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# BULK MODULUS OF AIR CONTENT OIL IN A HYDRAULIC CYLINDER

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## ABSTRACT

A model of oil with entrained air content is developed which considers fluid compression and the subsequent dissolving of mixed entrained air. According to the model the mixed entrained air affects the “gross” bulk modulus below some critical pressure, but has no effect above this value due to the complete dissolving of the entrained air into solution. The critical pressure is shown to be proportional to the square root of the amount of the initial mixed entrained air. The temporal pressure gradient has also a substantial effect on the critical pressure value and thus on the bulk modulus. The critical pressure value increases but tends towards an upper value with increasing temporal pressure gradient (a true dynamic condition); the opposite occurs when the pressure gradient decreases as the critical pressure converges to a lower value (essentially a static value). Thus regions of static and dynamic bulk modulus can be established. The model predicts that the upper critical pressure value is some 1.8 times that of the static one.

Experiments have been designed to verify the feasibility of the model by measuring the temporal pressure gradient against the variation of compressed oil volume. It is demonstrated that the model is verified not only for the case of positive pressures (above atmospheric pressure) but also for pressures less than atmosphere. Finally a comparison of the proposed model is made with those proposed in the literature. The bulk modulus predicted by the proposed model is a little larger than these given in literature. The reason for such difference is attributed to the result of air being dissolved into oil.

## 1. INTRODUCTION

Although fluid is incapable of sustaining an independent shape, it can demonstrate a considerable “volumetric stiffness” against any variations in fluid pressure. This property is defined as the fluid bulk modulus and it is a parameter that is known to influence significantly the dynamic response of the hydraulic actuators and servo-systems. Owing to the large numerical value of oil bulk modulus (typically, from 1.4 to 2.1 GPa) and subsequently, a high natural frequency, hydraulic servo systems demonstrate a fairly high level of performance compared to other types of control systems, such as electrical motor servo systems or electropneumatic servo systems. However, oil which is used as a power transmission media in conventional hydraulic system is often exposed to air in the reservoir and consequently, there is good chance that some air will be mixed into the oil in the form of either dissolved air or free bubbles (entrained air). While a lower percentage of dissolved air has little effect on the fluid bulk modulus value, the presence of entrained air will dramatically reduce this value and subsequently reduce the hydraulic natural frequency; in some extreme cases, this can lead to instability in the system.

Due to the significant role played by the bulk modulus in the transient response and natural frequency of hydraulic control systems, intensive investigations have been carried out to determine its value as a function of fluid pressure, for both oil without any air (dissolved or entrained) or with. Some of the first work on Bulk Modulus was done by Burton and Ukrainetz [1] in the early 70’s in which an effective “on line” bulk modulus was estimated which reflected actual operating conditions of the oil, but did not correlate the estimated value to the air content. Yu et al in 1994 studied the bulk modulus by measuring the speed of sound transmitted in a tube filled with a tested fluid; from the speed of sound value, they developed an empirical equation for the effective bulk modulus [2]. A similar experimental approach has also been used by Harms et al. [3], Kuss et al. [4] and a theoretical approach was presented by Nykanenet et al. [5] to study the bulk modulus of oil. These approaches can be classified as an "indirect method". Errors can be introduced in such approaches due to the process of converting the speed of sound into a bulk modulus value; the wave front, which, in reality is distorted from a straight plain wave front, is a factor in affecting accurate determination of the wave speed, when entrained or free is present in the fluid.

Kajaste, J. et al. (2005) [6] and Watton et al. [7] (1994) used direct methods to determine the value of the bulk modulus. The approach taken by Kajaste et. al. was to force a piston into a test fluid contained in a cylinder and by measuring the volumetric and pressure change inside, the bulk modulus was calculated; a model so developed was based on experimental results. A similar method to determine isothermal and tangent bulk modulus is defined in Standard ASTM D 6793 – 02 [8]. Watton et al. [7] (1994) used two flow meters to measure the differential between the inlet and outlet volumetric flow rate of a test rig, and established a relationship between the volumetric variation and the pressure.

The models given by above researchers were all experimentally-based empirical ones. In these models, it is possible that the volumetric change of any air content in the fluid could vary as entrained air becomes dissolved air as pressure increases. Hayward in 1961 [9] did consider this effect and did demonstrate that it could be significant as far as the estimation of the bulk modulus value was concerned. Not treating this effect effectively ignores that phenomenon of basic compression of air in a contained environment as defined by Henry's Law, that is, the volume of dissolved air in liquid is proportional to the pressure at a fixed temperature. It is well known that air dissolved in solution has little effect on the bulk modulus; if the entrained air presence is not directly accounted for in any model, the results so predicted can be in error. Thus it is very important to know when entrained air becomes dissolved air in any experiential study since the bulk modulus value can change significantly.

In many cases, models compute the bulk modulus as a function of air content in the oil and generally include variables such as the relative amount of air $X$ and the polytropic constant $k$. For many practical engineering applications, determining of the values $X$ and $k$ is extremely difficult, even more difficult that determining the bulk modulus itself. Another important factor is that most of the models developed in the literature cannot be applied under conditions of less than atmospheric pressure. For these reasons it was the motivation and hence the objective of this research to develop a more practical description of the process of the volumetric change of liquid containing free or entrained air as a function of pressure and to develop a model of bulk modulus which is in a convenient form for engineering applications. This, then, is the original contribution of this research. This study represents a first phase a very complex research area.

## NOMENCLATURE

| | | |
|---|---|---|
| $k$ | Polytropic constant | |
| $K_e$ | Bulk modulus of pure oil | [N/m$^2$] |
| $K_{ef}$ | Effective Bulk Modulus | [N/m$^2$] |
| $K_{eg}$ | Gross bulk modulus | [N/m$^2$] |
| $K_{em}$ | Measured bulk modulus | [N/m$^2$] |
| $p$ | Pressure | [N/m$^2$] |
| $p_a$ | Atmospheric pressure | [N/m$^2$] |
| $p_c$ | Critical pressure | [N/m$^2$] |
| $p_{c0}$ | Critical pressure of remnant entrained air | [N/m$^2$] |
| $p_{cT}$ | Critical pressure of isothermal process | [N/m$^2$] |
| $p_{cs}$ | Critical pressure of adiabatic process | [N/m$^2$] |
| $t$ | Time | [s] |
| $t_c$ | Response time arriving at critical pressure | [s] |
| $V_a$ | Volume of entrained air | [m$^3$] |
| $V_g$ | Volume of entrained air at atmospheric pressure | [m$^3$] |
| $V_o$ | Volume of oil without entrained air | [m$^3$] |
| $V_t$ | Total volume | [m$^3$] |
| $V_s$ | Volume of free air bubbles | [m$^3$] |
| $X$ | Relative amount of entrained air at atmospheric pressure | |
| $X_0$ | Relative amount of remnant entrained air | |
| $\alpha$ | Coefficient of dissolubility | |
| $\Delta p$ | Pressure increment | [N/m$^2$] |
| $\Delta V$ | Volumetric increment | [m$^3$] |
| $\gamma$ | Modifying coefficient elastic container shell | |

## 2 VOLUMETRIC VARIATION OF AIR CONTENT IN OIL

The mixture of air in oil generally exists in two forms: entrained and dissolved. The molecules of air dissolved in oil form an air-in-oil emulsified liquid [10]. Entrained or entrained air is known to drastically reduce the gross (also known as effective or equivalent) bulk modulus of oil; however, there is no evidence that dissolved air has any noticeable effect on the bulk modulus within the range normally experienced in classical hydraulic systems [11]. Entrained air can become dissolved air under certain conditions and vice versa. According to Henry's Law, the relative volume of air dissolved in oil is proportional to the absolute pressure at a given temperature. Thus depending on the location and movement of the fluid in conventional hydraulic systems, entrained air can dissolve into or separate from oil. Therefore, the relative volume of either entrained air or dissolved air is dependent on the pressure, in addition to the initial amount of air entrained.

### 2.1 Volumetric Change of Free Bubbles

Entrained air in an oil mixture exists in the form of bubbles due to interfacial tensions. The boundary of the bubble is the intermediate layer separating air and oil. Under "saturated" steady conditions the number of air molecules passing into this layer equals to that leaving the layer and maintains a dynamic balance. Hence, the oil nearby the boundary is considered saturated because the amount of air that can be dissolved reaches its maximum. However this balance can change with fluid pressure or temperature. When the pressure rises, for example, the average level of kinetic energy of air molecule will increase and the number of air molecules entering the layer will be greater than that coming out of the layer. Eventually, the number of air molecules passing through the boundary layer decreases, until they again equal to the number of molecules passing outwards across the boundary layer and a new balance is once again established. When the pressure decreases, the opposite occurs but again a new balance is established. This phenomenon has been also reported in [6].

The above mentioned process can be described analytically using Henry's Law which states that the maximum ratio of dissolved air to oil is proportional to the absolute pressure. With increasing pressure, the boundary can absorb more air. Therefore, the volumetric change of free air bubbles which are dissolved in oil as a function of pressure can be expressed as:

$$\Delta V_s = \alpha \Delta p \qquad (1)$$

Or in differential form,

$$dV_s = \alpha dp \qquad (2)$$

In the above equations, $\alpha$ is the coefficient of "dissolubility".

The initial state of the entrained air is assumed to be [ $p$ ,$V_a$ ]. As the pressure goes up to $\Delta p$, the volume of the entrained air a change of $\Delta V$ which includes volumetric variation of the free air bubbles (entrained air) $\Delta V_a$ and dissolved air volume $\Delta V_s$. The volumetric variation of the air bubbles satisfies following polytropic equation

$$pV_a^k = (p + \Delta p)(V_a + \Delta V_a + \alpha \Delta p)^k \qquad (3)$$

In the above equation, $k$ is the polytropic constant which takes a value within a range from 1 to 1.4, depending on the heat exchange between the air bubbles and the surrounding oil. For two extreme cases, isothermal and adiabatic, $k$ equals to 1 and 1.4 respectively. In the operation of a hydraulic system, these conditions occur under steady state conditions (isothermal) or transient – dynamic (adiabatic) conditions. However, under "ideal" adiabatic conditions the temperature would become infinitely high as the air bubbles are compressed to a volume close to zero (collapse). In reality, some heat exchange does occur and the pure adiabatic condition is no longer sustainable. Therefore, for a dynamic process, $k$ should take a value smaller than 1.4.

By Applying Tailor's Equation in Equation (3) and considering $\Delta V_a \to 0$ and $\Delta p \to 0$, the differential equation dominating volumetric variation of the air bubbles is obtained as:

$$kpdV_a + V_a dp = -k\alpha p dp \qquad (4)$$

For isothermal compression, that is $k = 1$, and thus Equation (4) becomes:

$$pdV_a + V_a dp = -\alpha p dp \qquad (5)$$

Initial conditions occur when $p = p_a$, and $V = V_g$. By solving the differential equation (5) and applying initial conditions, the relationship between the bubbles' volume as a function of absolute pressure is obtained as

$$V_a = \frac{p_a}{p} V_g - \frac{\alpha}{2} \left( \frac{p^2 - p_a^2}{p} \right) \qquad (6)$$

From Equation (6), it is observed that the volume of the bubbles varies with both isothermal pressure compression and with the coefficient of "dissolubility" $\alpha$. For isothermal compression, the volume of the bubbles change with pressure alone according to the expression $V_a = \frac{p_a}{p} V_g$. However, the actual volume of the bubbles experiences an additional reduction of $\frac{\alpha}{2} \left( \frac{p^2 - p_a^2}{p} \right)$, because of the oil being dissolved into solution, which increases as pressure increases. At some critical point, $p = p_c$, the air bubbles cannot maintain a free volume anymore and become dissolved in the liquid completely ($V_a = 0$) Therefore, the critical pressure $p_c$ can be obtained as:

$$p_c^2 = \frac{2 p_a V_g}{\alpha} + p_a^2$$

or

$$\alpha = \frac{2 p_a V_g}{p_c^2 - p_a^2} \qquad (7)$$

Substituting Equation (7) into Equation (5) yields:

$$V_a = \frac{p_a V_g}{p} \left( \frac{p_c^2 - p^2}{p_c^2 - p_a^2} \right) \qquad (8)$$

Equation (8) therefore represents the volumetric change of air mixed in oil, and reflects the point where entrained air enters into solution or dissolved air passes out of (a balance point). In this expression the variable is critical pressure $p_c$ instead of $\alpha$.

To illustrate this, consider the hydraulic cylinder shown in Figure 1. A volume of entrained air is shown mixed into oil at atmospheric pressure. As pressure rises but is still lower than the critical pressure, $p_c$, the volume of entrained air decreases according to Equation (8) because of the effect of both isothermal compression of the entrained air and the process of dissolving air into solution. As the pressure continues to increase beyond the critical value, $p_c$ all entrained air becomes dissolved and $V_a$ tends to zero. It should be noted that Equation (8) is applicable not only for positive pressures, $p > p_a$ (where $p_a$ is atmospheric pressure), but also to pressures less than atmosphere. $p \le p_a$, even to the case when cavitation occurs. At $p_c = \infty$, entrained air is never completely dissolved and as such, entrained air is compressed in an isothermal process. In this case, Equation (8) yields results similar to that derived by Hayward [9].

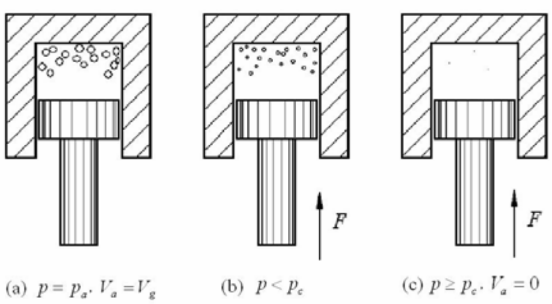


Figure 1 Volumetric change of entrained air in a hydraulic cylinder

The differential form of the relationship between the volume of the air bubbles and the absolute pressure can be obtained by differentiating Equation (8) with respect to $p$ to give:

$$dV_a = -\frac{p_a V_g}{p^2}\left(\frac{p^2 + p_c^2}{p_c^2 - p_a^2}\right)dp \qquad (9)$$

The polytropic process or the dynamic response of volumetric change with respect to pressure is now considered. The polytropic process for entrained air is defined by Equation (4). It is difficult to obtain an analytical relationship between the volume of entrained air and pressure by solving the Equation (4) directly. For this reason a numerical solution to Equation (4) is carried out by utilizing the Runge-Kutta method. The solution for the volume change as a function of the ratio between the pressure and the critical pressure is shown in Figure 2 and the critical pressure with respect to different values of the polytropic constant is also listed in Table 1. It is found from the table that the critical pressure increases with the polytropic constant. The critical pressure of the adiabatic process ( $p_{cs}$), for instance, is some 1.83 times the value of the isothermal process ( $p_{cT}$ ).

Even though there is such a variation of the critical pressure for the polytropic process, the volumetric variation vs pressure in a relative sense is fairly close; it can be clearly seen from Figure 2 that the relative volumetric variation $V_a / V_g$ with the pressure ratio $p / p_c$ is close for different values of the polytropic constant and is also fairly close to the isothermal process. A numerical computation demonstrates that the maximum difference of $V_a / V_g$ under the same pressure ratio $p / p_c$ between the adiabatic process and isothermal process is less than 2%. For this reason, as an approximation, the relationship between the volumetric variation of air and the absolute pressure given in Equation (8) or (9). This relationship is developed from the isothermal process, and is also applicable for the polytropic process, by noticing an appropriate value of critical pressure with respect to the peculiar polytropic process. The value of the critical pressure of the different polytropic process can be obtained through the polytropic constant in Table 1.

As discussed previously, when the process tends to be an isothermal one (the polytropic constant $k \to 1$), it can be regarded as a steady process; when the process tends to be an adiabatic one (the polytropic constant $k \to 1.4$), it is a dynamic response process. The above analysis implies that the relative change in volume due to a dynamic pressure response process is close to the static one for the conditions of air entrained in oil although they demonstrate a substantial difference in the value of the critical pressure $p_c$.

### 2.2 Volumetric Change of Oil (with no entrained air)

In examining the case of volumetric variation of oil for the hydraulic cylinder in Figure 1, it was assumed that the overall volumetric variation of oil due to dissolved air was small and negligible. This assumption was based on the fact that volume of the entrained and dissolved air is usually much smaller than that of oil and becomes even smaller when compressed. It was further assumed that the bulk modulus of oil without any air (pure oil) is approximately constant and independent of pressure variations. According the studies of Kuss et. al., the oil modulus of pure oil will increase about 1% over a pressure span of 10 MPa [4]. This 1% variation is much smaller than the variation in the overall volume due the presence of entrained air. This is quite true at low pressures, usually lower than 10 MPa. For this reason, a fundamental hypothesis is forwarded in this study that the bulk modulus of pure oil is constant and the volumetric change of oil is linearly proportional to the absolute pressure, that is

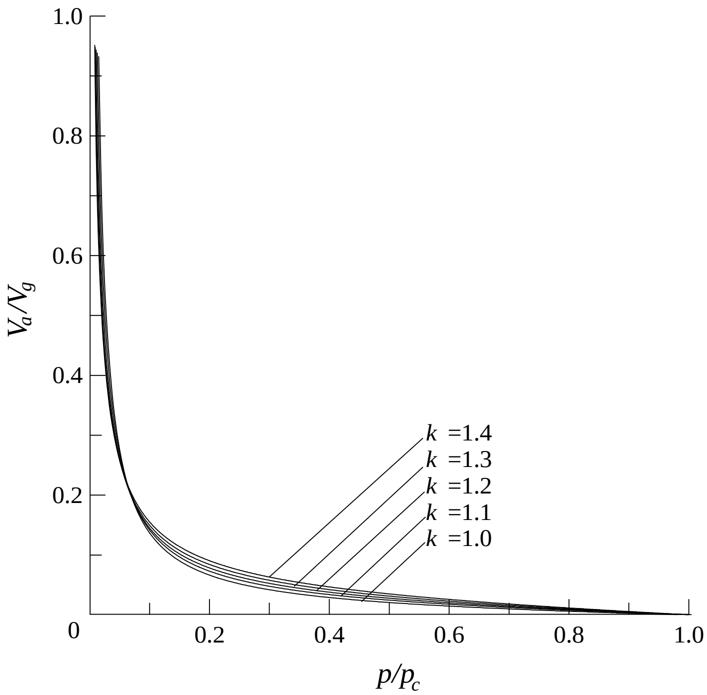


Figure 2 Volumetric variation with pressure for the polytropic process

Table 1 Critical pressure vs. polytropic constant

| $k$ | **1.0** | **1.1** | **1.2** | **1.3** | **1.4** |
|---|---|---|---|---|---|
| $p_{cT} / p_{cs}$ | 1.00 | 1.17 | 1.38 | 1.60 | 1.83 |

$$\Delta V_o = -\frac{V_o}{K_e}\Delta p = -\frac{V_t - V_a}{K_e}\Delta p \tag{10}$$

A simplification to Equation (10) is possible by noticing that the volumetric percentage of entrained air is always much smaller than the percentage volume of pure oil. Thus $V_o$ in equation (10) can be replaced by $V_t$, to yield:

$$\Delta V_o = -\frac{V_t}{K_e}\Delta p \tag{11}$$

or

$$dV_o = -\frac{V_t}{K_e}dp \tag{12}$$

**2.3 Overall (Gross) Bulk Modulus of Oil with Entrained Air**

The oil with entrained air can be considered as a mixture of the pure oil and entrained air bubbles at pressures below $p_c$. The fluid "effective" bulk modulus, $K_{eg}$, is introduced to reflect the presence of entrained air as:

$$K_{eg} = -\frac{V_t}{\Delta V_s + \Delta V_a + \Delta V_o}\Delta p \tag{13}$$

By substituting Equations (1), (7), (9) and (11) into (13), the relationship between the effective bulk modulus and pressure is obtained as:

$$K_{eg} = \frac{V_t}{\dfrac{p_a V_g}{p^2} - \dfrac{p_a V_g}{p_c^2 - p_a^2}\left(1 - \dfrac{p_a^2}{p^2}\right) + \dfrac{V_t}{K_e}} \quad \text{for } p < p_c \tag{14}$$

Assuming $X = V_g / V_t$ and noticing that the critical pressure, $p_c$, is generally much larger than atmospheric pressure, $p_a$, Equation (14) can further be simplified as:

$$K_{eg} = \frac{K_e}{p_a X K_e\left(\dfrac{1}{p^2} - \dfrac{1}{p_c^2}\right) + 1} \tag{15}$$

Finally the air content oil bulk modulus in a hydraulic cylinder can be expressed as

$$K_{ef} = \begin{cases} \dfrac{K_e}{p_a X K_e\left(\dfrac{1}{p^2} - \dfrac{1}{p_c^2}\right) + 1} & (p_c > p > p_a) \\ K_e & (p \geq p_c) \end{cases} \tag{16}$$

## 3. EXPERIMENTAL CONSIDERATIONS

In order to measure the bulk modulus of oil and to verify the feasibility of the proposed model, an experimental test rig was designed and constructed. In the experiment, the pressure variation inside the chamber of a hydraulic cylinder as a function of a "forced" incremental change in oil volume was investigated. The schematic of the experimental system is shown in Figure 3 and its operation is now described. A special digital pressure compensated valve designed by one of the authors was used to supply a constant flow rate to the chamber of the cylinder independent of the pressure in the cylinder (Ruan et al [12]). The operating characteristics and accuracy of this valve were well established in previous studies. With this arrangement, a known volume of oil over a prescribed time period was forced into the cylinder. This volume could readily be varied by changing the input signal to the digital control valve. It should be pointed out the time constant of the digital flow control valve was approximately 0.02s; thus the temporal transient response delay was considered negligible compared to the dynamic response times of the bulk modulus test ( 3s and above).

The measured flow rate at different outlet pressures of the throttling orifice is given in the Table 2. From Table 2 it is seen that the variation of the flow rate is less than 1% full scale within the pressure range of 0.3~30 MPa, so the flow rate in the experimental system can be regarded to be constant, or independent of the pressure of the cylinder.

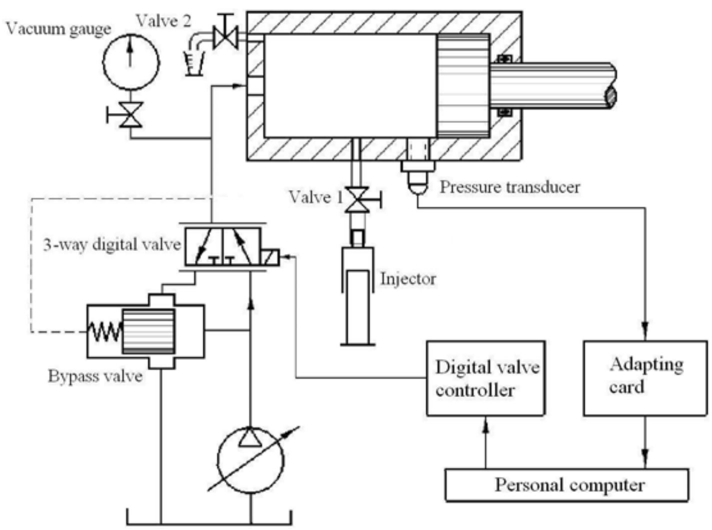


Figure 3 Arrangement of experimental system

Table 2 Flow rate vs. pressure of the cylinder

| **Pressure [MPa]** | **0.3** | **5** | **10** | **20** | **30** |
|---|---|---|---|---|---|
| Flow [ml/Min] | 15.00 | 14.98 | 14.96 | 14.93 | 14.90 |

The 3-way digital valve was controlled by a personal computer via a digital valve controller. The relationship between the output flow rate and input digital signal was calibrated in advance using a Calibration Cup and a stop watch, the accuracy of which is 0.2%. The pressure response of oil inside the cylinder was measured using a pressure transducer. The signal coming from the pressure transducer was sampled at the effective rate of 10Kb/s by an adapting card manufactured by National Instrument (Type: PCI-6035E). The pressure transducer was calibrated using a standard pressure gage and was accurate to 0.2% F.S. The data sampled by the adapting card was further processed in the personal computer using Matlab Realtime Workshop [13]. The oil used in the experiment was ISO VG46 mineral oil.

The pressure vessel used in this system was a standard cylinder with the piston displaced to full extension. Even though the cylinder shell was quite thick, (high stiffness) there was the possibility that the chamber could expand due to the elasticity of the shell when internally pressurized. The elasticity of the shell would cause the value of the measured bulk modulus of oil to be smaller than the true oil bulk modulus. The actual value of the oil bulk modulus was modified using the equation, $K_e = \gamma K_{em}$ ( where $\gamma$ is the modification coefficient).

The choice of the particular experimental configuration of the actuator for the test system was based on a desire to simulate a practical hydraulic control system application.

### 3.1 Estimations of Air content and Critical Pressure

It was deemed very critical to be able to estimate air content and critical pressure from the measured pressure response to implement the bulk modulus model as given in Equation (16). For the experimental system designed in Figure 3, the chamber of the cylinder was considered to be airtight and oil volume was the sum of the trapped fluid in the chamber, and pipe supplying the oil to the chamber. According to the continuity equation, the relationship between the pressure and the supply flow rate is

$$\Delta Q = \frac{V_t}{K_{eg}} \frac{dp}{dt} \tag{17}$$

Noticing that the flow rate is constant and independent of pressure and defining $\Delta V = Qt$ , Equation (16) is substituted into Equation (17). After integration, this equation becomes

$$\frac{\Delta V}{V_t} = p_a X \left( \frac{1}{p_a} - \frac{1}{p} \right) - p_a X \left( \frac{p}{p_c^2} - \frac{p_a}{p_c^2} \right) + \frac{(p - p_a)}{K_e}$$

for $p \le p_c$ (18)

In particular, at the critical point $p = p_c$ ,

$$\frac{\Delta V_c}{V_t} = \frac{X (p_c - p_a)^2}{p_c^2} + \frac{(p_c - p_a)}{K_e} \tag{19}$$

Defining $X(1 - p_a / p_c)^2 = X'$ , Equation (19) can be solved for $X'$ as:

$$X' = \frac{\Delta V_c}{V_t} - \frac{(p_c - p_a)}{K_e} \tag{20}$$

When $p \ge p_c$, the volumetric variation of oil changes with pressure according to a linear equation as:

$$\frac{\Delta V}{V_t} = \frac{\Delta V_c}{V_t} - \frac{(p_c - p)}{K_e} \tag{21}$$

From Equations (20) and (21), it can be shown that $X'$ can be interpreted as the volumetric change of Equation (21) at pressure $p = p_a$. $X'$ can be readily obtained at the intersection point between the curve given by Equation (21) and the line $p = p_a$ in a $\Delta V - p$ graph as shown in Figure 4. It should be noted that when $p_c >> p_a$, then $X' = X$. In most cases, the condition $p_c >> p_a$ is usually tenable.

It should be noted that the slope of the $p - \Delta V / V_t$ above the critical pressure, $p_c$ is nothing more than the bulk modulus of pure oil, as shown in Figure 4. These results are consistent with those found in [6].

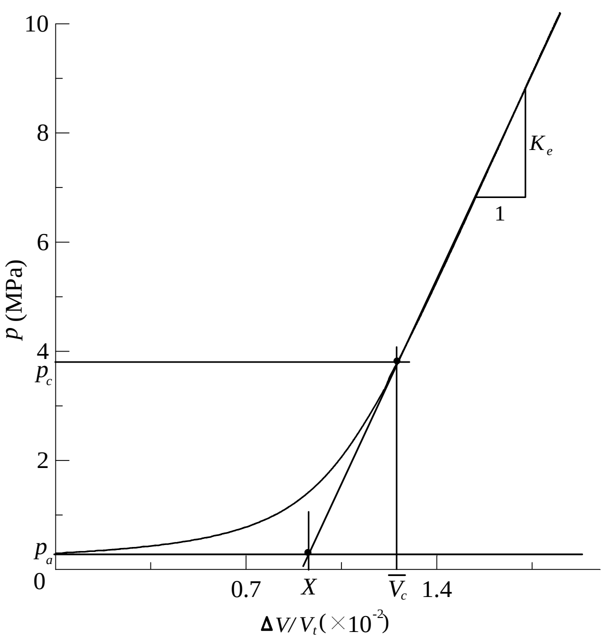


Figure 4 Relative amount of air in $p - \Delta V / V_t$ graph. (Note the volume change is a decrease)

In the measured $p - \Delta V / V_t$ curve, the critical pressure, $p_c$, is the lowest point where the straight line departs from the curved line. Thus, the critical pressure, $p_c$ can readily be obtained from the experimental results. However, there is some error introduced in determining the critical pressure, $p_c$, using the above mentioned method. As the pressure decreases, the actual point where the $p - \Delta V / V_t$ relationship deviates from the straight line is not very precise resulting in an estimation rather than the specification of a single point. To facilitate the estimation, the critical pressure $p_c$ was obtained at the point where the slope over a 5MPa pressure span reduced by 0.5%. This approach provided good repeatability and hence was adopted for all subsequent data processing in the tests.

It was also found that the bulk modulus model given in Equation (16) was less sensitive to the change of the critical pressure compared to the change in air content. Figure 5 shows the effect of varying critical pressures on the goodness of fit of the theoretical model to the measured result.

### 3.2 Gross Bulk Modulus of Oil with Entrained Air

#### 3.2.1 "Remnant" entrained air (air entrained at atmospheric pressures)

For the oil used in most hydraulic systems, usually there is always some free air (or air pockets) at atmospheric pressures and is called remnant entrained air. This appears as $X_0$ in Figure 6. It is believed that this remnant entrained air is an important factor affecting the value of the oil bulk modulus in hydraulic control systems. Experiments have shown that the remnant entrained air and the correspondent critical pressure are related to the temperature, as given in Table 3. It is also interesting that remnant entrained air can be observed by the human eye via a visual inspectional of the oil when drained to tank from the cylinder when valve 2 is opened.

#### 3.2.2 Bulk modulus under various entrained air conditions

By opening the valve 1 and 2 and extracting oil with the injector (see Figure 3) a gauge pressure less than atmosphere was created in the cylinder. The displaced oil volume of the extracted oil was then filled by entrained air coming through the valve 2. In this way the amount of entrained air fed into the cylinder was accurately controlled. The experimental results using the same procedures as outlined above are compared in Figure 7. It should be noted that the remnant entrained air has to be included in accounting for the mixed entrained air. As the mixed entrained air volume increased the pressure transient response to the input forced volume of oil slowed down considerably and the critical pressure $p_c$ increased accordingly. The variation of the critical pressure, $p_c$, with a relative volumetric change in the amount of air, $X$, is shown in Figure 8. From this Figure, an empirical curve was developed and is given as:

$$p_c^2 = p_{c0}^2 \sqrt{\frac{X}{X_0}} \tag{22}$$

Table 3 Remnant entrained air and critical pressure vs. temperature

| $T$ (C$^o$) | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|
| $X_0$ (%) | 0.31 | 0.32 | 0.34 | 0.36 | 0.39 |
| $p_c$ (MPa) | 2.80 | 2.95 | 3.12 | 3.3 | 3.45 |

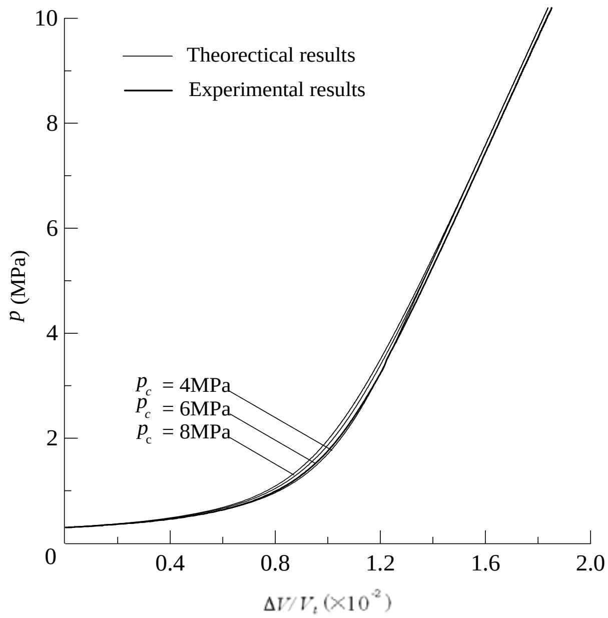

Figure 5 Fitness of varied critical pressures to measured results (Note that the change in volume is a decrease)

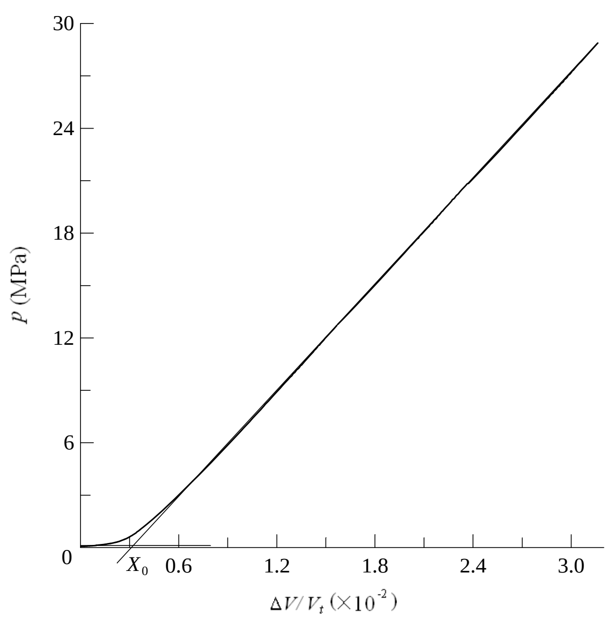

Figure 6 Determination of "Remnant" entrained air. (Note the volume change is a decrease)

### 3.3 Static and dynamic bulk modulus

Another important aim of this research was to investigate the dependence of the bulk modulus on different rates of forced volumetric variation to find out how the static and dynamic bulk modulus values change with temporal (time) changing pressure responses. A numerical solution to Equation (4) demonstrates that the critical pressure, $p_c$, increases substantially with the polytropic constant $k$ (noting that the theoretical model of the bulk modulus developed from the thermal process is still valid). Determining how the critical pressure, $p_c$, or the polytropic constant $k$ actually varies becomes crucial for the generality of the model. In the experiment, the magnitude of the step opening of the 3-way digital valve was adjusted to control the time constant of the transient response of the pressure as shown in Figure 9. It is evident that the smaller the time constant of the pressure change, the higher the critical pressure.

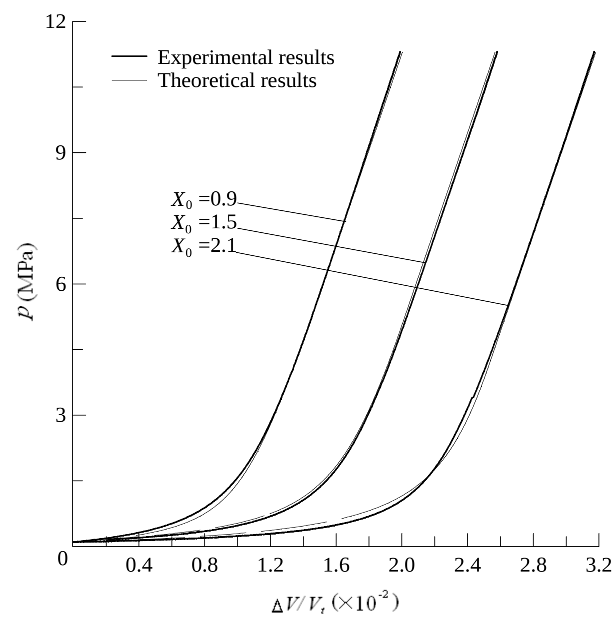

Figure 7 Relationship between $\Delta V/V_t$ and $p$ with various amount of remnant entrained air. (Note the volume change is a decrease)

The experimental relationship between the critical pressure, the polytropic constant and the response time is shown in Figure 10, in which the corresponding value of the polytropic constant is obtained by an appropriate interpolation of the results given in Table 1. It is observed from Figure 10 that both the critical pressure and the polytropic constant increase and converge towards an upper constant as response speed quickens (time constant decreases). Both the critical pressure and the polytropic constant decrease and converge to a lower constant value as the response speed slows down (time constant increases). Depending how close the value of the polytropic constant is to 1 and 1.4 the range of values of the response speed corresponding to the static and dynamic conditions can be established.

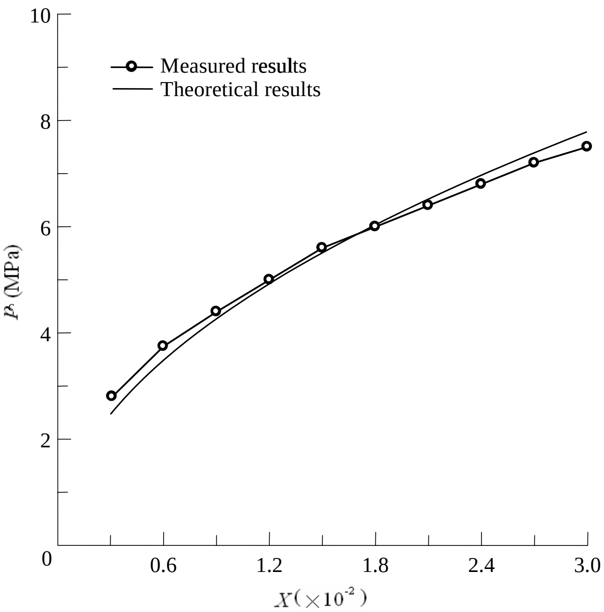

Figure 8 Variation of critical pressure with mixed entrained air

Since it takes an infinitely long time to fulfill a "complete" isothermal process for the entrained air, or ideal static pressure response and a pure adiabatic process, or ideal dynamic pressure response, corresponds extremely short of period, it is impossible to have an accurate definition of either static or dynamic response. Here, some approximation has to be made in the definition of static and dynamic response, depending on how close to isothermal or adiabatic process judged by the value of polytropic constant, $k$. In Figure 10 the static response is defined in the area where the polytropic constant is smaller than 1.01, or $1 \le k < 1.01$, and the dynamic response is defined in the area where the polytropic constant is larger than 1.39, or $1.39 \le k < 1.4$. If $k = 1.0$ is taken as 100% pure static process and $k = 1.4$ as 100% pure dynamic process, $k = 1.01$ could be considered as 97.5% close to the pure static process and 1.39 is 97.5% close to the pure dynamic process. In this way the static and dynamic bulk modulus can be acquired.

It should be noted that the value of dynamic modulus is 1.8 times larger than that of the static one. When the time, $t_c$, to reach the critical pressure, is less than 7 seconds for example, the pressure response is considered to be a dynamic one; When $t_c$ is larger than 60 seconds, the pressure response can be considered static. Since in most hydraulic control systems, the dynamic response is usually below 7 seconds, the bulk modulus to be applied for analysis should be based on a dynamic value.

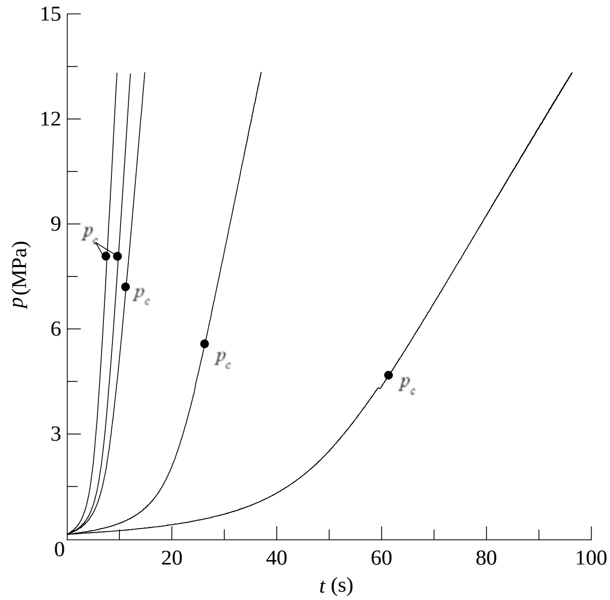

Figure 9 Pressure responses under different speeds

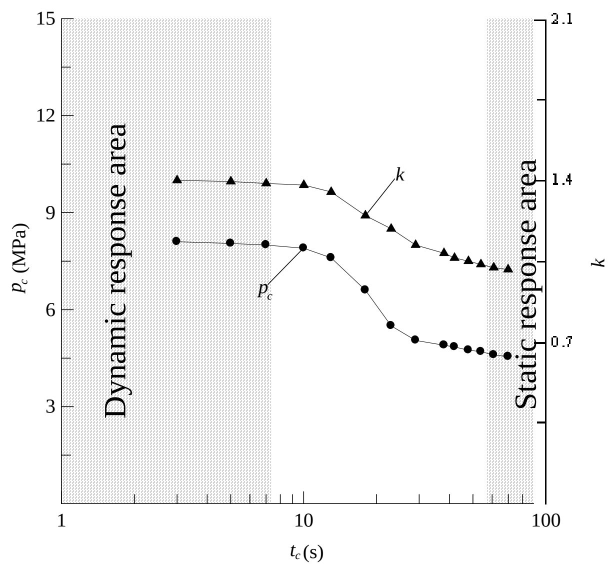

Figure 10 Static and dynamic response area

**3.4 Bulk modulus under pressures less than atmosphere**

The model given by the Equation (16) is also valid for the case of pressures less than atmosphere, i.e. $p < p_a$. An experiment was carried out to verify the feasibility of extracting the oil out of the cylinder with the injector to create a pressure less than atmosphere in the cylinder. For this study, the pressure variation was read manually from a vacuum gauge. The measured results are compared with their theoretical counterparts in Figure 11. It can be seen that the trend of theoretical results agree well with the measured ones.

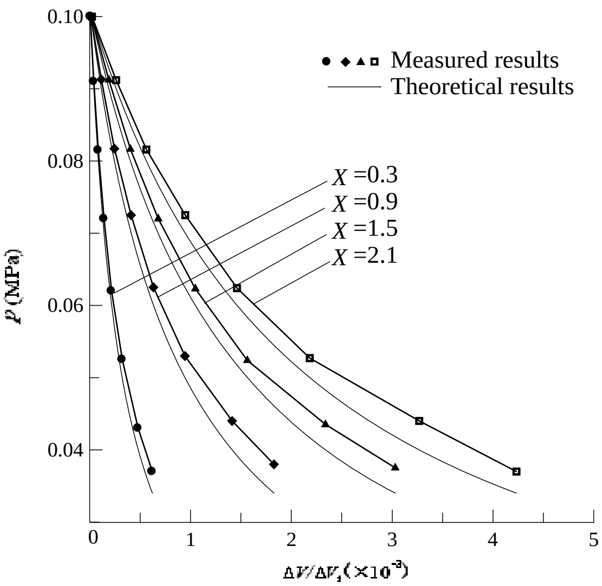


Figure 11 Pressure variation with pressures less than atmosphere (Note the volume change is a decrease)

## 4. COMPARISON TO OTHER MODELS OF BULK MODULUS

As a last step, it was desirable to compare the results of the model proposed in this study to those adopted in other studies, noticeable by Nykanen, Wylie and Yu. The equations developed by these authors are summarized as follows:

1. The model of bulk modulus by Nykanen [5]

$$K_{eg} = \frac{\left( \left(\frac{p_a}{p}\right)^{1/k} X + \frac{1-X}{1+\frac{1}{K_e}(p-p_a)} \right)}{\frac{\left(\frac{p_a}{p}\right)^{1/k} X}{kp} + \frac{1-X}{\left(1+\frac{p-p_a}{K_e}\right)^2 K_e}} \qquad (23)$$

2. The model of bulk modulus by Wylie [6]

$$K_{eg} = \frac{K_e}{1+X\left(\frac{p_a}{p}\right)^{1/k}\left(\frac{K_e}{kp}-1\right)} \qquad (24)$$

3. The model of bulk modulus by Yu [2]

$$K_{eg} = \frac{K_e\left(1+\frac{p-p_a}{p_a}\right)^{1+1/k}}{\left(\frac{p-p_a}{p_a}+1\right)^{1+1/k} + Xp_a\left(\frac{K_e}{k}-p\right)} \qquad (25)$$

It should be noted that the model by Wylie is very similar to that of Yu because in a hydraulic system the condition $p_a << p$, is almost always tenable. If $(p-p_a)/p_a$ in Equation (25) is replaced by $p/p_a$, Equation (25) will become identical to Equation (24). A comparison of the proposed model with Nykanen's model and Wylie's (Yu's) model is made at the conditions, $X = 0.01$, $k = 1.39$ (dynamic response), and is presented in Figure 11. The bulk modulus from the proposed model is a little bit larger than that predicted in the Nykanen's model and Wylie's (or Yu's) model. This difference can explained as follows: in Nykanen's model and Wylie's (Yu's) model, the air mixed in the oil is considered constant and hence no account is taken to reflect the dissolving of the entrained air into solution as pressure varies. In the model presented in this paper, entrained air and its transition (volume reduction) to dissolved air as pressure increases is considered. This is significant. .

## 5. CONCLUSIONS

The bulk modulus is one of the most important parameters in fluid power control systems. Oil used as the transmission media in a fluid power system is usually exposed to the air. In addition, the oil can experience large pressure variations in the different parts of the system due to circulation and of course, due to changes in loading conditions. As a result air can enter the fluid in a form of entrained (free air) or dissolved air and is very strongly influenced by this changing pressure. While the dissolved air has little effects on the bulk modulus value the entrained air drastically reduces the magnitude of the bulk modulus. The volume of the entrained air varies with the pressure in two ways: volumetric compression and volumetric reduction due to air dissolving into solution. Based on this fact a new model of bulk modulus is developed, which reflects how entrained air affects the gross bulk modulus in a region below a critical pressure. This critical pressure is proportional to the square root of the volume of the entrained air.

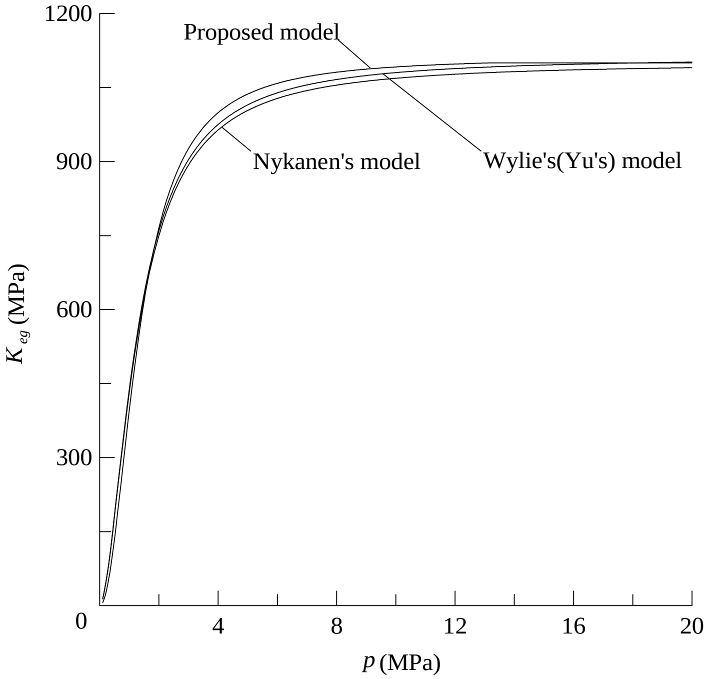


Figure 12 Comparison of different models

Another important factor which significantly influences the bulk modulus is the temporal pressure response of the pressure. The critical pressure increases and tends towards an upper constant as the pressure time response increases (time constant decreases). The critical pressure decreases and tends towards a lower constant as the pressure time response decreases (time constant increases). For this reason the area can be can be "laid out" with respect to the polytropic constant and the static and dynamic bulk modulus can be defined. The dynamic Bulk modulus is some 1.83 times that of the static value. From Figure 10, it can also be clearly seen that in a hydraulic control system the bulk modulus should be a dynamic one since the dynamic response time of the system is usually less that several seconds. It was observed from the experiment that the bulk modulus varied with the value of polytropic process, when pressure was lower than the critical pressure.

The feasibility of the model has been supported experimentally in which the pressure variation as a function of the variation of forced oil volume was measured. It was demonstrated that the model fits conditions where the system pressure was greater or less than atmospheric pressure. Finally a comparison of the proposed model is made with those established in the literature. It is found that the bulk modulus predicted by the proposed model is a little larger than that given in literature. The reason for this difference is the model proposed in this paper includes the fact that the volume of the entrained air decreases with both changes in pressure and the amount of air dissolving into solution (also pressure dependent).

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