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Experimental and Finite Element Modal Analyses of Planar Mechanisms With Three-Dimensional Geometries

This article presents the results of numerical and experimental modal analyses of a planar four-bar (4R) mechanism having three dimensional geometry due to the offsets between its links. An experimental mechanism is built and modern modal testing techniques are used. A five-node isoparametric finite element is developed and used to model the mass and flexibility of the links and joints, link offsets, and drive and load shafts. The results obtained from the finite element solution matched closely the experimental results, a testimony to the accuracy of the proposed element. The results indicated that link offsets have considerable influence on the dynamic characteristics of a mechanism system and should be considered in dynamic modeling of planar mechanisms.

Background and Objective

In designing a high-performance mechanism for proper dynamic characteristics, its natural frequencies should be a key design factor because they influence its operating speed and the amplitudes and settling times of its vibrations. Natural frequencies of a planar mechanism system is influenced by the inertia and flexibility of the links and joints, link offsets, and components comprising the drive and load shafts.

The geometry of a kinematically planar mechanism is essentially three dimension (3D) due to the inevitable presence of link offsets dictated by the task and geometric constraints. Link offsets increase the flexibility of the mechanism system, lowers its natural frequency and may cause out-of-plane vibrations to dominate. Furthermore, the inherent compliance and inertia of the drive and load train components such as the driving unit, couplings, transmission mechanisms, and connecting shafts may influence linkage response by lowering its natural frequencies. Therefore, a reliable model of a planar mechanism should include the influence of all its components rather than its links only.

Natural frequencies of a mechanism system can be estimated numerically by finite element techniques or experimentally by modal testing. Finite element studies available in the literature on the dynamics of planar mechanisms using finite element techniques dealt primarily with the effect of mass and flexibility of the links and joints of planar mechanisms and spatial mechanisms. A comprehensive list of references on the subject can be found in (Smaili and Bagci, 1996). Only a few experiments on mechanism dynamics have been reported in the literature, mostly on mechanisms with 2D geometries (Turcic et al., 1984; Masurekar and Gupta, 1987; Najarajan and Turcic, 1992; Liou and Erdman, 1989; Liou and Peng, 1993; Thompson and Sung, 1984). Stamps and Bagci (1985) conducted finite element and experimental investigations using strain gauges to estimate the fundamental natural frequency of a planar mechanism with 3D geometry.

The objective of the current study is to perform modal analysis of a planar 4R mechanism system with 3D geometry by finite element and experimental modal testing techniques. An experimental 4R mechanism is built for this purpose. A five-node space-frame finite element is developed to model the links, joints, link offsets and drive and load shafts. Modern modal testing technology is utilized to perform the experiment. The results from both studies are compared to validate the accuracy of the finite element results and affirm the importance of modeling the mechanism as a system.

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The 5-Node Isoparametric Element

The study conducted by Smaili et al. (1995) on the effectiveness of various Timoshenko elements in predicting modal response of structures and mechanisms proved that the five-node element is simple, accurate, and robust when compared with other available elements. Therefore, it will be used in the current investigation. Figure 1 shows the 5-node isoparametric space frame element and its nodal distribution in terms of spatial and natural coordinates, x and η. The local frame Σ_e $(x_e y_e z_e)$ originates at the initial node (node 1). The x_e axis represents the centroidal axis of a member in its undeformed position. A point P along the element has 6 displacements, 3 translations—u, v, and w along the x, y, and z axes, and 3 rotations—θ_x, θ_y, and θ_z about the x, y, and z axes.

Due to space limitations, details on how the element matrices are formulated will be omitted here. Interested readers may refer to the reference (Smaili and Bagci, 1996) for details. The final form of the mass and stiffness matrices are only given here. Using Gauss Quadrature, the mass matrix $[m_e]$ and the stiffness matrix $[m_e]$ of a spatial five node-element e take the following forms:

$$[m_e] = \rho|J| \sum_{j=1}^{n} R_j[H(\eta_j)]^T[Q(\eta_j)][H(\eta_j)] \tag{1}$$

and

$$[k_e] = |J| \sum_{j=1}^{n} R_j[B(\eta_j)]^T[P(\eta_j)][B(\eta_j)] \tag{2}$$

where n represents the number of Gauss sampling points and R_j is the jth weighting factor. $|J|$ is the determinant of the Jacobian of the element. Matrix $[H(\eta_j)]$ contains the shape functions that relate the displacements at a point on an element to the element nodal displacements. Matrix $[Q(\eta_j)]$ is a diagonal matrix that contains A, A, A, I_{xx}, I_{yy}, and I_{zz}, along its

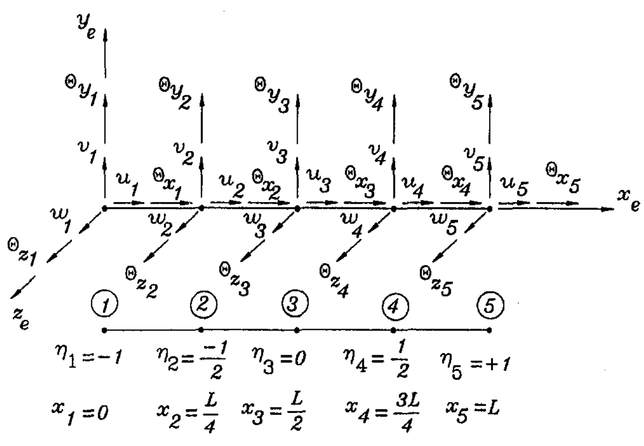

Fig. 1 Nodal distribution and DOF of the five-node isoparametric finite element

Table 1 Minimum natural frequencies (Hz) and the corresponding crank position

offset length (mm)	mode 1 (θ_2°)	Mode 2 (θ_2°)	Mode 3 (θ_2°)
planar	69.64 (350)	230.21 (320)	-
5.08	11.00 (330)	56.97 (320)	98.85 (320)
31.75	10.55 (330)	52.63 (320)	87.18 (320)
63.5	10.08 (330)	48.38 (330)	76.40 (320)
101.6	9.55 (340)	43.46 (330)	76.40 (320)

diagonal; matrix $[B(\eta_j)]$ relates the strains at a point on the element to the element nodal displacements; and matrix $[P(\eta_j)]$ is a diagonal matrix that contains EA, EI_{yy}, EI_{zz}, $GA\kappa_{xy}$, $GA\kappa_{xz}$, and EI_{xx} along its diagonal. A, I, and κ are the area, area moment of inertia, and Timoshenko coefficient of the cross section. E, G, and ρ are the Young's modulus, shear modulus, and density of the material.

Application Example

A planar 4R mechanism with 3D geometry is built to experimentally determine its natural frequencies by modal testing and compare them with those obtained by the 5-node element. The experimental 4R mechanism was the subject of the study by Stamps and Bagci (1985). It is being used here as the current case study for the following reasons: (1) The natural frequencies found by Stamps and Bagci (1985) were the result of an experiment that employed strain-gauges. However, using modern modal testing hardware and software in the current investigation would certainly lead to a more accurate results and, in turn, help realize a better finite element model for the mechanism system. (2) The finite element and experimental results obtained by Stamps and Bagci (1985) did not match. In an effort to minimize the difference between the two solutions, the authors used two different schemes to model the joints of the 4R mechanism, revolute and spherical. The average values of the natural frequencies obtained from the two models were then compared with the experimental values. This approach, however intuitive may be, does not reflect the true physical nature of link connections. Consequently, the conclusions made by Stamps and Bagci (1985) deserves another look. (3) The finite element model used by Stamps and Bagci (1985) is based on Euler-Bernoulli thin beam theory and shear deformations and rotary inertia of the links were not accounted for. Furthermore, lumped mass system instead of consistent mass system was used.

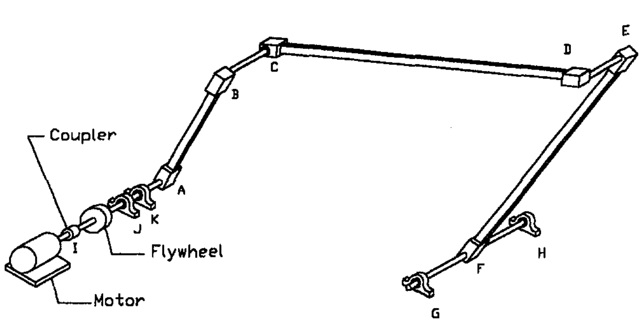

Fig. 2 Schematic of the experimental planar 4R mechanism with 3D geometry

Experimental 4R Mechanism System. Figure 2 shows a schematic of the experimental 4R mechanism. The dimensions (mm) of this mechanism are $IJ = 88.9$, $JK = 50.8$, $KA = 63.5$, $AB = 193.675$, $BC = 101.6$ or 5.08, $CD = 495.53$, $DE = 101.6$ or 5.08, $EF = 419.1$, $GF = 101.6$, and $FH = 25.4$. Link offsets BC and DE vary between 5.08 and 101.6 mm. Each link is comprised of three segments. The cross section of the main link segments is 31.75 mm × 7.9375 mm and the cross section of the shaft-support-blocks is 31.75 mm × 25.4 mm. The links are made of aluminum having $E = 71 \times 10^3$ N/mm^2, $G = 26.2 \times 10^3$ N/mm^2, and $\rho = 2.93 \times 10^{-6}$ kg/mm^3. The offsets are made of steel having $E = 207 \times 10^3$ N/mm^2, $G = 79.3 \times 10^3$ N/mm^2, and $\rho = 8.425 \times 10^{-6}$ kg/mm^3.

A 3D and a 2D finite element models of the mechanism system were developed. Table 1 gives the minimum values of the frequencies for the first three modes of the 2D and 3D models with several offset dimensions and the crank positions at which they occur. The second frequency of the 3D model encompasses a frequency range which contains the first in-plane frequency. This is similar to the findings reported by Stamps and Bagci (1985). These results underscore the importance of including the effect of link offsets when modeling a planar mechanism.

Experimental Modal Analysis. Modal testing of the 4R mechanism system was performed using impulse excitation hammer (PCB208A03), a pair of accelerometers

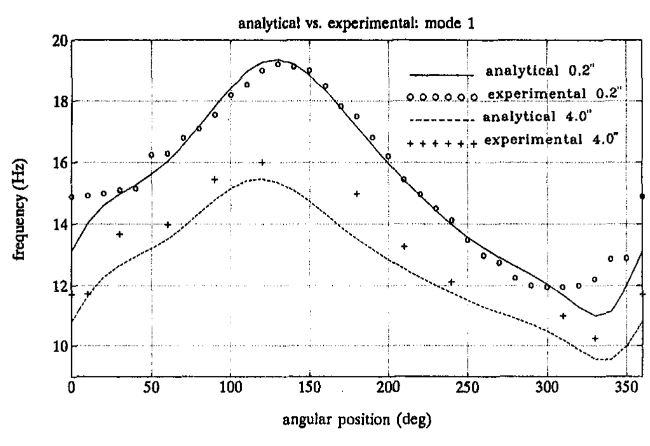

Fig. 3 Fundamental natural frequency of the experimental 4R mechanism with link offsets: Experimental vs. finite element results

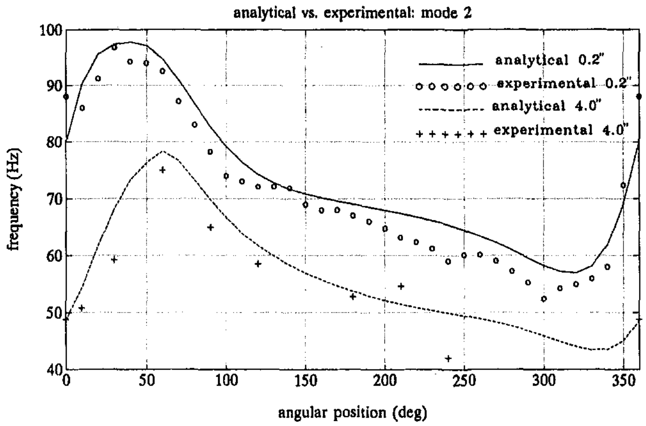

Fig. 4 Second natural frequency of the experimental 4R mechanism with link offset: Experimental vs. finite element results

(PCB353B67), Star Modal software, and a Data 6000 spectrum analyzer. The accelerometers were mounted at the midpoint of the follower link such that one accelerometer measured in-plane response and the other captured the response normal to the motion plane. The experimental mechanism was mounted on a 12.7 mm aluminum plate which in turn was fastened to a rigid heavy steel frame to eliminate the effects of foundation flexibility on the response of the mechanism. The mechanism was excited by the impact hammer at fifteen different points to collect enough data for accurate characterization in the Star Modal software. The frequency response functions (FRF) were determined by the Data 6000 for 10 deg increments of CCW crank rotation for the 5.08 mm offset and for 30 deg increments of CCW rotation for the 101.6 mm offset. The data 6000 was set for 2048 points and a sampling rate of 500 μs for a frequency resolution of 1.0 Hz.

The first three natural frequencies obtained from the experiment on the 5.08 and 101.6 mm offsets are depicted in Figs. 3–5, along with the frequencies obtained from the 5-node finite element model. Table 2 compares the finite element and experimental results for the 5.08 mm offset at 0 deg, 180 deg, and 250 deg crank positions. Figures 3–5 and Table 2 indicate

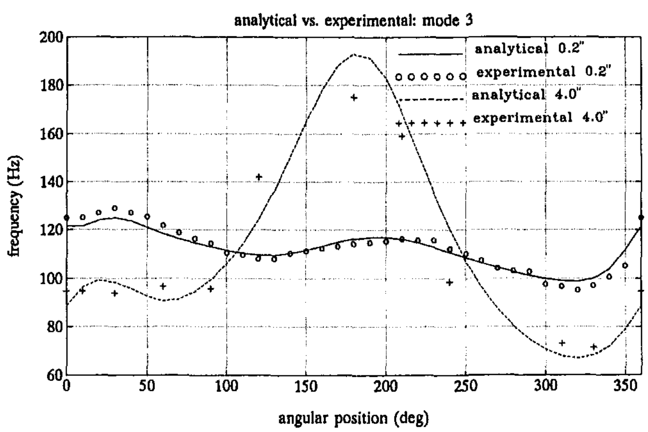

Fig. 5 Third natural frequency of the experimental 4R mechanism with link offsets: Experimental vs. finite element results

Table 2 Comparison of experimental and finite element natural frequencies (Hz) for 5.08 mm link offsets

crank position	mode	5-node element	experimental
	1	13.12	15.01
0°	2	99.84	96.10
	3	125.84	124.72
	1	17.18	18.22
180°	2	69.07	64.70
	3	116.31	113.69
	1	13.58	14.87
250°	2	37.33	32.51
	3	108.11	117.85

that the experimental and finite element results are in close agreement, a testimony to the accuracy of the proposed finite element model.

Conclusions

Numerical and experimental modal analyses of a planar 4R mechanism with 3D geometry were presented. An experimental mechanism was built and modern modal testing techniques were employed. A five-node isoparametric finite element was developed and used to model the mass and flexibility of the links and joints, link offsets and drive and load shafts of the experimental mechanism. The finite element and experimental results were in close agreement, a testimony to the accuracy of the proposed finite element. The results indicated that link offsets have considerable influence on the dynamic characteristics of a mechanism system and that a 2D model of a planar mechanism may not reveal its true response.

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