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MICROCHANNEL OPTIMIZATION FOR HEAT DISSIPATION FROM A SOLID SUBSTRATE

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ABSTRACT

A computational model was developed to analyze and optimize the convective heat transfer for water flowing through rectangular microchannels fabricated in a silicon substrate. A baseline case was analyzed by solving the nondimensional governing equations. Using a quasi three-dimensional computational model, the velocity and temperature distributions were obtained and the numerical results were then used to determine the overall dimensionless thermal resistance for the convective heat transfer from the substrate to the fluid. To validate the numerical model, the average Nusselt numbers as determined by the numerical model were compared with experimental results available in the literature, for channels with comparable hydraulic diameters. The procedure for arriving at an optimum geometric configuration and arrangement of microchannels on the substrate, subject to given design constraints, so that the thermal resistance is at a minimum, is described and demonstrated using the computational model.

Keywords: Microchannel, Microchannel Optimization, Electronics Cooling, Heat Sinks

NOMENCLATURE

A	- Area of cross section of flow (m^2)
C_f	- Skin friction coefficient
C	- Specific heat capacity (J/kg K)
D_H	- Hydraulic diameter (m)
H	- Substrate height (m)
H_c	- Channel height (m)
k_f	- Thermal conductivity of the fluid (W/m K)
K_s	- Thermal conductivity of substrate (W/m K)
L	- Channel length (m)
Nu	- Nusselt number
P	- Pressure (Pa)
Q	- Heat flux entering the substrate (W/ m^2)
r	- Dimensionless thermal resistance
R	- Overall thermal resistance (K/W)
Re	- Reynolds number
T	- Temperature (K)
T_b	- Bulk temperature (K)
u	- Velocity of the fluid (m/s)
U	- Non-dimensional velocity
W	- Pitch of the channel (m)
W_T	- Width of the substrate (m)
W_c	- Width of the channel (m)

Greek Symbols

θ - Non-dimensional temperature difference

μ_f - Dynamic viscosity of water (N/s m^2)

ρ_f - Density of water (kg/ m^3)

INTRODUCTION

Investigations of fluid flow and heat transfer in microscale passages have indicated that conventional correlations for large channels are not capable of accurately predicting the frictional pressure drop or heat transfer characteristics in either individual or parallel arrays of microchannels. The rationale for these variations, however, is not well understood and many researchers believe that the dimensions of these microchannels are sufficient in size to justify the use of assumptions and models based upon a continuum model. Though it would require extensive and careful experimental research to bring out conclusive results and observations regarding the suitability of the use of conventional correlations for predicting the fluid and thermal behavior in these channels, it is very instructive to perform modeling and theoretical studies on channels with microscale dimensions using conventional governing equations for fluid flow for comparative purposes. Overall performance characteristics from such analyses could serve not only to provide information and trends for comparison with existing experimental data, but could also lead to a better understanding of the significance of the deviations from conventional predictive techniques in these channel arrays. Moreover, such analyses could be used to optimize the channel dimensions and spacing for practical design applications.

In the current investigation, laminar single phase flow of water in rectangular micro channels, fabricated in silicon, is studied to determine the resulting heat dissipation effects. Local and average heat transfer coefficients and Nusselt numbers are calculated using the temperature distributions obtained through the solution of the non-dimensional governing equations, using a quasi-three dimensional approach. The results for a constant cross-sectional area are further utilized to obtain the optimum arrangement of the micro channels in the silicon substrate, based upon the minimum thermal resistance, subject to design constraints on the bulk temperature rise and pressure drop in the fluid, and the maximum channel depth.

REVIEW OF LITERATURE

The concept of using micro channels as an effective means of heat removal from micro electronic devices was first proposed and successfully demonstrated by Tuckerman and Pease [1]. By observing the variation in the thermal resistance with the fluid flow rate in the channels, it was also concluded that the flow was thermally fully developed. Wu and Little [2] reported measurements of friction factors and heat transfer in the flow of nitrogen gas through very fine channels used in miniature Joule-Thomson refrigerators. The deviation from conventional channel correlations was attributed to the relative roughness of the small channels being significantly larger than for conventional channels. Phillips et al. [3] demonstrated very good agreement between the predicted theoretical results in microchannel heat sinks and the experimental data obtained by Tuckerman for various flow rates in the laminar regime.

Choi et al. [4] experimentally investigated the flow of nitrogen gas in microchannels. It was observed that the experimental data in the turbulent regime fell above the predictions obtained using the Dittus-Boelter correlation for conventional sized tubes and channels. Flow and heat transfer measurements in micro channels etched into silicon were obtained by Rahman and Gui [5]. It was concluded by the authors that micro channels provided significant performance and operational advantages in terms of higher heat fluxes when compared to other cooling methods, such as jet impingement or forced convection. Peng et al. [6,7] experimentally investigated the forced convection of water through single rectangular microchannels. Experimental results indicated that the laminar to turbulent flow transition, occurred at Reynolds numbers of 200 to 700. The geometric parameters, hydraulic diameter, and height to width ratio were found to be the most important parameters affecting the flow behavior. Tso and Mahulikar [8] proposed the use of the Brinkman number as the parameter for correlating convective heat transfer in micro channels, based on a dimensional analysis of the variables influencing laminar forced convection. Adams et al. [9] investigated turbulent forced convection of distilled water in micro channels. A correlation for the Nusselt number in turbulent forced convection was proposed for circular micro channels of diameters 0.102 to 1.09 mm. Adams et al. [10] also extended the experiments to a non-circular micro channel of hydraulic diameter of 1.13 mm.

The observed apparent deviations from the results predicted using conventional large-channel predictive techniques for single phase flow in arrays of microchannels have been attributed to a number of possible causes. Most frequently, these variations or deviations are attributed to inaccuracies occurring in the data reduction and analysis process [11], or problems associated with the experimental design, which were not considered in the experimental test procedures, such as deviations resulting from ineffective header design and/or maldistribution of flow in the channel arrays brought about by variations in the flow in the individual channels [12]. Recent investigations using sophisticated measurement methods suggest that microchannel flows can be analyzed using conventional methods for large channels, as the physical dimensions encountered in such cases are generally valid for continuum analysis [13]; however, the reported data is rather limited. Because the existing experimental data for microchannels do not provide sufficient evidence upon which broad generalizations of the physical phenomena of heat and fluid flow can be made, it is instructive to compare various theoretical models with the limited experimental data to arrive as conclusive interpretations of observed phenomena in microchannel flows.

A computational study of integrated microchannels has been presented in the literature by Weisberg et al. [14]. In this work, a non-dimensional formulation has been utilized, based on a non-dimensional temperature defined in terms of the bulk fluid temperature, to solve the energy equation, while the non-dimensional velocity filed was obtained from classical expressions for fully developed laminar flow. The two dimensional analysis was used for an optimization study, which led to the determination of the conditions for minimizing the non-dimensional thermal resistance. The present investigation also proceeds on similar lines, making the analysis more realistic by incorporating a pseudo-three dimensional marching in the numerical scheme, as well as by solving the momentum equation to obtain the non-dimensional velocity distribution. An optimization study is also performed, using the non-dimensional thermal resistance defined in [14], following the computational analysis. Comparisons with existing experimental results [13] suggest that the continuum model utilized in the analysis predicts the channel performance reasonably well.

THE PHYSICAL MODEL

Figure 1 illustrates a characteristic arrangement of rectangular microchannels on a substrate, similar to that which is the focus of the current investigation. In this configuration, the channels have a uniform rectangular cross- section of width W_C, a height of H_C, and a length L. Further in this arrangement, it is assumed that the heat transfer into the system is uniform and is applied to the top surface of the wafer while the bottom surface is insulated. Ultrapure water is used as the cooling fluid, which is uniformly circulated through the channels.

The mathematical formulation used is non-dimensional. In order to better understand the physical nature of the numerical solution, a baseline case is studied, with the dimensions and operating conditions listed in Table 1.

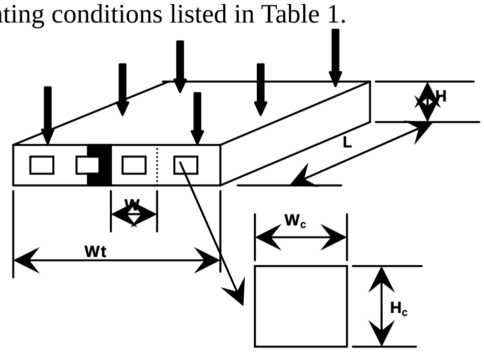

Fig. 1 The physical model showing the microchannel arrangement in the substrate and notations for the geometrical dimensions.

THE MATHEMATICAL MODEL

The domain of analysis is shown as the shaded portion in Fig. 1, which represents the channel and the substrate configuration. A non-dimensional formulation with the channel height, H_c, as the characteristic length dimension was utilized, and the governing equations were derived accordingly. The flow is assumed to be steady, laminar and hydrodynamically and thermally fully developed, as suggested by Tuckerman and Pease [1] based on their experimental observations.

A quasi three-dimensional approach was utilized, which solves the governing equations in two dimensions (cross-section) and utilizes a marching technique for the third dimension (length).

Parameter	**Value in the base-line case**	
Geometric dimensions	H_c= 0.3 mm W_c= 0.18 mm L= 25 mm H= 0.43 mm W = 0.36 mm (H_c= 0.3 mm W_c= 0.3 mm for Fig.3)	
Inlet fluid temperature	24°C	
Reynolds number	500 for base line case, 500-2400 for Fig.3	
Heat input	50 W/m^2	
Pressure drop	80 kPa	
Thermophysical Properties	**Silicon**	**Water**
Thermal conductivity	148 W/m K	0.628 W/m K
Density	2330 kg/m^3	995 kg/m^3
Specific Heat	712kJ/kg K	4178 kJ/kg K
Thermal diffusivity	89.2x 10^{-6}m^2/s	0.1511x10^{-6} m^2/s
Kinematic viscosity	--	0.657 x 10^{-6} m^2/s

Table 1: Details of the baseline case analyzed

Momentum Equation and Velocity:

Neglecting frictional dissipation effects, for steady, laminar flow, the governing momentum equation can be written as:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu}\frac{dP}{dz} \tag{1}$$

and

$$\frac{dP}{dx}=\frac{dP}{dy}=0 \tag{2}$$

As the flow becomes fully developed, the velocity, u, does not vary with the axial length, z, which implies that $\frac{dP}{dz}$ is a constant.

The momentum equation can be non-dimensionalized using the dimensionless variables given below:

$$X = \frac{x}{H_c},\ Y = \frac{y}{H_c}, U = \frac{u}{\bar{u}}$$

where $\bar{u}$ is a bulk mean velocity given by

$$\bar{u} = \frac{1}{W_c H_c} \int_0^{W_c} \int_0^{H_c} u \ dy\ dx \qquad (3)$$

To further simplify the momentum equation, a transformation can be applied as follows:

$$U^* = \frac{U\bar{u}}{\frac{H_c{}^2}{\mu}\left(-\frac{dP}{dz}\right)} \qquad (4)$$

which allows the momentum equation to be expressed as

$$\frac{\partial^2 U^*}{\partial X^2} + \frac{\partial^2 U^*}{\partial Y^2} = -1 \qquad (5)$$

subject to $U^* = 0$ at all of the boundaries of the channel cross-section.

The actual non-dimensional velocity U can be obtained from the solution for U^* as

$$U = \frac{u}{\bar{u}} = \frac{U^*}{\overline{U}^*} \qquad (6)$$

where $\overline{U}^* = \frac{2H_c}{W_c} \int_0^{\frac{W_c}{2H_c}} \int_0^1 U^* \ dY\ dX \qquad (7)$

Energy Equation and Temperature:

In order to non-dimensionalize the energy equation utilizing the bulk fluid temperature, the approach suggested in [14] was adopted. The method is outlined below:

The steady-state energy equation for laminar fully developed flow, neglecting viscous dissipation, can be written as:

$$C_f \rho_f u \frac{\partial T_b}{\partial z} = k_f \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \qquad (8)$$

For the substrate, the two-dimensional conduction equation is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (9)$$

Using the expressions for the bulk fluid temperature rise, and the definition of the non-dimensional temperature difference as obtained in [14], the above equations become

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{UH_c}{W_c} \textit{ (in the fluid)} \qquad (10)$$

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = 0 \quad \textit{(in the substrate)} \qquad (11)$$

where

$$\frac{dT_b}{dz} = \frac{QW}{\rho_f C_f \bar{u} W_c H_c} \qquad (12)$$

$$\theta = \frac{k_f}{QW}\left(T - T_b\right) \qquad (13)$$

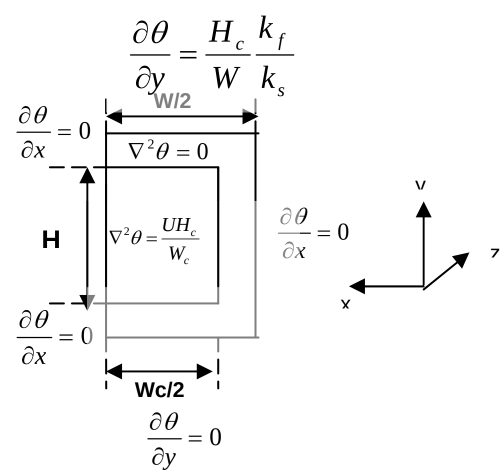

Fig. 2. The domain of computational analysis with boundary conditions for the heat transfer problem.

The definition of the non-dimensional temperature difference, θ, as expressed in equation 13, implies that

$$\int_0^{\frac{W_c}{H_c}} \int_0^1 U\theta \ dY\ dX = 0 \qquad (14)$$

This condition is utilized to check the computational results.

Boundary conditions:

The boundary conditions utilized for the fluid flow are a no-slip condition on the solid walls. The boundary conditions utilized for solving the energy equation includes symmetric boundary conditions in temperature, insulated boundary condition at the bottom boundary, as well as interfacial heat balance at the solid-liquid interfaces. The computational domain, along with the non-dimensional governing equations and boundary conditions for the heat transfer problem are illustrated in Fig. 2.

Heat Transfer Parameters:

From the temperature distributions in the flow field, two important heat transfer parameters can be determined, namely the average Nusselt number and the overall thermal resistance. The Nusselt number is used for validation of the model by comparison with the existing experimental results. The thermal resistance is utilized in the optimization study to arrive at the optimal channel arrangement for a given operating condition. These parameters are explained in the sections that follow.

Nusselt Number:

The temperature distribution, which can be deduced from the non-dimensional results in the field, were used to determine the local surface heat fluxes and heat transfer coefficients on the surfaces, and an average heat transfer coefficient at a given axial location. The local temperature difference between the surface and the bulk fluid is used in defining this heat transfer coefficient. This can also be used to calculate a local average Nusselt number around the periphery for any axial position. A further averaging can be done for the Nusselt number for the entire flow, from repeated calculations at various axial locations along the channel.

To compare the overall performance of the channel and validate the numerical model, the experimental results of Lee and Garimella [13] were plotted in Fig. 3, for channel hydraulic diameters of 300 μm in the present case and 318 μm in the experimental case reported in the literature. As shown in Fig. 3, the correlation between the predicted and measured values is fairly good at Reynolds numbers above 800 with a gradually increasing deviation with increasing Reynolds numbers.

At Reynolds numbers below 800, the Nusselt number decreases rather dramatically with decreasing Reynolds numbers. The deviation at lower Reynolds numbers may, in part, be due to the constraints in measurement at relatively low flow rates.

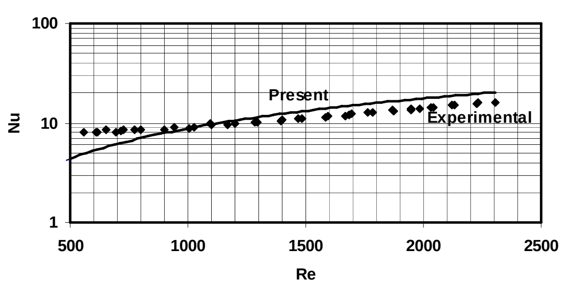

Fig. 3 Comparison of the predicted average Nusselt number and the experimental data presented in the literature [13].

Thermal Resistance:

Determination of the optimal channel arrangement requires that the resistance to heat flow from the substrate-fluid interface into the bulk fluid flowing through the channel, be minimized. Following the analysis presented in [14], the dimensionless thermal resistance was used as follows:

$$r = \frac{W_T k_f LR}{H_c} = \left(\frac{W}{H_c}\right)\bar{\theta}_{surf} \qquad (15)$$

Minimization of this quantity leads to the optimum design.

COMPUTATIONAL RESULTS

Utilizing a fully implicit scheme for solution, it was possible to obtain convergence with a fairly coarse computational domain grid size (30 x 20). As the present optimization study is based on averaging of surface temperatures, the use of coarse grids is justified, also taking into account the savings in computation time. However, the analysis can be carried out using much finer grid structures to produce more accurate local results.

The cross-sectional distributions of the non-dimensional velocity and non-dimensional temperature at a typical axial location, are shown in Figs. 4 and 5. In Fig. 4, the velocity distribution is shown for the entire channel cross-section. The temperature distribution shown in Fig. 5 is for the computational domain, which is half the width of the channel. As shown, the fluid temperature has considerable variation within the channel, though the substrate temperature variation around the channel is not very significant.

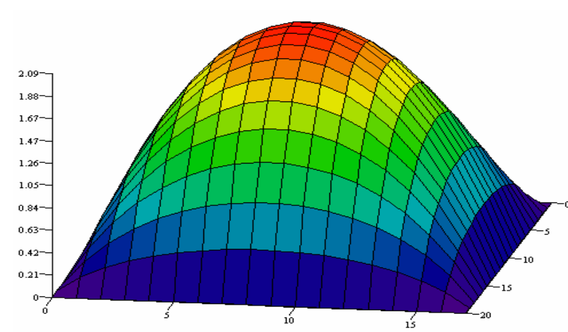

Fig. 4. Distribution of the non-dimensional velocity in the channel for the entire channel cross-section.

Figure 6 illustrates the variation in the dimensionless thermal resistance with respect to the ratio of the channel width to the channel pitch, W_c/W, for various aspect ratios (H_c/W_c). The operating conditions and design constraints under which this plot was constructed is described in the later section on optimization. As shown, for a constant value of aspect ratio, the dimensionless thermal resistance decreases with increases in the width to pitch ratio. This is expected, as the larger the surface area for heat transfer, the more closely the bulk fluid temperature will approach the surface temperature, thus reducing the thermal resistance. The variation of the thermal resistance is steeper for smaller values of the aspect ratios, which represent deeper but narrow channels. With a small aspect ratio, the effect of increasing the width of the channel is significantly more pronounced, as a result of a comparatively large increase in the surface area that lies directly under the applied heat flux, with respect to the total channel surface area. This introduces a relatively large reduction in the thermal resistance with respect to the width to pitch ratio, particularly when compared to cases with larger aspect ratios. The results are in good qualitative agreement with those presented in [14], and the differences could be attributed to the fact that in the present study, a solution of the momentum equation in the flow domain was used to obtain the velocity field, rather than an assumption of a velocity distribution.

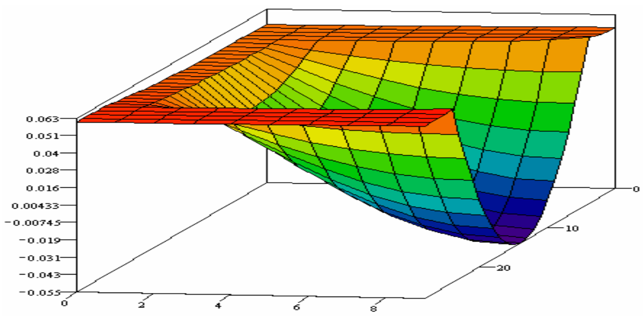

Fig. 5. Distribution of the non-dimensional temperature difference in the channel section for the computational domain, which comprises of half the channel cross-section.

OPTIMIZATION STUDY

Based on the field solution and computation of the thermal resistance previously presented, a study was performed to obtain the optimal arrangement of channels on the substrate.

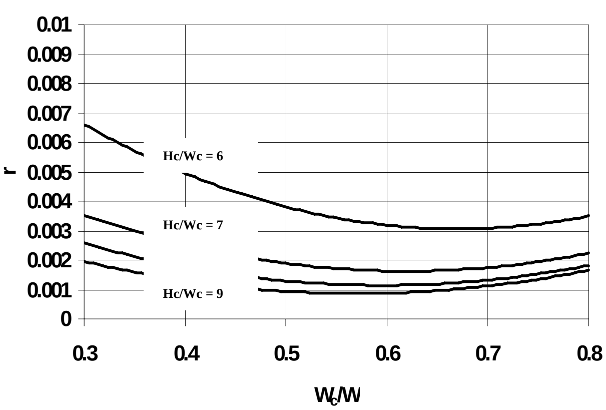

Fig. 6 Variation of the dimensionless thermal resistance with respect to the width to pitch ratio of the channel, for different aspect ratios of the channel.

The design constraints selected to demonstrate the procedure are, a maximum rise in bulk temperature of the fluid of 40°C, an allowable maximum pressure drop in the channel of 80 kPa, and a maximum channel depth of 300 μm.

The optimization procedure involves arriving at the minimum thermal resistance subject to the given constraints. Optimization is done by minimizing the dimensionless thermal resistance with respect to the width to pitch ratio and the height to width ratio [14]. The combination of these parameters that will produce the minimum thermal resistance can be determined, by performing a parametric study of the computational model. The optimization procedure for a typical case with the operating conditions and the channel length given in Table 1, and the design constraints stated above, can be best understood by examining Figs. 6 and 7.

The steps involved in selecting the optimal channel arrangement are given below:

1. A plot of the dimensionless thermal resistance with respect to the width to pitch ratio (W_c/W) is made, for various values of aspect ratios as given in Fig. 6.
2. A plot of H_c/W_c, for various values of the width to pitch ratio (W_c/W) is made, for specified values of pressure drop and bulk mean temperature rise (Fig. 7).
3. The value of (W_c/W) is obtained for various values of H_c/W_c, from step 2.
4. The value of the dimensionless thermal resistance for each aspect ratio is noted, corresponding to the value of H_c/W_c, from the plot in step 1 (Fig. 6).
5. The channel dimensions and arrangement corresponding to the minimum value of the dimensionless thermal resistance is selected.

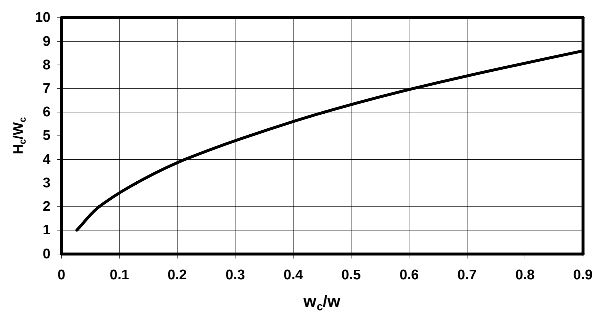

Fig. 7 Comparison of the height to width ratio (H_c/W_c) and the width to pitch ratio (W_c/W).

Table 2 lists the dimensionless thermal resistances for various aspect ratios and width to pitch ratios from the optimization study as performed on the design constraints listed earlier. From Table 2, the solution corresponding to the minimum thermal resistance involves an aspect ratio of 8 and a width to pitch ratio of 0.644. Using this, the design geometric parameters can be calculated and are shown in Table 3.

CONCLUSIONS

A computational model was developed to analyze the convective heat transfer for laminar flow in rectangular microchannels fabricated as an integral part of silicon substrates. The computational results for the average Nusselt number were compared with experimental results from the literature and were found to be in reasonably good agreement with the reported data.

Table 2 Feasible solutions: Dimensionless Thermal Resistances

Design constraints:
Maximum rise in bulk temperature = 40°C
Allowable pressure drop in the channel = 80 kPa
Maximum channel depth = 300 μm.

H_c/W_c	Width to Pitch Ratio (Wc/W)	Dimensionless resistance (r)
6	0.372	0.0053
7	0.5	0.00185
8	0.644	0.00131
9	0.805	0.00149

Table 3: Selected Microchannel Arrangement from Optimization Study

H_c/W_c	Width to Pitch Ratio (Wc/W)	Dimensionless resistance (r)
6	0.372	0.0053
7	0.5	0.00185
8	0.644	0.00131
9	0.805	0.00149

Using the computational results, an overall thermal resistance was calculated and the influence of the geometrical parameters was examined. A procedure to arrive at optimal combinations of the geometric configuration and microchannel arrangement in the substrate, subject to the specified design constraints was then described and demonstrated.

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