# BEHAVIOR OF REGULAR TRIANGULAR JOINTS UNDER CYCLIC SHEARING

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## ABSTRACT

Synthetic regular triangular joints made of gypsum plaster were cyclicly sheared to measure asperity degradation and mathematical behavior of regular triangular joints. Laboratory cyclic shear tests were conducted for three joint types under three different normal stresses. Asperity degradation and shear strength of joints were found to be a function of joint roughness, normal stress, shearing displacement and number of loading cycles. Based on the experimental results, mathematical models were developed for evaluation of shear strength in cyclic loading conditions. Comparison of the test results with that of the proposed model, it was found a good agreement was observed.

## I. INTRODUCTION

Joints can significantly affect the mechanical behavior of rock masses. More specifically, joint properties such as roughness, strength of asperities, separation, matedness, gouge and even the spacial distributions make the behavior of jointed rock masses more complicated. Most of laboratory experiments for the mechanical properties of rock joints have been focused on determining the peak shear strength and the stress-displacement relations under unidirectional shear loading. The comprehensive behavior of rock joints under the cyclic loading condition where the direction of shear load is repeatedly reversed has been rarely reported. A correct evaluation of dynamic behavior of rock joints plays an important role in the design of deep underground openings in jointed rocks, stability analysis of rock slopes and risk assessment of underground waste disposal for earthquake and explosive safety.

Hutson and Dowding [3] performed some cyclic tests on real rock granite and limestone joints sawn with numerical control technique. They found that asperity degradation is a function of work, joint roughness, normal stress and unconfined compressive strength of the joint wall. Based on the experimental results, they presented a wear equation for joint asperity. Huang *et al.* [2] also performed cyclic tests on saw-tooth samples to evaluate the degradation law proposed by Plesha [8]. Divoux *et al.* [1] performed a mechanical constitutive model based on the results of cyclic shear tests. Lee *et al.* [7] studied influence of asperity degradation on the mechanical behavior of rough joints under cyclic shear loading. Based on the experimental results, an elasto-plastic constitutive model, which can consider the degradation of second order asperities, was proposed. Jafari *et al.* [5] found that the shear strength of joints is related to rate of displacement, number of loading cycles and stress amplitude. Consequently, mathematical models were developed by Jafari *et al.* for evaluation of shear strength under cyclic loading conditions.

In this paper the results of an experimental investigation carried out on artificial joints will be presented. A series of cyclic shear tests were conducted using gypsum plaster specimens. Based on the experimental results, mathematical models were developed for evaluating the cyclic shear strength of rock joints.

## II. APPARATUS AND TEST METHOD

The shear box apparatus used in this study was designed and built by GCTS company (as shown in photo 1). Specimens of rock with a maximum inner diameter of 152 mm can be tested under normal loads of up to 5 tons and cyclic shear loads of up to 10 tons.

The machine consists essentially of an arrangement to accommodate the specimen to be tested, a mechanism to apply different constant vertical loads on the specimen and a mechanism to apply cyclic shear loads, in a direction perpendicular to the normal load. The shear box assembly consists of two different parts, a lower half and a top half. Both halves have the same inner diameter of 152 mm and height of 70.62 mm. During cyclic test, the vertical and horizontal displacements are measured by 2 LVDT (one vertical and one horizontal). The maximum vertical displacement is 12.7 mm, and the maximum shear displacement is 25.4 mm.

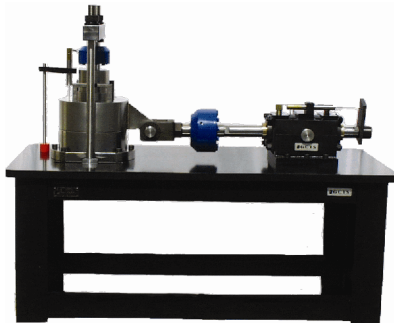Gypsum plaster is used to make ideal soft rock joints, as

**Photo 1.  The GCTS shear test machine.**

this material is universally available and is inexpensive.  It is easily molded into any shape when mixed with water, and the long term strength does not change significantly once the chemical hydration is completed.

The engineering properties of the model material are determined by conducting tests on specimens cured for two weeks.  The cured material has uniaxial compressive strength ($\sigma_c$) of 11.1 MPa and Young's modulus (E) of 6.63 GPa for plaster mixed with water in the ratio of 2:1 by weight.  Based on rock classification determined by ISRM [4], model material is a weak rock.

After dismantling the top and bottom moulds from the shear apparatus, the bottom mould together with the collar on top was then filled with this mixture, and left undisturbed for one hour to ensure adequate hardening prior to casting the upper specimen.  The top of the collar was subsequently shaped according to the predetermined surface profile.  Based on Kodikara *et al*. [6] suggestion, triangular asperities with asperity heights of 6 mm, 4.5 mm and 3.0 mm, respectively, and inclination angle of 22.5°, 17.5° and 12.5°, respectively, were cast as shown in Fig. 1.  In order to cast fully mated joints, the top mould was placed over the bottom mould and filled with the plaster mixture, then the whole assembly was air-cured for one hour to complete initial setting.  Thin polythene paper was inserted between the two moulds to prevent bonding between the mated joints.  During specimen preparation, mild vibration was applied to the mould externally to eliminate any entrapped air.  Before testing, the specimens were cured at 30°C for two weeks and subsequently air cooled to room temperature.

The cyclic shear behavior of soft rock joints is studied under constant normal load (CNL) by testing the gypsum plaster regular triangular joints.  All the tests were performed under different initial normal stress $\sigma_n$ of 0.5, 1.0 and 1.5 MPa, respectively.  Specimens were cyclicly sheared at a frequency of about 0.003 Hz.  Total 9 specimens were tested, as summaried in Table 1.

## III. EXPERIMENTAL RESULTS

The cyclic shear tests were performed on saw-tooth joint samples to study the behavior of the joint replicas.  Tests were

**Table 1.  Summary of test specimens.**

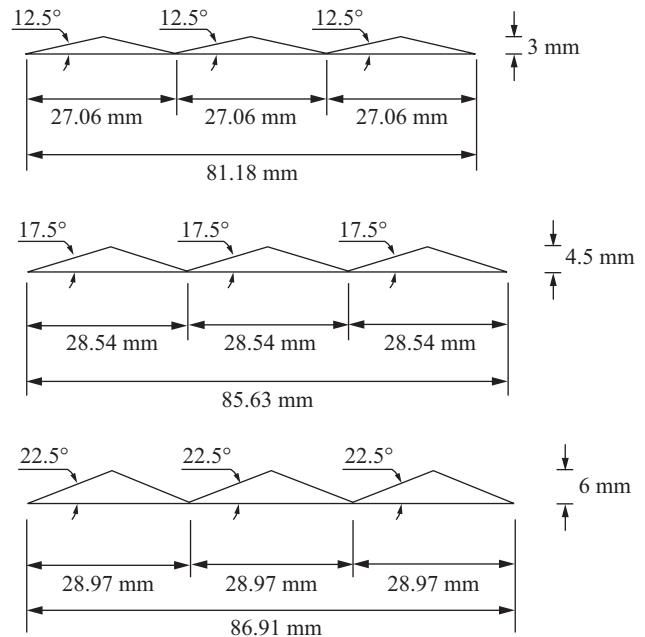| Normal stress (MPa) | 0.5 | | | 1.0 | | | 1.5 | | |
|---|---|---|---|---|---|---|---|---|---|
| Asperity height (mm) | 6 | 4.5 | 3 | 6 | 4.5 | 3 | 6 | 4.5 | 3 |
| Inclination angle ($\theta°$) | 22.5 | 17.5 | 12.5 | 22.5 | 17.5 | 12.5 | 22.5 | 17.5 | 12.5 |



**Fig. 1.  Profiles of saw-tooth plaster joints.**

performed at three different levels of normal stress and the variations in shear strength as well as the degradation of the asperities were studied.

Test results for degradation of dilation angle $i$ ($i$ = vertical displacement/horizontal shear displacement) for all joint samples performed under three different normal stresses are presented in Fig. 2.  Degradation ratio $D_n$ of asperity height ($D_n$ = height degraded D/initial height h) performed under three different normal stresses are presented in Fig. 3.

As shown in Fig. 2, dilation angle decreases as number of shearing cycles increases.  For samples with smaller asperity height (h = 3.0 mm) and inclination angle ($\theta$ = 12.5°), degradation of asperities happens mainly during the first cycles if samples were sheared under high normal stress of 1.5 MPa (Fig. 2(a)), resulting clearly observed degradation of dilation angle during the first shearing cycles.  After the first cycle, only small dilation can be occurred.  It is believed that at high levels of normal stress, smaller asperities will be broken Jafari *et al*. [5], and the dilation angle for the rest of the cycles was nearly constant.
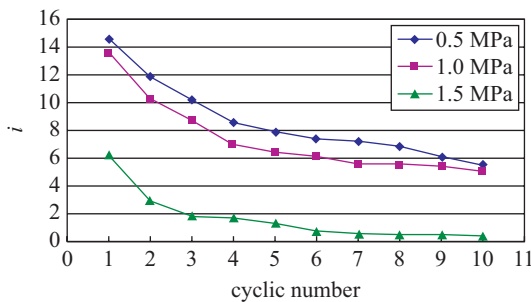
At low levels of normal stress (0.5 MPa), the main shearing mechanism during cyclic shearing is sliding along the asperities.  The degradation of asperity continues during cyclic shearing.
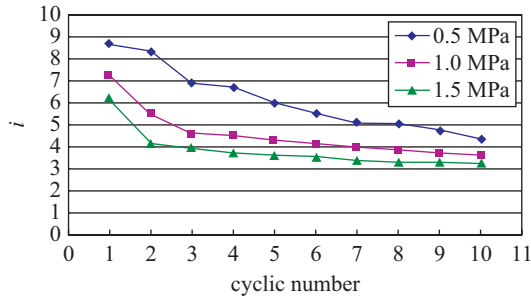
(a) $h = 3.0$ mm and $\theta = 12.5°$
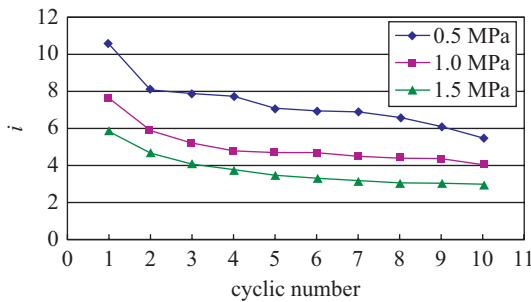


(b) $h = 4.5$ mm and $\theta = 17.5°$



(c) $h = 6.0$ mm and $\theta = 22.5°$

**Fig. 2. Variation of dilation angle *i vs.* number of cyclic loading.**



(a) $h = 3.0$mm and $\theta = 12.5°$



(b) $h = 4.5$ mm and $\theta = 17.5°$



(c) $h = 6.0$ mm and $\theta = 22.5°$

**Fig. 3. Variation of asperity degradation ratio $D_n$ *vs.* number of cyclic loading.**

However, for sample with big asperity height and inclination angle (Fig. 2(c)), even at high normal stresses, asperity was not totally broken in the first cycle. All the teeth were cut until the second shearing cycle.
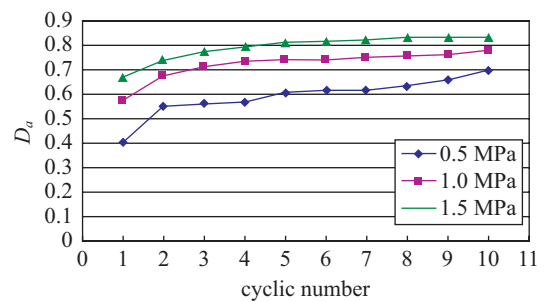
In contrast to dilation, as shown in Fig. 3, degradation of asperity height ratio increases as number of shearing cycles increases. At higher level of normal stress, degradation of asperity height ratio also happens mainly during the first shearing cycle. In the other cycles, only small degradation of asperity occurred and the asperity height ratio approaches nearly constant.

Fig. 4 shows the results of shear strength *vs.* number of shearing cycles performed under three different normal stresses. Like the results of dilation degradation shown in Fig. 2, at high level of normal stress, the shear strength appears to decrease more clearly during first two or three cycles.
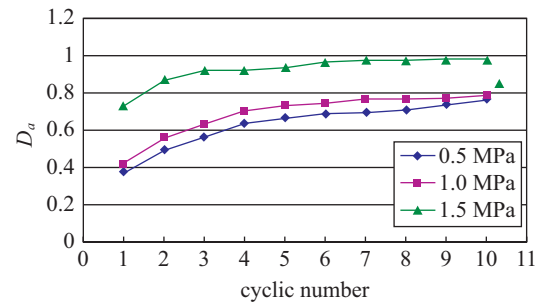
As the shear strength is directly related to the dilation angle (i), shear strength will be reduced when the dilation angle decreases. At high level of normal stress, asperities will be broken during shearing in the first or two cycles and considerable dilation may not occur in the rest cycles. During forward in the first two cycles, almost all the teeth were cut and the shear strength for the rest of the cycles was nearly constant.

At low level of normal stress, the main shearing mechanism during cyclic shearing is sliding along the asperities. During sliding, degradation continues to occur and the shearing surface is smoothed. The shear strength of the joint samples diminishes gradually in each cycle to reach a constant level after experiencing a few shearing cycles.

## IV. MATHEMATICAL MODEL OF CYCLIC SHEAR STRENGTH

Based on the results of the tests performed and on the trends of the data in each group discussed, Jafari *et al*. [5] developed the following mathematical model to evaluate the shear strength of the jointed samples during cyclic shearing:
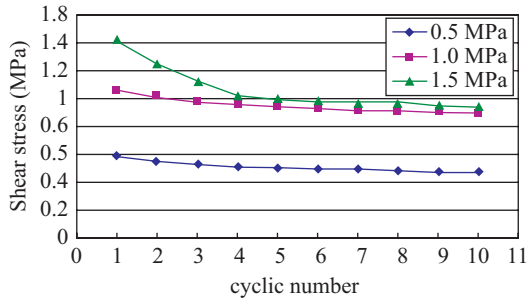
(a) $h = 3.0$ mm and $\theta = 12.5°$



(b) $h = 4.5$ mm and $\theta = 17.5°$
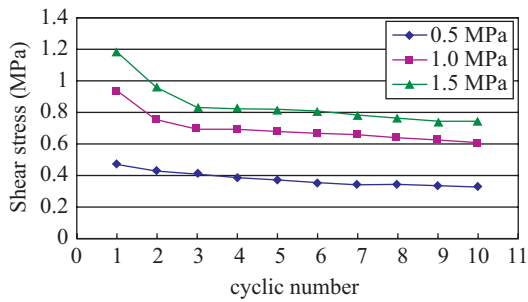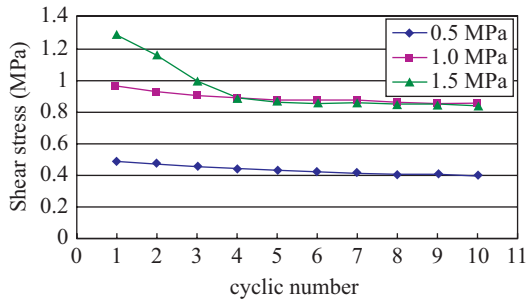


(c) $h = 6.0$ mm and $\theta = 22.5°$

**Fig. 4. Variation of shear strength *vs.* number of cyclic loading.**

$$\frac{\tau}{\sigma_n} = \frac{b(NC_d)^p(i_n)^q + c}{1 + b(NC_d)^p(D_n)^q} \qquad (1)$$

where $\tau$ is shear strength, $\sigma_n$ is normal stress, $NC_d$ is number of shearing cycles, $i_n$ is the normalized dilation angle (normalized by the maximum angle measured before the test ), $D_n$ is normalized degradation (normalized by maximum value of asperity amplitude). The parameters $b$, $c$, $p$ and $q$ in the relation presented by Jafari *et al*. can be obtained by model calibration, as follows:

$$b = -0.33, c = 1.44, p = 0.12, q = 0.3 \qquad (2)$$

Jafari *et al*. showed that the parameters b and c are related to the mechanical properties of the sample tested, such as friction angle ($\psi$) and also the geometrical features of the joint surface. Wearing and asperity degradation are functions of the number of cycles, and may have important effects on the shearing

**Table 2. Dilation angle (ratio), asperity degradation (ratio) and shear strength (ratio) under loading cycles.**

| Cycle number | $i$ (°) | $i_n$ | $D$ | $D_n$ | $\tau$ | $\tau/\sigma_n$ |
|---|---|---|---|---|---|---|
| 1 | 8.670801 | 0.693664 | 0.936675 | 0.312225 | 0.4755 | 0.951 |
| 2 | 8.312105 | 0.664968 | 1.023267 | 0.341089 | 0.43507 | 0.87014 |
| 3 | 6.887955 | 0.551036 | 1.365576 | 0.455192 | 0.4156 | 0.8312 |
| 4 | 6.712829 | 0.537026 | 1.407519 | 0.469173 | 0.3941 | 0.7882 |
| 5 | 6.028093 | 0.482247 | 1.571232 | 0.523744 | 0.3732 | 0.7464 |
| 6 | 5.517652 | 0.441412 | 1.693002 | 0.564334 | 0.3589 | 0.7178 |
| 7 | 5.097293 | 0.407783 | 1.793124 | 0.597708 | 0.3486 | 0.6972 |
| 8 | 5.0575 | 0.4046 | 1.802595 | 0.600865 | 0.3476 | 0.6952 |
| 9 | 4.733293 | 0.378663 | 1.879716 | 0.626572 | 0.3403 | 0.6806 |
| 10 | 4.374607 | 0.349969 | 1.964955 | 0.654985 | 0.333 | 0.666 |

**Table 3. *JRC*, *b* and *c* values.**

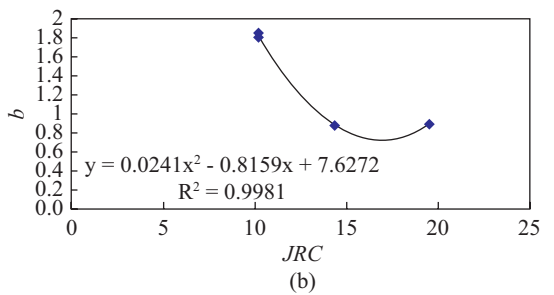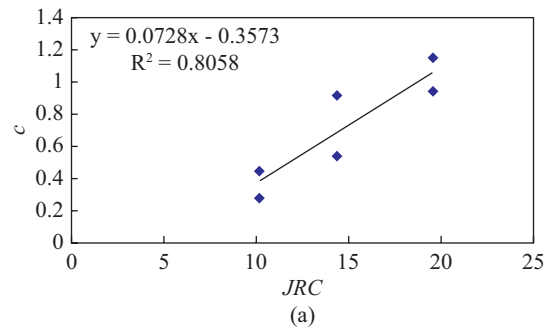| Sample | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| *JRC* | 10.18 | 14.37 | 19.52 | 10.18 | 14.37 | 19.52 |
| *b* | 1.85 | 0.88 | 0.90 | 1.79 | 0.89 | 0.88 |
| *c* | 0.45 | 0.54 | 0.94 | 0.28 | 0.92 | 1.15 |



(a)



(b)

**Fig. 5. Relationship of *b* and *c* with *JRC*.**

strength of jointed samples. Dilation angle and asperity degradation are also two related parameters that control the shear strength of rock joints during cyclic shearing. If asperity degradation increases, the dilation angle and the shear strength will decrease. In addition, Jafari *et al*. pointed out that for a better evaluation of these parameters, it is necessary to perform similar tests on different kinds of joint surfaces. In other words, in Jafari *et al*. model, equations were not developed for estimation of *b* and *c*.
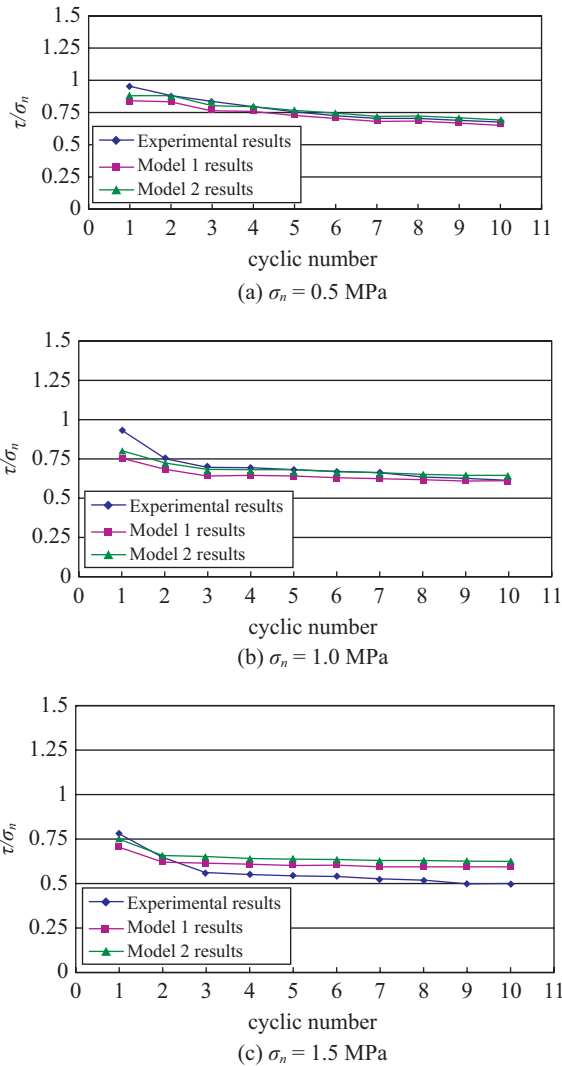
(a) $\sigma_n$ = 0.5 MPa

(b) $\sigma_n$ = 1.0 MPa

(c) $\sigma_n$ = 1.5 MPa

**Fig. 6. Comparison of the measured and evaluated shear strength for joint with asperity height of *h* = 3.0 mm.**



(a) $\sigma_n$ = 0.5 MPa

(b) $\sigma_n$ = 1.0 MPa

(c) $\sigma_n$ = 1.5 MPa

**Fig. 7. Comparison of the measured and evaluated shear strength for joint with asperity height of *h* = 4.5 mm.**

In Jafari *et al*. studies, range of normalized normal stress (normal stress normalized by compressive strength) is between 0.8455 and 0.0218. The range of normalized normal stress in this study is between 0.1349 and 0.0798, that falls in the range of Jafari *et al*. To simplify the calculation of Eq. (1), it can be assumed that effects of normalized dilation angle ($i_n$) and asperity degradation ($D_n$) on shear behavior in this study be the same as that of Jafari *et al*. In addition, number of shearing cycles in this study is 10, that is equal to that of Jafari *et al*., so that the effects of shearing cycles can also be assumed to be the same. Therefore, *p* and *q* values in Eq. (1) is used in this study, i.e. *p* = 0.12 and *q* = 0.3, and Eq. (1) can be rewritten as follows:

$$\frac{\tau}{\sigma_n} = \frac{b(NC_d)^{0.12}(i_n)^{0.3} + c}{1 + b(NC_d)^{0.12}(D_n)^{0.3}} \tag{3}$$

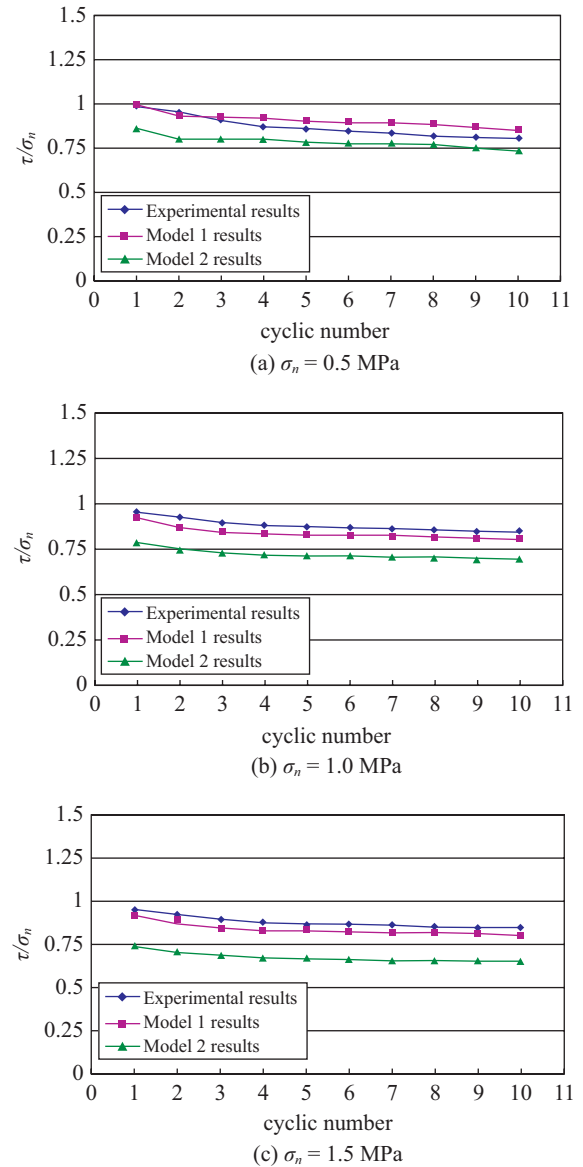Parameters *b* and *c* can be calculated by least square

method based on experimental data. Table 2 presents one of test data for specimen with asperity height of 3.0 mm and inclination angle of 12.5° under normal stress of 0.5 MPa. In this study, it is found that b and c have close relationship with *JRC* (Joint Roughness Coefficient) as shown in Table 3 and Fig. 5. *JRC* is calculated as Tse *et al*. [9]:

$$\tau = \sigma_n \tan\left[ JRC \log\left( \frac{JCS}{\sigma_n} + \varphi_b \right) \right] \tag{4}$$

where JCS is joint compressive strength, $\psi_b$ is basic friction angle. Therefore, *b* and *c* parameters can be calculated by regression analyses as follows:

(a) $\sigma_n = 0.5$ MPa



(b) $\sigma_n = 1.0$ MPa
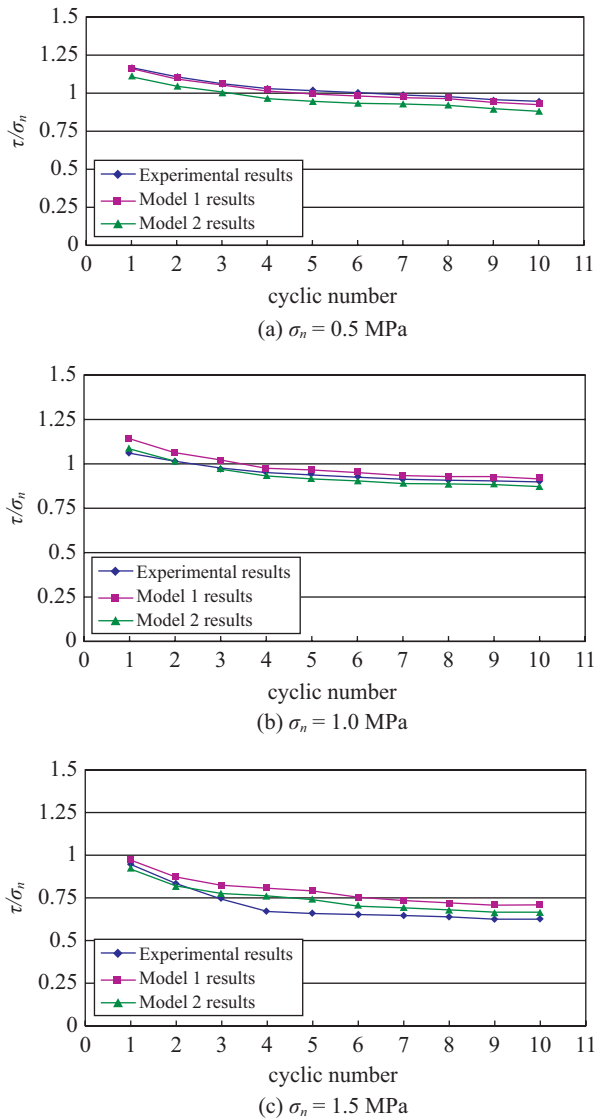


(c) $\sigma_n = 1.5$ MPa

**Fig. 8. Comparison of the measured and evaluated shear strength for joint with asperity height of *h* = 6.0 mm.**

$$b = 0.024JRC^2 - 0.816JRC + 7.627 \qquad (5)$$

$$c = 0.028JRC - 0.3574 \qquad (6)$$

Substituting *b* and *c* values obtained from Eqs. (5) and (6) into Eq. (3), shear strength ratio $\tau/\sigma_n$ can be estimated. Figs. 6-8 shows the test results compared with the proposed models. In Figs. 6-8, model 1 indicates *b* and *c* values are calculated by experimental data, while model 2 indicates *b* and *c* values are calculated by regression analysis from *JRC* value. It is ob-

served that good agreement is observed.

## V. CONCLUSION

Fundamental mechanism of joint asperity degradations, and the variation in shear strength of joint replicas under cyclic loading conditions were studied. Typical features of regular triangular joint behavior under cyclic shear loading such as peak shear strength was significantly affected by the degradation of asperity, dilation angle and number of loading cycles. In addition, degradation of asperity, dilation angle and shear strength are affected by the asperity height and inclination angle. Based on the experimental results, mathematical models were developed for evaluation of shear strength in cyclic loading conditions. Regression equations for calculation of parameters *b* and *c* were also developed such that cyclic shear tests on different kinds of real joint surfaces are not necessary to perform for evaluation of parameters *b* and *c*. Compared the test results with the proposed model, it is found that good agreement is observed.

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