A Theoretical Study, With Experimental Verification, of the Temperature-Dependent Viscosity Effect on the Forced Convection Through a Porous Medium Channel

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Introduction

The problem of forced convective flow in a channel is a topic of fundamental importance, and one which features prominently in textbooks on convective heat transfer such as Bejan (1984/1995) and in articles in handbooks such as those by Kays and Harnett (1973) or Shah and Bhatti (1987). However, it appears that there is little accessible information available on the practically important but complex case where the effect of the dependence of the viscosity on temperature is included. In this case, the velocity profile depends on the temperature field and the latter also depends on the former, so in general a system of coupled differential equations must be solved.

Considering specifically the case of flow through a porous medium, an analytical treatment of convection by a temperature-dependent fluid was presented recently by Ling and Dybbs (1992). However, their model is restricted to convection through a porous medium adjacent to an isothermal flat plate.

Our purpose is to present a theory, based on a perturbation approach, which permits the determination of the effect of viscosity variation (with temperature) on the pressure drop of a convectively cooled, porous medium channel. The need for such a theory is not only fundamental but also practical. New designs of microporous enhanced cold plates for cooling airborne microelectronics rely on brazed metallic porous inserts for improved thermal efficiency (Lage et al., 1996; Antohe et al., 1996; Antohe et al., 1997). These devices are to be cooled with PolyAlphaOlefin (PAO), a very common synthetic oil used for cooling military avionics. This oil has viscosity strongly dependent on temperature,

$$\mu(T) = 0.1628\, T^{-1.0868}, \quad 5°\mathrm{C} \le T \le 170°\mathrm{C}, \tag{1}$$

where T is the temperature in °C, and μ is the dynamic viscosity in kg/ms (Chevron, 1981). The prediction of the thermohydraulic behavior of these devices, particularly the pressure drop increase imposed by the porous insert, is critical for design optimization.

Our theoretical analysis provides a detailed explanation of the way in which the temperature effect comes into play. The validity of the model prediction is checked against experimental data obtained from tests of PAO flowing through a typical microporous cold plate. Obviously, our theory is not limited to PAO, but to any coolant presenting temperature-dependent viscosity (with other properties being much less affected by temperature). PAO was chosen as our test fluid because it presents immediate application interest to the avionics industry.

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Theoretical Model and Discussion

We consider the flow of an incompressible fluid in a channel between plane parallel plates at $y = \pm H$ (i.e., H is the half-spacing of the channel, y being a Cartesian coordinate transverse to the plates). The thermal boundary conditions are symmetrical ones of uniform heat flux. The Péclet number is assumed to be sufficiently high for the axial thermal conduction to be neglected. Fluid properties other than the viscosity μ are assumed to be constant (a good approximation for most fluids, including PAO). The energy equation in this case is

$$\frac{u}{\alpha}\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial y^2} \tag{2}$$

where α is the thermal diffusivity, x is the longitudinal coordinate, and u is the longitudinal fluid speed. Equation (2) is valid when the fluid velocity is uniform in y (slug flow), which is appropriate to a porous medium when the Darcy's Law is valid, i.e., when

$$u = \frac{K}{\mu} G \tag{3}$$

where K is the permeability of the medium and G is the applied pressure gradient. Even though the slug flow model is also appropriate for the hydrodynamically undeveloped flow of a low Prandtl number fluid, in the following analysis just the porous medium situation is considered, in line with our practical application.

For the present case (uniform wall heat flux) the variables x and y of Eq. (2) can be separated in an additive fashion (compare Eqs. (3.56) and (3.59) of Bejan (1984/1995)) and the First Law of Thermodynamics leads to

$$\frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \frac{\alpha q''}{kHu_m} \tag{4}$$

where u_m is the bulk (mean) longitudinal speed, q'' is the boundary heat flux, k is the thermal conductivity, and T_m is the bulk temperature defined generally as

$$T_m = \frac{1}{u_m H}\int_0^H uT\,dy. \tag{5}$$

From Eq. (4) it follows that the order of magnitude of the ratio of $\partial T/\partial x$ and $\partial T/\partial y$ is equal to the reciprocal of the Péclet number, and so this ratio is small under an assumption already made. Hence, variations of the viscosity with x are neglected in this analysis. It follows that one can also assume that the speed u is independent of x.

Using Eq. (4), Eq. (2) gives

$$\frac{\partial^2 T}{\partial y^2} = \frac{q'' u}{kHu_m}. \tag{6}$$

The boundary conditions can be rewritten using the wall temperature T_w, as

$$\partial T/\partial y = 0 \quad \text{at } y = 0, \quad T = T_w(x) \quad \text{at } y = H. \tag{7}$$

A linear approximation (based on a truncated Taylor series expansion) for the variation with temperature of the reciprocal of the viscosity is made:

$$\frac{1}{\mu} = \frac{1}{\mu_0}\left\{1 - \frac{1}{\mu_0}\left(\frac{d\mu}{dT}\right)_0 (T - T_0)\right\} \tag{8}$$

where the suffix 0 indicates evaluation at the reference temperature T_0. A "viscosity variation number," N, is now defined as

$$N = \frac{q''H}{k}\frac{1}{\mu_0}\left(\frac{d\mu}{dT}\right)_0 \tag{9}$$

The following perturbation analysis is made on the assumption that N is a small parameter. A process involving successive approximations is carried out. (A more formal procedure, which leads to the same results, is to expand the dependent variables and the viscosity in powers of N, substitute in the equations, equate the various expressions involving the same power N^i of N, and proceed to solve in turn the system of equations for $i = 0, 1, 2, \ldots$. In fact, here the process is stopped at $i = 1$.)

The zeroth-order solution is the familiar one corresponding to constant viscosity, $N = 0$. For this case, $u = u_m$. One readily finds from Eqs. (5), (6), and (7)

$$T = T_w + \frac{q''}{2kH}(y^2 - H^2) \tag{10}$$

$$T_m = T_w - \frac{q''H}{3k} \tag{11}$$

$$\mathrm{Nu} = \frac{2Hq''}{k(T_w - T_m)} = 6 \tag{12}$$

where Nu is the Nusselt number. Proceeding to the next approximation, one returns to Eqs. (3) and (8) with T given approximately by Eq. (10), and with the reference temperature identified with T_w. (Since T_w is taken to be a slowly varying function of x, there is no harm done in doing this. To determine the precise value of N from Eq. (9) one has to specify some particular value of x.) This gives

$$u = \left(\frac{K}{\mu_0}G\right)\left[1 + \frac{1}{2}N\left(1 - \frac{y^2}{H^2}\right)\right] \tag{13}$$

$$u_m = \left(\frac{K}{\mu_0}G\right)\left[1 + \frac{N}{3}\right]. \tag{14}$$

With these expressions for u and u_m, one returns to Eqs. (6) and (7). Working to first order in N, one gets in turn, after some algebraic manipulation,

$$\frac{\partial^2 T}{\partial y^2} = \left(\frac{q''}{kH}\right)\left[1 + N\left(\frac{1}{6} - \frac{1}{2}\frac{y^2}{H^2}\right)\right] \tag{15}$$

$$T - T_w = \left(\frac{Hq''}{2k}\right) \times \left\{\frac{y^2}{H^2} - 1 + N\left[\frac{1}{12}\left(1 - \frac{y^4}{H^4}\right) - \frac{1}{6}\left(1 - \frac{y^2}{H^2}\right)\right]\right\} \tag{16}$$

$$T_m - T_w = -\left(\frac{Hq''}{k}\right)\left[\frac{1}{3} + \frac{2}{45}N\right] \tag{17}$$

$$\mathrm{Nu} = 6\left(1 - \frac{2}{15}N\right). \tag{18}$$

The conclusion is that the effect of variation of viscosity with temperature is to the Nusselt number being multiplied by a factor of approximately $(1 - 2N/15)$, where N, defined by Eq. (9), represents the proportional change of viscosity, across the half-channel-spacing H, corresponding to that hypothetical linear conduction temperature gradient in the cross-channel $(y-)$ direction which would produce a heat flux q''. Looking at Eqs. (13), (15), and (17), we see that the change in the value of Nu is a result of changes to the velocity profile, the curvature of the temperature profile, and the difference between T_m and T_w, in turn.

For the case of a fluid whose viscosity decreases with increase of temperature, N is negative. In moving from the case of zero N to that of nonzero negative N, the velocity near the walls is increased relative to that in the center of the channel. As a result, the curvature of the temperature profile is increased near the walls and decreased in the middle of the channel (which has the net effect of flattening the overall profile), and this leads to a decrease in the magnitude of $T_w - T_m$, which in turn leads to an increase in the magnitude of the Nusselt number Nu.

The previous analysis has been carried out for just the isoflux case. The isothermal case could be treated similarly, but the algebraic manipulation would be messier because then trigonometric functions, as well as polynomials, would be involved. We leave this extension to another opportunity, and now we concentrate on the experimental verification of the model.

Experimental Hydraulic Verification

For testing the theory a microporous cold plate was manufactured. Electric heaters generating a heat flux $q'' = 0.59\ V^2$, in W/m^2, where V is the supply voltage in Volts, were used to heat the 0.001 m apart channel plates. The total PAO pressure drop across the cold plate, $\Delta p_e = p_i - p_o$, was calculated from the inlet p_i and outlet p_o pressure measurements. Details on the experimental apparatus and procedure are found in Porneala (1998).

Using Eq. (14) we can define an expression for the theoretical pressure drop Δp across the porous insert when the cold plate is heated as

$$\Delta p = \left(1 + \frac{N}{3}\right)\Delta p_{e0} \tag{19}$$

where Δp_{e0} is the pressure drop across the cold plate when the heaters are off and the coolant flows at a reference temperature T_0 (taken as the coolant bath temperature, in °C). Using q'', H, Eq. (1), and $k_{\mathrm{PAO}} = 0.1454$ W/m°C, we can simplify Eq. (9) to: $N = -2.2 \times 10^{-3}\ V^2/T_0$.

The verification of Eq. (19) involves measuring Δp_{e0} with the heaters off and the coolant flowing at a certain reference (bath) temperature, and then using it for comparing the Δp value predicted by Eq. (19) against the Δp_e measured experimentally under different heating conditions.

The uncertainties of the PAO flow rate Q and of the experimental pressure drop Δp_e are estimated following the recommendations of Kim et al. (1993). A conservative estimate for the uncertainty of the experimental volumetric flow rate reported in this work, U_Q/Q, is to five percent.

Because both precision pressure gages are calibrated by the manufacturer using the same equipment and procedure, the resulting bias limit of the pressure difference is zero. Therefore, the uncertainty of the pressure drop across the cold plate becomes equal to the precision limit $P_{\Delta p_e}$. This precision limit is estimated as being equal to twice the standard deviation of several measurements, or approximately three percent.

Results

Figure 1 presents a comparison between the pressure drop predicted by Eq. (19) and the pressure drop measured experimentally, versus the coolant volumetric flow rate, for a reference coolant temperature $T_0 = 21$°C, and $V = 46.9$ V ($q'' = 1.3$ kW/m^2). In this case, $N = -0.23$. It can be seen that the analytical estimate obtained from Eq. (19) compares very well against the experimental measurements when the flow rate is small, deviating to slightly smaller values when the flow rate increases beyond 3×10^{-5} m^3/s.

As indicated previously, Eq. (19) is expected to be precise for $N \ll 1$ only. To test this limitation, we have performed additional hydraulic tests increasing the voltage to $V = 114.9$ V ($q'' = 7.8$ kW/m^2). In this case, $N = -1.383$. The pressure drop predicted by Eq. (19) and the experimental pressure drop values are shown in Fig. 2. Observe that the agreement between the prediction from Eq. (19) and the measured pressure drop Δp_e deteriorates rapidly as the volumetric flow rate increases.

Also plotted in Fig. 2 is the pressure drop measured without heating the cold plate, Δp_{e0}. Observe how the measured pressure drop, when heating the cold plate, Δp_e tends to the pressure drop with no-heating, Δp_{e0} as the coolant volumetric flow rate in-

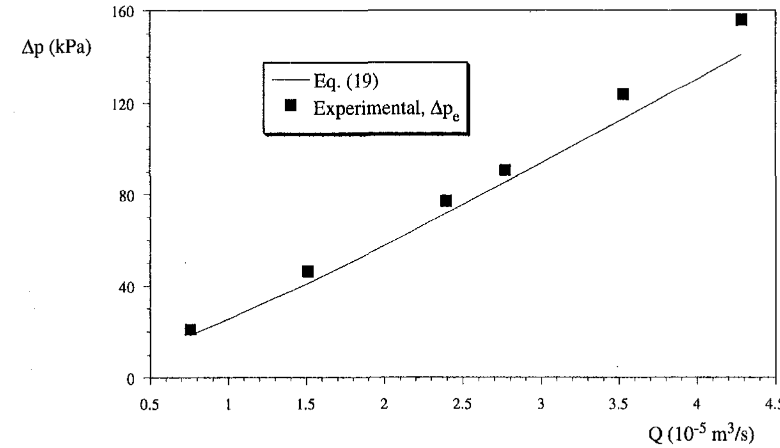

Fig. 1 Pressure drop versus volumetric coolant flow rate for T_0 = 21°C and V = 46.9 V. (Uncertainties: $U_{\Delta p_e}/\Delta p_e$ = 3 percent, U_Q/Q = 5 percent).

creases. This is explained by the departure of the flow from a linear velocity regime (the Darcy regime) dominated by viscous drag, to a quadratic velocity regime dominated by the form drag. Recall that the form drag is independent of the fluid viscosity. This means that when the fluid velocity (or the volumetric flow rate) is increased, the temperature effect on the viscosity of the coolant will affect less and less the pressure drop across the cold plate. Because Eq. (19) was derived under the assumption of zero form drag, the analytical pressure drop estimate of Eq. (19) is limited to linear (viscous drag dominated) flow regimes. One can expect the analytical estimate to deteriorate when the flow rate increases, i.e., when the form drag becomes increasingly important.

For the case shown in Fig. 2, it is relatively easy to determine the effective permeability K, and the form coefficient C of the porous insert, in m^{-1}, by fitting the experimental no-heating results with a function of the type

$$\Delta p_0 = \frac{L\mu_0}{K}\frac{Q}{A_f} + L\rho C\left(\frac{Q}{A_f}\right)^2 \tag{20}$$

where L is the cold plate length equal to 0.076 m, A_f is the flow cross-section area equal to 5.08×10^{-4} m^2, and μ_0 and ρ are the PAO viscosity and density at 21°C, respectively, 5.95×10^{-3} kg/ms and 789.2 kg/m^3. The values obtained by curve fitting the results of Fig. 2 are $K = 3.28 \times 10^{-10}$ m^2 and $C = 89.2 \times 10^3$ m^{-1}. As indicated by Lage (1998), the ratio between the form drag D_C (responsible for the quadratic flow rate term of Eq. (20)) and the viscous drag D_μ (responsible for the linear flow rate term of Eq. (20)) can be estimated using

$$\frac{D_C}{D_\mu} = \frac{\rho C K}{\left(1 + \frac{N}{3}\right)\mu_0}\frac{Q}{A_f} \tag{21}$$

where the denominator of the RHS term represents the first-order approximation (see Eq. (14)) of the temperature effect on the fluid viscosity.

After substituting the proper values in Eq. (21), one obtains $D_C/D_\mu = 1.42 \times 10^4\, Q$. So, when $Q \sim 7 \times 10^{-5}$ m^3/s then $D_C \sim D_\mu$. This criterion indicates that the results of Figs. 1 and 2 are in the range of the transition regime. This is why the analytical results deviate from the experimental results as Q increases.

One of the reviewers has asked for a justification of the high-Péclet number assumption invoked when writing Eq. (2). For the present experimental tests, we can write Pe = $QL/A_f\alpha$. Using the values listed previously and $\alpha_{PAO} = 8.68 \times 10^{-5}$ m^2/s we obtain a minimum Pe equal to 8,617 establishing that all results satisfy the high-Pe assumption.

Conclusions

A theory for the thermohydraulic prediction of the single-phase convective cooling of a porous medium enhanced enclosure using a temperature-dependent viscous fluid is presented. The theory

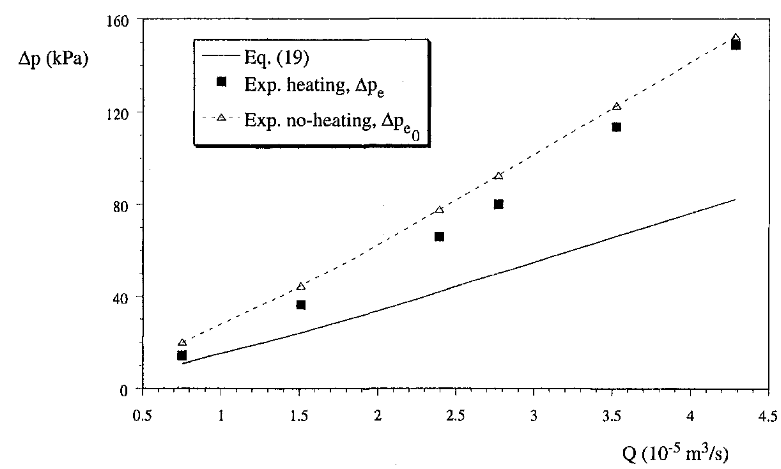

Fig. 2 Pressure drop versus volumetric coolant flow rate for T_0 = 21°C and V = 114.9 V. (Uncertainties: $U_{\Delta p_e}/\Delta p_e$ = 3 percent, U_Q/Q = 5 percent).

considers the first-order effect of viscosity variation on the Darcy flow regime valid for low fluid speed.

When applied to the prediction of the pressure drop across a microporous enhanced cold plate, the theory anticipates quite well the experimental results as long as the viscosity variation number is small compared with unity and the flow regime, under heating conditions, is linear in velocity, i.e., when the form drag effect is negligible.

Comparisons with experimental results when N is not small compared with unity, and the form drag effect becomes comparable to the viscous drag effect, indicates a consistent departure of the theoretically predicted pressure drop values from the experimental values. Obviously, when the form drag predominates, the pressure drop can be estimated analytically by neglecting the viscous drag, and thus neglecting the heating effect on the fluid viscosity.

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