a complex control constant. Clearly, alternative controls can be imposed in a similar manner.

The remaining boundary conditions for the case of active specification of the cross-stream ($\hat{v}$) velocity at the boundaries are the vanishing of the streamwise ($\hat{u}$) and spanwise velocity ($\hat{w}$) components at the walls. Consequently at the channel wall we have the following boundary conditions:

$$\left.\frac{d\hat{v}}{dy}\right|_{y=\pm 1} = 0 \quad , \quad \hat{\eta}(\pm 1) = 0\,. \tag{5}$$

The perturbation evolution equation (1) together with boundary conditions (4) and (5) form a linear system with homogeneous boundary conditions and the imposition of feedback control constitutes a change in the boundary conditions of the flow. Therefore the control action can not be understood using arguments about cancellation or reinforcement of perturbations that exist in the unmanipulated flow. Instead, suppression of turbulence occurs because control parameters alter the boundary conditions so as to constrain the perturbations to exhibit reduced growth compared to that found with the standard boundary conditions in the unmanipulated flow.

We will determine the magnitude and phase of the control C and the observation level Y_o that reduces the growth of perturbations. Plane Poiseuille flow with $U = 1 - y^2$ is used as an example.

In order to proceed it is necessary to have a measure of perturbation growth. We choose the perturbation energy and we denote with **M** the energy metric. We transform (1) into generalized velocity variables $\psi = \mathbf{M}^{1/2}\,\phi$ so that the usual L_2 norm corresponds to the square root of the mean energy. Under this transformation a perturbation ψ_0 at $t = 0$ evolves to time t according to:

$$\psi^t = e^{\mathbf{A}t}\,\psi_0 \tag{6}$$

in which the dynamical operator has been transformed to $\mathbf{A} = \mathbf{M}^{1/2}\,\mathbf{B}\,\mathbf{M}^{-1/2}$.

An appropriate measure of perturbation growth at time t is the square of the Frobenius norm of $e^{\mathbf{A}t}$. This quadratic measure is equal to the sum of the squares of the singular values of $e^{\mathbf{A}t}$. This measure is proportional to the growth over an interval t of the mean perturbation when all perturbations are forced equally initially. The time integral of this measure is proportional to the perturbation variance maintained in the channel flow under white noise forcing i.e. the accumulated variance over an interval t for unit forcing of each degree of freedom is given by:

$$< E^t > = \text{trace} \left(\int_0^t e^{\mathbf{A}t} \, e^{\mathbf{A}^\dagger t} \, dt \right) , \tag{7}$$

where the brackets denote ensemble averaging. The steady state maintained variance $< E^\infty >$ is given for asymptotically stable systems as the limit of (7) as $t -> \infty$.

The maintained variance for asymptotically stable flows is found by solving the Liapunov equation for the correlation matrix $\mathbf{V}^\infty$:

$$\mathbf{A} \, \mathbf{V}^\infty + \mathbf{V}^\infty \, \mathbf{A}^\dagger = - \mathbf{I} , \tag{8}$$

with $\mathbf{I}$ the identity matrix corresponding to unitary forcing. The asymptotic variance can be identified with the trace of the correlation matrix, $< E^\infty > =$ trace $(\mathbf{V}^\infty)$.

The variance maintained by unbiased forcing in an unmanipulated Poiseuille flow peaks at the roll axis ($k = 0$). For $R = 2000$ there is a broad maximum at $K = O(1)$. For large Reynolds number ($R > 1000$) the peak wavenumber increases linearly with Reynolds number. Oblique harmonic perturbations also build energetic streaks and maintain substantial variance. Consequently, in our investigation of optimal control parameters we include oblique perturbations.

Effective controls, C, are those that minimize

$$\frac{< E_C^\infty >}{< E_0^\infty >} , \tag{9}$$

in the complex C plane where $< E_0^\infty >$ is the variance maintained under stochastic forcing with no control applied ($C = 0$). We investigate the magnitude of the variance suppression as a function of the amplitude $|C|$ and phase Θ of the control for roll and oblique perturbations and for observation at various distances from the wall, Y_o. An effective control must lead to robust suppression of both roll and oblique perturbations.

3 In phase and out of phase control

We first constrain the control parameter C to be real. As expected, in phase control ($C > 0$) leads for small control amplitudes to increased variance. In the vicinity of $C = 1$ the flow becomes unstable (this instability is diffusive in nature and occurs also in the absence of flow for any finite Reynolds number). We find further that, remarkably, higher amplitudes of in phase control lead to robust variance suppression. This surprising suppression will be referred to as overdriving suppression. For example in phase overdriving at an amplitude $C \approx 2$ leads to variance suppression of the order of 60-70 % when observations are made at $Y_o = 0.2$ from the wall. This control robustly suppresses the variance of both roll and oblique perturbations.

Out of phase control ($C < 0$) of roll perturbations leads to robust reduction of variance with the suppression becoming more effective the farther the observation level is located from the controlled boundary (at least for $Y_o < 0.5$). Maximum suppression requires amplitudes $|C| > 4$ and is of the order of 90%. Unfortunately, this promising control strategy does not generalize to oblique perturbations. For distant observation levels i.e. $Yo > 0.2$ and with out of phase control of amplitude $|C| \approx 1$ oblique perturbations become unstable leading to variance increase. This instability appears at low Reynolds numbers (typically $R = O(500)$) and analysis of the energetics of the instability reveals that the control injects only a small amount of energy while the predominant energy source is the downgradient Reynolds stress term. The most unstable perturbations occurs for purely 2-D perturbations in agreement with predictions of Squire's theorem. The direct numerical simulation experiments of Choi *et al.* (1994) showed that out of phase control of unit amplitude ($C = -1$) leads to drag reduction for observations at locations less than 20 wall units and to drag increase for observations at greater distances from the wall. The cause of this drag increase is presumably inception of the instability described above. We performed the same stability analysis on the Reynolds-Tiederman profile. The results of the stability analysis of the Orr-Sommerfeld operator for this profile indicates that inception of the instability occurs for observations located at 30 wall units. The calculations reported here and the experiments of Choi *et al.* (1994) were carried out in a channel flow so that the possibility remains that the instability occurring in the vicinity of out of phase control ($\Theta = 180$) would not occur in boundary layer flows. To check this a stability analysis was performed on the one sided Reynolds-Tiederman profile. Although the instability occurs at a higher value of observation locations (for observations located at 40 wall units from the boundary) it is qualitatively similar to that found in channel flow.

4 Boundary control in quadrature with observations

We consider control actions in which the boundary response is in quadrature with the observation ($\Theta = \pm 90^o$). For roll perturbations ($k = 0$) a robust suppression of variance as a function of the control amplitude is found for various observation levels near the wall i.e. $Y_o < 0.4$. For example for $Y_o = 0.4$ the variance suppression reaches 70−80% for control amplitudes $|C| \approx 4$. For observation levels $Y_o > 0.5$ roll perturbations show an increase in variance. Despite robust suppression of variance at $\Theta = 90^o$ the greatest suppression for roll perturbations occurs at $\Theta = 180^o$ corresponding to exactly out of phase control. Unfortunately, as we have seen, this out of phase control fails to similarly suppress oblique perturbations because of the existence of an unstable mode. Analysis of the resulting flow shows that the constraint imposed by the control leads to the development of a much weaker doublet of opposing streaks.

We turn now to variance suppression for oblique perturbations. We find that for observations at $0.3 < Y_o < 0.7$ robust suppression of the order of $60 - 70\%$ occurs at $\Theta = 90^o$.

We have already seen that out of phase control suppresses variance optimally for streamwise roll perturbations. However, even slightly oblique perturbations may become unstable for out of phase controls. Consequently, the most robust strategy is out of phase control of roll perturbations and $\Theta = 90^o$ control of oblique perturbations.

Physically the variance reducing control action inhibits the formation of streaks in the vicinity of the wall where high shear would lead to substantial build up of streak amplitude. The control boundary condition induces a time varying cross-stream velocity near the boundaries which inhibits the formation of the energetic near wall streaks. Analysis of energetics confirms that the energy growth arising from the Reynolds stress is reduced in the controlled flow.

An implication of these results is that the rms amplitude of the streamwise and cross-stream velocities in controlled flows peak at greater distance from the wall. This can also be seen to be the case in the numerical simulations presented by Choi *et al.* (1994).

Acknowledgement

This work was supported by NSF ATM-96-25453.

References

Butler, K. M., and B. F. Farrell, 1992: Three dimensional optimal perturbations in viscous shear flow. *Phys. of Fluids A*, **4**, 1637–1650.

Choi, H., P. Moin, and J. Kim, 1994: Active turbulence control for drag reduction in wall bounded flows. *J. Fluid Mech.* , **262**, 75–98.

Farrell, B. F.,1988: Optimal excitation of perturbations in viscous shear flow. *Phy. Fluids*, **31**, 2093–2102.

Farrell, B. F., and P. J. Ioannou, 1993a: Optimal excitation of three dimensional perturbations in viscous constant shear flow. *Phys. Fluids A* , **5**, 1390–1400.

Farrell, B. F., and P. J. Ioannou, 1993b: Perturbation growth in shear flow exhibits universality. *Phys. Fluids A* , **5**, 2298–2301.

Farrell, B. F., and P. J. Ioannou, 1996: Turbulence suppression by active control. *Phys. Fluids A* , **8**, 1257–1268.

Gustavsson, L. H., 1991: Energy growth of three-dimensional disturbances in plane
Poiseuille flow. *J. Fluid Mech.* , **224**, 241–260.

Reddy, S. C., and D. S. Henningson, 1993: Energy growth in viscous channel flows. *J. Fluid Mech.* , **252**, 209–238.

Trefethen, L. N., A. E. Trefethen, S. C. Reddy, and T. A. Driscoll, 1993: Hydrodynamic stability without eigenvalues. *Science*, **261**, 578–584.